

**Zeitschrift:** IABSE congress report = Rapport du congrès AIPC = IVBH  
Kongressbericht

**Band:** 9 (1972)

**Artikel:** Three remarks on viscoelasticity and inelasticity of concrete

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**DOI:** <https://doi.org/10.5169/seals-9543>

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### Three Remarks on Viscoelasticity and Inelasticity of Concrete

Trois remarques sur la viscoélasticité et inélasticité du béton

Drei Bemerkungen zur Viskoelastizität und Unelastizität des Betons

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The intent of this discussion is to make three remarks calling attention to some recent developments in inelasticity of concrete extending the exposition of this subject in the Introductory Report to Theme Ia.

(1) In the Introductory Report it is stated that the theory of linear viscoelastic bodies with age-dependent properties can be applied to concrete structures. It would be, however, more accurate to say that no better theory is available at present. The theory of viscoelasticity can accurately describe the behavior of concrete under low stress only when its specific water content and temperature are constant, which is a rare condition in actual structures. Otherwise concrete is not viscoelastic but exhibits a more general behavior in which the strains are functionals not only of the history of stress but also of the histories of specific water content and temperature. As a consequence, the stress problem is coupled with a diffusion problem. Taking this fact into account, the number of unknown material parameters is immensely increased so that their determination from the available creep and relaxation data alone would hardly be possible. It is therefore necessary to turn attention to the physical processes in the microstructure and try to determine the form of the stress-strain relations on this basis. The main source of shrinkage, delayed thermal dilatation and creep of concrete at working stress levels is presently believed to be some kind of diffusion processes in the microstructure of cement paste, involving hindered adsorbed water, interlayer hydrate water, calcium ions and capillary water [3]. This approach has recently received a good deal of attention and the latest advances can be found in references [1] and [2]. The constitutive equation which follows from the above mechanism appears to be amenable to structural analysis with the help of electronic computers [1].

(2) Even if the behavior of concrete is assumed to obey the linear theory of viscoelasticity of age-dependent bodies, the structural analysis is not easily accomplished. Among the simplified stress-strain relations used in practical problems, the relatively best ones are those of the type used by Arutyunyan, as quoted in the Introductory Report. These are able to reflect both the reversible and irreversible deformations and also the so-called aging. However, for simplicity of analysis, the time shape of the creep curve in these relations is being assumed as a simple exponential, which implies only a single retardation time. In reality, the shape of the creep curves observed is quite different, which can be seen in the logarithmic time scale; the creep curves continue to rise significantly over many decades of the time elapsed and have thus a very broad retardation spectrum, with many exponential components. Therefore it is important to carry out the structural analysis for the actual unit creep curves. This can be done only numerically, e.g. by the methods described in references [4] and [5].

If the structure is large (i.e. requires too many nodes), this approach runs into difficulties because the need of storing the complete history of the stress state in the structure and evaluating from it the hereditary integrals overtaxes the storage capacity of the computers presently available and requires too much machine time. These requirements can be circumvented by characterizing the entire stress history with a few suitably defined hidden state variables of the material, as has been proposed in reference [6].

(3) Alternatively, the stress-strain law can be also represented by a spring-dashpot model. The Introductory Report refers to this approach in conjunction with the applications of the finite element method by Zienkiewicz, Watson and King. For an accurate representation of the shape of creep curves, as discussed above, a relatively long chain of Kelvin elements or Maxwell elements is needed. In pursuing this approach, there are, however, two obstacles. First the method of determining the model parameters from creep data has not yet been clarified, for the case when aging is involved. The second obstacle arises in numerical application. Namely, the usual numerical algorithms (of Euler, Runge-Kutta or predictor-corrector type) become numerically unstable when the time step is increased beyond the value of the shortest retardation time. Because this time is for concrete quite short, analysis of the long-term response would require an impractically high number of small time steps (about  $10^6$ ), regardless of the fact that under steady conditions the solution must vary very slowly after long times elapsed. This obstacle may be overcome and an arbitrary increase of the time steps (without causing numerical instability) can be made feasible by introducing a certain special set of hidden material variables (see Eqs. 70 to 79 in Ref. [1]).

Acknowledgement. The developments which are herein briefly mentioned have been achieved at Northwestern University mainly under National Science Foundation Grant GK-26030.

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#### SUMMARY

The discussion presented calls attention to some recent developments which include (1) the theory of creep of concrete based on diffusion processes in the microstructure, (2) the numerical integration methods based on superposition of unit creep curves and (3) those based on an equivalent rate-type formulation.