Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 9 (1972)

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DOI: https://doi.org/10.5169/seals-9586

Analysis of a Prestressed Cable-Roof Anchored in a Space-Curved Ring Beam

Analyse d'une couverture précontrainte suspendue sur un cordon tridimensionnel courbe

Analyse eines Hängedaches mit vorgespanntem Seilnetz, verankert in einem räumlich gekrümmten Ring

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The structure

In May 1971 the arena "Scandinavium" in Gothenburg, Sweden, was completed. With space for 14000 spectators it is the largest covered arena in northern Europe and has already been utilized for various activities as ice-hockey, concerts and opera performances.

The roof consists of a prestressed cable net carrying corrugated steel plates with thermal and water insulation. Its weight is 60 kg/m^2. All cables are anchored in a space-curved reinforced concrete ring whose projection on a horizontal plane is almost circular with a diameter of 108 m. The ring is carried by 40 slender columns of circular sections and four stiff ones each formed by two walls connected by beams. The surface of the roof, Fig 1, deviates but little from a hyperbolic paraboloid.

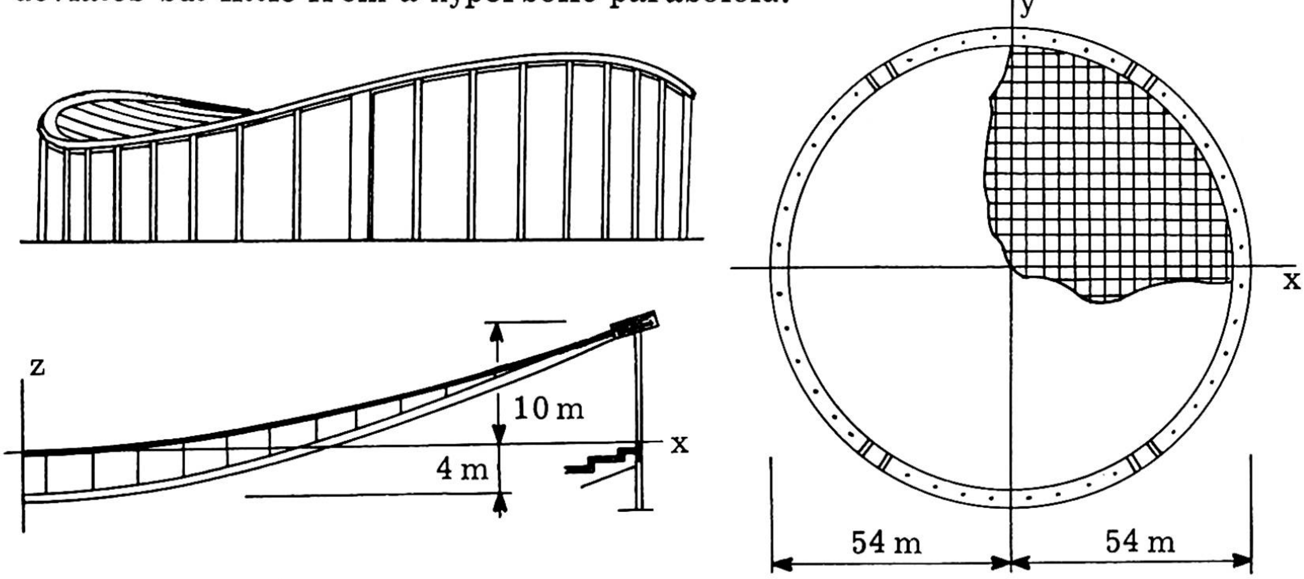

Fig 1. View of the arena and dimensions of the roof

From the center point of the roof the main cables rise 10 m to the top and the perpendicular cables fall 4 m to the valley of the ring. The distances between the cables are nearly constant and equal to 4 m in both directions.

Preliminary calculations

Preliminar dimensions of the ring and the cables were estimated by analysis of a shear-free membrane model. The stiffness of the ring was taken as the stiffness of a plane ring with the same dimensions as the real one supported horisontally at the four stiff columns. The deflection of the roof was approximated by polynomials and the membrane stresses were approximated by sectionally constant values in each direction. The unknowns were determined from equations expressing vertical equilibrium and compatibility between membrane and ring. Section forces and moments in the ring due to snow and wind loads calculated from the membrane forces were modified with respect to the inclination of the ring. Comparison with the more accurate analysis presented below showed a difference of at most 10 % in bending moments in the ring. Accurate values of the twisting moments could not be obtained by the approximate method.

Finite element method, general

The more accurate analysis was performed by applying a mixed finite element method. The structure was then divided into two substructures, the network of cables and the ring beam on columns. In studying the effect of vertical live load on the roof and arbitrary live load on the ring the substructures were analysed by the stiffness method and connected by the flexibility method.

The symmetry of the roof was utilized by making the calculations for only a quarter of the roof. Since the analysis was non-linear superposition was possible only in combination with iteration.

Form load condition

The analysis for live load was made for deflections and forces measured from a reference position defined by vertical positions z and the corresponding vertical dead load P^0 on the cable joints. In matrix form the vertical equilibrium of the cable joints can be expressed as

$$(1) \qquad -X_{H0}\, z = P^0/H_G + \text{boundary terms}$$

where H_G is a reference force and

$$(2) \qquad H_G X_{H0} = H_G \alpha^0 X_x + A^T H_G \beta^0 X_y A$$

Here X_x and X_y are second-order difference operators, see ASPLUND, and $H_G \alpha_i^0$, $H_G \beta_k^0$ the horizontal components of the dead load cable forces in the x- and y-cables i and k. The first term on the right hand side of eq (2) yields the contribution from the x-cables and the second from the y-cables. The matrix A is an ortho-normal renumbering matrix. The minus sign on the left hand side of eq (1) annihilates minus signs in the diagonals of X_x and X_y, thus making the set of equations positive definite.

With the chosen form of the roof the forces $H_G \alpha_i^0$ and $H_G \beta_k^0$ were constant giving a nearly moment-free concrete ring under dead load.

Live load

For dead load P^0 plus live load P vertical equilibrium of the cable joints requires

(3) $$-X_{(H0+H)}(z+p) = (P^0+P)/H_G + \text{boundary terms}$$

where

(4) $$H_G X_{(H0+H)} = H_G \alpha X_x + A^T H_G \beta X_x A$$

The difference between eq (3) and eq (1) can be written

(5) $$-X_{(H0+H)}\, p + ZH/H_G = P/H_G$$

where H is a column matrix with $H_G(\alpha - \alpha^0)$ followed by $H_G(\beta - \beta^0)$ and Z is a rectangular matrix built up by the second-order difference operators multiplied by the vertical distances between the anchors and the cable joints.

The second order difference operator X is built up by quotients $\Delta(z+p)/\Delta(x+u)$. Here the changes Δ u in horizontal movement u can be neglected or be considered approximately e. g. as indicated by ASPLUND.

The other set of equations needed expresses compatibility between the cable net and the ring. This set should be expressed by the same unknowns as eq (5). Combined with eq (5) the two sets of equations can be written

(6) $$\begin{bmatrix} -X_{(H0+H)} \cdot H_G & Z \\ -Z_p^T & (L/EA + B^T e B) \end{bmatrix} \begin{bmatrix} p \\ H \end{bmatrix} = \begin{bmatrix} P \\ h_0 \end{bmatrix}$$

The minus sign in the second line is typical for the mixed formulation. The matrix Z_p is equal to Z in a linear theory. A more accurate formulation is here needed. A second and satisfactory approximation of Z_p is obtained if z is replaced by $(z+p/2)$.

In $(L/EA + B^T eB)$ the first term gives the elastic elongations of the cables. The effective length of a cable can here be approximated with good accuracy as

(7) $$L_{eff} = L_H + (3/2)\, z^T(-X)\, z$$

where L_H is the horizontal distance between the anchors.

The second term $B^T eB$ is the flexibility of the ring on columns loaded by cable forces. The matrix e is the flexibility matrix of the ring on columns loaded by general forces and moments. For the calculation of this matrix the ring was divided into elements, the straight parts between the columns. The matrix e can be obtained by first or second order theory from a standard finite element system program.

The column matrix h_0 on the right hand side is zero in general. It can, however, be used for the complementary solution to a particular solution. Load on the ring and temperature changes in the ring were included in this way.

Iteration

The set of equations (6) was solved by iteration starting with a guess on

H, solving for p, establishing Z_p^T, solving for H and so on. Even with a poor guess on H the convergence in all practical cases was rapid (3 to 4 iterations were sufficient). On an IBM 360/65 the central processing time for one iteration with 105 unknowns was about one minute.

Pretensioning stages

After the main cables had been hanged out, four cables in the other direction located symmetrically around the valley were laid out and tensioned. Repeatedly four and four cables were laid out and tensioned until all cables were on place. After that the roof plates were laid out. In the analysis this procedure was followed backwards from the reference state by eliminating the dead load and the forces in some cables. In eq (6) this means that P was set equal to $-P^0$ and that some elements in H were set equal to zero. The calculations were checked against measurements of the vertical position of the line from valley to valley and the horizontal movement of the valley. Maximum discrepencies between calculated and measured values of the vertical position at full pretensioning amounted to 7 cm. The horizontal movement of the valley of the ring during pretensioning was calculated and measured to 7.5 cm. The corner strains of the ring were also measured during pretensioning. Comparison with theoretical values gave maximum discrepencies of 1 MN/m^2 in corner stresses of 10 MN/m^2 at full pretensioning.

Behaviour under live load

With a concrete ring of $3.0 \cdot 1.2\,m^2$ in section area all cables were tensioned by uniform snow load because the valleys of the ring moved outwards considerably. How the flexibility of the ring affects the forces in the cables is illustrated in Table 1.

Stiffness of ring	A	B
K	105	51
3K	99	28
10K	92	-32

Table 1. Cables forces in kN/m due to snow load calculated for rings of stiffness K, 3K and 10K where K is the actual stiffness.
A Cables anchored at the top
B Cables anchored at the valley

The chosen flexibility of the ring gave a favourable distribution of cable forces. This became relatively uniform both for downward snow load and upward wind load, thus giving ring moments of moderate magnitude. The upward wind load was after wind-tunnel tests taken to -400 N/m^2. The snow load was 750 N/m^2 according to Swedish norms. The maximum vertical movement of the net due to snow load was calculated to 68 cm.

Natural vibrations

The roof was also analysed with regard to natural modes of vibration. Only small vibrations superposed on deflections under dead load and dead load plus uniform snow were considered. For vibration calculations a pure stiffness formulation is suitable. With variables

(8) $$p = p_a \sin \omega t, \quad h = h_a \sin \omega t$$

where h is the horizontal component of the cable movement in the cable direction at the ring, the homogeneous equations become

$$(9)\quad \begin{bmatrix} (-X_{(H+H0)}H_G + Z(L/EA)^{-1}Z^T - M_n\omega^2) & Z(L/EA)^{-1} \\ (L/EA)^{-1}Z^T & ((L/EA)^{-1}+(B^TeB)^{-1}-M_r\omega^2) \end{bmatrix}\begin{bmatrix} p_a \\ h_a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For small vibrations Z_p^T is equal to Z^T and $X_{(H+H0)}$ is constant so eq (9) is linear. In the diagonal matrix M_n the mass of the roof and load on it lumped to the cable joints is arrayed. In M_r the mass of the ring and the columns increased by some contributions from the roof is lumped to the cable anchors.

For vibration modes antisymmetric in both directions h_a is zero so eq (9) can for this case be simplified. Symmetrical modes, however, induce bending of the ring and for such cases the acceleration of the ring should be considered. Results from some calculations are given in Table 2.

Mode	A	B
Antisym.	0.96	1.45
Sym.	0.85	1.27

Table 2. Period times in sec. for lowest antisymmetric and symmetric modes of vibrations superposed on deflections due to dead load (A) and dead load + snow (B)

Reference:

S.O. Asplund: Structural Mechanics, Ch N and S, Prentice-Hall, Englewood Cliffs, 1966.

Summary

The roof structure of the arena "Scandinavium" in Göthenburg consists of a prestressed cable net anchored in a space-curved ring beam. It was analysed by a non-linear mixed finite element method with the cable joint deflections from a reference position and the live load cable forces as variables.

The comparatively high flexibility of the ring caused a favourable distribution of cable forces due to snow and wind. Still the system was sufficiently stiff for ensuring acceptable dynamic properties.