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Interaction of Postcritical Plate Buckling with Overall Column Buckling of Thin-Walled Members

Interaction du voilement post-critique de plaques et du flambement de colonnes aux parois minces

Wechselwirkung von überkritischem Plattenbeulen und Knicken des ganzen dünnwandigen Stabes

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I. Introduction

The interaction of postcritical plate buckling with overall column buckling in thin-walled members is a complex phenomenon which is very important in many situations. Thin-walled steel construction in buildings has increased greatly in the past two to three decades; thin-walled members have always been used extensively in aircraft construction.

In thin-walled members plate buckling is of major importance and constitutes one of the chief design criteria. The classical critical plate buckling stress for the component plates, which is the stress at which local bifurcation buckling occurs, is often regarded as the chief design criterion. However, this is by no means the maximum load which the component plate can carry. The plate will usually continue to take increasing load, often more than twice the critical local buckling load. This postcritical plate buckling strength can be used to achieve substantial economies.

This paper concerns itself with postcritical plate buckling and its effect on the overall buckling of columns. For most thin-walled columns of low and medium slenderness, critical plate buckling of one or more of the component plates occurs first, and is then followed by overall column buckling at some point in the postcritical plate buckling range. The prior plate buckling lowers the overall capacity of the column, but the load at which local bifurcation buckling occurs is less than the actual carrying capacity of the member and may not be used as a reasonable indication of the overall capacity.

The investigation reported herein has been sponsored at Cornell University by the American Iron and Steel Institute. It is aimed at developing information on this interaction between postcritical plate buckling and overall column buckling. Thirty-three tests have been conducted on columns in which both the postcritical plate buckling strength and the overall column buckling strength were varied systematically. The results of these tests illustrate clearly the interaction effects between postcritical plate buckling and overall column buckling.

II. Survey of Previous Work

Many researchers have investigated plate buckling and a number have done work in the postcritical plate buckling range; also, overall column buckling has been extensively investigated. However, little has been done in the area of the interaction of postcritical plate buckling with overall column buckling. In fact, postcritical plate buckling by itself needs further clarification. This refers particularly to the later stages, where relatively large plate deflections interact with nonlinear materials' behavior.

One researcher, T.R.G. Smith in England[1], has made a very important contribution in the field which considers both of the above nonlinearities in the postcritical plate buckling range as they interact with overall column buckling. Unfortunately his analysis is limited to tubes and cannot be applied to other shapes without repeating his lengthy derivations for each type of section.

Some work has been done toward a simpler method of treating different types of common sections such as by Uribe [2] and Wang [3] at Cornell University, though their work was primarily involved with things other than interaction effects and was not very extensive. Others [4-7] have made contributions to the field of interaction, but their work has not involved both types of nonlinearities as has T.R.G. Smith. There appears to be no thorough set of tests utilizing different column shapes upon which a general method of design of thin-walled columns subject to the interaction of postcritical plate buckling with overall column buckling can be based.

III. Testing

Since thin-walled members are made of essentially two types of elements, elements with one edge stiffened by a web, flange or stiffener and elements with both edges stiffened, two types of sections were chosen in order to test each type of element separately. One section was composed entirely of stiffened elements; it was made of two channels connected together at the flanges to form a rectangular tube (see Fig. 1a). The other was composed primarily of unstiffened elements (the stiffened element was designed so that local buckling would not occur); it was made of two channels connected back to back along the webs to form an "H" type section (see Fig. 1b).

Four sections of each type were fabricated. The dimensions of the sections were chosen so that the critical and postcritical plate buckling strength were varied by varying the element width-thickness ratios over a wide range. The dimensions are given in Table 1.

For each section, the overall column buckling strength was varied by varying the slenderness ratio L/r. In addition to a stub column test (no column buckling), three different column lengths were chosen to cover the region in which local buckling had an effect on the ultimate load a column will support.

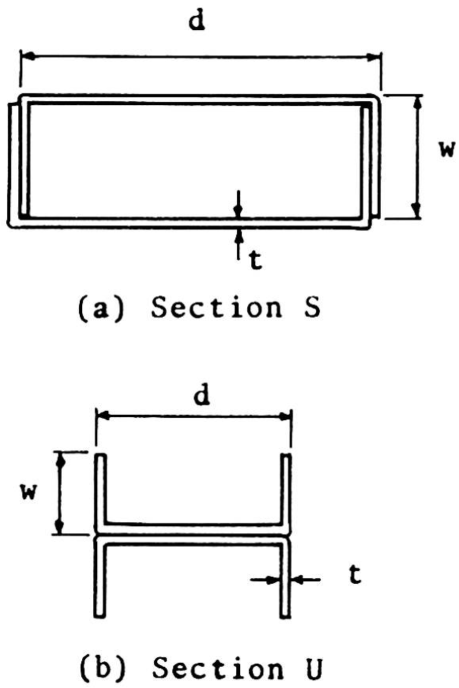

Fig. 1 Column Cross-Sections

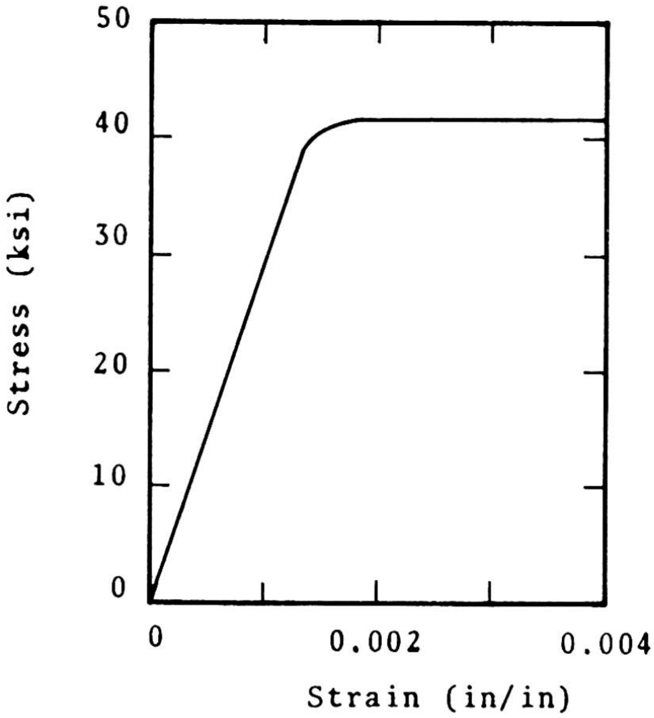

Fig. 2 Material Stress-Strain Curve

The testing arrangement was such that concentric loads were achieved. This was done by loading to approximately twenty percent of the expected failure load and then unloading and making adjustments in the centering as necessary. Dial gages were used to measure the lateral deflections and strain gages to obtain the strain distributions, as well as to determine approximately when local buckling occurred in the plate elements. It is easily verified from the strain gage results that nearly concentric loads were achieved in all of the columns, except for three tests which were eccentric and whose results are ignored.

Measurements of the out-of-plane deflections were taken for all of the unstiffened plate elements. It was found that these deflections were minimal until failure, being no more than a few thousandths of an inch.

The initial portion of the stress-strain curve for the material used, a carbon steel of a structural quality, is shown in Fig. 2. The average yield stress was 41.9 ksi., and the average ultimate stress was 53.8 ksi. Strain hardening occurred at an average strain of 0.014. The average percentage of elongation of a two-inch gage length at rupture was 37 percent.

IV. Results

(a) General

The results of these tests illustrate clearly the interaction between postcritical plate buckling behavior and overall column

Table 1

DIMENSIONS OF SECTIONS

(a) Sections S

Specimen	w (in)	d (in)	t (in)	Width/Thickness Single Thickness Element (d/t)	Width/Thickness Double Thickness Element (w/2t)
S-1	2.0	3.5	0.058	57.2	16.7
S-2	2.0	5.0	0.058	83.0	16.7
S-3	2.0	7.0	0.058	117.4	16.7
S-4	2.0	9.0	0.058	151.8	16.7

(b) Sections U

Specimen	w (in)	d (in)	t (in)	Width/Thickness Single Thickness Element (w/t)	Width/Thickness Double Thickness Element (d/2t)
U-1	1.0	3.0	0.058	16.2	24.8
U-2	1.25	3.0	0.058	20.5	24.8
U-3	1.5	3.0	0.058	24.8	24.8
U-4	1.75	3.0	0.058	29.1	24.8

buckling. The ultimate carrying capacities for the columns are given graphically in Figs. 3 and 4. The figures compare the experimental carrying capacity with the expected strength (1) when local instability is entirely neglected; (2) when the effect of local instability is based on the critical bifurcation stress, neglecting postcritical strength; (3) when postcritical buckling is included approximately by means of an effective width concept. These will now be discussed separately.

(b) Local Instability Neglected

A concentric, perfect column which is elastic will fail at the critical Euler stress or at the yield stress, provided that local instability and residual or cold-forming effects are nonexistent. For this type of column behavior, column curves, i.e. slenderness ratio vs. critical stress, are given as curves (1) in Figs. 3 and 4 for each of the different sections.

As mentioned, this neglects the effects of local instability, which were present in the test specimens. Additionally, the specimens were cold-worked so that cold-forming effects were present. However, since cold-forming effects are related to the area of the corners as a fraction of the total area (8), and since the ratio of the corners to the total area was less than one percent for all of the sections, the cold-forming effects were negligible. The columns

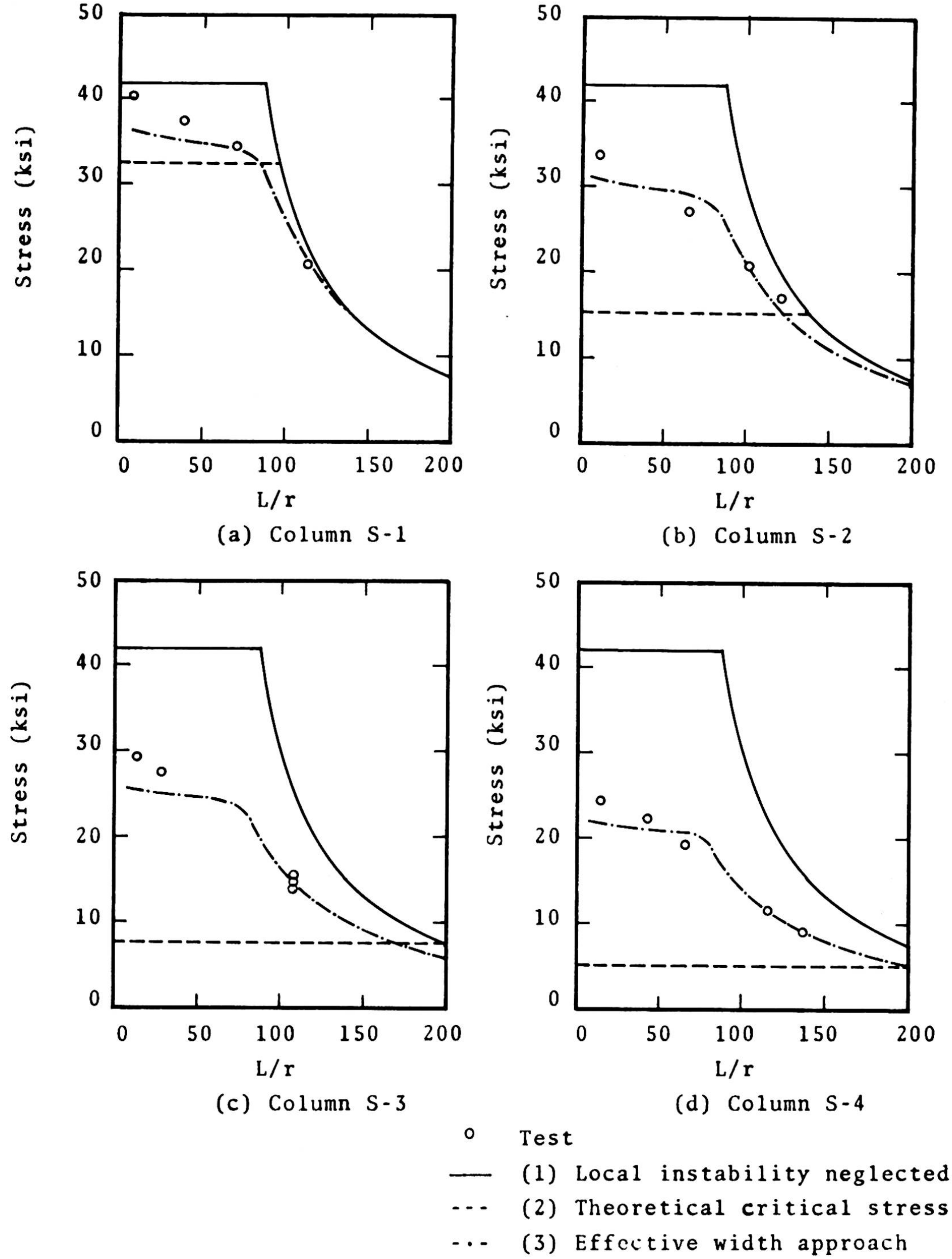

Fig. 3 Comparison of Test Results and Column Curves For Column Specimens S

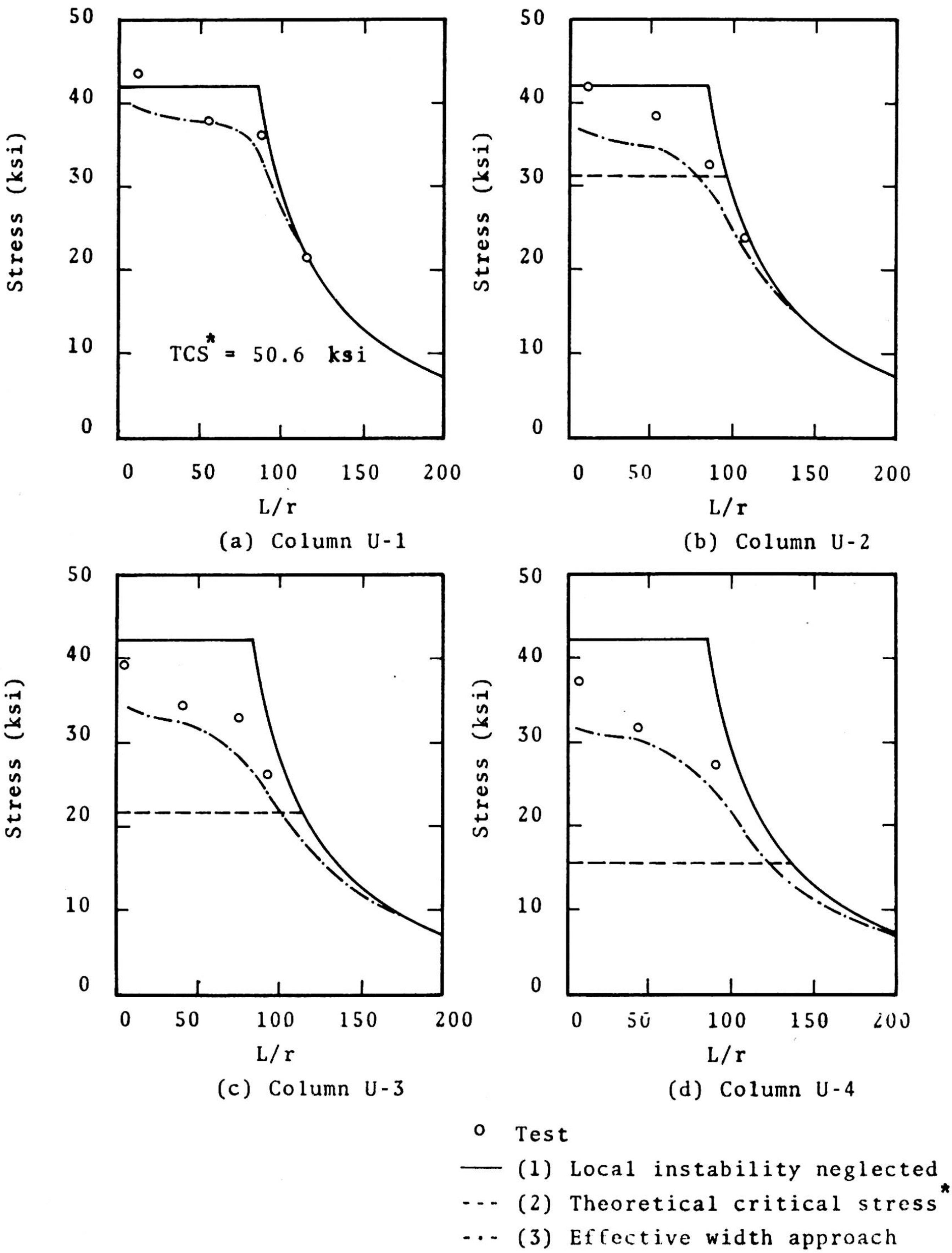

Fig. 4 Comparison of Test Results and Column Curves For Column Specimens U

were thus assumed to have a uniform stress-strain curve which is approximated by the material stress-strain curve in Fig. 2.

It is seen that almost all test points fall significantly below curves (1) which indicates that local instability had a great effect on strength. This effect became greater as the width/thickness ratios of the elements were increased.

(c) Theoretical Critical Stress

The theoretical critical bifurcation stress is that stress at which a perfect plate begins to buckle. Here it refers to the widest elements for the closed tubular sections and to the unstiffened flanges for the "H" sections. The theoretical critical stress by definition is the stress at which postcritical strength begins.

The classical critical stress for a plate element that buckles elastically is calculated from:

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1 - \mu^2)(\frac{w}{t})^2} \qquad 1$$

where E is the modulus of elasticity, μ Poisson's ratio, w the plate width, t the plate thickness, and k depends on the edge conditions, chiefly along the longitudinal edges parallel to the compression stress. Values of k which are often used in practice, and which in some cases will be conservative, are 0.5 for elements with one edge supported by a web and the other edge unsupported, and 4.0 for elements with both edges supported by a web.

For the purpose of comparing the test results to a theoretical stress, these two values of k were used for the sections under study. The results are given in curve (2) in Figs. 3 and 4. With the exception of Fig. 4a the theoretical critical stress was below the highest load at failure, indicating that postbuckling strength existed. For Fig. 4a, the theoretical elastic critical stress was above the yield stress, though the section showed local waving prior to failure during the tests. It is seen that, except for Fig. 4a, almost all test points fall significantly above curve (2) indicating that postcritical strength does add considerably to the carrying capacity. This effect increases with increasing width/thickness ratios.

(d) Effective Width Approach

In order to consider the postbuckling strength, an effective width approach is now under development. At present, this approach is only an approximate method for considering postbuckling strength, and in its present form is not always satisfactorily accurate. The concepts of the method are these:

The bifurcation stress of a column is well accepted to be given by the Engesser-Shanley tangent-modulus equation:

$$\sigma_t = \frac{\pi^2 E_t}{(\frac{KL}{r})^2} \qquad 2$$

where E_t is the tangent modulus for the material, L is the column length, r is the radius of gyration, and K is the effective length factor. Dividing the cross-section into j elements, one can define:

$$E_t r^2 = \frac{\sum_{i=1}^{j} E_{ti} I_i}{A} \qquad 3$$

Then, for K = 1.0, by substitution, Eq. 2 becomes:

$$L^2 = \frac{\pi^2 \sum_{i=1}^{j} E_{ti} I_i}{\sigma A} \qquad 4$$

where, for any selected strain, E_{ti} is the tangent modulus of the i^{th} element, I_i is the moment of inertia of the i^{th} element about the weak principal axis, A is the full area of the cross-section, and σ_{cr} is given by:

$$\sigma = \frac{\sum (A_{eff})_i \sigma_i}{A} \qquad 5$$

where $(A_{eff})_i$ is the effective area of the i^{th} sub-element at the stress σ_i corresponding to the assumed strain ε, and A is the full area of the section. Essentially, Eq. 5 considers the effects of local buckling and postbuckling strength by reducing the area to an effective area, using the effective widths of the elements.

The general equation for the effective width on which American specifications are now based is:

$$\frac{b}{w} = \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}} \left(1 - 0.22 \sqrt{\frac{\sigma_{cr}}{\sigma_{max}}}\right) \qquad 6$$

which constitutes a slight revision of the equation first given by Winter [9]. Here b is the effective width, w is the real width of the particular plate element, σ_{cr} is the classical plate bifurcation buckling stress for the given edge conditions, and σ_{max} the edge stress in the postbuckling range. This equation can be transformed into the more convenient form:

$$b = C_1 t \sqrt{\frac{E}{\sigma_{max}}} \left(1.0 - C_2 \frac{t}{w} \sqrt{\frac{E}{\sigma_{max}}}\right) \qquad 7$$

where t is the plate element thickness, E the modulus of elasticity, and C_1 and C_2 are constants which depend on the edge conditions. For elements with one side supported by a web and the other side unsupported, approximately $C_1 = 0.672$ and $C_2 = 0.148$. For elements with both edges supported, approximately $C_1 = 1.9$ and $C_2 = 0.415$.

The ultimate loads predicted using this effective width approach are given in curves (3) in Figs. 3 and 4. It is seen that the test results agree best with the last of the three methods (curve 3), which includes the effect of postcritical plate buckling strength on column capacity. At the same time, it is also seen that the accuracy of this approach is inadequate. Further development of this method is needed, and is under way at the present time.

V. Conclusions

On the basis of extensive test results and theoretical comparisons, it is shown that:

(1) The strength of thin-walled columns can be considerably reduced by local plate buckling.

(2) Column capacities calculated on the basis of the classical critical plate buckling stress considerably underestimate the actual column strength.

(3) The effect of the postcritical strength of the component plates increases the column strength significantly over that determined by the critical plate buckling stress.

(4) A tentative method, based on the postcritical effective width of plates, is now under development and promises to furnish a satisfactory tool for calculating the strength of thin-walled columns.

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Summary

Experimental data are given which illustrate clearly the interaction of postcritical plate buckling with overall column buckling in thin-walled members. The experimental carrying capacity is compared with the expected strength when (1) local instability is entirely neglected; (2) the effect of local instability is based on the critical bifurcation stress, neglecting postcritical strength; (3) postcritical buckling is included approximately by means of an effective width concept.