Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 9 (1972)

Artikel: The influence of plasticity and viscosity on the strength and deformation of structures

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DOI: https://doi.org/10.5169/seals-9540

The Influence of Plasticity and Viscosity on the Strength and Deformation of Structures

L'influence de la plasticité et de la viscosité sur la résistance et la déformation des constructions

Der Einfluss der Plastizität und der Viskosität auf die Traglast und die Verformung von Tragwerken

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1. Material Behavior

1.1 Elasticity

Of the various mechanical properties that determine the usefulness of a structural material, its elastic modulus is undoubtedly the single most important one since it alone determines wether the material is rigid enough to satisfy the limitations on deformability that are imposed on any structure by its function and that usually delimit its "serviceability." Although the elastic modulus is also directly proportional to the cohesive strength of the material[1] since, theoretically at least, both can be related to the action of interatomic or intermolecular forces, it has long been recognized that materials that attain a significant portion of their theoretical cohesive strength while strained elastically are not usable as structural materials, because the unavoidable inhomogeneities in their microstructure are bound to cause premature "brittle" fractures that are not only the more explosive the higher the level of elastic strain energy stored in the volume element and suddenly released at fracture, but also the less predictable or reproducible. Therefore, a useful structural material, expected to deform elastically within the limit of serviceability of the structure, must be prevented from explosively failing elastically beyond this limit by energy disipation mechanisms in its microstructure that prevent the build-up of potentially destructive elastic strain energy as soon as the forces acting on the structure exceed its serviceability limit. The avoidance of brittle failure by providing structural materials the deformational response of which to forces beyond the limit of serviceability deviates from elasticity not only sufficiently but also sufficiently fast to achieve this aim under all loading and environmental conditions which structures must, with an adequate level of confidence, be expected to withstand, is one of the foremost tasks of the material .producing industry.

1.2 Plasticity and Viscosity of Structural Materials.

Plasticity and viscosity are the phenomenological expressions of the two basic strain energy disipation processes in the metallic microstructure: the weakly strain-rate and temperature sensitive transgranular (plastic) slip and the strongly strain-rate and temperature sensitive intergranular (quasi-viscous)flow. While in all metals both mechanisms are simultaneously present as soon as the limit of technologically elastic deformation is exceeded (the true physical elastic limit is lower and depends essentially on the limit of accuracy with which deviation from linear elastic behavior can be observed), one or the other will predominate as the loading and environmental conditions change, with low temperatures or high strain-rates suppressing the viscous response, while high temperatures and low strain-rates amplify it. The post-elastic response of structural metals at room and low temperatures is therefore reasonably well represented by plastic flow with constant or strain-dependent (hardening) yield-limit, while the operational stress level in the structural members is governed by the elastic response, although it is well-known that the full utilization of the elasticity of the members depends on sufficient constrained plastic deformation in the overstrained connections being available to prevent localized elastic (brittle) failures through overstrain.

In the non-metallic microstructures of ceramics and concrete almost linear quasi-viscous flow characterizes the binder already at very low stresses[2], so that a real "elastic limit" does not exist, while a highly nonlinear viscosity which is almost indistinguishable from "plasticity" is the expression, at stress-levels closer to the unconfined compressive (shear) strength, of processes of progressive destruction of the microstructure. Thus, structural concrete is a linear visco-elastic material at any stress above zero, so that the limit of serviceability of concrete structures, particularly at relatively high sustained compressive operational stresses, cannot be identified with the complete absence, but only with a specified limiting amount of irreversible deformation, a limitation that is made possible by the asymptotic increase of the apparent coefficient of viscosity of the concrete in the course of the hardening process that transforms the initial visco-elastic into the fully hardened elastic compound. Experiments[3] have shown that the range of linear visco-elasticity does not extend beyond a stress level of 25 to 30 percent of the compressive strength, which is the usual range of operational stresses. Beyond this stress-intensity the viscous response of the material becomes increasingly non-linear until, at about 80 percent of the compressive strength it is, on first loading, indistinguishable from plastic response since the associated destruction proceeds at constant stress. Thus, the linear visco-elastic response of the concrete governs the range of serviceability of the structure. As the viscous component becomes increasingly non-linear in stress at stress levels beyond this range, stress relaxation processes governing prestress levels become increasingly rapid, with the result that the higher the prestress, the more difficult to sustain it. In heavily overreinforced beam sections the effect of the highly non-linear viscous redistribution of stresses as well as of moments produces the deceptive appearance of "plasticity" of the concrete. However, this plasticity cannot, as in metals, be depended upon to produce a reliable "overloading capacity" that could be sustained under repeated loading or that could lead to a "shakedown" condition under load repetition, since the increasing local destruction of the microstructure that is reflected in the increasing non-linearity of the apparent viscosity of the concrete leads to rapid large-scale destruction of the overstrained compressive zone after a relatively small number of load-cycles. Only in underreinforced beams does the plastic yielding of the metal reinforcement at low concrete stresses produce conditions resembling "plastic hinges",[4] however, at the usually unacceptable price of large, concentrated cracks in the concrete. The extension of rigid-plastic limit analysis to redundant reinforced concrete is therefore not without serious problems.[5]

2. Design Criteria

The influence of plasticity and of viscosity on the strength and deformation of structures and thus on their analysis and design can hardly be adequately assessed or even discussed without carefully considering the implications of the dual design requirements of serviceability and failure resistance as well as of the relations to these requirements, of plasticity or near plasticity on the one hand (highly non-linear viscosity) and of plasticity and quasi-linear viscosity on the other. Unfortunately the Introductory Report misses these critical implications almost completely and presents, instead, a simplistic rigid-plastic limit analysis of one-parametrically loaded steel structures as well as a simplified version of linear-viscoelastic analysis applicable to reinforced concrete structures, presumably as representative illustrations of the "state of the art" in the field of research and engineering practice encompassed by Theme Ia. And this in spite of the fact that every single one of the eight preceding International Congresses of IABSE has devoted part of its attention to some aspect of this theme, and that in the Proceedings of these eight Congresses as well as in the Memoir volumes published by the Association many of the pioneering papers on the various aspects of the influence of plasticity and viscosity on strength and deformation of structures have been published.

2.1 Plastic Limit Design

However, the reader of this Introductory Report which is intended, presumably, to prepare the ground for the discussion of this theme at the Ninth Congress would never suspect this preoccupation over many years on the part of IABSE. Nor is he made aware of the general perspective that has emerged from this preoccupation and that reflects a balance between the applied mechanics, the materials engineering, and the structural design view-points relating to this theme, which would seem to preclude juxtapositions like that between the pre-1940 "pessimistic" elastic theory and the post-1940 "optimistic" plastic theory[6], as if anybody concerned with the ultimate carrying capacity of structures, from Galileo and Mariotte to the research workers in the nineteen forties, had ever seriously considered the elastic theory as a procedure for the determination of a "failure load" or carrying capacity, or as if the purpose of the plastic theory were to replace the elastic theory that "refers to a physically unrealistic limit state" by a method "that is in agreement with experimental results" and makes it possible to "refer the safety of statically determinate and indeterminate structures to the uniform base of a real limiting state (failure mechanism)" (p. 9).

This exaggerated assessment of the relevance of the theory of rigid-plastic limit design is obviously compatible neither with the dual aspect of design for serviceability and failure nor with those experimental results that contradict even for structures of low redundancy the assumption of this theory that the full plastic carrying capacity associated with the fully developed failure mechanism can actually be attained in spite of the fact that the large plastic hinge rotations required for its development are frequently preceded by local instability phenomenon that cause premature failure below the full plastic moment. In a basic paper Stüssi[7] has in fact demonstrated by an extremely simple experiment that the real carrying capacity of a redundant structural beam of mild steel lies somewhere above the limiting elastic but below the fully plastic carrying capacity. The failure of redundant metal structures to attain their full plastic carrying capacity has also been demonstrated in many other experiments[8], and the rules that have to be followed to ensure this carrying capacity[9] can be successfully applied only to a very narrow range of structural types.

The orthodox adherents of plastic limit design also chose to discount the practical relevance of the results of experiments on structures loaded by movable or reversed loads or by repeated loads or loads of variable intensity and/or configuration which have conclusively shown[10] that the limiting loads reached in plastic shake-down processes are much closer to the elastic than to the plastic limit loads, so that the range of applicability of the simple plastic limit load analysis is, in fact, severely restricted and becomes doubtful even in the case of multiple floor frame structures of heights at which the wind stresses in the principal members attain intensities comparable to the load-stresses. The small differences between the results of shake-down analysis and of elastic analysis makes it, in fact, appear that under the many conditions under which shake-down analysis would be necessary, elastic analysis with limiting conditions derived from low-cycle fatigue tests is fully adequate.

2.2 Elastic-Plastic Analysis.

In their assessment of the rigid-plastic limit analysis for metal structures as "more realistic than the elastic analysis" (p. 9) the authors of the Introductory Report[6] subscribe to the thesis that rigid-plastic analysis, being superior to the elastic analysis, makes the latter superfluous and can therefore completely replace it, independently of the nature of the phenomena arising in the transition from the assessed rigid (in reality elastic) into the fully plastic state. Most of the relevant references of the Introductory Report subscribe to this point of view, from which it follows that theoretical and experimental investigations of this transition have been and are, from an engineering point of view, unnecessary; their results are, therefore, best relegated to oblivion being irrelevant: they either support the assumptions of plastic limit-analysis, in which case they are "self-evident", or they contradict it, in which case they are unwelcome. The list of references of the Introductory Report reflects this point of view clearly, though perhaps not deliberately: after the usual courtesy to Kazinczy (1914) and Kist (1917)[11], the next reference to plastic analysis dates from 1951. The period of the really pioneering experimental and analytical engineering research relative to plasticity and structural design between 1920 and 1940, in the course of which the basis of the plastic limit analysis has been carefully established, and many of the important results of which can be found in the IASBE Congress Proceedings has, therefore, been deleted from memory. As far as the reader of the Introductory Report is concerned, the basic research on the subject of Theme Ia by J. Fritsche[12], E. Melan[13], H. and F. Bleich[14], Chwalla[15], Stussi[7], and others[16] might have never been done, nor might this theme have ever attracted the attention of a previous IABSE Congress.

This curious distortion of perspective, reinforced by the statement that "until 1940 the only method taught and applied was elastic theory" reflects the suprisingly widely held belief that plastic limit design somehow originated in England at about 1950. This belief may have arisen because at the Fourth IABSE Congress (1952) the English group not only completely dominated the proceedings on Theme I3 (Plastic Design), but also consistently omitted to refer to any work done before 1950, except for an oblique reference to Meier-Leibnitz as "having first introduced the concept of the plastic hinge," although in his classic survey of the experimental work in structural plasticity[8], Meier-Leibnitz rather than "introduce" the concept of the plastic hinge has scrutinized the experimental evidence and, on this basis, carefully discussed and specified the conditions limiting the application of this concept in design.

2.3 Decision Rules.

The Introductory Report puts considerable emphasis on the fact that problems of plastic limit analysis can be formulated as problems in linear programming, so that computerized plastic analysis and automatic minimum weight design on this basis can be expected to replace most of the effort of the designer. It should be remembered, however, that apart from the physical limitations of the validity of such analysis, the question of when or whether a minimum weight or minimum material cost criterion provides a valid decision rule for structural design, or whether such a rule leads to a unique answer has never been even formulated. In the absence of a clear answer it appears that a minimum weight or minimum material cost criterion may not produce a minimum cost design, in view of the fact that in structures the cost of labour (fabrication, erection) is inversely rather than directly proportional to the weight of the material.

2.4 Linear Visco-elastic Analysis of Concrete.

The Introductory Report recommends the theory of linear visco-elasticity as a basis for the analysis of reinforced concrete structures, referring to creep experiments that were limited to low compressive stress levels for support of this recommendation. While the application of linear-visco-elastic analysis within the range of applied operational stresses can thus be justified, the difficulty arises of selecting visco-elastic models relevant for loads of long duration on the one hand, and loads of relatively short duration and dynamic loads on the other. Attempts to cover the whole time-range with a single model are futile since differential models would involve differential quotients of too high order to permit complete specification of initial conditions for the solution of problems, while the experimental determination of a memory function for a Boltzman integral representation presents practically unsurmountable difficulties.[18] Obviously, the Kelvin model, suggested in the Introductory Report, is identical with the elastic medium for loads of long duration and completely useless for dynamic loads because of the unlimited increase of its damping with frequency. It has been shown that Burgers model[19] with parameters derived from tests of long duration containing a nonlinear "dashpot" that may become linear for low stresses is the simplest representation of the long-term mechanical response of concrete, to be used for analysis of dead load stresses and deformations, as well as of other long-term phenomena such as shrinkage and temperature stresses as well as of the effect of creep and relaxation on prestress-level and deformation of prestressed structures. For operational stresses of short duration and dynamic effects the Standard Solid[20] with short-time parameters seems adequate.

It appears that students nowadays do not spend enough time on exploring the literature in the field of their intended research before starting their own work, as otherwise the student referred to in the Introductory Report (p. 12) could have found that problems of increase, with time, of "second order" moments (moments in the deformed structure) due to creep, particularly in flat, long span arches where this increase tends to lead to instability[21] ("creep-buckling"), as well as of unexpectedly large relaxation of prestress due to the non-linearity of the creep[3] have been analytically treated quite some time ago. This knowledge might have induced him to expand his research beyond the limits of previous research efforts, considering, for instance, that it has been shown that even within the stress-range within which the use of linear visco-elastic theory is applicable, the fact of the increase with time of the coefficients of viscosity of the model due to time-hardening of the concrete, which requires the replacement of these parameters by time functions, severely curtails the usefulness of the correspondence principles, since it leads to differential equations with variable coefficients that have no correspondence in elastic theory and can usually not be solved in closed form. These difficulties, which arise from the discrepancy between the assumptions of linear visco-elastic theory and the behavior of a real material like concrete, must be carefully considered in the application of this theory as a method of structural anaylsis.

2.5 Non-linear and Failure Analysis of Reinforced Concrete.

Problems of non-linear analysis of reinforced concrete, as distinct from failure analysis, are of limited practical significance and arise only when compressive stresses in the concrete attain between one third and two thirds of the unconfined compressive strength while the reinforcement remains elastic. Such conditions, which are rarely considered in design, cause, where they occur, moderate redistribution of elastic stresses and moments and can be repeatedly applied without producing deterioration of the micro-structure followed by destruction of the concrete or abnormal cracking in the tension zone.

Failure of reinforced concrete sections subject to transient, sustained or repeated bending moments is initiated either when the compressive stress attains not less than 80 to 85 percent of the compressive strength under single load application, or when the stress in the reinforcement attains the yield limit or the fatigue strength associated with the number of load cycles, or both. While a substantial "plastic" redistribution of stresses and of bending moments may accompany the gradual crushing of the concrete or the plastic yielding of the reinforcement, the state attained is one of incipient or progressive destruction; it is not comparable, in its implications, to the "plastic resistance" of metals which is accompanied by a capacity for substantial, irreversible, but non-destructive deformation. This is the reason that the early analysis of similar conditions in reinforced plates was not introduced as plastic design, but as failure or rupture design and the traces of destruction as "rupture lines", in accordance with the realistic assessment of this condition, which is missing in the more recent attempts to apply plastic limit analysis to reinforced concrete. The objections to these attempts have been clearly summarized by G. Winter at the Miami Symposium[22] and repeated in the Introductory Report, with implied dissatisfaction at the "reluctance of the engineers to exploit the inelastic phenomena in beams". This reluctance is, however, well founded, and can hardly be compensated by the "advantages" of mathematical optimization of the design in terms of "economic functions" of similarly dubious engineering relevance as in the case of plastic design of metal structures for minimum weight or minimum material cost.

The fact that design for failure[23] utilizing lines of rupture, can be successfully applied to the prediction of the carrying capacity of plates does not imply that this procedure represents a kinematic limit design as understood in the theory of plasticity. Such plates are usually under-reinforced and fail simultaneously within the span and along the (fixed or continuous) supports by yielding of the reinforcement, with heavy fissurization of the concrete; problems of moment-redistribution and capacity of hinge, rotation do not arise.

Extension of this method to shells is difficult and uncertain, particularly because of the effect of creep due to the sustained compressive forces. The recommendation of the Introductory Report to obtain the necessary information for creep failure analysis of reinforced concrete shells from model tests using "a material that represents as closely as possible the real material" is, in fact, a call for full-scale testing since the rules of similitude preclude meaningful structural model tests with reduced geometric dimensions but unchanged material rigidity and creep response.

3. Design Procedures.

It appears from the Introductory Report that there is a divergence in the approach to "limit-design" between those concerned with steel structures and those concerned with reinforced concrete structures. While the former seem to consider limit-design for plastic collapse as the unique design procedure, to be applied to the exclusion of any other and in particular of elastic design, thus completely disregarding the dual aspect of design for operation (serviceability) and design against failure (reliability), the latter, becoming increasingly conscious of the fact that between the stages of the first appearance of fine cracks in the concrete and final structural failure (usually by collapse due to a mixture of total and partial destruction of the resistance of a relatively small number of critical section) there are many intermediate stages, seem to have developed the belief that engineering design should, ideally, consist in matching of the structural resistance at several intermediate stages with their "corresponding loads" and associated probabilities of occurence[24]. This latter point of view as much over states the complexity of the design-problem as the former under states it. In the Preliminary Publication of the Eighth IASBE Congress an attempt was made to present the principal aspects of the probabilistic approach to safety[25], which provides the only rational way to a balanced design procedure, and to discuss the effect of the deformational response of the structural material on this procedure[26]. In the light of this discussion it can be concluded that the consideration of "intermediate" stages between the limits of serviceability and of failure, which would have to include consideration of their respective statistical dispersions, complicates the procedure unnecessarily, since only in the cases of very costly structures subject to purely stochastic destructive loads, such as sea-walls, break-waters and off-shore platforms as well as, perhaps, tall buildings subject to earthquakes does it become necessary to consider conditions of partial damage, so that the cost of repair of structural damage associated with such pre-failure (intermediate) conditions which, in decision theory, are known as conditions of "success-loss", may strongly affect the design decisions[27]. This is, however, not the case in the design of the majority of engineering structures for operational loads for which, therefore, the recommendation of CEB to design for the two criteria of serviceability and of failure seems fully adequate. It would be desirable to use the same approach also in the design of metal structures.

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