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### IIIb

#### **Space Structures**

Structures tridimensionnelles

Raumtragwerke

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#### Introduction

Space structures have received a great deal of publicity during the last decade. This noticeable trend towards three-dimensional structures is partly due to a reaction from the linear systems of the previous decades, but largely due to the realization of the structural advantages of space systems, their inherent lightness combined with great stiffness.

Space structures are especially suitable for large-span roof systems and during the last few years they have been used frequently for covering exhibition halls, assembly rooms, churches, swimming pools and industrial buildings in which large unobstructed areas are required. If properly designed, space structures require less material than ordinary linear systems and can be highly economical in cost.

The first international conference on space structures was organized by the University of Surrey and held in London in 1966. This provided a comprehensive survey of the main developments in the field of skeleton three-dimensional structures up to 1966. (Ref. 1)

In this report the author is concentrating his attention almost exclusively on the trends and further developments which have taken place during the last five years. The opportunity is also being taken, however, to reexamine the predictions made during the conference in 1966 to see which have been fulfilled and which did not materialise.

International exhibitions stimulate progressive designers and provide them with the opportunity to try out new forms of construction, to use new building materials and to apply new construction techniques. Expo '67 in Montreal and Expo '70 in Osaka are typical examples. They illustrate technological progress and technical innovation. In each of these exhibitions space structures provided the domineering features emphasizing the tremendous impact exerted by three-dimensional structures upon modern architecture and structural engineering.

Out of many space structure pavilions erected at Montreal, three examples of prefabricated space systems are really outstanding – the double-layer grid dome over the American pavilion, the sophisticated multi-layered Canadian "theme" pavilions and the original space frame building for the Netherlands pavilion constructed in the Triodetic System.

In Osaka's exhibition the list of space structures would be very long indeed – the prefabricated double – and multi-layered grid structures and various types of braced domes provide proof of the enormous popularity of these systems.

Kenzo Tange's splendid space frame for the Festival Plaza is the centre piece of the whole exhibition. This gigantic space frame is astonishing in its scale and fascinating in its detail.

It is a classical example of a rectangular two-way double-layer grid, 292 m long, 108 m wide and 30 m high. The total weight of steel used for this grid is 4800 tons – the roof was assembled on the ground and lifted by 48 jacks and supported finally by six three-dimensional pillars.

The main tubular members forming the top and bottom layers are 500 mm dia, and the inclined diagonal members are 400 mm dia. The members are connected by means of spherical balls 800 mm dia. The grid is divided into square bays each 10.8m long.

The trend towards prefabrication is gaining momentum. Space frames can be built from simple prefabricated units, in many cases of standard size and shape.

Such units, mass produced in the factory, can be easily and rapidly assembled on site by semi-skilled labour. The small size of the components greatly simplifies the handling, transportation and erection, as no heavy hoisting equipment is required on site. As a rule, the high quality control in the factory results in a high standard of workmanship and a good finish.

A review of the recent developments in the field of space structures shows clearly that the most remarkable progress has been made in prefabricated double-layer grids.

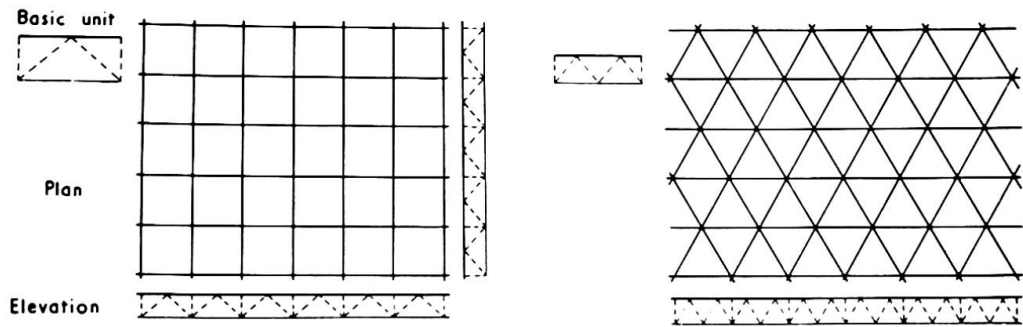
### Double-layer grids

Double-layer grids are of special importance as they are frequently used in roof and, more recently, in floor construction. They consist of two plane grids forming the top and bottom layers, parallel to each other, and interconnected by diagonal members. Plate I shows six main types of double-layer grids used by various commercial firms all over the world. Basically there are two main types of double-layer grids – lattice (or truss) grids, consisting of intersecting vertical lattice girders and true space grids consisting of a combination of tetrahedra, octahedra or skeleton pyramids having square or hexagonal bases.

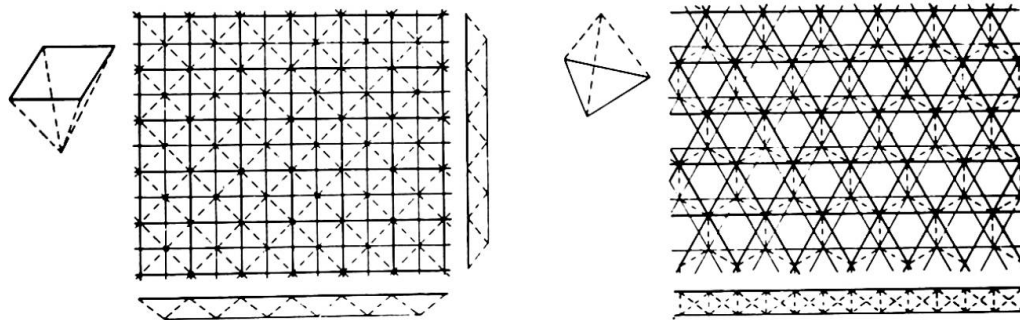
A good example of a two-way lattice grid is provided by the recently finished (January 1970) space frame roof, the largest in the United States, for Chicago's McCormick Place convention centre. The total weight of steel is 9500 tons. Fig. 1 illustrates the grid during construction.

## Plate I

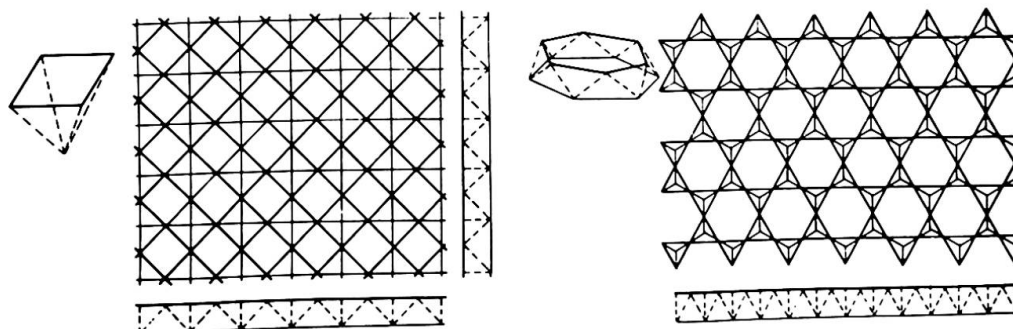
## Double - layer grids



Lattice grids



Space grids





The structure consists of 4.7 m deep trusses at 45 m centers, supported by 36 steel plate cruciform columns. The top chords of the trusses are 20 m above the main floor level. The total area covered by this huge structure is 410 m by 180 m.

The two way grids can be arranged either in rectangular or diagonal fashion. In the first case the elements run parallel to the sides of the opening, in the second, they form normally an angle of  $45^\circ$ . Tests show that diagonal grids possess much greater rigidity and therefore deflect less than the rectangular grids.

The roof structure covering the huge hangar erected in 1970 at London Airport for the Boeing 747 aircraft constitutes the largest diagonal lattice grid in the world. The hangar roof structure contains about 1,500 tons of steel, of which some 1,100 tons is in structural hollow sections. The hangar is 34 m high and has an overall length of 171 m. The clear door opening is 138 m wide by 23 m high.

The roof is supported by only 8 columns. The preliminary studies showed clearly that, for large spans, deflection limitations become vital and the overall rigidity is extremely important. This was especially so because the roof of the hangar had to be capable of supporting almost 700 tons of equipment, of which 300 tons could be applied at any number of positions. A comparison of ten different solutions showed that space frames performed more efficiently than conventional systems and therefore a grid structures was chosen for the final design. The roof consists of four main parts :

- (a) the horizontal low level diagonal grid built from prefabricated units, supported by four columns and interconnected along one side with a vertical spine girder
- (b) the spine girder which is 166 m long 15 m deep and 3.75 m wide, supported by two columns
- (c) the high level diagonal grid of identical modular latticed units as the low grid. It is supported along one side by the spine girder and along the entrance to the hangar by the fascia girder
- (d) the fascia girder is 189 m long and consists of two parallel V-trusses (3.25 m high and 1.6 m apart), laced together to form a torsionally stiff unit.

The horizontal diagonal grids are constructed of prefabricated tubular trusses 8.5 m long and 3.66 m high. The grid trusses were shop welded and then bolted on site to one another and to the main girders. The boom members are steel tubes 168 mm dia, the diagonals are of 140 mm dia and the vertical member of 114 mm dia. The spine and fascia girders are of all-welded construction. The main chords of the girders are tubes 457 mm dia, the wall thickness for the most heavily loaded units being 29 mm. The diagonals are tubes of 356 mm dia. All the main members are of high grade steel with a minimum yield stress of  $45 \text{ kg/mm}^2$

Fig. 2 shows the grid under construction. Fig. 3 illustrates the external appearance the diagonal grid. Reference (2) gives full technical description of the structure.

The stress distribution under unsymmetrical loading is more uniform in the three-way latticed grids than in the two-way grids. Several examples of such structures have been built during recent years in the U.K. by Booth & Co. (Steel Structures) Ltd. in their

Met-Ram system. In France, S. du Chateau has covered several churches, swimming pools and industrial buildings with this type of construction using his Tridimatec system. A recent example is provided by a steel three-way lattice grid covering the hydraulics research station in Madrid in Spain. The roof area is over 5,000 sq.m. divided into two sections by an expansion joint. The unit weight of the structure is  $55 \text{ kg/m}^2$  which is quite low taking into account the span which is up to 60 m. The whole structure was assembled at ground level and then raised into position. (Ref 3)

The detailed analysis of the stress distribution in this structure was carried out on an Elliott 503 electronic computer on behalf of the Spanish consultant by the Space Structures Research Centre of the University of Surrey. Full details of the analysis are given in reference 4.

From a structural point of view the true space grids are superior to lattice grids because of their greater rigidity. On the other hand, however, the transport and erection of lattice grids may be somewhat simpler since they consist of flat lattice trusses which can be stored and transported very easily. The flat space grids can provide a column free roof system with a depth/span ratio of about  $1/20$  to  $1/25$ .

Fig. 4 illustrates the gradual evolution of the classical two-way double-layer grid frequently used in the past in systems, like Mero, Oktaplatte, Unistrut, Space Grid, Varitec, Space Deck, Nenk, Pyramitec, Triodetic, etc.

The most significant recent development is the type of bracing shown at the bottom of Plate I. This new type of space grid has been produced by arranging the top layer in a diagonal fashion and leaving the bottom grid forming a two way rectangular grid. This novel system can be built from identical prefabricated skeleton pyramids. Detailed analysis and comparison of various systems shows that this system has several remarkable advantages. Since 1963 the Space Structures Research Centre of the University of Surrey has carried out a comprehensive research investigation on the analytical and experimental stress distribution in such grids as part of a research programme sponsored by the British Iron and Steel Research Association. A detailed computer programme for the structural analysis developed at the University of Surrey gives a precise determination of forces in all members of such structures. Fig. 5 shows the forces and deflections for a building supported only at four corners. This was built in 1970 for the Northern Gas Board in Killingworth, the constructional engineers being Robert Frazer & Sons Ltd., of Hebburn Co. Durham, a firm which takes special interest in this form of prefabricated steel space construction. Fig. 6 shows the erection of the structure. A similar configuration was used in 1968 in Northern Ireland for covering the refectory and games hall at the new University of Ulster. The structures consist of prefabricated pyramids. The upper layer is formed using angles but tubes were chosen for the bottom layer, partly on the grounds of appearance and partly for fabrication reasons since a simple connection could be made at the nodal points by flattening the tubes. High-strength friction grip bolts were used to connect the various components.

An identical configuration has been used during the past five years for many steel space structures by Takanashi - a Japanese designer, for exhibition pavilions, bowling alleys and industrial buildings. It is known in Japan as the Takanaka truss. This system has now been introduced into the U.S.A., one of the recent examples being the roof over the Roosevelt Memorial Hall at the American Museum of Natural History. (Ref 5)

Dr. Max Mengerhausen, the inventor of the well-known Mero system, seems to be one of the first designers to use this configuration in Europe. Employing Mero joints a factory in Bath, England, has been covered with this system in 1967. The total area is  $117 \text{ m} \times 44 \text{ m}$  made up as an arrangement of eight bays by three bays. The roof is designed as a

continuous flat double-layer grid. The diameters of the circular tubular members used for this structure vary from 60 mm to 114 mm.

The same configuration forms the basis of a French patent by S. du Chateau a well-known designer of steel space structures who has built many large-span steel grids in France. His new system is known as Unibat and has been used already with great success not only for industrial buildings but also for multi-storey schools and blocks of flats. Fig. 7 illustrates the erection in 1970 of a multi storey block of flats in Gonesse.

One must refer also to the extremely interesting system developed in 1967-68 in Japan by Yamashita, Kannon and Kanazawa, known as the Obayashi truss H-1 system. It is a modified and simplified version of the classical three-way space grid. The structure consists of prefabricated tetrahedral units which are interconnected to form a double-layer grid. The top layer is a combination of regular hexagons and triangles, whilst the bottom layer has a regular hexagonal pattern. Several flat roof structures, as well as braced domes and barrel vaults, have been constructed in this system in Japan - details are given in reference 6.

With all these new developments taking place it is interesting to see that several systems which were already well established in the past are intensifying their activities and continuing to flourish.

In England, Space Deck's popularity is increasing in a visible way. It is a two-way double-layer grid consisting of prefabricated inverted steel pyramids which are bolted together at the top along their common edges and have their lower apices interconnected by tie bars fitted with turnbuckles.

It has been used in well over four hundred contracts for schools, hospitals, museums, assembly halls, bowling alleys, factories, etc. not only in the U.K. but also abroad. For example, the exhibition hall at the 14th Triennale in Milan has been covered in 1968 by a Space Deck roof which has a clear span of 45 m.

The Space Deck units are manufactured using a fully automatic conveyor production technique. After cutting to the required length, the components are degreased by immersion in trichlorethylene. Scale and rust is then removed by shop blasting before they are assembled and welded on special jigs into complete pyramids. The installation plant includes large dip tanks in which the units are automatically painted by immersion, and also a stoving oven, through which all the units pass before reaching the unloading bay. The maximum span which can be achieved using the standard Space Deck unit 1 m high is about 40 m, but much larger spans can be obtained using high tensile steel or units having a greater depth.

The two-way double-layer rectangular grid has been used for a number of large-span structures in the U.S.A. mainly for assembly halls, combined auditorium-sports arenas etc. A typical example is the recently finished Edwin W. Panley Pavilion for the University of California at Los Angeles, over an area of 91 m x 122 m. In this case the steel space frame consists of 108 four-sided pyramids with their tops connected by members running parallel to the sides of the building.

The interconnected pyramids are identical in plan, but vary in height, so that roof slopes from a 9 m height at the centre to 5.5m at the perimeter. This creates a hip roof configuration, and provides drainage.

A very similar example of the same trend towards the use of two-way double-layer grids for sports halls is the steel space frame for the University of South Carolina Coliseum,

erected in 1967-68.

To provide a totally column-free interior at maximum economy, the engineers designed a grid covering an area of 100 m x 100 m. Their investigations showed that the use of a space frame resulted in considerable savings over conventional structural systems.

An extremely interesting two way double-layer grid using tubular steel has been constructed in 1968-69 for the Amstell Hall at Amsterdam with plan dimensions of 62 m x 196 m.

A similar construction was selected for the 41 m square storage building at Bornem, Belgium, built for Brown Boveri.

The interest in two-way double-layer grids continues to increase and steel roof systems of this type are now being used in many continental countries, including Hungary, Poland, Czechoslovakia and East Germany, where the steel is still a "deficit" material and as such is normally replaced by concrete which is cheaper. However, in many instances the use of steel grids proved to be highly competitive.

The developments in East Germany are particularly interesting. The research institute of the Weimar Technological University devoted special attention to the problem of skeleton space structures and several large span double-layer grids erected in East Germany are the direct outcome of these activities. One has to mention especially the sports stadium in Halle covered in 1968-69 by a prefabricated steel double-layer grid over an area of 57.6m x 72 m. A modified version of the Mero connector was used in this case. (Ref. 7)

Although double-layer grids have been used successfully in various countries for numerous roof structures of large span, it has been assumed that such systems would not be economical for smaller spans. However, development work carried out by the Directorate General of Research and Development of the Ministry of Public Building and Works in the U.K., has shown that double-layer grids can compare with conventional systems even for moderate spans and can be used both for roofs and floors in multi-storey buildings. The Nenck method of building used for the War Office barracks at Maidstone is based on the principles of the Space Deck.

The French GEAL system demonstrated in a very convincing way that space frames form an important step forward in reducing the material content of modern construction. The GEAL system uses three-dimensional floor elements forming very rigid space slabs.

The Unibat system of S. du Chateau used with great success in 1970 for the construction of several multi-storey buildings shows clearly that even for spans less than 8 - 9 m double-layer prefabricated grids provide an economical solution. (Ref. 8)

#### Braced barrel vaults

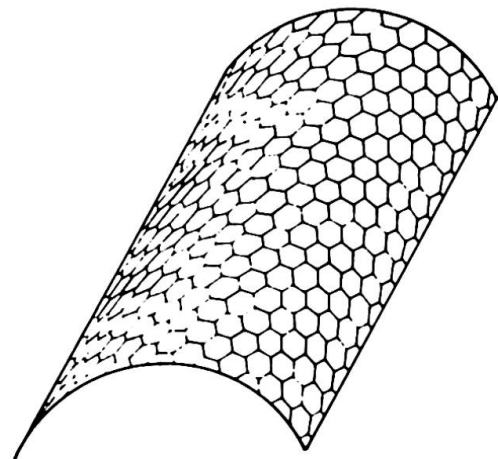
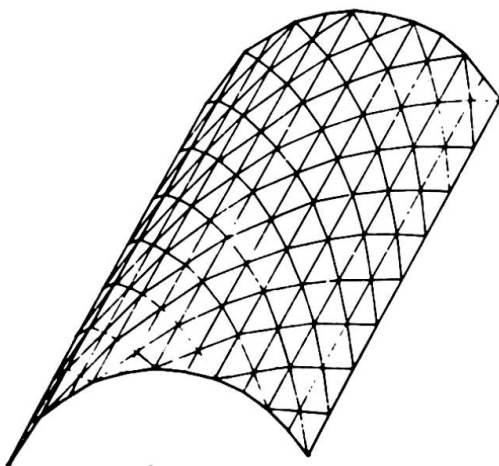
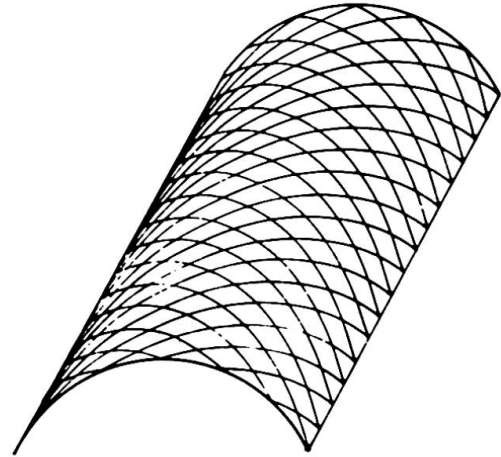
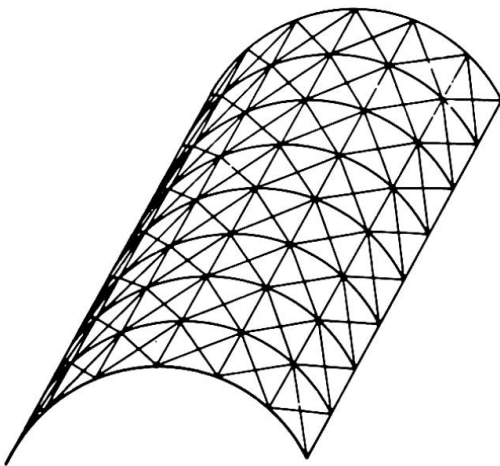
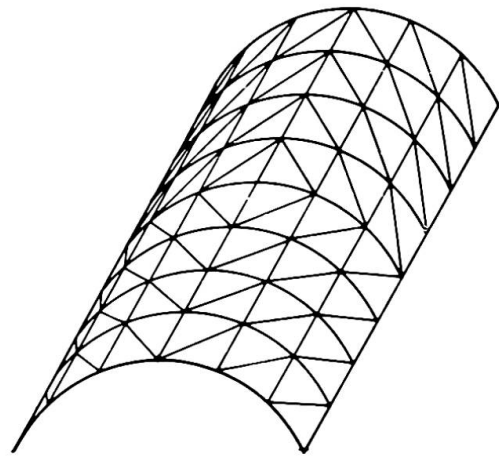
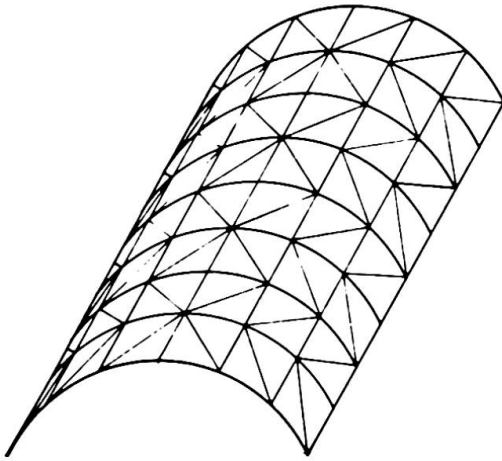
Braced barrel vaults form another type of space system often used to cover industrial buildings, swimming pools, tennis courts etc. The structure is similar in configuration to a shell but it is not homogenous, being an assembly of bars.

Plate II shows six principal types of bracing used. Tests and numerical analysis show that the three-way grid type of bracing, provides very uniform stress distribution (Ref 9) and, because of its inherent rigidity, this type is frequently used in practice.

A very recent case and a convincing proof of the economy of such structures is provided by the roof over the multi-storey dock transit building erected in London during 1966 - 67.

## Plate II

## Braced barrel vaults





Seven barrel vaults built in steel rectangular hollow sections and with the three-way grid type of bracing cover an area of 56 m x 175 m. The barrel vaults are supported by walls along three sides and columns along the fourth side, leaving an unobstructed area in the centre. The structure has been built from flat tubular tresses which were welded on site to form seven barrel vaults each of 25 m span with a rise of 5 m and a radius of 17 m. Fig. 8 illustrates the building.

When discussing steel barrel vaults, one must mention especially the work of Joseph Zeman. This engineer has been responsible for the design and construction of many steel barrel vaults built within the past five years in Czechoslovakia and East Germany. Indirectly he is also responsible for similar structures built in Poland. His work shows clearly that impressive economies in cost and material consumption can be obtained for large span buildings constructed as prefabricated space frames. Zeman covered several sports and public halls in Czechoslovakia with tubular steel segmental barrel vaults. The structures consist of prefabricated 8 m long units, weighing 200 kg, interconnected into a system of diagonally arranged arches. All the units are of identical dimensions built on a specially prepared rig - this enables them to be produced with a high degree of accuracy and to minimum tolerances.

The main system of segmental arches is supplemented with another system of load-bearing members spaced at approximately 2.5 m. and arranged in the longitudinal direction of the barrel vault.

These members act primarily as purlins, stiffening the whole structure and converting it into a three-way spatial grid. Fig.9.

The prefabricated units lend themselves easily to stacking, require little area, both on railway wagons during transport and in storage on site, are light in weight and can be handled manually. There are eight high-tensile bolts at every joint connecting not only the four main units, but also the chords of the purlin members. Typical examples of such structures are the winter sports stadium in Kladno (60 m x 60 m) and the sports hall in East Berlin (59 m x 75 m). The dead weight of the Kladno barrel vault is only 18 kg/m<sup>2</sup>. (Ref 10)

The same type of structural steel framework has been used in Poland to cover a sports hall in Sosnowiec over an area of 53.6 m x 78 m. Circular hollow sections have been used for the main members of the barrel vault.

### Braced domes

The dome is the oldest structural form and is a typical example of a space structure. It encloses a maximum amount of space with a minimum surface and provides one of the most efficient structural shapes, permitting the covering of very large areas in an economical way.

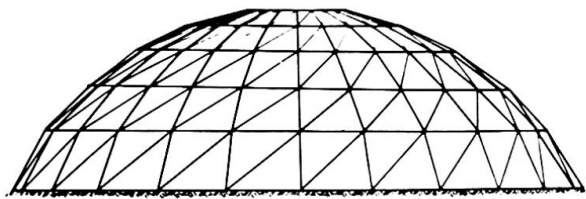
The classification of braced domes is very difficult owing to the great variety of possible forms (Ref 11)

Plate III shows five most popular types frequently used in practice.

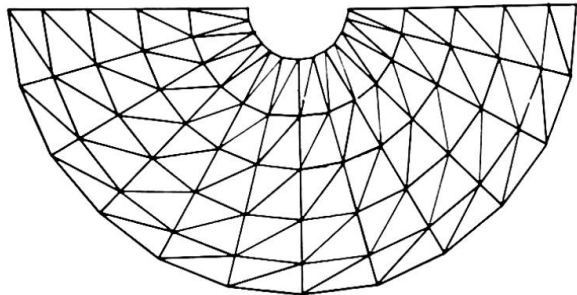
These are the	Schwedler, network, three-way grid, parallel lamella and geodesic domes.
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Braced domes

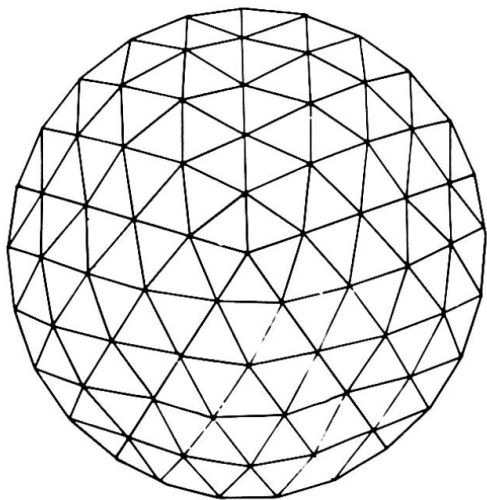
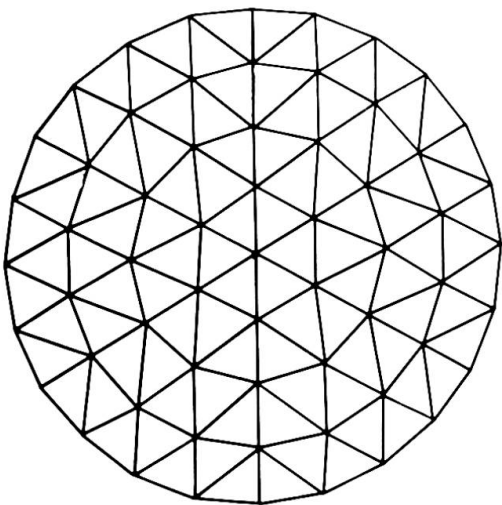
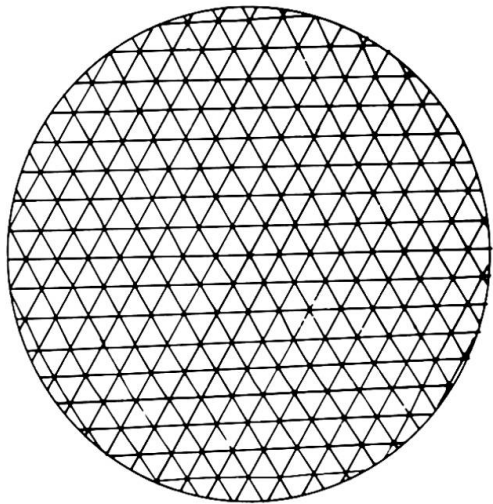
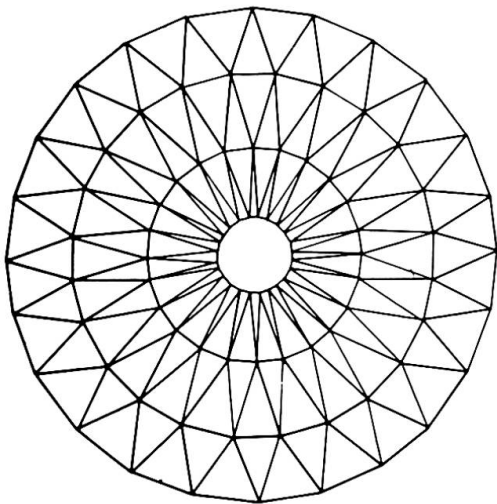
Plate III



Elevation



Plan



The earlier work of Buckminster Fuller on geodesic domes proved to be instrumental in reviving the interest of architects and engineers in braced skeleton domes. The recent work of D. G. Emmerich (Ref 12) on the morphology of skeleton space structures and K. Critchlow (Ref 13) on the general topology of three-dimensional subdivisions clearly illustrates the recent interest of architects in space frames.

The United States pavilion at Expo '67 designed by Buckminster Fuller and his associates, is the most impressive example of a geodesic dome. It is a three-quarter sphere of 76 m dia and contains some 6000 connectors and 24000 tubular members arranged into a double-layer space grid. A three-way grid forms the outer layer (Fig. 10) and a hexagonal grid the inner layer. The consulting engineers who tried to analyse the three-dimensional grid as a skeleton space frame, admit that the resulting configuration proved to be too complex for direct analysis by existing large-capacity computer programmes. (Ref 14) Instead, approximate stresses have been obtained using shell analysis for eight different loading conditions.

Because the pavilion is a double-layer space frame with a different configuration for the inner and the outer layers, the transformation of stress resultants to maximum member forces in the actual frame is quite involved; the situation is further complicated by the fact that the geometry of the inner layer is not fully triangulated but hexagonal and can support only fully symmetrical loading.

The computer analysis has been carried out using a programme designated as MAST (Membrane Analysis of Structures). The structural consultants stated that the extent of the participation of the inner layer in bearing the shell stress resultants is not constant and depends on the type of external loading.

For design purposes, the stress resultants from membrane analysis were apportioned approximately 55 to 75% to the outer layer and 25 to 45% to the inner layer depending upon the type of loading considered.

It is of interest to mention that the factor of safety of this dome against buckling failure, based on an equivalent thickness of a homogenous shell, was 4. The calculations also proved that the load capacity of this dome is governed by the stability of individual members and not by the overall stability of the dome.

The framing members are steel tubes 88.9 mm dia for the outer layer and 73 mm dia for the inner layer and for the diagonal web members. Though the outside diameter of the tubes is constant, the wall thickness varies depending on their location.

Two types of connectors were used one for twelve members and the second for six members, meeting at a joint.

The Montreal dome is an example of a double-layer type, which is suitable for very large spans, much greater than the span actually used. In fact, single layer domes have been built during recent years for spans exceeding 90 m. A good example is the geodesic steel dome designed by Synergetics Inc. for Electro Minerals Division of the Carborundum Co. N.Y. which has a span of 90 m. It consists of steel I sections, bolted at each end with four bolts through small circular vertex plates provided above and below the joint. The structure proved to be very simple to erect and highly economical in cost. One should not create the impression that for such large spans, as mentioned above, only steel is suitable. In fact, the University of Utah's special events centre features a timber braced dome of 107 m dia.



It has been designed as a single-layer triangulated dome in the Triax system and manufactured by Timber Structures Inc. The designers claim that the timber dome design permitted a substantial saving over alternative metal systems. The type of bracing follows the three-way grid pattern. This type is also very popular in several European countries. There are several recent examples of braced tubular steel domes built in this configuration by S. du Chateau. The swimming pool covered with a three-way tubular dome at Drancy near Paris, is an excellent example of a single-layer grid dome having a dia of 45 m. Special connectors of weldable cast steel were used at Drancy and the joints were fully rigid. Du Chateau used the three-way type of grid for many of his domes including the Agadir dome in Morocco which is supported at only three points spaced 32 m apart.

In the U.S.A. the Schwedler type domes are still very popular and are frequently used. Two especially interesting examples are the domes built in 1968 - 69 at the Notre Dame University, each measuring 100 m dia and having a 12,500 seating capacity for basketball games, convocations and stage show productions.

The domes have Schwedler type of bracing and are of welded construction. There are 36 ribs in each dome spanning from the tension ring to the compression ring. The ribs were fabricated in two pieces and bolted together in the field with high-strength bolts. The erection was very simple, a temporary steel tower being used to support the crown compression ring while the ribs were placed in tandem across the dome with two cranes.

The Ohio University has one of the lightest Schwedler domes in the United States over their convocation centre erected in 1968. The dome has a dia of 110 m and is supported on 48 columns.

In Europe, Professor F. Lederer influenced to a great extent the development of network domes. An interesting and a very recent example is a steel tubular dome having a dia of 70 m built at Opole in Poland. A special type of universal connector has been used, which allows simple adjustment in the length of the members.

### Connectors

The review of the last five years shows clearly that the search for an "Ideal" connector for prefabricated space structures continues. Many new types of connector have been developed, but most of them are too complex and therefore too expensive. As a result, very few have survived the test of time. The interest shown in this field is probably best illustrated by the competition organized in 1964 by the French Chambre Syndicale des Fabricants de Tube d'Acier for the development of efficient connectors for tubular space structures. Reference 15 shows details of the various proposals submitted during this competition. There are several articles discussing the relative advantages and disadvantages of numerous connectors.

Extensive testing of various types of mechanical connectors suitable for prefabricated tubular space structures was carried out during 1969 - 70 by Stewarts & Lloyds Co. Ltd. Fig.11 shows the specially constructed jig for testing full-size connectors under three-dimensional systems of loading.

One of the more successful connectors is the Triodetic joint developed by a Canadian firm, originally for aluminium space structures, but nowadays used mainly for steel systems. It uses specially extruded hubs, provided with slots to receive the pressed ends of the structural members. It must be noted that in forming the ends metal is not removed but only displaced. This results in the formation of a tapered thread with a gradual transfer of load and high structural efficiency. Assembly is carried out by slipping the ends of the member into the hub.

The introduction of the Triodetic connector influenced to a marked degree the use of steel and aluminium tubular members for triangulated hyperbolic paraboloidal structures. A classical example of this trend is the Olympic Games Sports Palace constructed in 1968 in Mexico City. The structure is covered by a domed roof 132 m dia. The roof is a spherical shell formed by a grid of steel trusses forming arches. The 12 m square areas between the arches are covered by a triangulated grid of aluminium tubes in the shape of hyperbolic paraboloids. The whole roof is covered therefore with 144 aluminium HPs weighing only 3 kg/m<sup>2</sup>. Fig. 12 illustrates the interior of this most unusual structure. All the aluminium tubes are interconnected by means of the Triodetic joint.

Several steel triangulated hyperbolic paraboloidal structures have been built in Japan, some of very considerable span.

A large hangar at the Minneapolis - Saint Paul International Airport in the U.S.A. has been covered in 1969 with a steel hyperbolic paraboloidal structure. The HP measures 38 m along each rear side, 50 m along each front side and spans 58 m between two supporting buttresses.

These examples show that even in the field of shell structures (which many engineers regard as the province of reinforced concrete), steel and aluminium are steadily being introduced by reason of economy and structural efficiency.

### Analysis

A decade ago the analysis of a complex space structure was frequently an almost impossible task. In 1970, the designer can obtain with reasonable accuracy the assessment of stress distribution in his structure, assuming that it behaves elastically. Tests show that double-layer grids can be analysed by elastic methods and that the rigidity of the joints does not change the stresses by more than some 10 to 15% for the two-way arrangements. In the case of three-way grids the difference between the analyses for pin-connected and rigidly connected members is normally even less, because of the greater overall rigidity of three-way grids.

However, the elastic analysis of single-layer barrel vaults and domes still provides only an approximation. Such structures behave elastically but not in a linear manner. The possibility of buckling for double-layer grids is minimal, but very real for single-layer triangulated shells.

One of the main reasons for the rapid acceptance of space frames and their striking development within the past few years has been the introduction of electronic computers. Its use has radically changed the whole approach to the analysis and design of space frames. It has also been realised that matrix methods of analysis developed for use on high-speed digital computers provide an extremely efficient means for rapid and accurate treatment of many types of space structures.

Matrix algebra is ideally suited for automatic computation and great interest has been expressed during recent years in the formulation of general matrix equations for three-dimensional structures. In these methods the digital computer is now used not only to solve many simultaneous equations, but techniques have been developed to generate the input data, the analysis of the structure, the determination of the required stress resultants and the production of the finite output. (Ref 16)

Most of the programs now existing for the analysis of space structures are based on the stiffness method. Electronic computers are better suited to perform fully automatic operations,

which can be followed blindly by the computer regardless of the nature of the framework. This condition is satisfied in the case of the stiffness approach in which the final equations of structural analysis are formulated with deflections as unknowns and the computer is not required to make arbitrary choices of unknown quantities as is the case for the flexibility method. However, theoretically, it is possible to use the matrix formulation, either in the flexibility or in the stiffness methods and to obtain from the electronic computer the stresses and the displacements - in practice, in the past, the flexibility approach proved to be more complicated than the stiffness approach. It is very interesting to note that during recent years techniques have been developed for the automatic selection of redundancies and for the generation of the self-equilibrating force systems. This allows the flexibility analysis to be used and is changing the earlier preference for the stiffness method. Przemieniecki in his book (Ref 17) shows that the selection procedures for flexibility methods based on the Jordanian elimination techniques lead invariably to well conditioned equations.

With the increased interest in computer analysis, the designers soon found that the analysis of complex space frames required very large core memory capacity in the electronic computer. To overcome the practical limitations of the storage capacity, methods have been formulated, in which advantage is taken of the band form of the main stiffness matrix of the system and also partitioning techniques have been perfected in which the analysis is done in interconnected steps, analysing the structure divided into smaller units of manageable size.

Soon, other difficulties have been observed, even using the above mentioned techniques the round-off errors can reduce the accuracy of the results.

Tezcan (Ref 18) proposed several modifications in the matrix treatment of space frames using the transformed member stiffness matrices in connection with the code number approach. This results in a greatly increased speed of generation of the main stiffness matrix and leads to a considerable saving in data preparation and computer storage. The use of code numbers makes the programming easier and the computation much faster.

The problem of ill-conditioning of stiffness matrices has received a great deal of attention. The influence of truncation errors on the accuracy of the numerical solutions using the stiffness matrix formulations can be considerable. This influence depends upon the characteristics of the stiffness matrix, namely its eigen values and the eigen vectors. These determine the conditioning of a given matrix and the extent of coupling among its eigen vectors. Shah (Ref 19) shows that one of the measures of conditioning of a matrix is the ratio between the largest and the smallest eigen values.

Another characteristic trend which became very noticeable during the past five years are the attempts to apply the finite difference and the finite element methods to the analysis of various types of space frames. The finite difference methods have been used successfully in the past in the stress analysis of plates. However, it is only during the last decade that these techniques have been extended to reticulated shells and especially to double-layer grids. (Ref 20 - 23)

The finite difference methods lead to a system of algebraic equations, determining the values of the function at isolated points. Obviously they are approximate but their application is general and does not suffer from the usual limitations of the differential calculus methods.

The distinction between the finite difference and the finite element techniques lies in the method used to determine the system of partial differential equations.

In the first method the equivalent differential equations are approximated by difference operators which require an assumption of the displacement form between the adjoining node points. In the finite element method the field is divided into various small elements each connected to its neighbouring elements at their node points. (Ref 24)

Several attempts have been made to apply the finite element technique to the analysis of elastic buckling of structures using a digital computer.

In spite of the availability of digital computers, there are frequent cases when the complexity of the framework makes the analysis of space structures either very tedious and expensive, or simply impossible because of the very large number of bars and joints in the structure.

In such cases certain types of reticulated shells and double-layer grids can be analysed by treating them as continua and applying the shell or plate analogies. Recent work of D. Wright has been of fundamental importance in this field. (Ref 25 - 26) He determined elastic properties of the analogous shells and showed how to use them in the general shell equations. This enables an approximate determination to be made of the stresses in reticulated shells.

The determination of the instability behaviour of space structures still produces considerable difficulties - the existing data allow only an approximate assessment of this very important phenomenon. Whereas the bar stability problem is covered in great depth, the local buckling or so called snap-through buckling is just beginning to receive the attention of research workers. Little (Ref 27) produced recently an interesting review of the reliability of shell buckling predictions and several of his conclusions can be applied directly to reticulated shells. Aguilar (Ref 28) investigated the joint stability of braced domes, and took the vanishing of the first order variation of the total potential of the system as the criterion for equilibrium.

Interesting experimental work of Buchert (Ref 29 - 30) demonstrated that shell edge conditions play an important role in the capability of a shell-like structure to resist buckling.

Whereas the computer analysis of elastically linear space frames has received a great deal of attention, there are only very few papers dealing with the matrix formulation for dynamic analysis or calculation of vibration frequencies and modes. (Ref 31)

### Summing-up

- (1) Great advances in prefabricated double-layer grids have taken place in many countries and these systems are now widely accepted.
- (2) There has been a remarkable increase in the popularity of tubular space structures. Hollow steel sections now compete very successfully with conventional rolled steel sections and are used very frequently for three-dimensional structures.
- (3) The search for an improved connector for space structures still continues.
- (4) The expected major break through in the structural use of aluminium in space structures has not taken place, the high cost of aluminium being the main reason. The vast majority of space structures are built in steel.

- (5) The elastic analysis of space structures is no longer a major problem for the designer. The readily available programs for high-speed electronic computers provide rapid stress analysis and, in some cases, even the design of the structures.
- (6) Whereas the dead and even live loads can be determined with reasonable accuracy the assessment of wind loading is still based on very approximate assumptions. Very little is known about the distribution of wind loading on domes, barrel vaults and hyperbolic paraboloidal structures. Wind tunnel research is urgently required.
- (7) Space structures can be very economical in the use of material and, due to their light weight, they often have to be designed for reversal of stresses due to wind suction. Unsymmetrical loading can produce instability in single-layer space structures. The recent collapse of three aluminium braced domes and one steel barrel vault under a heavy unsymmetrical snow loading emphasises the importance buckling, a phenomenon still not fully understood.
- (8) No information exists on the influence on composite action of the framework and the roof deck. The buckling behaviour of many types of space structures is influenced by the cladding.

### Conclusion

Interest in space structures is growing constantly – the large number of such systems built all over the world shows clearly that space structures can compete very successfully with more conventional structures.

### References

1. R. M. Davies (editor) "Space Structures"  
Proceedings of the first international conference  
on space structures  
Blackwell Scientific Publications,  
Oxford & Edinburgh, 1967.
2. K. J. Joyner, R. G. Taylor  
and Z. S. Makowski "The Boeing 747 Hangar 01 Heathrow, London"  
Tubular Structures No. 15,  
March 1970, pp. 2 – 32.
3. J. M. Montero Rodriguez and  
J. C. Contreras Carrillo "Roofing the hydrographic research centre –  
Madrid (Spain)"  
Acier, No. 6 1969 pp. 277 – 280
4. Z. S. Makowski and H. Nooshin "The structural analysis of a large three-way grid"  
Proceedings of the Space Structures International  
Conference  
Blackwell Scientific Publications, Oxford, 1967  
pp. 327 – 342.
5. D. H. Geiger "A cost evaluation of space trusses of large span"  
AISC Engineering Journal  
April 1968, pp. 49 – 61.

6. N. Yamashita, N. Kannai and M. Kanazawa  
 "Design and construction of Obayashi Truss H-1"  
 (in Japanese)  
 Column No. 25, pp. 43 - 48  
 Published by Yawata Iron & Steel Co. Ltd.  
 Tokyo, Japan.
  
7. H. Stenker  
 "Entwurf der Stahlkonstruktion für Eissportstadion Halle"  
 Bauplanung - Bautechnik  
 November 1969.
  
8. S. du Château  
 "Les structures tridimensionnelles dans l'industrialisation du bâtiment"  
 Recherche & Architecture, No. 2, 1970, pp. 23 - 30.
  
- 9a. H. Beer  
 "Einige Anwendungen der Schalenbauweise im Stahlbau"  
 Der Bauingenieur,  
 May 1966, pp. 200 - 208.
  
- 9b. Z. S. Makowski  
 "Analytical and experimental investigations of stress distribution in steel space frames"  
 Proceedings of the Steel Congress 1964  
 European Coal and Steel Community,  
 pp. 581 - 606.
  
10. J. Zeman  
 "A remarkable steel structure for roofing a large sports hall in Czechoslovakia"  
 Acier, No. 4, 1968, pp. 181-188.
  
11. Z. S. Makowski  
 "Steel space structures"  
 Michael Joseph, London, 1965.
  
12. D. G. Emmerick  
 "Géométrie Constructive"  
 Ecole Nationale Supérieure des Beaux-Arts,  
 Paris, 1970.
  
13. K. Critchow  
 "Universal space families"  
 Architectural Design,  
 October 1965, pp. 514 - 517.
  
14. S. Sadae and F. J. Heger  
 "The United States Pavilion at Expo '67"  
 Tubular Structures, No. 9, 1967  
 pp. 2 - 9.
  
15. Chambre syndicale des fabricants de tubes d'acier  
 "Le tube d'acier dans la construction métallique - noeuds et assemblages"  
 CSFA, Paris, 1966.
  
16. K. Eiseman, L. Woo and S. Namyet  
 "Space frame analysis by matrices and computer"  
 Journal of the Structural Division,  
 Proceedings of the A.S.C.E.  
 December, 1962, pp. 245 - 278.
  
17. J. S. Przemieniecki  
 "Theory of matrix structural analysis"  
 McGraw-Hill, Inc. 1968.



18. S. S. Tezcan "Computer analysis of plane and space structures"  
Journal of the Structural Division,  
Proceedings of ASCE, Vol. 92, No. ST2,  
April 1966, pp. 143 - 173.
19. J. M. Shah "III-Conditioned stiffness matrices"  
Journal of the Structural Division,  
Proceedings of ASCE, Vol. 92, No. ST6,  
December, 1966, pp. 443 - 457.
20. J. D. Renton "The related behaviour of plane grids, space grids  
and plates"  
Space Structures (Edited by R. M. Davies)  
Blackwell Scientific Publications,  
Oxford & Edinburgh, 1967, pp. 19 - 32.
21. M. V. Soare "Application of finite difference equations to  
shell analysis"  
Pergamon Press Ltd., London, 1967.
22. M. V. Soare "Application des equations aux différences finies  
au calcul des réseaux spatiaux planaires carrés"  
Rev. Roum. Sci. Techn. - Méc. Appl.  
Tome 14, No. 3, pp. 595 - 628, Bucarest, 1969.
23. M. V. Soare "Contribution à l'étude des réseaux spatiaux planaires  
simples par la méthode des différences finies",  
Rev. Roum. Sci. Techn. - Méc. Appl.,  
Tome 14, No. 5, pp. 949 - 984, Bucarest, 1969.
24. G. W. Hicks "Finite-element elastic buckling analysis"  
Journal of the Structural Division,  
Proceedings of ASCE, Vol. 93, No. ST6,  
December, 1967, pp. 71 - 86.
25. D. T. Wright "Membrane forces and buckling in reticulated shells"  
Journal of the Structural Division, Proceedings of the  
A.S.C.E.  
February 1965, pp. 173 - 201.
26. D. Wright "A continuum analysis for double-layer space frame  
shells"  
Memories, I.A.B.S.E.  
Vol. 26, 1966, pp. 593 - 610.
27. W. A. Little "Reliability of shell buckling predictions"  
Research Monograph No. 25,  
M.I.T. Press, Cambridge, Massachusetts
28. R. J. Aguilar "Snap-through buckling of framed triangulated domes"  
Journal of the Structural Division  
Proceedings of the A.S.C.E.  
April 1967, pp. 301 - 317.

29. K. P. Buchert "Effect of edge conditions on buckling of stiffened framed shells"  
Engineering Experiment Station Bulletin Series No. 65  
University of Missouri, Columbia, October, 1967.
30. J. O. Crooker and K. P. Buchert "Reticulated space structures"  
Journal of the Structural Division  
Proceedings of the A.S.C.E.  
March 1970, pp. 687 - 699.
31. W. D. Whetstone & C. E. Jones "Vibrational characteristics of linear space frames"  
Journal of the Structural Division,  
Proceedings of the A.S.C.E.,  
October 1969, pp. 2077 - 2091.



Fig. 1. Two-way latticed grid over the McCormick Plate in Chicago



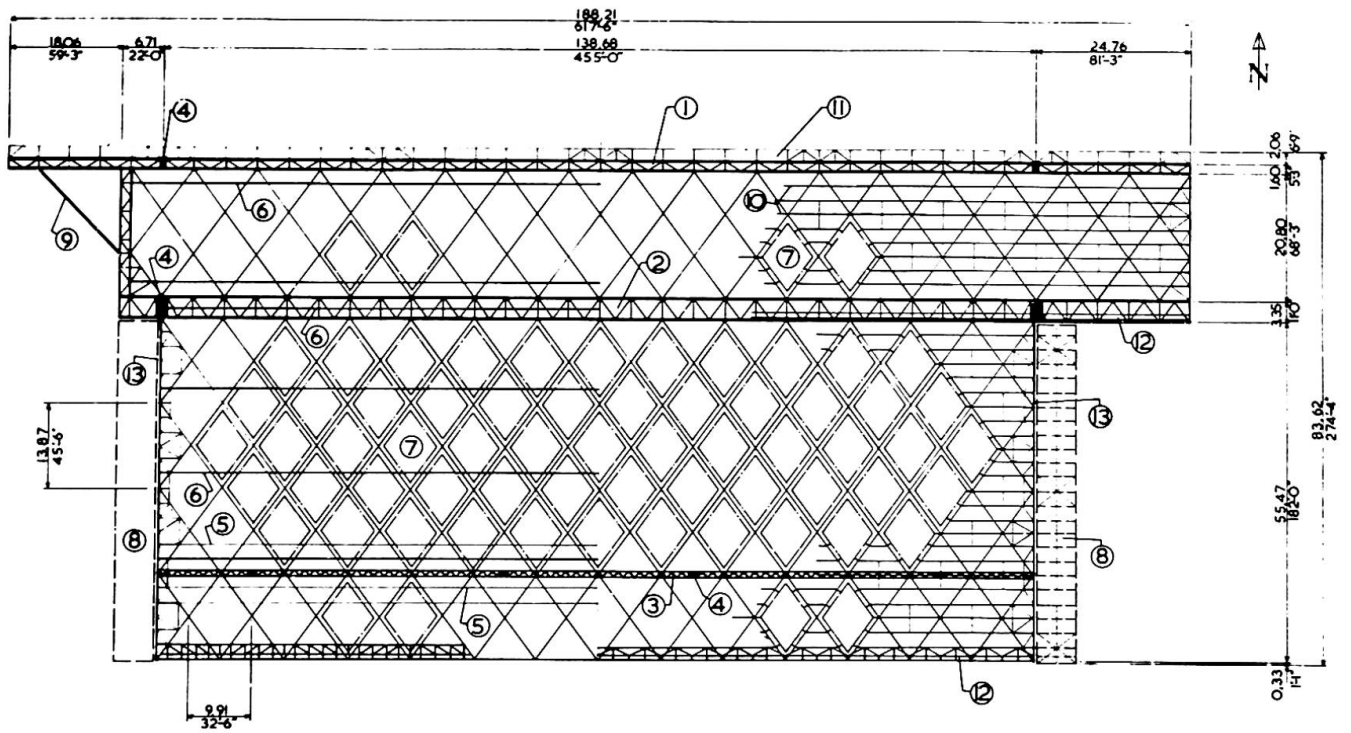


Fig. 2. Plan of the diagonal grid covering the hangar at London Airport

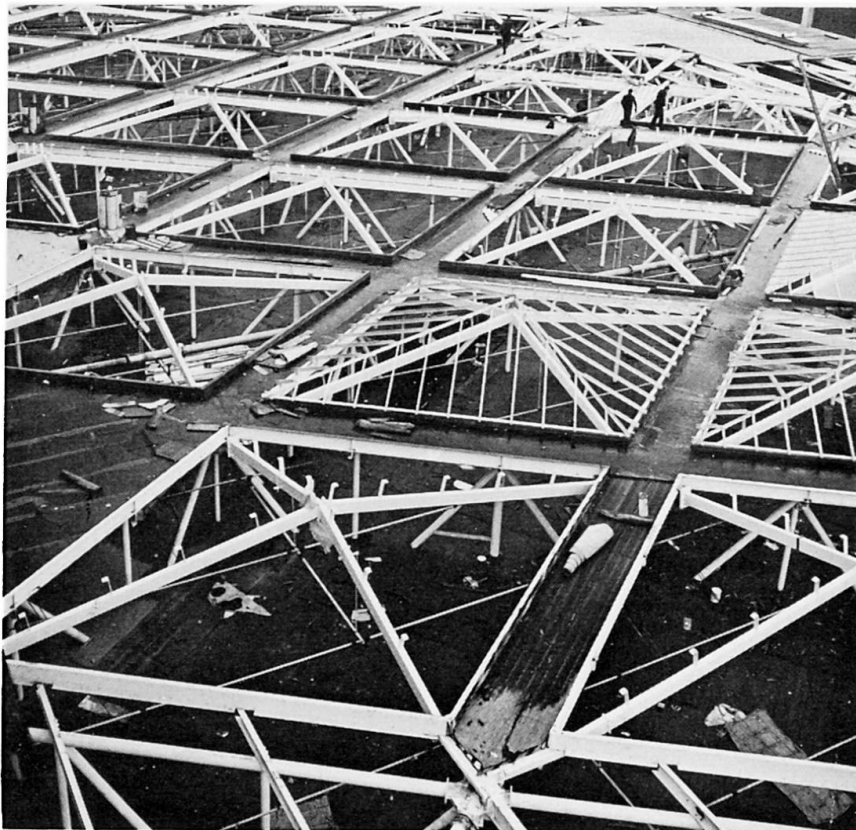


Fig. 3. The external view of the hangar

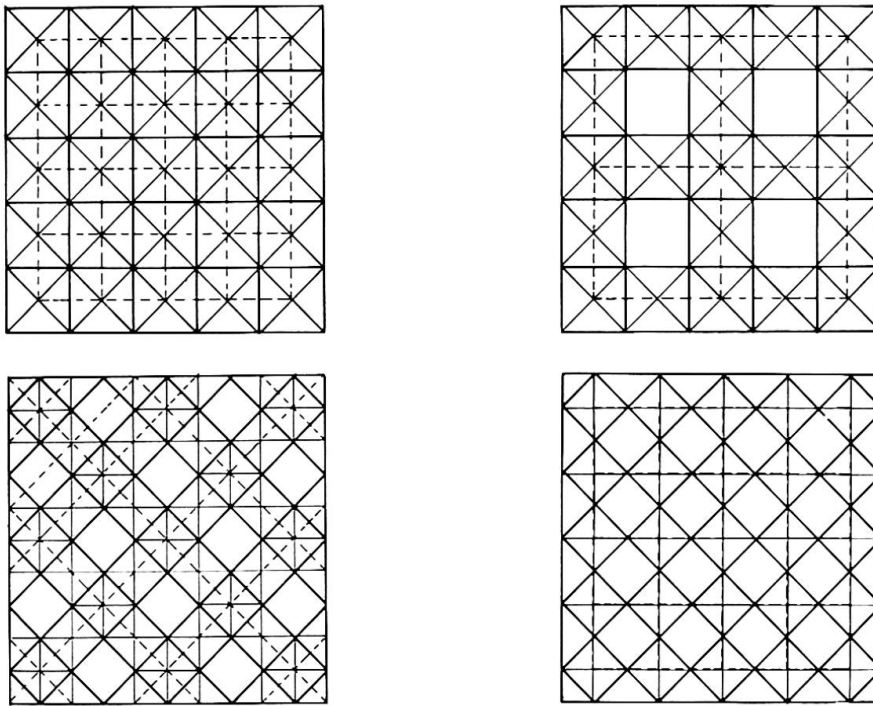
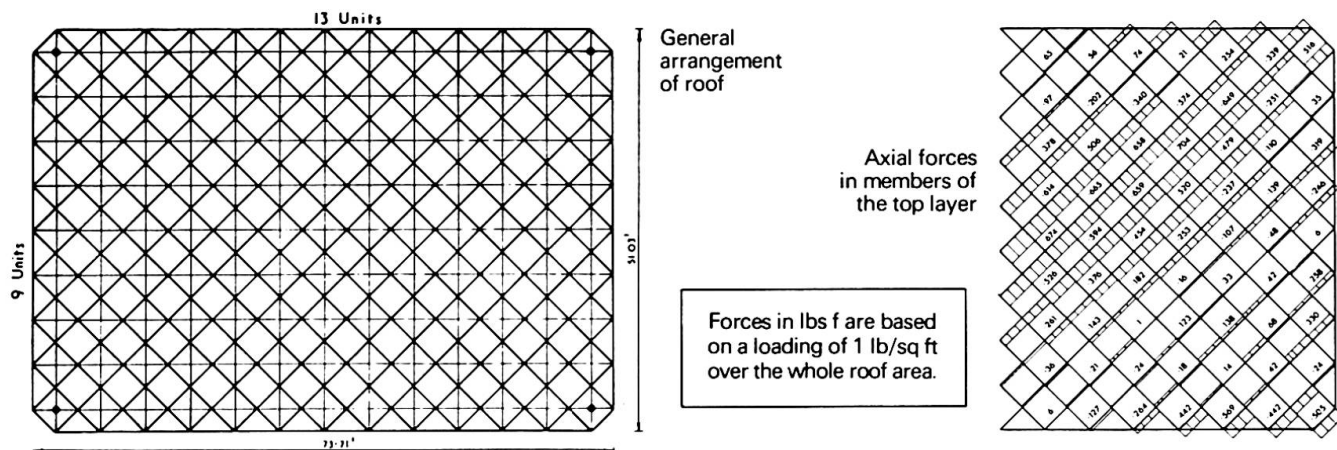


Fig. 4. Evolution of two way double-layer grids



Analysis of UNIBAT grid supported only at the 4 corners for training area roof - Northern Gas Board

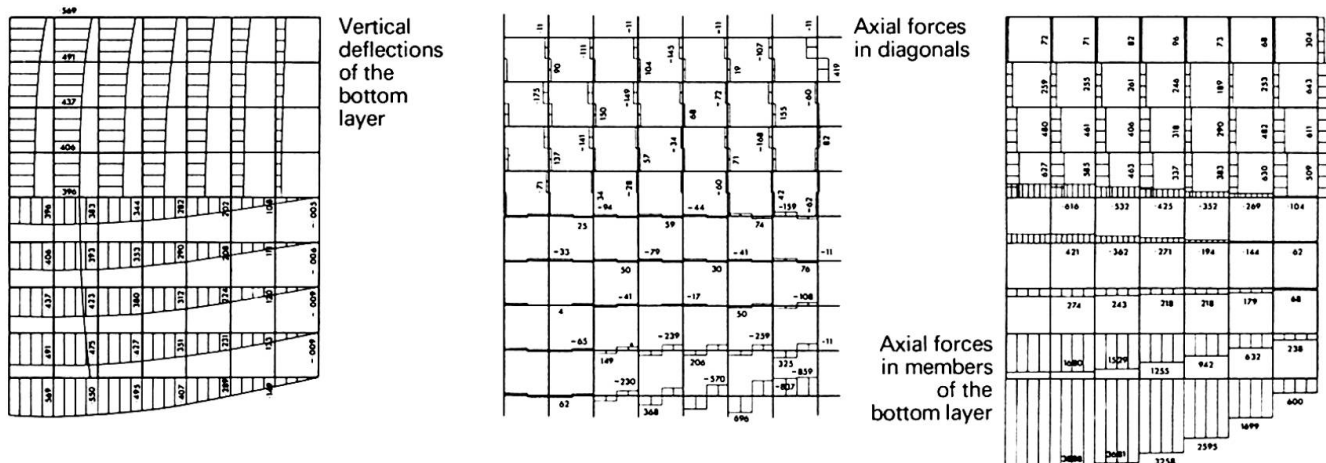


Fig. 5. Stress distribution in a two-way double-layer grid supported only at four corners

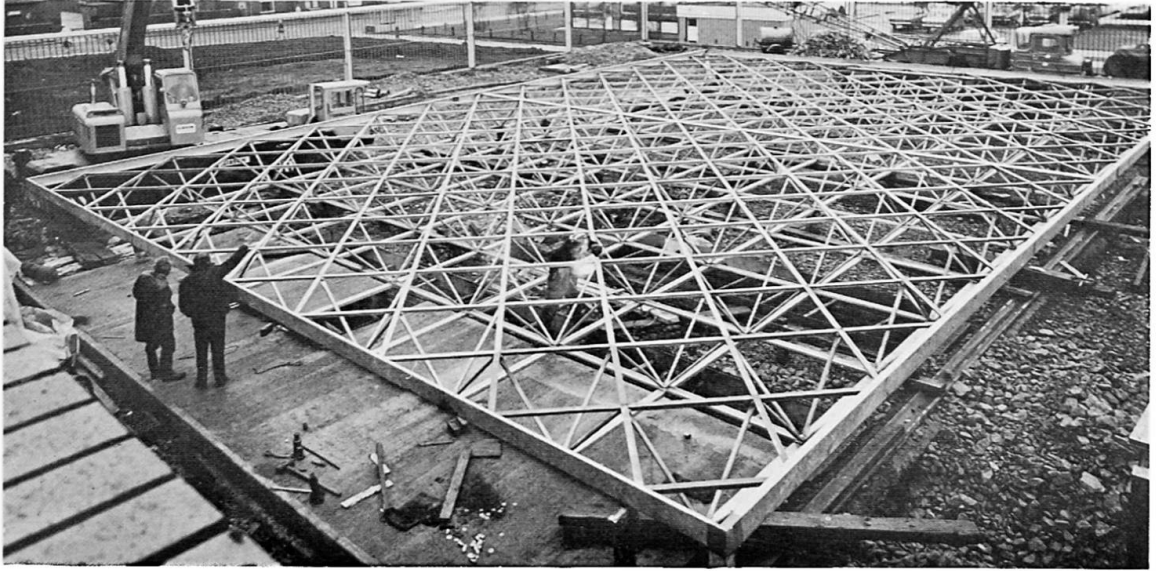


Fig. 6. Erection of the double-layer grid at Killingworth

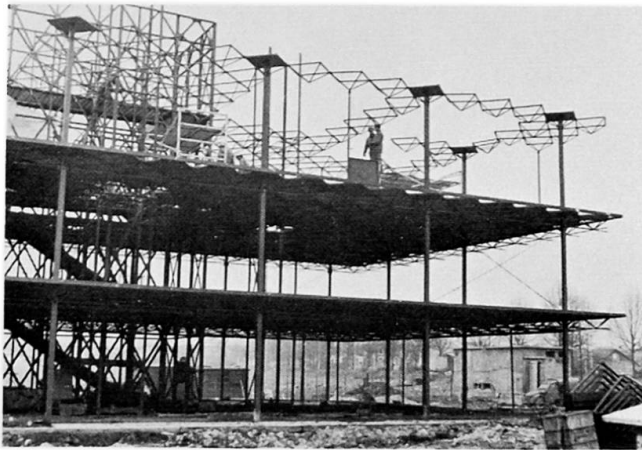


Fig. 7. Construction of a multi storey building in France using the Unibat system



Fig. 8. The external view of seven barrel vaults built for the Port of London Authority

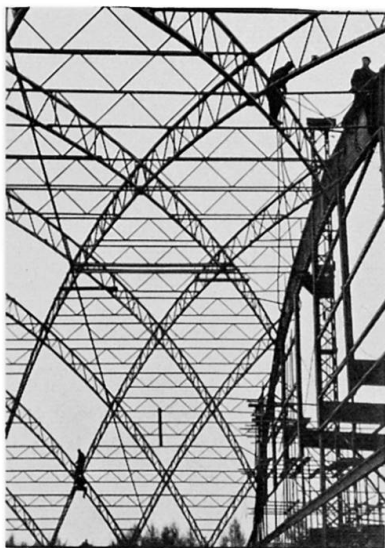


Fig. 9. Erection of a braced barrel vault at Kladno, Czechoslovakia



Fig. 10. Geodesic steel dome erected for the American Pavilion at Expo '67

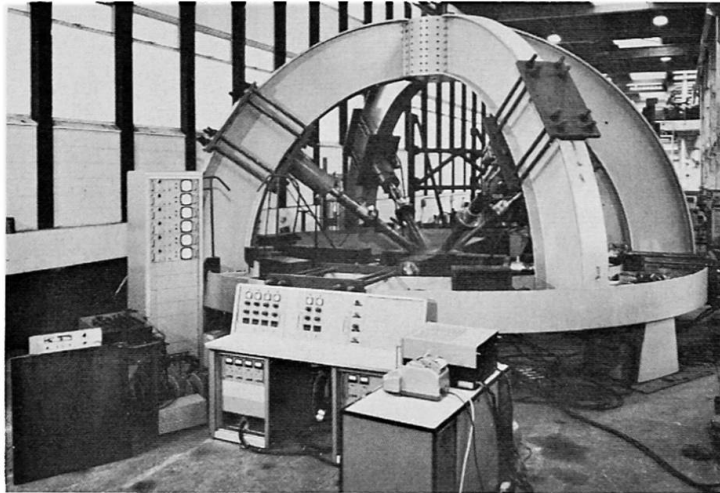


Fig. 11. Rig for testing joints in space structures



Fig. 12. Space grid roof over the Olympic Games Sports Palace in Mexico City



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