Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 9 (1972)

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DOI: https://doi.org/10.5169/seals-9568

Influence of Diaphragms on Behaviour of Box Girders with Deformable Cross Section

Influence d'entretoises sur le comportement de poutres en caisson à section déformable

Einfluss der Querträger auf das Verhalten von Kastenträgern mit deformierbarem Querschnitt

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1. INTRODUCTION

Thin-walled structures, which have recently been applied very often in the field of civil engineering, have got big-sized and complicated remarkably. In designing such structures three-dimensional analyses have been getting essential to harmonize safety with economy. At present, however, we cannot say that the interaction between their structural elements is fully considered in their design.

One of interaction problems is the distortional deformation of box-girders and the influence of diaphragms on it. The conventional theory of thin-walled beams, based on the assumption of non-distortional deformation, cannot be applied to the above problem. Therefore diaphragms are designed from experience rather than on a theoretical basis, and there seems to be no clear specification on their design in any country. 1),2)

The subject of this paper is to analyze theoretically and experimentally the distortional behaviour of box-girders and the interaction between box-girders and diapharagms.

There are two methods in analyzing thin-walled structures. One is the finite element method (F.E.M.), and the other is the folded plate theory.*) In this paper, we modefied Vlasov's folded plate theory and applied it to the analyses of straight box-girders with rectangular and ribbed trapezoidal cross section and curved box-girders. Theoretical results obtained agreed well with experimental ones. We also developed the finite strip method, which is a kind of F.E.M., 10) and compared results from both theories. The comparison shows that there is not so marked difference between both theories with respect to primary stresses and deformations. 11)

2. FUNDAMENTAL EQUATIONS OF A STRAIGHT SINGLE BOX GIRDER 12)

2.1 Assumed displacement functions

We consider a ribbed trapezoidal cross section, as shown in Fig. 1, and assume the followings:

1. The cross section is uniaxially symmetrical.
2. The ribs exist concentratedly on the contour line of the cross section.

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*) For example, the references 3),4),5),6),7),8) and 9)

3. The shear stresses in the ribs are negligible.
4. Each plate element resists only in-plane forces.*)

Let us the displacements in the axial direction and in the direction tangent to the cross-section contour line as follows (Fig. 2):

$$u(z,s)=\sum_{i=1}^{4}\varphi^{(i)}(s)\,U_i(z) \qquad (1\text{-a})$$

$$v(z,s)=\sum_{j=1}^{4}\psi^{(j)}(s)\,V_j(z) \qquad (1\text{-b})$$

where $\varphi^{(i)}$ and $\psi^{(j)}$ are generalized coordinates, and U_i and V_j are generalized displacements. If the generalized coordinates are given as shown in Fig. 3 and 4, the generalized displacements represent the following physical meanings:

U_1: Axial displacement	V_1: Twist angle
U_2: Rotation about x axis	V_2: Deflection in x direction
U_3: Rotation about y axis	V_3: Deflection in y direction
U_4: Warping rate	V_4: Distortion rate

In Fig. 3.(d) α_1 and α_2 are determined from the following orthogonal condition:

$$\oint \varphi^{(3)}\varphi^{(4)}dF=0 \qquad (2)$$

In Fig. 4.(a) y_s are determined from the following orthogonal condition:

$$\oint \psi^{(1)}\psi^{(3)}tds=0 \qquad (3)$$

Similarly β_1 and β_2 in Fig. 4.(d) are obtained from the following assumption that a unit shear flow of pure torsion does not work for the displacement $\psi^{(4)}$:

$$\oint \psi^{(3)}\psi^{(4)}tds=0 \quad , \quad \oint \psi^{(4)}1tds=0 \qquad (4)$$

where d_w is the length of a perpendicular from the shear center to a web.

2.2 Fundemental equations

Applying the principle of virtual work for the assumed displacements u and v, we obtain the following fundamental equations:

$$EFU_1''+p_1=0 \qquad (5)$$

$$EJ_x U_2''-GF_w(U_2+V_2')+p_2=0 \qquad (6)$$

$$EJ_y U_3''-GF_f(U_3+V_3')+p_3=0 \qquad (7)$$

$$EJ_w U_4''-G\{B_{44}U_4-B_{34}(U_3+V_3')-C_{41}V_3' \;-C_{44}V_4'\}+p_4=0 \qquad (8)$$

$$G(C_{41}U_4'+D_{11}V_1''+D_{14}V_4'')+q_1=0 \qquad (9)$$

$$GF_w(U_2'+V_2'')+q_2=0 \qquad (10)$$

$$G\{F_f(U_3'+V_3'')+B_{34}U_4'\}+q_3=0 \qquad (11)$$

$$G(C_{44}U_4'+D_{14}V_1'+D_{44}V_4'')+q_4=0 \qquad (12)$$

where E and G are elastic constants, F, I_x, ... and D_{44} are cross-sectional constants, and p_1, p_2, ... and q_4 are generalized loads.

When it is necessary to take into consideration the influence of transverse plate-bending, we must add the term $\oint M_s\rho_s^{(4)}ds$ to the left side of the equation (12), where $\rho_s^{(4)}$ is the curvature due to the displacement $\psi^{(4)}$.

Now for a distortional load shown in Fig. 5,

*) In the case of very a few diaphragms the influence of transverse plate-bending is significant, as mentioned in 2.2.

$$p_1 = p_2 = p_3 = p_4 = q_2 = q_3 = 0 \tag{13}$$

then we have only to deal with the simaltaneous equations (7), (8), (9), (11) and (12).

2.3 Cross-sectional forces

The relation between stresses and displacements is as follows:

$$\sigma_z = E\sum_{i=1}^{4}\varphi^{(i)}U_i' \tag{14-a}$$

$$\tau_{zs} = G\sum_{j=1}^{4}(\psi^{(i)}V_j' + \varphi^{(j)}U_j) \tag{14-b}$$

We define cross-sectional forces as N,Mx,My,B,H,Qx,Qy and Q. Then stresses are related with cross-sectional forces as follows:

$$\sigma_z = \frac{N}{F} + \frac{M_x}{I_x}y + \frac{M_y}{I_y}x + \frac{B}{I_w}\varphi^{(4)} \tag{15-a}$$

$$\tau_{zs} = \frac{D_{44}H - D_{14}Q}{D_{11}D_{44} - D_{14}^2}\psi^{(1)} + \frac{Q_y}{F_w}\psi^{(2)} + \frac{Q_x}{F_f}\psi^{(3)} + \frac{D_{14}H + D_{11}Q}{D_{11}D_{44} - D_{14}^2}\psi^{(4)} \tag{15-b}$$

2.4 Diaphragms

We assume that diaphragms cannot resist forces out of their own planes. Diaphragm stresses, as shown in Fig. 6, are obtained as follows:

$$\tau_h = \frac{Q_d / t_d}{(b_l\beta_1 + b_u\beta_2 + 2h)d_w}, \quad \tau_u = \tau_h\frac{b_l}{h}, \quad \tau_l = \tau_h\frac{b_u}{h} \tag{16}$$

Where Q_d is a distortional reaction from the girder to the diaphragm.

3. THE CASE OF RECTANGULAR(BIAXIALLY SYMMETRICAL) CROSS SECTION 13)

3.1 Fundamental equations and solutions

In the case of biaxially symmetrical cross section, $b_u=b_l=b$ and $t_u=t_l=t$ results in:

$$y_s = 0, \; d_w = b/2, \; \alpha_1 = \alpha_2 = hb/4, \; \beta_1 = \beta_2 = h/b \tag{17}$$

Therefore $B_{34}=0$ and the system of the equations (7) and (11) is independent of the system of the equations (8),(9) and (12) for distortional loads.

When we must take into consideration the influence of transverse plate-bending, the equation (12) becomes as follows:

$$G(C_{44}U_4' + D_{14}V_1' + D_{44}V_4'') - E\left(\frac{8}{b/t_b^3 + h/t_h^3}\right)V_4 + q_4 = 0 \tag{18}$$

The equation system (8), (9) and (18) coincides with the equations which V. Z. Vlasov derived from his closed beam-shell theory.

Numerical calculations were made by using Fourier series and the transfer matrix method. The transfer matrix method is efficient in the cases of variable cross section, deformable diaphragm and complicated boundary condition such as continuous girders. In Table 1 and 2 are shown the field matrix and the point matrix.14)

3.2 Model test

The above-mentioned theory were checked with a model test. The models are three simple-supported box-girders with 6 mm plate thickness, 50 cm x 30 cm cross section and 5 m span length. One and five intermediate diaphragms are placed at two of them, which are called A501 and A505. No intermediate diaphragm is placed at the other, A500.

Fig. 7 and 8 show illustrations of the comparison between the theoretical and experimental results. Fig. 7 shows the displacements in the perpendicular direction due to a distortional load. The fine lines correspond to the c=0 (the equation (12)) and the thick lines correspond to the case c≠0 (the equation (18)). Fig. 8 shows the warping stress and the transverse membrane stress. It can be seen from both figures that the membrane equations and Vlasov's equations are confirmed by the experiment.

3.3 Considerations on diaphragm designing

There is close relation between spacing and rigidity in diaphragm designing. Spacing and rigidity should not be determined independently, but in this paper we consider these problems separately.

3.3.1 Spacing

We investigate the influence of diaphragm spacing by the following numerical examples under the condition that infinitely rigid diaphragms are placed at a uniform interval:

Cross sections: (1) 1.0 m x 1.0 m, (2) 1.0 m x 1.5 m,
(3) 1.0 m x 2.0 m, (4) 1.0 m x 2.5 m,
(5) 1.0 m x 3.0 m

Span length: l=10 m - 80 m

Numbers of diaphragm: 0 - 10

Fig. 9 shows the relationship between maximum values of warping moment and numbers of diaphragm when the same quantity of uniform distortional load is distributed along each span. It indicates that we can consider two critical points with respect to diaphragm spacing.

1. The number of diaphragms needed so that stresses or displacements do not exceed limited values
2. The number of diaphragms needed so that the membrane equations are valid

3.3.2 Rigidity

We investigate the influence of diaphragm rigidity by a numerical example of box-girder with variable cross section, as shown in Fig. 10. The relationship between maximum values of warping moment and rigidity (thickness) of diaphragms is shown in Fig. 11, which indicates that diaphragms may be considered as infinitely rigid even if they are comparatively thin.

4. THE CASE OF RIBBED TRAPEZOIDAL CROSS SECTION

4.1 Characteristics of trapezoidal box-girders

We investigate the characteristics of trapezoidal box-girders by comparing numerical results of rib-stiffened simple-supported girders with three types of cross section in Fig. 12. The thickness of their webs and bottom flanges is adjusted so that each cross section has the same area and moment of inertia about the x axis.

The results obtained show that as the number of diaphragms increases, warping stresses approach gradually zero and shear stresses approach gradually Bredt's values in each girder, but as a trapezoid of cross section becomes sharp, stresses increase. Diaphragm stresses are shown in Fig. 13, from which it is seen that as a trapezoid of cross section becomes sharp, the stresses become greater if the diaphragms have the same thickness.

4.2 Considerations on diaphragm designing

We investigate the stresses of girder and diaphragm on the basis of numerical results in the case that infinitely rigid diaphragms are placed at a uniform distance on a girder of the cross section shown in Fig. 12.(b). σ_b and τ_B , which are normal stresses due to bending about the x axis and shear stresses due to pure torsion (Bredt's values) respectively, are used in order to compare absolute values of stresses due to the distortional load.

For a uniform distortional load, as shown in Fig. 14, the influence of distortion decreases rapidly as the number of diaphragms increases. For a concentrated distortional load, as shown in Fig. 15 and 16, it depends on load positions. If the load acts at a diaphragm point, the influence of distortion almost vanishes, but if the load acts away from diaphragm points, comparatively great stresses are produced due to the influence of distortion, although diaphragm spacing is close.

Diaphragm stresses are illustrated in Fig. 17 and 18, They have the same tendency as girder stresses. When the load acts at a diaphragm point, the shear stress of the diaphragm is almost constant regardless of diaphragm spacing, and almost equal to the value calculated on the assumption that a distortional moment due to the load is resisted only by the diaphragm. This fact indicates that we may estimate shear stresses of diaphragms acted upon by a concentrated distortional load, if they are rigid, on the above assumption. This estimation is on the safety side for deformable diaphragms in practical application.

5. CURVED BOX- GIRDERS

5.1 Fundamental equations

In curved girders F.E.M. is applicable, too. 15),16),6) In this paper we apply fundametal equations of curved box-girder based on the folded plate theory.

In curved box-giders, it is tedious to choose generalized coordinates satisfying orthogonal conditions, as seen in straight box-girders. This is appreciable, for example, from the fact that the neutral axis does not pass through the center of cross section. The influence of initial curvature enforces us to consider three-dimensional displacement functions, because axial strains are produced by the displacement in the radial direction.

Let us assume the displacements in the directions of the coordinate α , s and r (Fig. 19) as follows: 17), 18)

$$\left.\begin{aligned} u(\alpha,s) &= \sum_{i=1}^{4} \varphi^{(i)}(s)\, U_i(\alpha) \\ v(\alpha,s) &= \sum_{i=1}^{4} \psi_v^{(i)}(s)\, V_i(\alpha) \\ w(\alpha,s) &= \sum_{i=1}^{4} \psi_w(s)\, V_i(\alpha) \end{aligned}\right\} \qquad (19)$$

where $\varphi^{(i)}$, $\psi_v^{(j)}$ and $\psi_w^{(j)}$ are generalized coordinates, and U_i and V_j are generalized displacements.

Applying the principle of virtual work for the assumed displacements, we obtain the fundamental equations as follows:

$$\left.\begin{aligned} \sum_{j=1}^{4} EA_{ij}\frac{U_j''}{R_o^2} - \sum_{j=1}^{4}(EB_{ij}^{*} + GB_{ij})\frac{V_j'}{R_o} + \sum_{j=1}^{4} GC_{ij}\, U_j + p_i &= 0 \\ \sum_{j=1}^{4} GD_{ij}\frac{V_j''}{R_o^2} + \sum_{j=1}^{4}(EE_{ij}^{*} + GE_{ij})\frac{U_j'}{R_o} + \sum_{j=1}^{4} EF_{ij}\, V_j + q_i &= 0 \end{aligned}\right\} \quad (i=1,2,3,4) \qquad (20-a,b,c,d,e,f,g,h)$$

where A_{ij}, B_{ij}, ... and F_{ij} are cross-sectional constants and p_i and q_i are generalized loads.

When we must take into consideration the influence of transverse plate-bending, the term $\oint M_s \rho_s^{(4)} ds$ is added to the left side of the equation (20-h).

5.2 Model test

We compare theoretical and experimental results by a model test. Model were supported simply. The theoretical results were obtained after consideration of the influence of transverse plate-bending and stretching. The results showed that the influence of transverse plate-stretching is not important to primary stresses and the influence of transverse plate-bending vanishes as the number of diaphragms increases, in the same manner as straight girders.

Fig. 20 shows the deflections due to an eccentric load at the centers of the inner and outer web. The inner loading produces greater deflection for the case of no intermediate diaphragm, while the outer loading produces greater deflection for the case of intermediate diaphragms. Fig. 21 shows the normal stress distribution due to an eccentric load at the 1/4 point of the span. The normal stress for the case of no intermediate diaphragm is twice as great as the one for the case of intermediate diaphragms, and if there is no intermediate diaphragm, distortion affects curved box-girders to the same extent as longitudinal bending.

6. CONCLUSIONS

We come to the following conclusions from the above theoretical and experimental results:

1) The cross-sectional distortion has a great influence on the behaviour of a box girder, when a distortional load acts upon it.

2) The characteristics of a diaphragm for distortional loads are considered as follows:

a) Decrease of stresses and increase of rigidity in a girder
b) Keeping cross-sectional shapes unchangeable
c) Reduction of distortional loads into torsional loads
d) Distinguishing plate membrane actions from plate bending actions
e) Introduction and distribution of loads

3) There are almost the same tendencies in curved box giders as in straight box girders with respect to distortional behaviours and diaphragm characteristics. The inner loading produces more severe condition than the outer loading for a curved box girder with very a few diaphragms.

4) Diaphragm spacing should be determined so that warping stresses are within a limited value, for example 10%, of the maximum bending stress for a uniformly distributed load, providing that it needs to check by calculations for a concentrated load.

5) Diaphragms may be considered as rigid, even if they are comparatively thin. When a usual diaphragm is acted upon by a concentrated load, it is practical to calculate the shear stress on the assumption that it is infinitely rigid.

7. ACKNOWLEDGEMENT

This study is much owing to Mr. Katsuhito Aono in the service of Japan Highway Corporation, who was a graduate student at University of Tokyo. The authors wish to thank him for his cooperation in experiments and calculations.

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SUMMARY

The subject of this paper is to analyze the distortional behaviour of box girders and the girder-diaphragm interaction and to investigate the effects of diaphragms. The fundamental equations of straight rib-stiffened trapezoidal box girders and rectangular curved box girders are presented, based on the folded plate theory. The experimental results agree with the theoretical ones and confirm the assumption of the theory. Some investigations are carried out on diaphragm designing, that is, diaphragm rigidity and spacing.

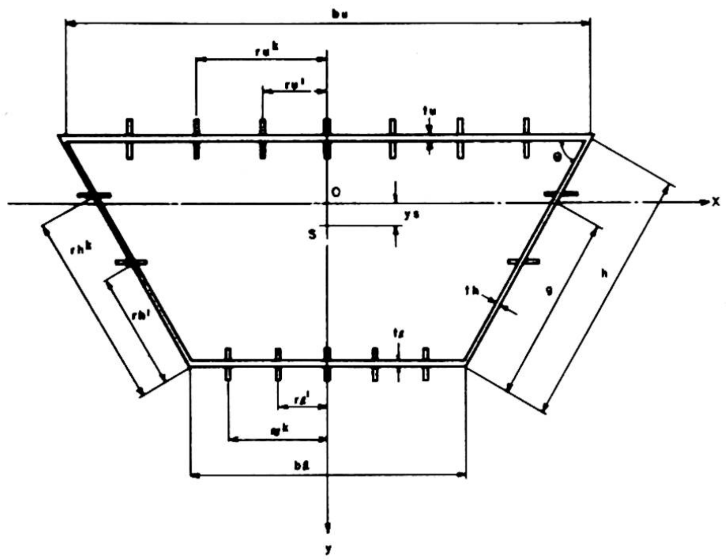


Fig. 1 A Ribbed Trapezoidal Cross Section.

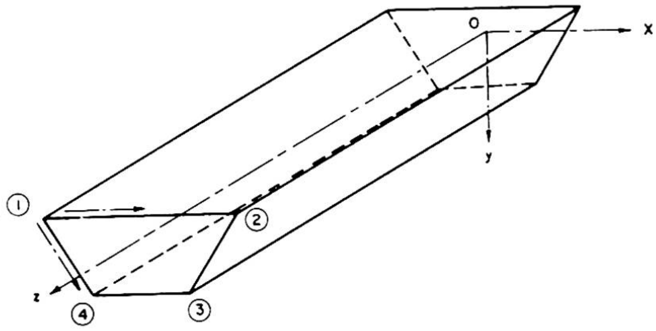


Fig. 2 Coordinates

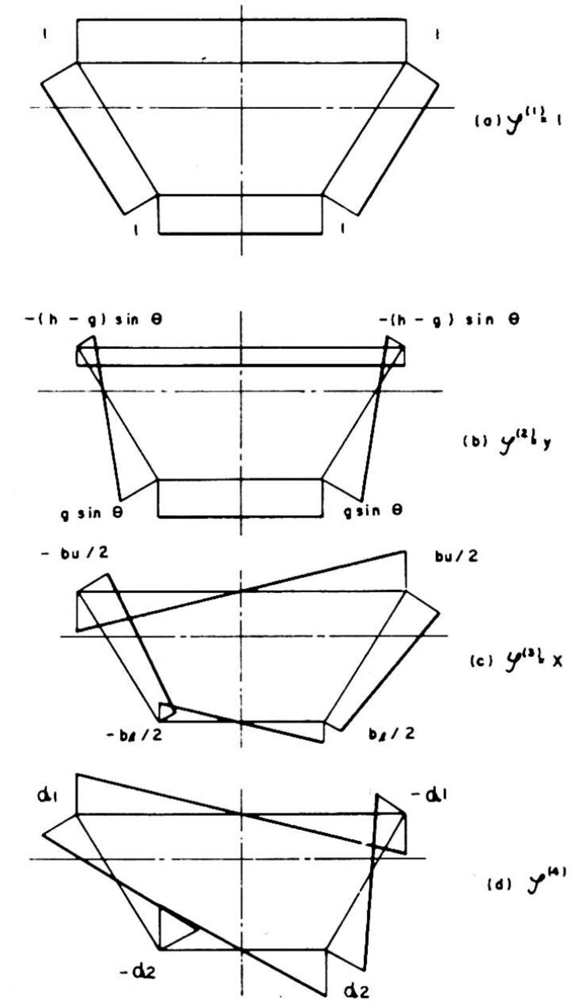


Fig. 3 Generalized Coordinates φ(i)

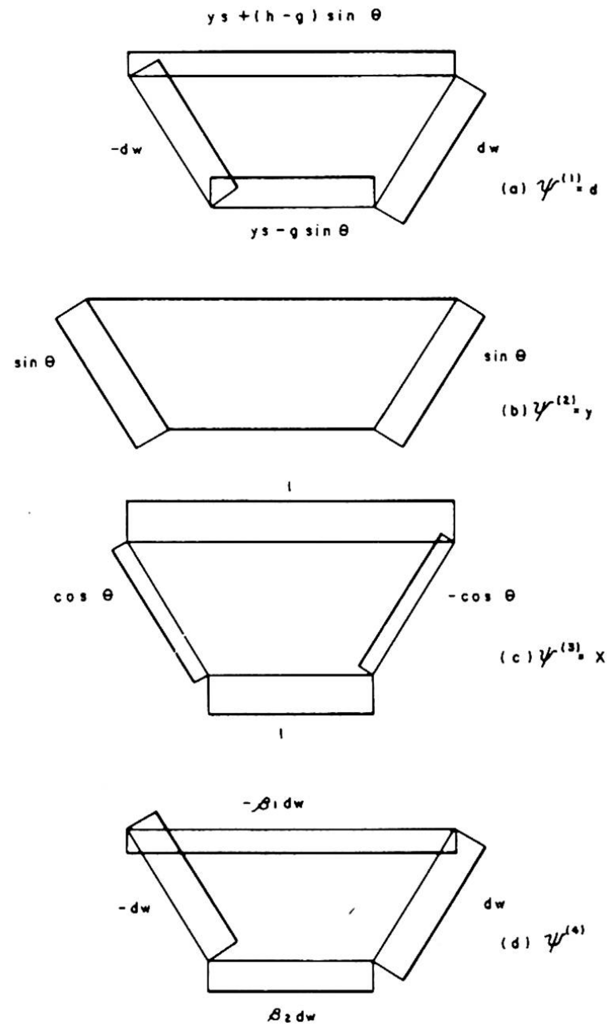


Fig. 4 Generalized Coordinates (i)

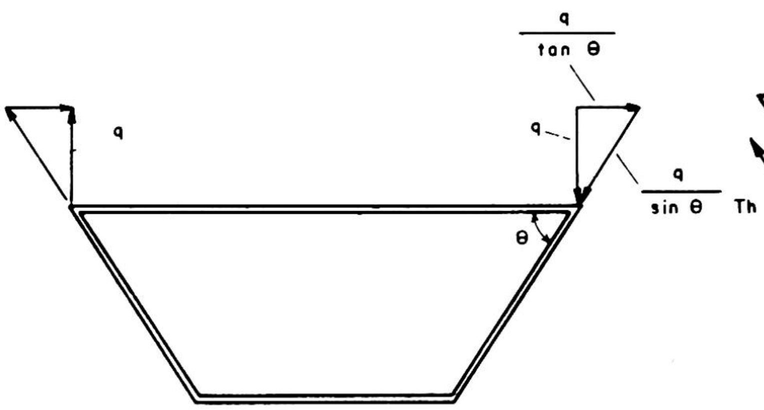


Fig. 5 A Distortional Load

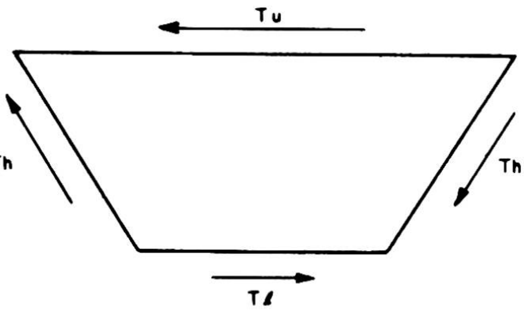


Fig. 6 Diaphragm Shear Stresses

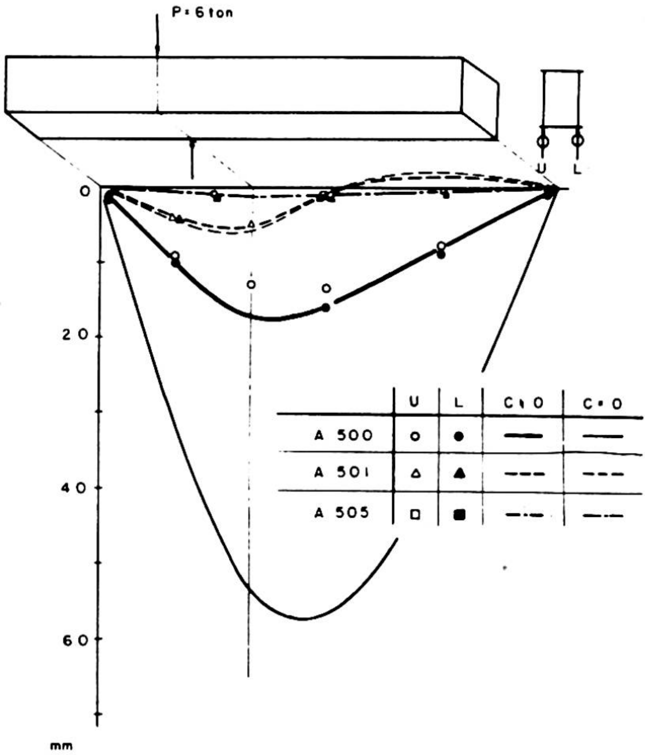


Fig. 7 Comparison of Displacements

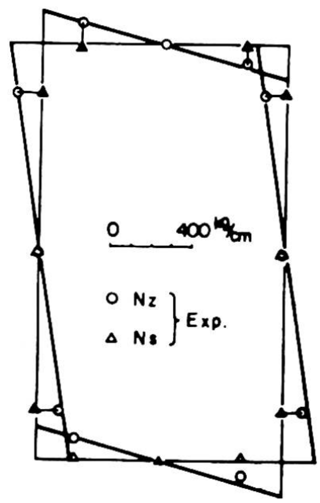


Fig. 8. Warping Stress at the Center

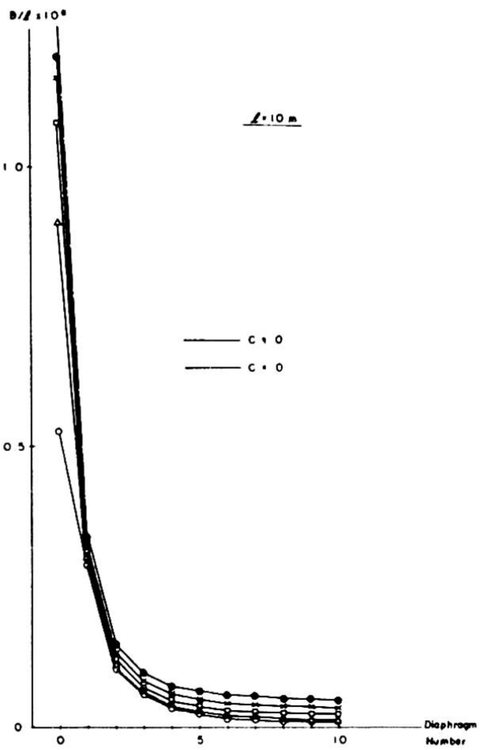


Fig. 9. (a) The Iufluence of Diaphragm Number

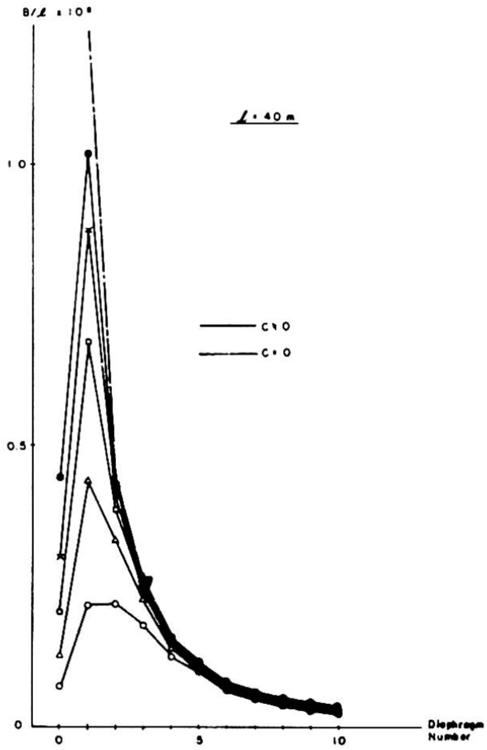


Fig. 9. (b)

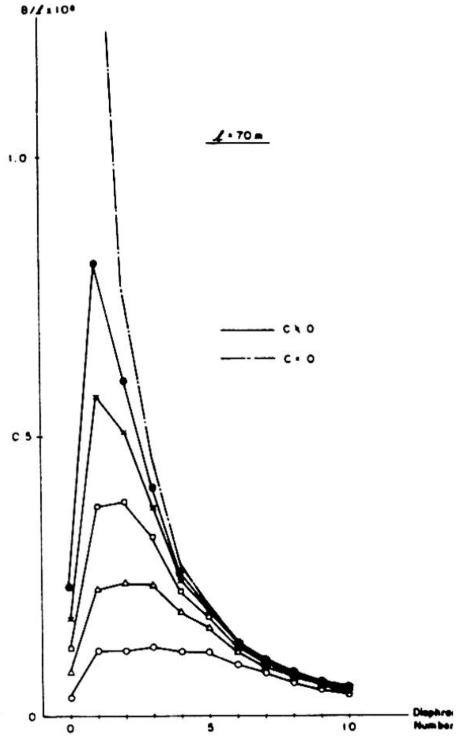


Fig. 9. (c)

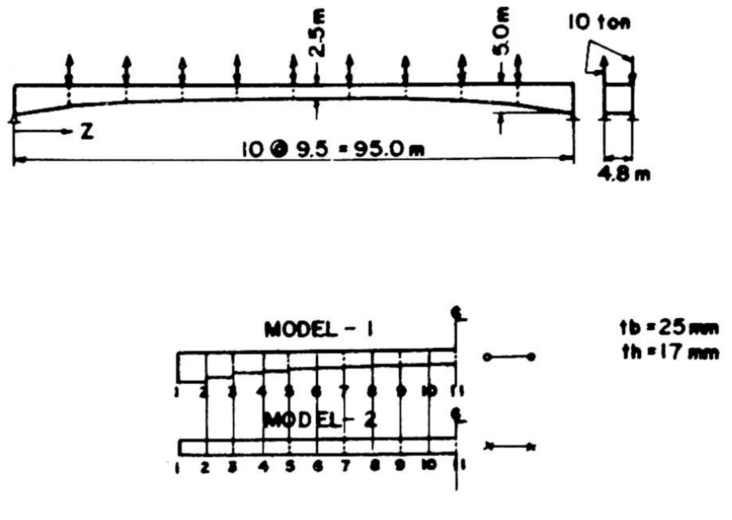


Fig. 10 A Numerical Model of Variable Cross Section

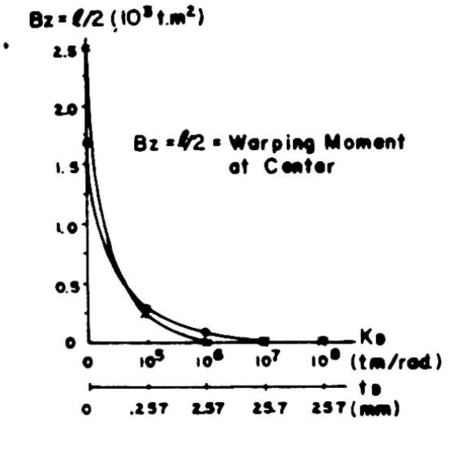
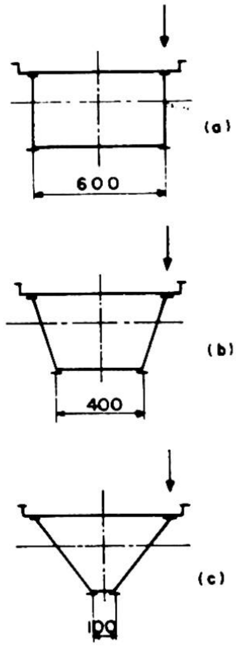
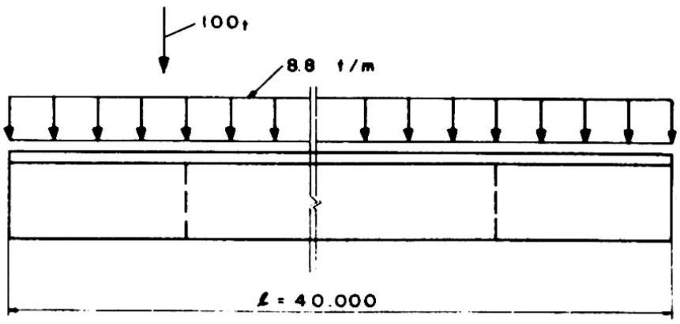


Fig. 11 Iufluence of Diaphragm Rigidity





	F (cm²)	Ix (cm⁴)	Iy (cm⁴)	Iw (cm⁶)
(a)	$\times 10^4$	$\times 10^8$	1.83×10^8	3.72×10^{12}
(b)	0.360	0.841	1.39	2.88
(c)			1.01	2.55

Fig. 12 Cross Sections for Comparison

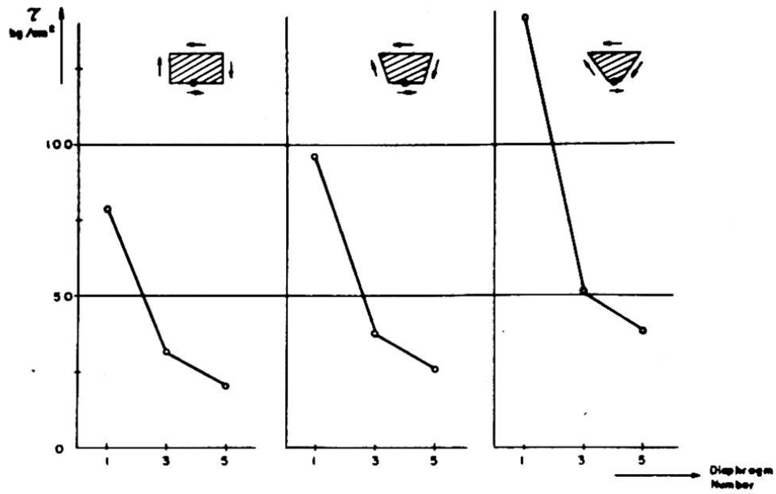


Fig. 13 Maximum Diaphragm Stress (ts = 20 mm)

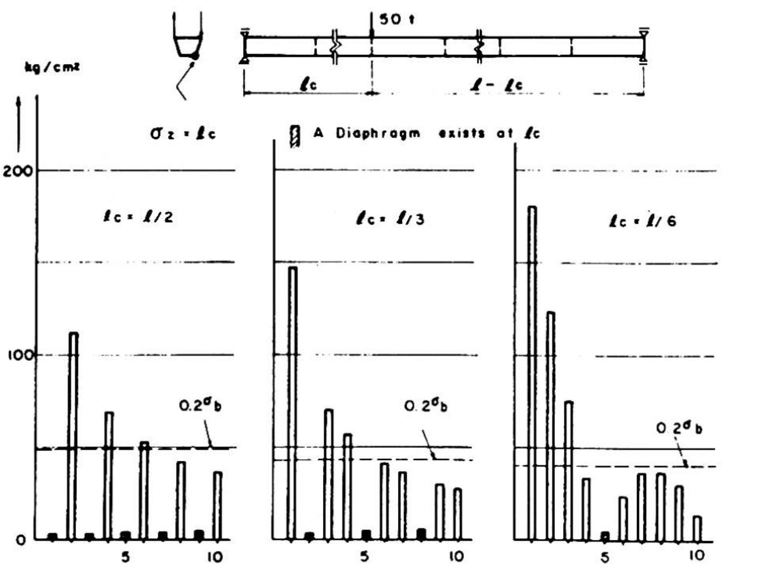


Fig. 15 The Influence of Diaphragm Number

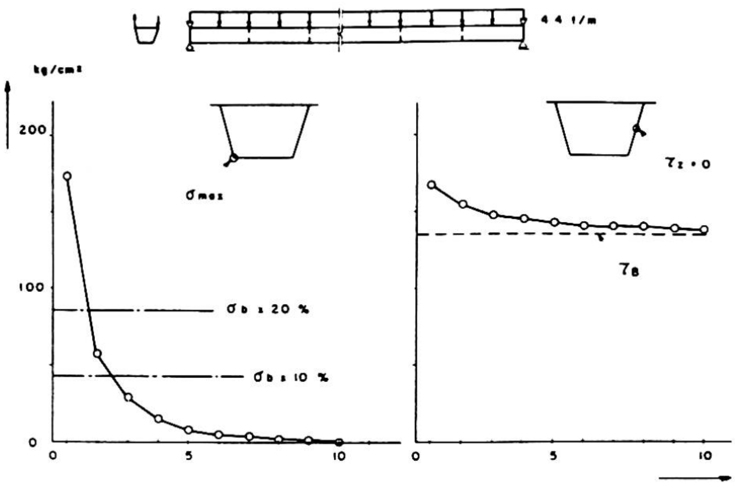


Fig. 14 The Relation between Diaphragm Number and Girder Stresses

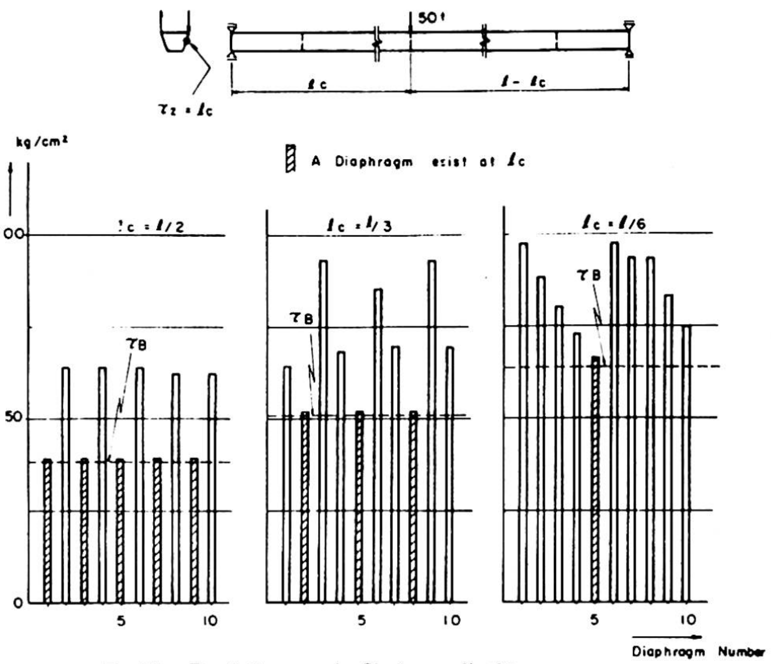


Fig. 16 The Iufluence of Diaphragm Number

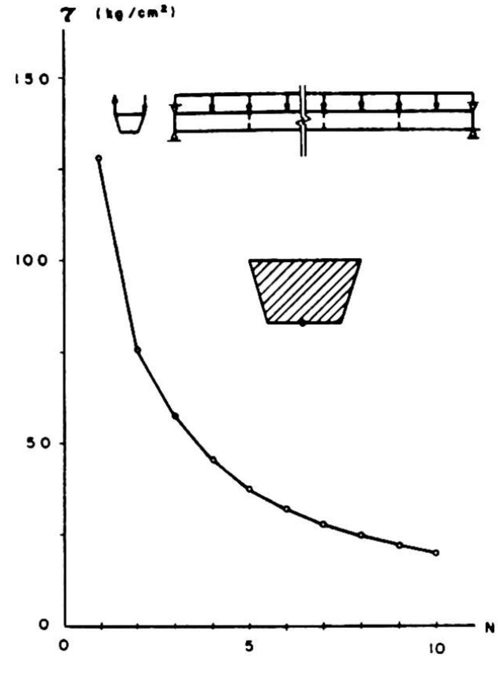


Fig. 17 The Influence of Diaphragm Number on Diaphagm Stress

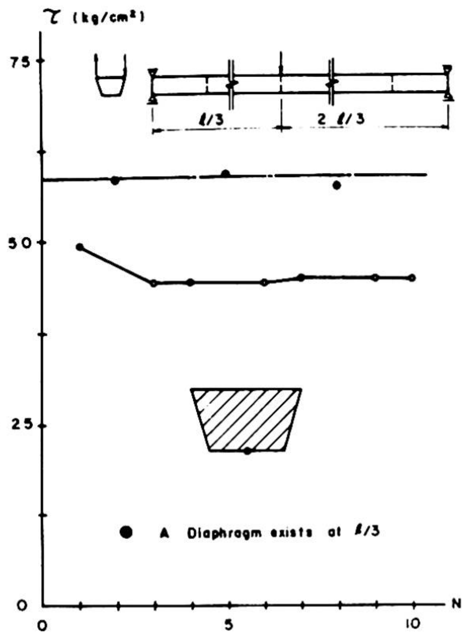


Fig. 18 The Iufluence of Diaphragm Number on Diaphragm Stress

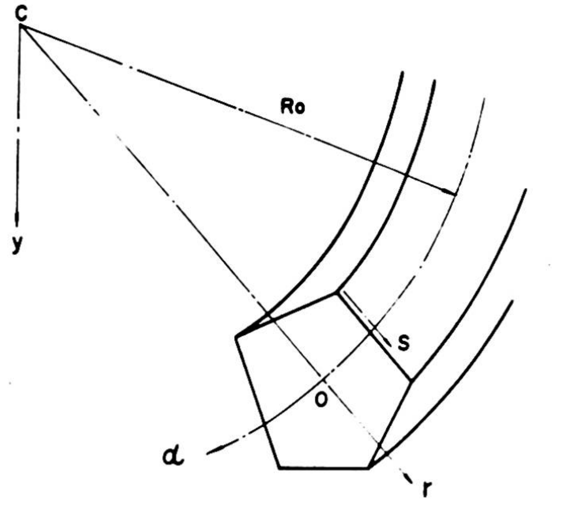


Fig. 19 Coordinates for Curved Box Girders

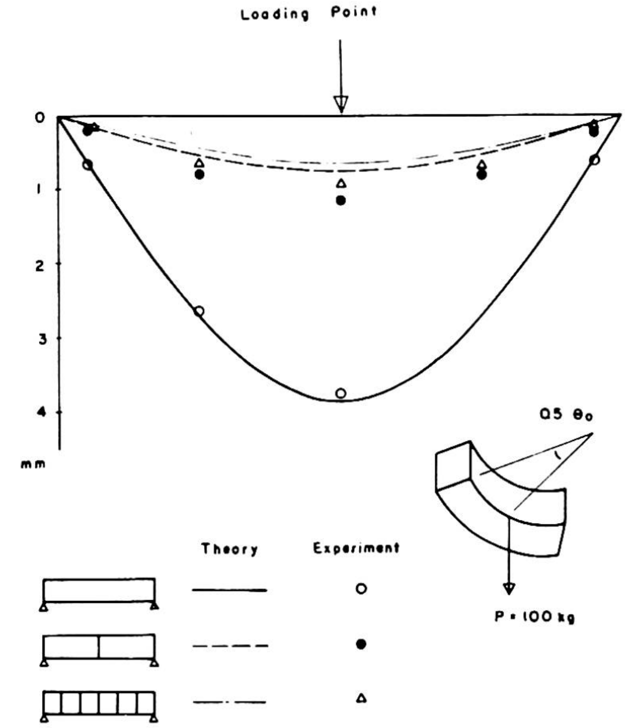


Fig. 20 (a) Deflection by Inner Loading

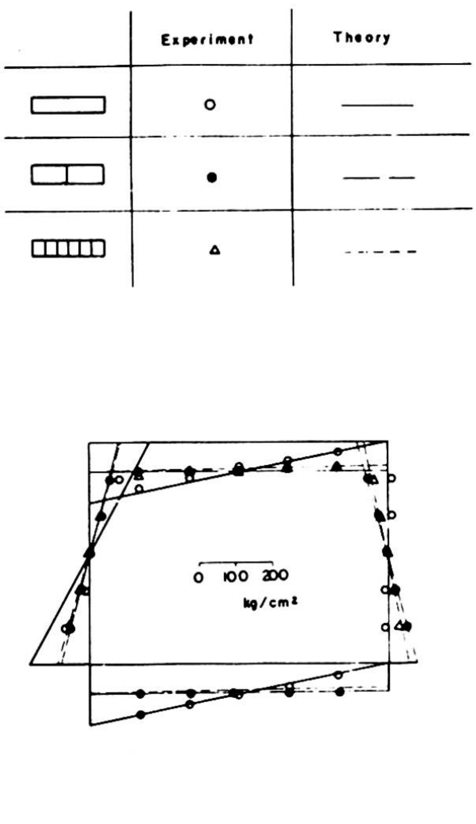


Fig. 21 Warping Stress at 5/12 α_o
(Outer Loading at 1/4 α_o)

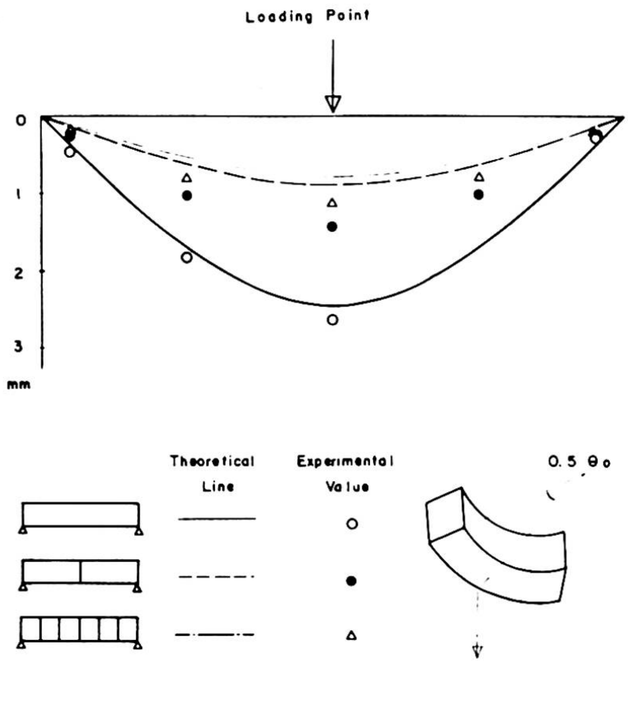


Fig. 20 (b) Deflection by Outer Loading

$$
\begin{bmatrix}
1 & -\ell & 0 & -\gamma_2 \ell & \frac{\ell^2}{2a} & -\frac{\ell^3}{6a}+\gamma_1 \ell & P_X \\
 & 1 & 0 & 0 & -\frac{\ell}{a} & \frac{\ell^2}{2a} & P_U \\
 & & 1 & \gamma_1 \ell & 0 & \gamma_2 \ell & P_B \\
 & & & 1 & 0 & 0 & P_H \\
 & 0 & & & 1 & -\ell & P_B \\
 & & & & & 1 & P_Q \\
 & & & & & & 1
\end{bmatrix}
$$

$$1 = \frac{b_1}{b_1^2 - b_2^2} \quad , \quad 2 = \frac{b_2}{b_1^2 - b_2^2}$$

Table 1. Field Matrix

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -K_H & 1 & 0 & 0 & H^{(i)} \\
0 & -K_B & 0 & 0 & 1 & 0 & B^{(i)} \\
-K_Q & 0 & 0 & 0 & 0 & 1 & Q^{(i)} \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Table 2. Point Matrix