Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 9 (1972)

Artikel: Lateral buckling strength of girders with bracing systems

Autor: Fukumoto, Yuhshi / Kubo, Masahiro

DOI: https://doi.org/10.5169/seals-9569

Lateral Buckling Strength of Girders with Bracing Systems

Résistance de voilement latéral de poutres raidies

Seitliche Beulsteifigkeit ausgesteifter Träger

YUHSHI FUKUMOTO
Associate Professor

MASAHIRO KUBO
Research Assistant

Department of Civil Engineering
Nagoya University
Nagoya, Japan

INTRODUCTION

The girder-bracing structural systems which consist of two or more parallel main girders and lateral and/or sway bracings between them are commonly used in steel girder bridges. The characteristics required of the bracing systems are to counteract lateral buckling of the girders and to give adequate bracing in order to provide the required lateral support condition at the bracing points.

The load carrying capacity of braced beams and the bracing requirements for the plastically designed beams are investigated extensively at Lehigh University[1), 2), 3)]. Elastic lateral buckling of beams with lateral restraint at the intermediate supports are treated in Refs. 4)-8) as the bifurcation problem and in Refs. 9),10),11) including the initial imperfections. Diaphragm-braced columns and beams are studied extensively at Cornell University[12), 13), 14)] and their studies are cited by Professor Finzi in his Introductory Report[15)].

In this paper the lateral buckling strength of two parallel girders with bracing system between them is solved in bending and the bracing effects on the buckling strength of main girders are discussed using stiffness parameters in the bracings. The optimum relative stiffnesses of the bracings which provide full bracing to girders are determined in which full bracing is defined to possess effectiveness to immovable supports in lateral and torsional at the bracing points. Lateral buckling strength of main girders is obtained in the inelastic range with assumed distributions of residual stresses while the bracing remains in elastic.

Tests are also conducted in this study. Total of eleven specimens are tested under uniform bending and the results of critical moments, buckled configurations, effective length, are compared with the theoretical ones. Bracing forces are also measured during test.

THEORETICAL ANALYSIS

Theoretical elastic and inelastic lateral buckling solutions are obtained by considering the total energy in the system as shown in Fig. 1. Girders with constant cross section are either simply supported or clamped at the both ends.

The total potential energy T in the system is

$$T = V + U + B \qquad (1)$$

in which V=the internal strain energy stored in the main girders, U=the potential energy of the external loads, and B=the internal strain energy stored in the bracing system. In the lateral bracing as shown in Fig. 2 the strain energy B[16] is

$$B = \frac{F_a}{2}\sum(\bar{u}_{Ai}-\bar{u}_{Bj})^2 + 2F_m\sum(\beta_{Ai}^2+\beta_{Ai}\beta_{Bj}+\beta_{Bj}^2) + 2F_{\bar{m}}\sum(\bar{u}_{Ai}'^2+\bar{u}_{Ai}'\bar{u}_{Bj}'+\bar{u}_{Bj}'^2) \qquad (2)$$

in which $F_a=E_bA_b\cos^2\theta/L_b$, $F_m=E_bI_{b\xi}\cos^2\theta/L_b$, $F_{\bar{m}}=E_bI_{b\eta}/L_b$, A_b=cross sectional area of one member, $I_{b\xi}$, $I_{b\eta}$ =moments of inertia of one member about ξ and η axes, respectively. η-ζ plane is in the vertical plane including the member ends i and j.

Horizontal displacement $\bar{u}_{Ai}$ at the bracing point is equated to the displacements u_{Ai} and β_{Ai} about the shear center as,

$$\bar{u}_{Ai} = u_{Ai}-\frac{kh}{2}\beta_{Ai}, \quad \bar{u}_{Bj} = u_{Bj}-\frac{kh}{2}\beta_{Bj} \qquad (3)$$

In the sway bracing as shown in Fig. 3 the strain energy B is,

$$B = F_{ad}\sum\beta_i^2 + 12F_{ms}\sum\beta_i^2 + 12F_{\bar{m}s}\sum\bar{u}_i'^2 \qquad (4)$$

in which $F_{ad}=E_bA_bh^2\cos^2\alpha/L_d$, $F_{ms}=E_bI_{b\xi}/L_s$, $F_{\bar{m}s}=E_bI_{b\eta}/L_s$, L_d =length of a strut, and L_s =length of a diagonal member.

The critical moment M_{cr} can be obtained by assuming the buckled configurations as trigonometric series which satisfy the boundary conditions at the both ends and solving the characteristic equations of the coefficients which extremize the total potential energy.

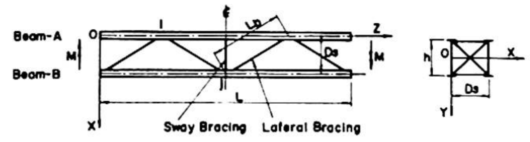


Fig. 1

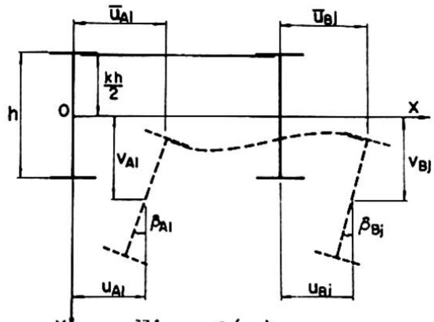


Fig. 2(a)

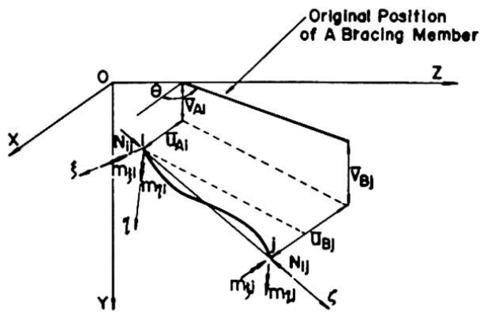


Fig. 2(b)

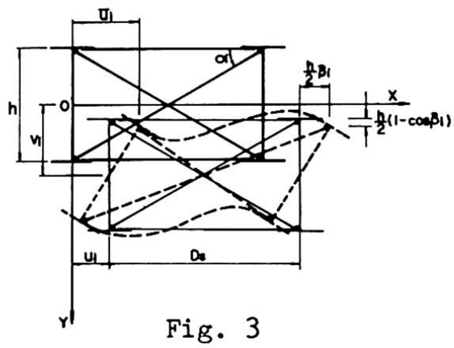


Fig. 3

Fig. 4 shows the elastic and inelastic lateral buckling strength curves for three different residual stress patterns with the maximum compressive stress of σ_{rc} $=0.3\sigma_y$[17,18]. The curves for H-200×100×5.5×8mm beam represent the basic strength curves when unbraced. The following numerical examples are carried out for the same cross section having the residual stress pattern (1).

NUMERICAL EXAMPLES

The authors derived the relationships of the critical moment with the slenderness ratio and the stiffness parameters δ and γ in the bracing system, where δ and γ are defined as follows,

$$\delta = A_b/A_c, \quad \gamma_\xi = I_{b\xi}/I_c, \quad \gamma_\eta = I_{b\eta}/I_c \qquad (5)$$

in which A_b= area of one bracing member
$A_c=A_f+\frac{1}{6}A_w$= area of a compression flange, and web of girder, respectively. I_c =moment of inertia of the compressive area A_c, $I_c \cong I_f = bt^3/12$.

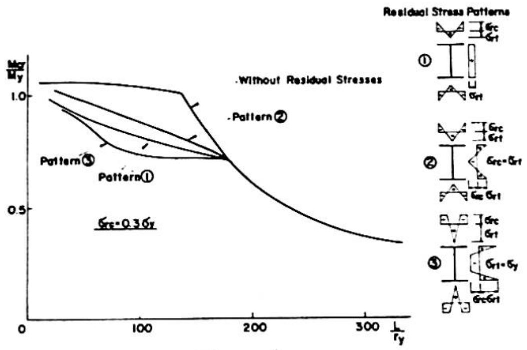


Fig. 4

Lateral Bracing : Fig. 5 shows the buckling strength curves with lateral bracing along the compression flange for four different δ and γ values. Bracing deformation is considered in the vertical plane, that is, $\gamma=\gamma_\xi$, and γ_η-effect on the buckling strength is almost negligible compared to γ_ξ-effect. Distance between two parallel girders in this case is D_s=50cm. In this figure fully braced B-curve indicates the buckling strength when beam B is fully supported at the bracing points in order to meet $u_i=\beta_i=0$ and fully braced A-curve is for beam A. The fully braced B-curve is, in this case, practically important for the design purpose. Fig. 6 is another presentation of M_{cr}-L/r_y-δ-γ curves. In this figure horizontal lines M_{cr}/M_y=0.86, 0.76, 0.69 and 0.52 represent the buckling strength for the fully braced B-curve with the specified L/r_y values, respectively. And γ-values which will be read at the intersections of the horizontal and curved lines give the optimum relative stiffness γ_{opt}=0.07,0.13, 0.23 and 0.28, respectively for the specified δ=0.01. Fig. 7 shows the relationship between the buckling strength and the braced point in the web height. k=1, 0, and -1 mean the beam braced at the tension flange,shear center (=centroid in this case) and compression flange, respectively. The bracing effect against the buckling strength is almost the same if the bracing points are in the compression zone of the main girders due to bending.

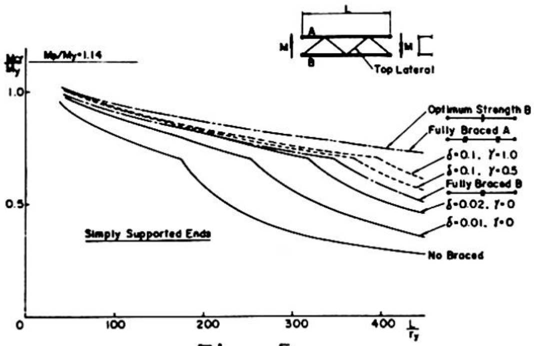


Fig. 5

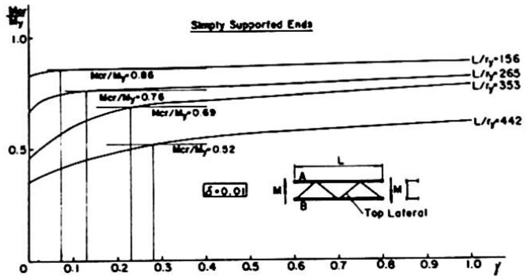


Fig. 6

Sway Bracing : Fig. 8 shows the buckling strength curves with sway bracing at the span center. The curves are bounded by the unbraced and full bracing limits. Since the sway bracing possesses the effectiveness in rotation at the braced section, angle of rotation β_i=0 is only required to meet the full bracing condition.

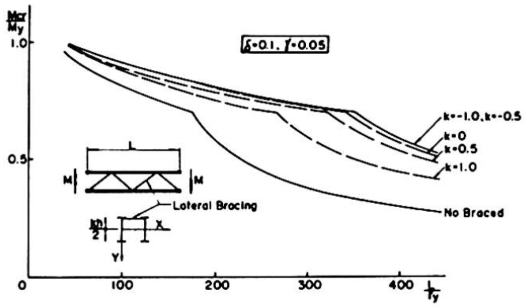


Fig. 7

Effective length : Fig. 9 shows the relations between the unbraced length λ L and the effective length λ_eL which delivers the same critical moment between the simply supported pinned ends. The curves are for full bracing strength in three different

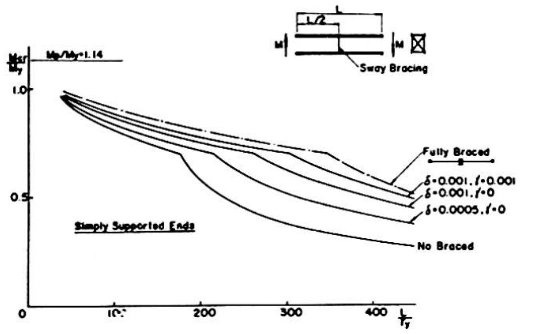


Fig. 8

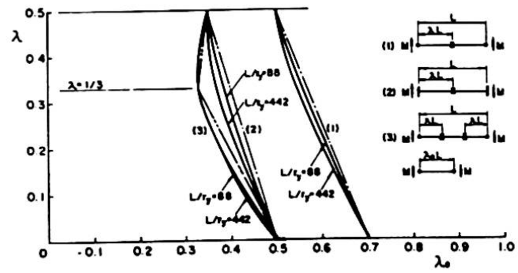


Fig. 9

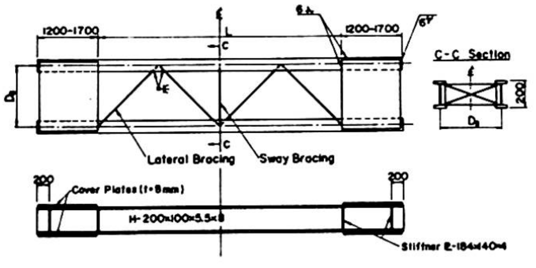


Fig. 10

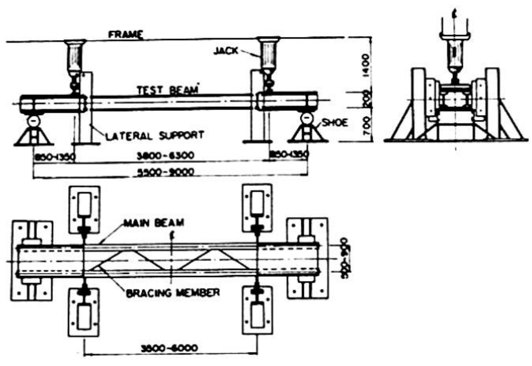


Fig. 11

cases. Theoretical solid lines for L/r_y=88 t0 442 may be approximated with sufficient accuracy by chain lines in each case as,

(1) $\lambda_e = -0.4 \quad + 0.7$ for $0 \leq \lambda \leq 0.5$
(2) $\lambda_e = -0.3 \quad + 0.5$ for $0 \leq \lambda \leq 0.5$
(3) $\lambda_e = -0.5 \quad + 0.5$ for $0 \leq \lambda \leq 1/3$
$\lambda_e = \quad 0.1 \quad + 0.3$ for $1/3 \leq \lambda \leq 0.5$

TEST PROGRAM

Test Specimens and Test Setup : Total of eleven specimens are tested in bending under clamped end conditions against lateral buckling. Rolled beams of H-200×100×5.5 ×8mm section are used as main girders throughout and steel round bars are as bracing members. The detailed dimensions of the test specimens are given in Fig. 10 and Table 1, and the test setup is shown in Fig. 11 and Photo 1.

Load-Deformation Curves : A typical example of load-deformation curves is shown in Fig. 12 where M versus u, v and β at span center are plotted for Type B11. From this figure it is observed that u_A and β_A remain unchanged during test and the full bracing condition is, thus, ensured at the bracing point.

Test Results : All of the test results are

Table 1

Specimens	Dimension of Test Beams	Bracing Systems
TYPE A1	H-200 × 100 × 5.5 × 8	Not Braced
TYPE B11		Lateral(D=19mm)
TYPE B12	D_s= 50cm	Lateral(D= 9mm)
TYPE C1	L= 350cm	Sway(D= 9mm)
TYPE D1	L/r_y=158	Lateral(D= 9mm) Sway(D= 9mm)
TYPE E1		Lateral(D=19mm) Sway(D= 9mm)
TYPE F1		Strut(D=19mm)
TYPE A2	H-200 × 100 × 5.5 × 8	Not Braced
TYPE B2		Lateral(D=19mm)
TYPE C2	D_s= 90cm L= 600cm	Sway(D= 9mm)
TYPE D2	L/r_y=270	Lateral(D= 9mm) Sway(D= 9mm)

D : Diameter of Steel bars used for bracing members

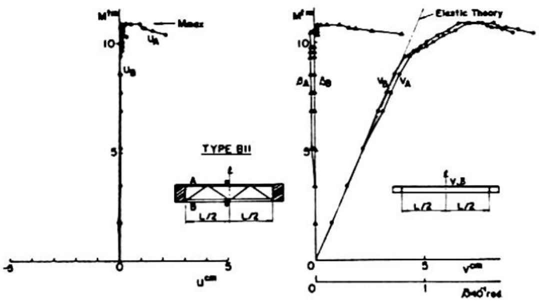


Fig. 12

Table 2

Specimens	$L=350^{cm}$ ($L/r_y=158$), $D_s=50^{cm}$					Specimens	$L=600^{cm}$ ($L/r_y=270$), $D_s=90^{cm}$				
	Test		Theory		$\frac{M_{test}}{M_{theo.}}$		Test		Theory		$\frac{M_{test}}{M_{theo.}}$
	M_{test} (t-m)	Effect of Bracing	$M_{theo.}$ (t-m)	Effect of Bracing			M_{test} (t-m)	Effect of Bracing	$M_{theo.}$ (t-m)	Effect of Bracing	
TYPE A1	4.95	1.00	4.83	1.00	1.02	TYPE A2	4.05	1.00	4.23	1.00	0.96
TYPE B11	5.40	1.09	5.27	1.09	1.02	TYPE B21	5.13	1.27	4.87	1.15	1.05
TYPE B12	5.35	1.08	5.26	1.09	1.02	TYPE B22	-	-	4.84	1.14	-
TYPE C1	5.25	1.06	5.06	1.05	1.04	TYPE C2	4.79	1.18	4.51	1.07	1.06
TYPE D1	5.55	1.12	5.27	1.09	1.05	TYPE D2	5.23	1.29	4.87	1.15	1.07
TYPE E1	5.50	1.11	5.27	1.09	1.05	TYPE E2	-	-	4.87	1.15	-
TYPE F1	5.23	1.06	5.06	1.05	1.03	TYPE F2	-	-	4.51	1.07	-

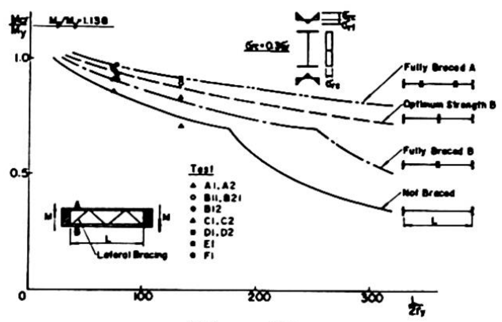
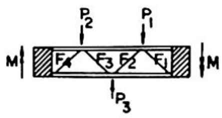


Fig. 13

Table 3

Specimens	F (kg)		F/F_{by}	P (kg)		P/P_f	Remarks
TYPE B11	1	-806	0.089	1	432	0.019	 $P_f = \frac{M_{max}}{W_x} A_f$
	4	-819	0.091	2	417	0.018	
	2	-80	0.009	3	113	0.005	
	3	-142	0.016				
TYPE B12	1	-565	0.278	1	470	0.021	
	4	-598	0.294	2	296	0.013	
	2	-178	0.088	3	138	0.006	
	3	-74	0.036				
TYPE B21	1	-604	0.067	1	445	0.020	
	4	-666	0.074	2	445	0.020	
	2	-62	0.007	3	50	0.002	
	3	-80	0.009				

plotted in Fig. 13 together with the theoretical buckling curves of unbraced and full bracing beams. Test points for the braced specimens B11-F1 are all above the fully braced B-curve.

A summary of test results is given in Table 2. The bracing effect which is defined by the strength ratio of each braced specimen to the unbraced beam is compared between theory and tests. Since all of the specimens are buckled in the inelastic range, the bracing effect may not be expected as it would be in the elastic range. Photo 2 illustrates several examples of specimens after test showing the buckled patterns of beams with full bracing.

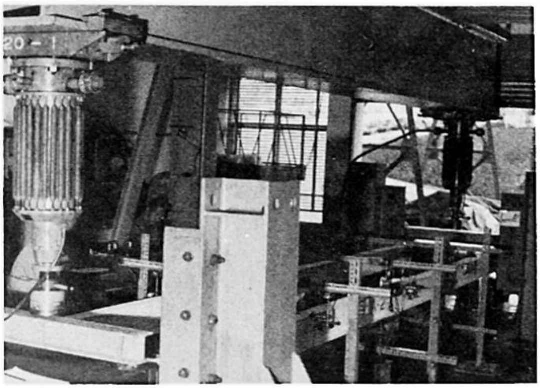


Photo 1

Bracing Forces : Bracing forces such as axial force F_{α} and bending moments M_{ξ} and M_{η} are measured during test. Table 3 summarizes the measured bracing forces F's and the resultant lateral forces P's which are composed of F's at the ultimate load. The maximum value of bracing force P is obtained as 2% of the compressive flange force P_f.

CONCLUDING REMARKS

Followings are the main subjects which are discussed in this paper.

(1) Elastic and inelastic lateral buckling strength of beams with lateral and/or sway

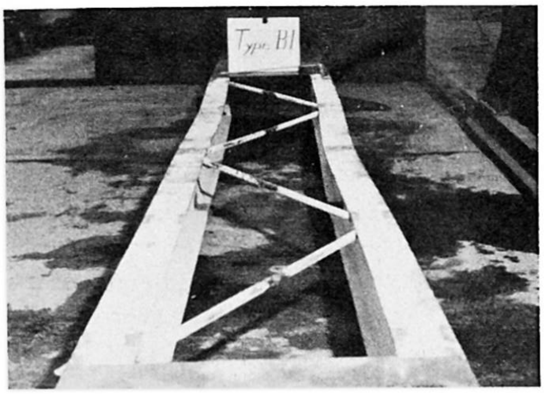


Photo 2(a)

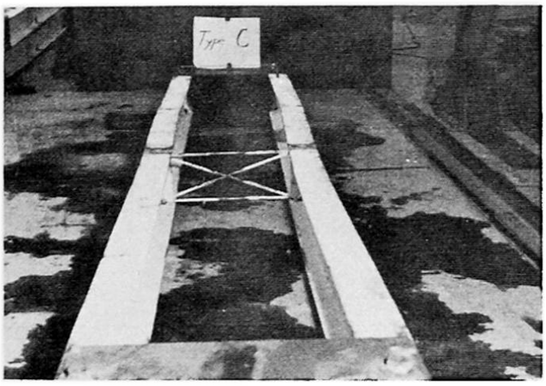


Photo 2(b)

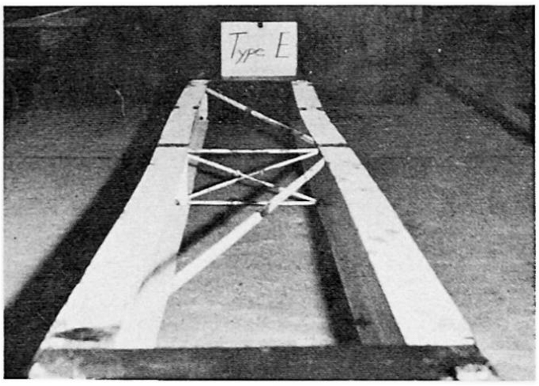


Photo 2(c)

bracings is determined theoretically including an arbitrary distributed residual stresses.
(2) Optimum relative stiffnesses δ and γ in bracing members are defined and the bracing effect of bracing systems are discussed.
(3) Tests are conducted for eleven specimens with different bracing systems and bracing effects are compared with the theoretical results.

REFERENCES

1) Lay, M.G. and Galambos, T.V., Proc. ASCE, Vol. 92, No.ST 2, April, 1966.
2) Lee, G.C. and Galambos, T.V., Proc. ASCE, Vol. 88, No.EM 1, February, 1962.
3) Lee, G.C., Ferrara, A.T. and Galambos, T.V., Bulletin No. 99, Welding Research Council, New York, N.Y., September, 1964.
4) Hartmann, A.J., Proc. ASCE, Vol.93, No.ST 4, August, 1967.
5) Hartmann, A.J., Proc. ASCE, Vol.96, No.ST 7, July, 1970.
6) Klöppel, K. und Unger, B., Der Stahlbau, Heft 7, 1969.
7) Schmidt, L.C., Proc. ASCE, Vol. 91, No.EM 6, December, 1965.
8) Taylor, A.C. and Ojalvo, M., Proc. ASCE, Vol.92, No.ST 2, April, 1966.
9) Massey, C., Proc. ASCE, Vol.88, No.EM 6, December, 1962.
10) Winter, G., Proc. ASCE, Vol.84, No.ST 2, March, 1958.
11) Zuk, W., Proc. ASCE, Vol.82, No.EM 3, July, 1956.
12) Apparao, Errera and Fisher, Proc. ASCE, Vol.95, No.ST 5, May, 1969.
13) Errera, S.J., Pincus, G. and Fisher, G.P., Proc. ASCE, Vol.93, No.ST 1, February, 1967.
14) Pincus, G. and Fisher, G.P., Proc. ASCE, Vol. 92, No.ST 2, April, 1966.
15) Finzi, L., 9th IABSE Congress Introductory Report, Amsterdam, 1972.
16) Fukumoto, Y., Kubo, M., Proc. JSCE, No. 196, December, 1971.
17) Galambos, T.V., "Structural Members and Frames," Prentice-Hall, Inc., Englewood Cliffs, N.J. 1968.
18) Fukumoto, Y., Fujiwara, M. and Watanabe, N., Proc. JSCE, No.189, May, 1971.

SUMMARY

Elastic and inelastic lateral buckling strength of two parallel girders with lateral and/or sway bracing systems between them is determined theoretically and the bracing effects on the buckling strength are discussed using stiffness parameters in the bracing. The optimum relative stiffnesses of the bracing which provide full bracing at the bracing points are obtained. Tests are also conducted in this study. Total of eleven specimens with different bracing systems are tested in bending, and the results of critical moments, buckled configurations and effective length are compared with the theoretical ones.