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### Strength of Hybrid Steel Columns

Résistances de colonnes hybrides en acier

Festigkeit hybrider Stahlstützen

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#### INTRODUCTION

The hybrid steel column is a special type of structural member built up of component plates having different grades of steel. The concept of hybrid shapes has been applied to structural members in bending, by placing a stronger material in a position where it can resist higher stresses, thus using materials according to their strength, and effecting significant material economy. This concept has been extended to columns in this study. The study provides insight also to the interesting problem of the reinforcing of old columns to carry heavier loads by adding higher strength cover plates.

This paper summarizes the analysis and results of a theoretical and experimental investigation to determine the strength of hybrid steel columns. The theoretical analysis treats hybrid shapes based on the tangent modulus and maximum strength concepts where the actual material properties, residual stresses, and initial imperfections are taken into consideration. Local buckling is also considered. The experimental study includes four different hybrid shapes fabricated from flame-cut or universal-mill plates of three different steel grades. The test program consists of tension tests, residual stress measurements, stub column tests, and pinned-end column tests.

A close correlation was shown between the theoretical predictions and experimental results. The strength of hybrid columns is defined, and some economy may be expected in their use in the lower stories of multi-story frames.

#### THEORETICAL ANALYSIS

##### Basic Column Strength

The strength of a column may be defined either by its bifurcation load or by its maximum load. For any column the strength

depends on the material properties, the magnitude and distribution of residual stresses, and on the initial out-of-straightness. A perfectly straight column with concentric load application remains straight under increasing load until the tangent modulus is reached. Real columns, however, show an initial out-of-straightness and unsymmetrical distributions in material properties and residual stresses which will cause the column to deflect immediately upon loading. The deflection will increase under increasing load up to the maximum load, as shown in Fig. 1.

In determining the strength of hybrid steel columns, the major problems are caused by the different levels in yield strength of the component materials and by the residual stresses. In this study the H-shaped hybrid column is considered consisting of higher-strength flanges and a lower-strength steel web. The material properties of the component plates are assumed to be elastic-perfectly-plastic as shown in Fig. 2.

#### Tangent Modulus Strength

The strength of a centrally loaded column based on the tangent modulus concept [1] may be written in the form

$$P_{tm} = \frac{\pi^2 E}{L^2} \int_A \left(\frac{E_t}{E}\right) y^2 dA \quad (1)$$

where  $P_{tm}$  is the tangent modulus load,  $E$  the elastic modulus,  $A$  the total cross-sectional area,  $L$  the effective length of the column, and  $E_t$  the effective tangent modulus of the shape. The tangent modulus load is computed based on the measured residual stresses using the computer.[2] The tangent modulus curves obtained for various shapes are shown in Fig. 3.

The tangent modulus load may also be computed based on a stub column test result as described in Ref. 3.

#### Maximum Strength

The calculations become more complex for maximum strength predictions even though the underlying basic concepts are rather simple. In Ref. 4 an approximate method is developed based on simplifying assumptions which is suitable for computer programming and applicable to initially straight centrally loaded columns as well as to columns with initial out-of-straightness.

The method is based on the assumption that the initial as well as deflected shape under load is described by a half-sine wave. This shape has a favorable feature since differentiation of the function twice, yields a simple relationship between the deflection  $\delta_m$  and the curvature  $\phi_m$  at the mid-height of the column as follows,

$$\phi_m = \frac{\pi^2}{L^2} \delta_m \quad (2)$$

Thus, for any arbitrary value of  $\delta_m$  the corresponding curvature  $\phi_m$  is determined directly. The equilibrium condition at the mid-height

cross section may be written in the form

$$P_{int} = \int_A E \epsilon dA = \frac{1}{\delta_m} \int_A E \epsilon y dA = \frac{1}{\delta_m} M_{int} \quad (3)$$

where  $\epsilon$  is the strain distribution in the cross section which is a function of  $y$ . By assuming plane sections to remain plane, Eq. 3 is solved employing a numerical iterative procedure.

The maximum load is determined when the rate of resisting internal moment of the column approaches zero. Under this load the column is in a state of neutral equilibrium. The maximum strength column curves have been determined using the computer [2] and the results for columns with initial out-of-straightness  $e = 0$  and  $e = L/1000$  are shown in Fig. 3.

#### Local Buckling

In hybrid shapes, web buckling may be of some concern because the web may be partially or wholly inelastic at working load. Flange buckling can be considered without difficulty, since it will be essentially elastic. The width-thickness ratio of the web must be such that the web does not buckle even if it has yielded completely.

The buckling of plates containing residual stresses has been analyzed using the incremental flow theory and the total strain theory on the basis of the finite difference approximation of a differential equation. [5,6] Figure 4 shows the plate buckling curves for webs of welded shapes in terms of the non-dimensionalized width-thickness ratio, defined as

$$\lambda = \frac{b}{t} \sqrt{\frac{\sigma_y}{E} \frac{12(1-\mu)}{k\pi^2}} \quad (4)$$

where  $\mu$  is Poisson's ratio and  $k$  is the plate buckling coefficient. [7] According to these curves, local buckling in the plastic range is prevented when  $\lambda \leq 0.62$  if the ends are fixed and  $\lambda \leq 0.68$  if the plates are simply supported.

#### EXPERIMENTAL STUDIES

The experimental study included four hybrid shapes which were fabricated from flame-cut or universal-mill plates. The columns were not subjected to cold straightening. The tests conducted on these shapes were: tension tests, residual stress measurements, stub column tests, and pinned-end column tests. The experimental study is described in greater detail in Ref. 2.

#### Supplementary Tests

Supplementary tests, which included tension tests and residual stress measurements, are conducted to determine the properties of the specimen so as to enable theoretical prediction of column strength. The mechanical properties of the shapes were obtained from tension tests conducted in accordance with the ASTM specification. The results of the tests are summarized in Table 1.

Residual stress measurements were made by the method of sectioning. The results for one quarter of each section are shown in Fig. 3

#### Column Tests

Stub column tests were performed in order to obtain for each cross section the average stress-strain curve which takes into account the effects of residual stresses. The proportional limit, the elastic modulus, the tangent modulus, and the average yield strength were the important data furnished by the stub column tests.

Pinned-end columns with a slenderness ratio of 65 were tested to verify the prediction of the behavior and strength of each column based on the measured residual stresses or stub column test. The test results for the stub columns and pinned-end columns are given in Table 1.

#### Discussion of Results

As shown in Fig. 3, there is a reasonably good agreement between the theoretical and experimental column strengths. The non-dimensionalized tangent modulus and maximum load column curves for the hybrid shapes show that the strength of these columns are much higher than that of welded homogeneous A36 steel columns. For columns with A514 steel flanges  $P_m/P_{tm} \approx 1.05$  and for columns with A36 steel  $P_m/P_{tm} \approx 1.25$ .

#### PRICE-STRENGTH RELATIONSHIP

A relationship between the price and strength of columns may be established for different steel grades with the aid of column curves and cost data. Figure 5 shows comparison curves for columns of three different steel grades based on average mill price ratios of 1969.[8] A price comparison of hybrid shape No. IV with the homogeneous shape counterparts is shown in Fig. 6. This indicates that hybrid shapes may be economical for columns with low slenderness ratios. For a reliable comparison, further factors should be taken into consideration such as fabrication, transportation, and erection, any of which may change the curves shown in Figs. 5 and 6.

#### CONCLUSIONS

This paper summarizes the analysis and results of a theoretical and experimental investigation to determine the strength of hybrid steel columns. The theoretical analysis treats hybrid shapes based on the tangent modulus and maximum strength concepts. Local buckling is also considered. The experimental study includes four different hybrid shapes. A close correlation was shown between the theoretical prediction and the experimental results. The strength of hybrid columns is defined, and some economy may be expected in their use in the lower stories of multi-story frames.

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Summary

The analysis and results of a theoretical and experimental investigation to determine the strength of hybrid steel columns is presented. The theoretical analysis treats hybrid shapes based on the tangent modulus and maximum strength concepts; the experimental study includes four different hybrid shapes, and a close correlation was shown between theory and experiment. Some economy may be expected in the use of hybrid columns in the lower stories of multi-story frames.

Table 1 : Summary of Test Results

SHAPE	COMPONENT PLATE	STEEL GRADE	TENSION SPECIMEN		STUB COLUMN	PINNED-END COLUMN ( $L/r = 65$ )				
			$\sigma_y$ Tens. (N/mm <sup>2</sup> )	$\sigma_{ult}$ (N/mm <sup>2</sup> )		$P_y$ (N)	$(\sigma_y)_{stab}$ ( $\sigma_y$ Tens.)	$\delta_0$ (mm)	$\frac{P_{max}}{P_y}$	$\frac{P_{max}}{P_{theor}}$
I	Flange	A441 (UM)	345	550	1750	1.01	1.5	1260	0.72	0.96
	Web	A36	269	461						
II	Flange	A441	361	575	1780	1.02	0.5	1480	0.83	1.05
	Web	A36	260	455						
III	Flange	A514	760	843	3440	1.03	1.5	2580	0.75	1.01
	Web	A36	275	461						
IV	Flange	A514	740	810	3450	1.04	2.5	2570	0.75	1.01
	Web	A441	338	490						

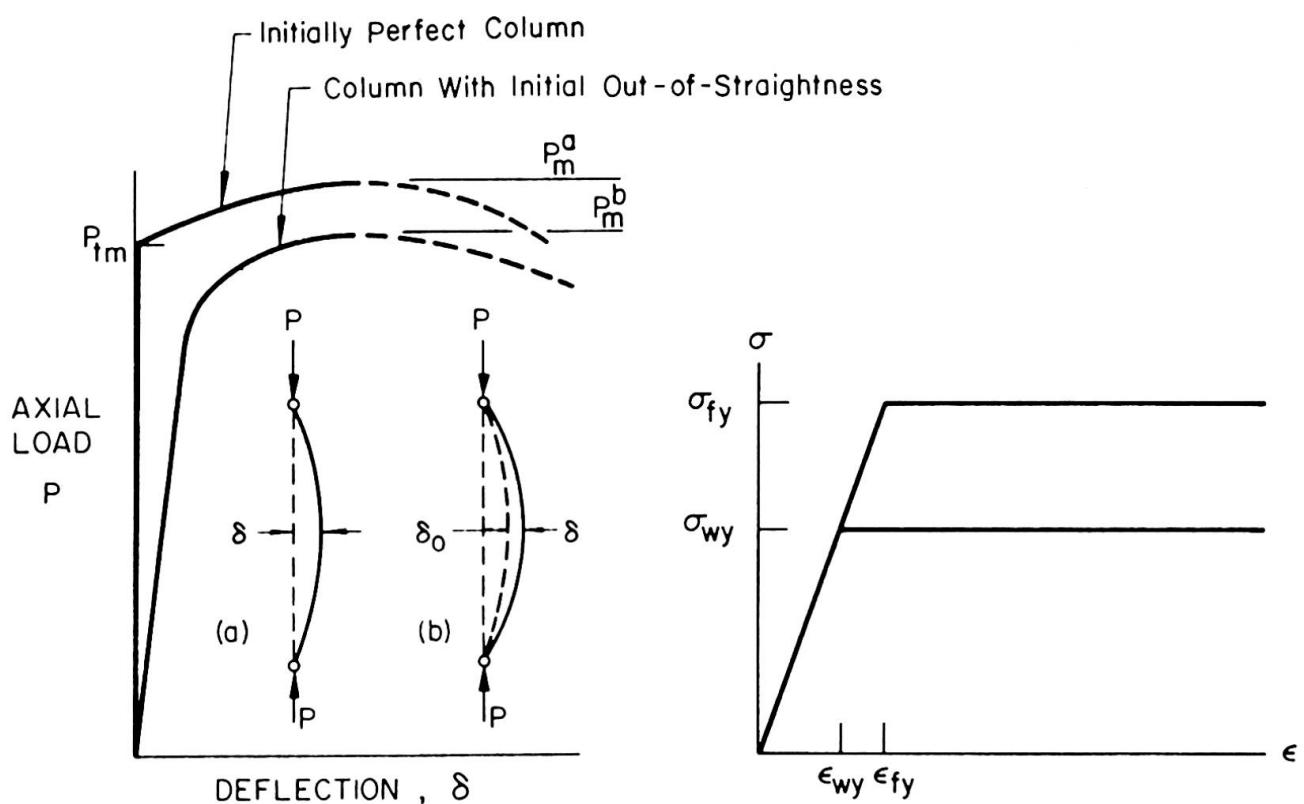
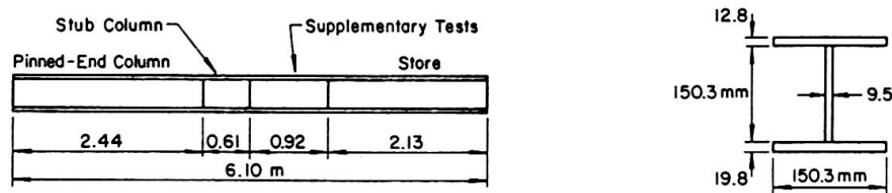


Fig. 1

Load - Deflection Curve for a Centrally Loaded Column

Fig. 2

Idealized Stress-Strain Curves

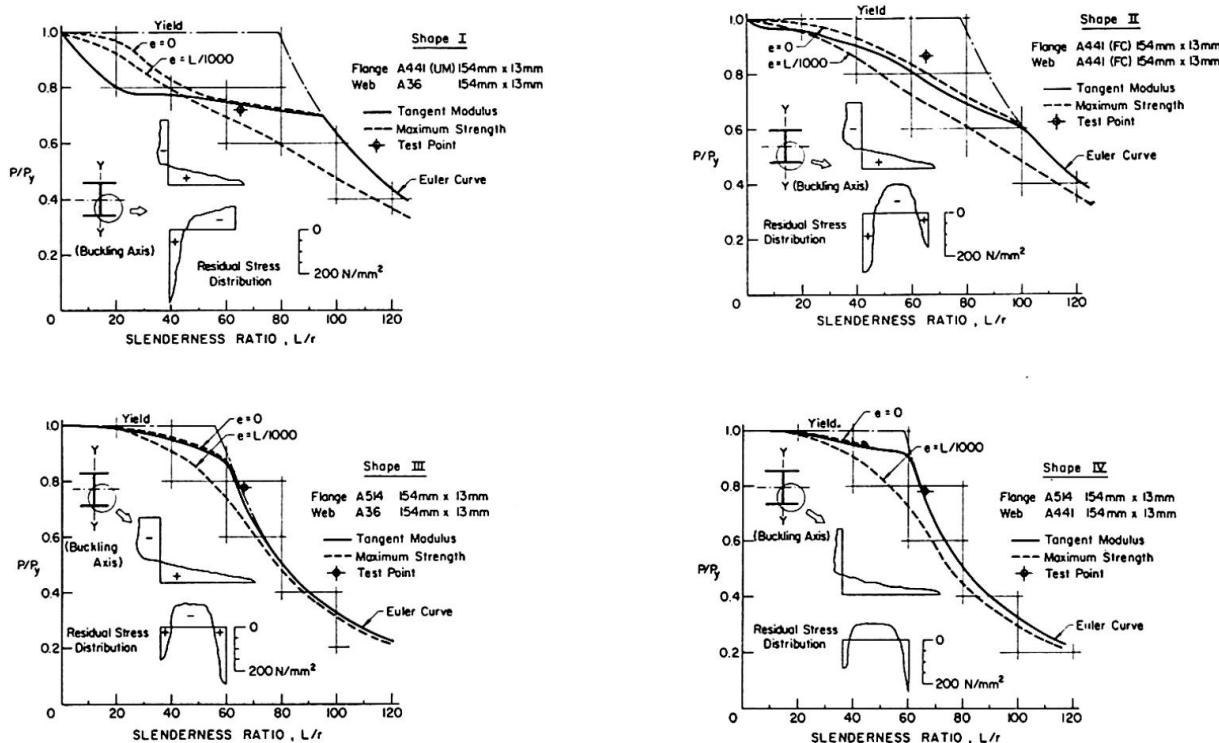


Fig. 3 Tangent Modulus and Maximum Strength Column Curves

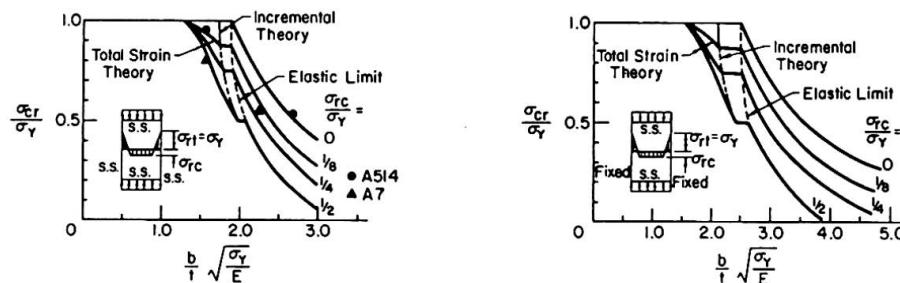


Fig. 4 Plate Buckling Curves

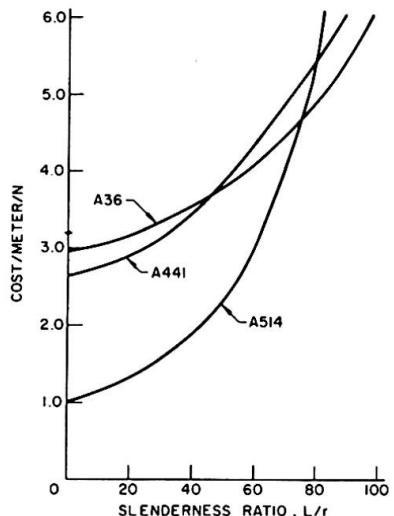


Fig. 5 Relative Cost vs. Slenderness Ratio of Homogeneous Shapes

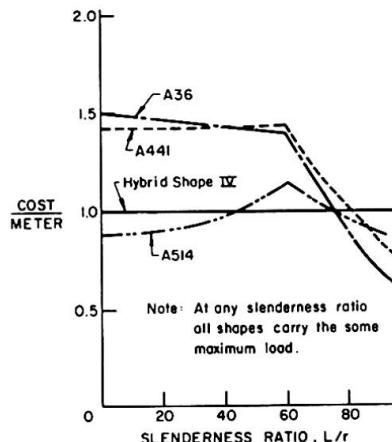


Fig. 6 Comparison of Price of Hybrid Shape No. IV and Homogeneous Shapes

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