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Discussion of Structural Lattices with Particular Reference to the Paper by S. Shore and B. Chandari, entitled "Free Vibrations of Cable Networks"

Discussion sur les treillages structurels, compte tenu en particulier de la contribution de S. Shore et B. Chandari, intitulée "Free Vibrations of Cable Networks"

Diskussion der Gitterwerkkonstruktionen unter besonderer Berücksichtigung des Beitrages von S. Shore und B. Chandari, betitelt "Free Vibrations of Cable Networks"

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I have a few comments to make regarding the analysis of 'Structural Lattices' with some reference to cable networks and grids discussed today. Structural systems with repetitive configurations and physical quantities with a definite sequential relation between them at regular intervals can be analysed by the Calculus of Finite Differences (1,2,3). The method, which is not to be confused with the numerical analysis of Finite Difference equations, is also useful in establishing the equivalence of interconnected cables to membranes and grids to plates (4).

A structural net analysed by Dean (5) is shown in Fig. 1.

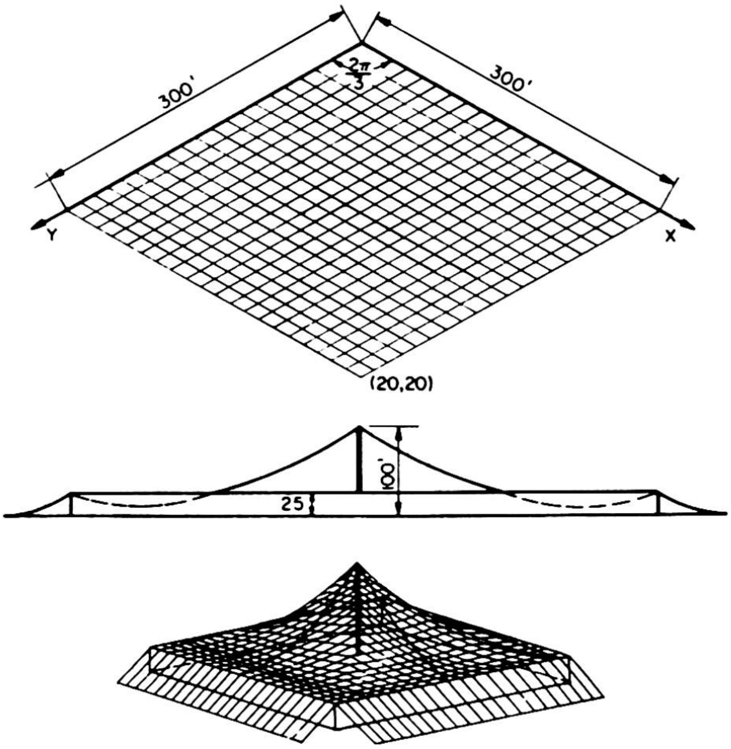

Fig. 1 Doubly threaded structural net with centre pole From Dean (5)

The method can be illustrated by the following two examples:

<u>Example 1</u>

<u>Support Moments of a Continuous Beam (Fig. 2)</u>

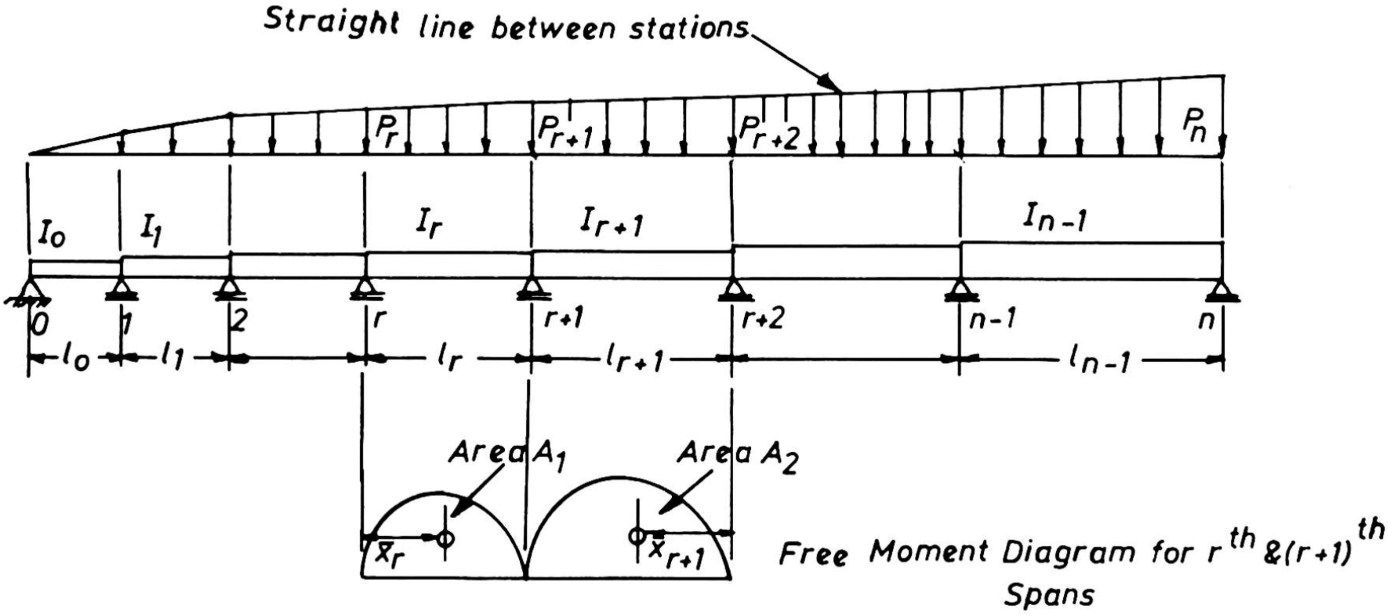

Fig. 2

The general three-moment equation for the rth and (r+1)th spans is

$$M_{r+2}\left(\frac{\ell_{r+1}}{I_{r+1}}\right) + 2M_{r+1}\left(\frac{\ell_{r+1}}{I_{r+1}} + \frac{\ell_r}{I_r}\right) + M_r\left(\frac{\ell_r}{I_r}\right)$$

$$= -\frac{6\,A_r\,\bar{x}_r}{\ell_r\,I_r} - \frac{6\,A_{r+1}\,\bar{x}_{r+1}}{\ell_{r+1}\,I_{r+1}} \qquad (1)$$

Assuming that (i) $\frac{\ell_r}{I_r} = k_1$, a constant, $r = 0, 1, 2 \ldots n-1$

(ii) $\frac{\ell_r}{\ell_{r-1}} = k_2$, a constant, $r = 1, 2, \ldots n-1$

and (iii) $P_r = P_n(\frac{r}{n})$, $r = 0, 1, 2, \ldots. n$

Eqn. (1) reduces to

$$M_{r+2} + 4M_{r+1} + M_r = \frac{1}{k_1\,k_2\,I_r}\left(\frac{-6\,A_r\,\bar{x}_r\,k_2}{\ell_r} - \frac{6\,A_{r+1}\,\bar{x}_{r+1}}{\ell_{r+1}}\right) \qquad (2)$$

The above can be rewritten in terms of ℓ_0 and I_0 as

$$(E^2 + 4E + 1)\,M_r = \frac{-P_n\,\ell_0^3\,k_2^{2r}}{60_n\,I_o\,k_1}\left[15r(1+k_2^2) + (22\,k_2^2 + 8)\right] \qquad (3)$$

Solving the above difference equation and applying the boundary conditions $M_1 = M_n = 0$ gives

$$M_r = C\left[D\beta^r + \left(\frac{\beta^{n+r} - \beta^{n-r}}{\beta^{2n} - 1}\right)(G - D\beta^n) - k_2^{2r}\left\{15r(1+k_2^2)+D\right\}\right] \qquad (4)$$

where

$$C = (P\,\ell_0^3) / \left[60n\ I_0\ k_1\ (1 + 4k_2^2 + k_2^4)\right]$$

$$D = (-8k_2^6 + 6\ k_2^4 - 6\ k_2 + 8) \ / \ (1 + 4k_2^2 + k_2^4)$$

$$G = k_2^{2n}\ \left[15n(1 + k_2^2) + D\right] \quad \text{and}$$

$\beta = (-2 + \sqrt{3})$, one of the roots of the auxiliary equation $(a^2 + 4a + 1) = 0$

Taking $k_2 = 1$ Eqn. (4) reduces to the form

$$M_r = \frac{P\,\ell^2}{12}\left[\frac{\beta^{n+r} - \beta^{n-r}}{\beta^{2n} - 1} - \frac{r}{n}\right] \qquad (5)$$

Example II

Frequency Analysis of a Grid (Fig. 3)
from Ellington and McCallion (6)

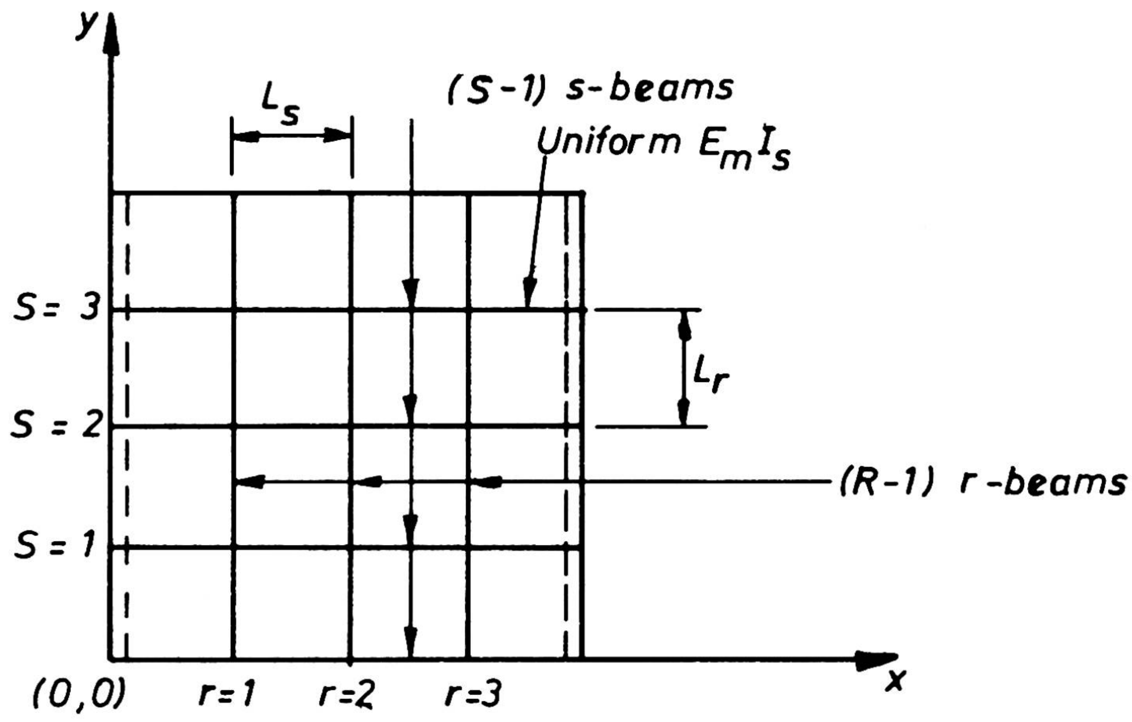

Fig. 3

Considering the grid shown, the two moment equations in the orthogonal directions and the shear equation at the node give the three fundamental equations as follows:-

r direction

$$M_{r+2,s} + 4M_{r+1,s} + M_{r,s} = \frac{6\ E_m I_r}{h_r^2} (y_{r+2,s} - 2y_{r+1,s} + y_{r,s}) \qquad (6)$$

s direction

$$M_{r,s+2} + 4M_{r,s+1} + M_{r,s} = \frac{6\ E_m I_s}{h^2} (y_{r,s+2} + 2y_{r,s+1} + y_{r,s}) \qquad (7)$$

Equilibrium Eqn. at (r+1, s+1)

$$\frac{M_{r+2,\ s+1} - 2M_{r+1,\ s+1} + M_{r,\ s+1}}{h_r} + \frac{M_{r+1,\ s+2} - 2M_{r+1,s+1} + M_{r+1,s}}{h_s}$$
$$= \frac{W}{g}\ \omega^2\ y_{r+1,\ s+1} \qquad (8)$$

Applying the finite difference operators E_r and E_s defined by $E_r{}^x\ y_{r,s} = y_{r+x}$ and $E_s{}^x\ y_{r,s} = y_{r,s} = y_{r,s+x}$ and combining the eqs. (6), (7) and (8) the following basic equation is obtained.

$$\left[\frac{K_r}{2} \left(\frac{\xi_r^2}{\xi_r + 6}\right) + \frac{K_s}{2} \left(\frac{\xi_s^2}{\xi_s + 6}\right) - \frac{W}{g}\ \omega^2\right]\ y = 0 \qquad (9)$$

where $\xi = \frac{(E-1)^2}{E}$ and $K = 12\ E_m I/h^3$

The deflection function satisfying the particular case of simply-supported edge conditions at r = 0 and r = R is

$$y = A\ \mathrm{Sin}\ \left(\frac{p\pi r}{R}\right) e^{\sigma s} \qquad (10)$$

where (R-1) = total number of 'r' beams

A = an arbitrary constant

p = an integer and σ has to be determined from the boundary conditions.

When the two edges are free the frequency equations obtained are as follows:

$$\frac{\text{Cos h } \alpha(Z+1)}{\text{Cos h}\alpha z} - \frac{\text{Cos } \beta(Z+1)}{\text{Cos } \beta Z} = 0 \qquad (11)$$

and

$$\frac{\text{Sin h } \alpha(Z+1)}{\text{Sin h}\alpha Z} - \frac{\text{Sin } \beta(Z+1)}{\text{Sin } \beta Z} = 0 \qquad (12)$$

for symmetric and antisymmetric modes respectively where (2Z + 1) = number of 's' beams

$$\left.\begin{matrix}\text{Cos h}\alpha \\ \text{Cos } \beta\end{matrix}\right\} = (1+\lambda) \pm \sqrt{\lambda^2 + 6\lambda}$$

in which $\lambda = \frac{W\omega^2}{2gK_s} - \frac{K_r}{2K_s}\left[\frac{\{\text{Cos}(p\pi/R) - 1\}^2}{\text{Cos }(p\pi/R) + 2}\right]$

It is hardly necessary to add that the method is very powerful for discrete models.

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SUMMARY

The presentation describes the application of Finite Difference Calculus to obtain analytical solutions for structural systems, like cable networks, involving repetitive configurations and physical quantities with a definite sequential relation between them at regular intervals. Two illustrative examples are presented:
1) Analysis of the support moments of a continuous beam and
2) Frequency analysis of a grid.

RESUME

Ce travail décrit l'application du calcul aux différences finies pour obtenir les solutions analytiques pour des systèmes structurés tels que les résilles de câbles, possédant une configuration qui se répète et des grandeurs physiques liées entre elles par une relation continue définie à intervalles réguliers. Deux exemples explicatifs sont présentés:
1) Analyse des moments d'appui d'une poutre continue, et
2) Analyse des fréquences d'une grille de poutres.

ZUSAMMENFASSUNG

Die vorliegende Arbeit beschreibt die Anwendung der endlichen Differenzenberechnung zur Erzielung analytischer Lösungen für strukturelle Systeme, wie Kabelnetzwerke unter Einschluss wiederholter Konfigurationen und physikalischer Mengen mit bestimmter aufeinanderfolgender Beziehung untereinander in regelmässigen Intervallen. Es werden zwei illustrative Beispiele gezeigt:
1) Analyse der Stützmomente eines durchlaufenden Balkens und
2) Frequenzanalyse eines Netzwerkes.

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