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I b

Instabilité dans le domaine post-critique

Instabilität im überkritischen Bereich

Post-critical Buckling

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DISCUSSION LIBRE • FREIE DISKUSSION • FREE DISCUSSION

Nouvelle théorie et essais sur la résistance ultime des poutres en caisson raidies en acier, soumises à flexion pure

Neue Theorie und Versuche über die Traglast ausgesteifter Stahlkastenträger unter reiner Biegung

New Theory and Test on the Ultimate Strength of Steel Stiffened Box Girders under Pure Bending

R. MAQUOI

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Belgique

CH. MASSONNET

Professeur à l'Université de
Liège

Une série d'accidents spectaculaires (Vienne, 6 novembre 1969, Milford Haven, 2 juin 1970, Melbourne, 15 octobre 1970, Coblenz, 10 novembre 1971) a polarisé l'attention des ingénieurs sur les méthodes de dimensionnement des grands ponts en caisson en acier raidis.

Nous voudrions, dans les quelques minutes dont nous disposons, attirer l'attention des participants au Congrès sur certaines des recherches en cours dans ce domaine.

La plupart des ponts du type considéré ont été dimensionnés, ces dernières années, en se basant sur la théorie linéaire du voilement. Les coefficients de sécurité généralement adoptés étaient les mêmes que pour les âmes des grandes poutres en double té, à savoir

- 1,35 pour le cas I de sollicitation
- 1,25 pour le cas II de sollicitation.

L'estimation de la contrainte critique se faisait souvent en utilisant les abaques contenus dans les deux livres du Professeur KLÖPPEL et de ses collaborateurs, MM. SCHEER et MÖLLER.

Les résultats expérimentaux sur des modèles de poutres en caisson fléchies, présentés par le Professeur P. DUBAS au Colloque de Londres "Design of Plate and Box Girders for Ultimate Strength" (25-26 mars 1971) ont montré que la contrainte moyenne de ruine d'une plaque comprimée uniformément et raidie par des raidisseurs théoriquement strictement rigides ($\gamma = \gamma^*$) était normalement inférieure à sa contrainte critique de voilement linéaire, c'est-à-dire qu'on ne pouvait compter sur aucun effet postcritique, et que les coefficients de sécurité actuels étaient par conséquent absolument insuffisants.

Nous basant sur les travaux théoriques de SKALoud et NOVOTNY, nous avons montré au même Colloque, mon collaborateur R. MAQUOI et moi-même, que ce résultat imprévu de DUBAS aurait pu être prédit théoriquement.

Ceci nous a conduit à développer une théorie non linéaire de la résistance à la ruine des grandes poutres en caisson raidies, qui a paru dans le volume 31-II des Mémoires de l'A.I.P.C. (pp. 91 à 140) et dont nous voudrions vous présenter brièvement ici les principes essentiels:

Nous admettons pour simplifier que nous pouvons nous borner à étudier la résistance limite de la membrure comprimée du caisson (Fig. 1). A la ruine, la partie ABCD de cette membrure limitée entre les deux lignes nodales successives AB et CD se comporte comme une plaque membrane orthotrope à raidisseurs dissymétriques.

Vu le grand nombre de raidisseurs longitudinaux, nous supposons leurs rigidités réparties continûment c'est-à-dire "tartinées"; nous avons développé la théorie non linéaire de ces plaques membranes en généralisant la théorie linéaire de PFLUGER et nous aboutissons au système de deux équations aux dérivées partielles du 4ème ordre non linéaires et couplées ci-dessous:

$$\frac{\phi^{(4)}}{D_x} + 2 \frac{\phi^{(3)}}{\bar{D}} + \frac{\phi^{(2)}}{D_y} =$$

$$(1 - \bar{\nu}^2) [(w_0' + w'')^2 - (w_0'' + w'')(w_0' + w'') - w_0'^2 + w_0'' w_0'],$$

$$\bar{B}_x w^{(4)} + 2\bar{C} \dot{w}^{(3)} + \bar{B}_y w^{(2)} =$$

$$\phi'' (w_0'' + w'') + \phi'' (w_0' + w'') - 2\phi'' (w_0' + w'').$$

Ces équations donnent le déplacement transversal w et la fonction de contrainte d'Airy ϕ .

Nous supposons que le panneau a une déformée initiale sinusoïdale

$$w_0 = f_0 \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}$$

Pour les déplacements additionnels w , nous devons nous contenter d'admettre une expression approchée affine au mode de voilement linéaire (et à w_0) c'est-à-dire

$$w = f \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}.$$

En supposant que les bords latéraux AD et BC du panneau sont libres de contraintes de membrane, à cause de la déformabilité des âmes du caisson perpendiculairement à leur plan et que les bords transversaux AB et CD doivent rester rectilignes, nous pouvons intégrer exactement l'équation donnant la fonction de contrainte ϕ .

La flèche supplémentaire f croît avec l'effort de compression, qui n'est pas réparti uniformément et on obtient la loi de croissance de f en employant la méthode de GALERKIN.

Finalement, nous admettons, pour ne pas devoir nous livrer à une analyse élasto-plastique extrêmement complexe, que la ruine de la plaque se produit quand la contrainte de compression membranaire moyenne $\bar{\sigma}_x$ le long des bords non chargés AD et BC atteint la limite élastique du métal. Cette condition nous permet d'établir la relation non linéaire donnant la flèche additionnelle f de la plaque en fonction de sa flèche initiale f_0 et des autres données du problème f étant connu, il est aisé d'établir l'expression du rendement d'ensemble de la plaque à raidisseurs "tartinés".

$$\rho_t = \frac{\text{Effort réel à la ruine}}{\text{Effort plastique maximum } \Omega R_e}$$

où R_e est la limite élastique.

Nous trouvons pour ρ_t une expression relativement simple.

Il faut cependant tenir compte de ce que la plaque réelle a un nombre fini de raidisseurs et que la tôle se voile dans tous les champs partiels compris entre deux raidisseurs successifs. Cela donne au diagramme des contraintes de compression à la ruine une allure festonnée. Il est possible de tenir compte de la perte de rendement supplémentaire correspondante en introduisant la notion de rendement local de la tôle

$$\rho_l = \frac{\text{Effort réel}}{\text{Effort maximum}} .$$

L'expression de ρ_l découle directement de l'expression de la largeur effective de cette tôle, pour laquelle nous avons adopté une formule déduite de nombreux essais de compression jusqu'à la ruine sur des tôles raidies qui est très voisine de celle de WINTER.

Le rendement global de la tôle raidie à raidisseurs discrets est donné par l'expression

$$\rho_g = \rho_t \cdot \rho_l .$$

Un pont en caisson devrait se dimensionner, selon nous, en exprimant que, sous la charge de ruine, la contrainte moyenne de compression dans le panneau ABCD ne dépasse pas la contrainte moyenne de ruine

$$R_r = \rho_g R_e$$

Notre théorie explique l'accident survenu au Praterbrücke à Vienne et est en assez bon accord avec les résultats expérimentaux du Professeur DUBAS.

Cependant, afin de tester cette théorie, nous entreprenons actuellement dans mon laboratoire à Liège une série d'essais jusqu'à la ruine sur des modèles de poutres en caisson munies de 7 raidisseurs longitudinaux. Les figures 2 et 3 donnent quelques précisions à ce sujet.

En conclusion, nous voudrions insister sur le fait que calculer les raidisseurs de la plaque comme des barres comprimées indépendantes, comme plusieurs chercheurs l'ont proposé tout récemment, reviendrait à ignorer l'effet stabilisant des contraintes de membrane et conduirait, selon nous, à un gaspillage d'acier excessif.

Ib

Stresses in Thin-Cylindrical Webs of Curved Plate Girders

Contraintes dans l'âme mince de poutres courbées cylindriquement

Spannungen in dünnen, zylindrischen Stegen von gekrümmten Vollwandträgern

RYSZARD DABROWSKI

Poland

See Colloquium London 1971, page 337, Volume 11 of the Reports of the Working Commissions
Voir Colloque Londres, 1971, page 337, Volume 11 des Rapports des Commissions de Travail
Siehe Seminar London 1971, Seite 337, Band 11 der Berichte der Arbeitskommissionen

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Failure Load and Effective Width of Compressed Steel Plates with Initial Stresses and Initial Deflections

Bruchlast und tatsächliche Breite gedrückter Stahlplatten mit Eigen- spannungen und Anfangsdurchbiegung

HENRIK NYLANDER
Sweden

The author has studied the plate buckling in the overcritical range using a model of calculation, which enables to consider the initial stresses and the initial deflection in a relatively simple manner.

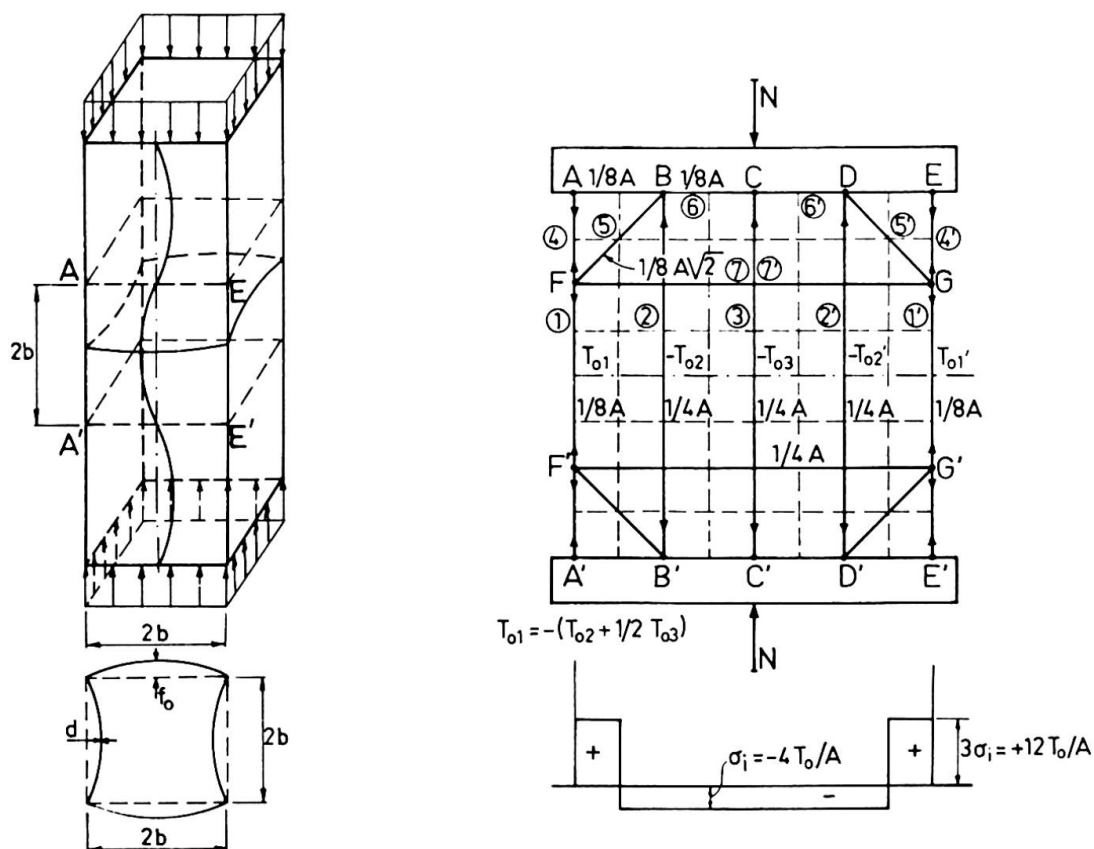


Fig. 1 a and 1 b Model of Calculation

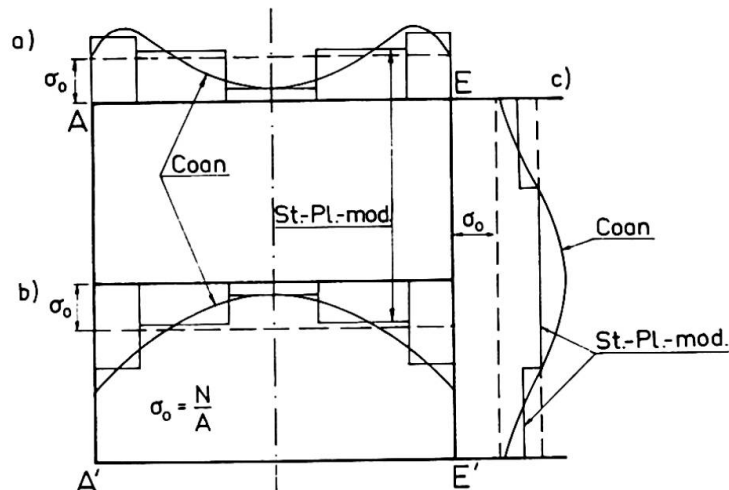


Fig. 1 c
Distribution of compressive stresses in the direction of the load N at the edge A-E (a)) at the line of symmetry (b)) and at the edge E-E' (c)). Comparison with solution by Coan $\sigma_o/\sigma_{el} = 1,74$.

The investigation is part of a research project regarding the carrying capacity of welded hollow columns, built up by thin plates. The project is carried out at the Department of Building Statics and Structural Engineering at the Royal Institute of Technology, Stockholm and at the Swedish Institute of Steel Construction, Stockholm.

The model of calculation consists of a plate acting only in plate bending and of the strips 1-7 and 1'-7', taking the membrane stresses only. (Fig. 1) The strips are connected to the plate at the points A-G and A'-G'. The areas of the strips are shown in Fig. 1, where $A = 2b \cdot d$ is equal to the area of the cross section of the plate. The normal forces in the strips are caused by 1) the initial stresses, 2) the normal force N in the plane of the plate, which gives forces in the different strips in proportion to their areas and 3) of forces which are caused by the changes of length of the strips as the bending deformation of the strips follows the bending deformation of the plate.

The mathematical treatment is omitted in this connection. It is the author's intention to publish the theory and the rather comprehensive results in a near future.

A treatment of the problem starting from the fundamental Eq. by von Kármán and Marguerre adjusted to take into account the influence of the initial stresses is in the author's opinion very difficult. In fig. 1 c a comparison is made with a solution by Coan [1] for a case where $\sigma_i = 0$. The membrane stresses in the direction of the compressive load N at the supports at the middle of the plate and along a free edge are considered. It is seen from the Figure that it is a good agreement between Coan's results and the results from the calculations for the model in Fig. 1 b both regarding the maximum values and the distributions of stresses.

It is hardly possible to precise adequate criteria of failure for the highly statically indeterminate system in question where the elastoplastic state of stresses must be considered. The author has instead of trying to give a complex theory started from a relatively detailed study of the stresses in different parts of the elastic plate caused by bending and torsional moments

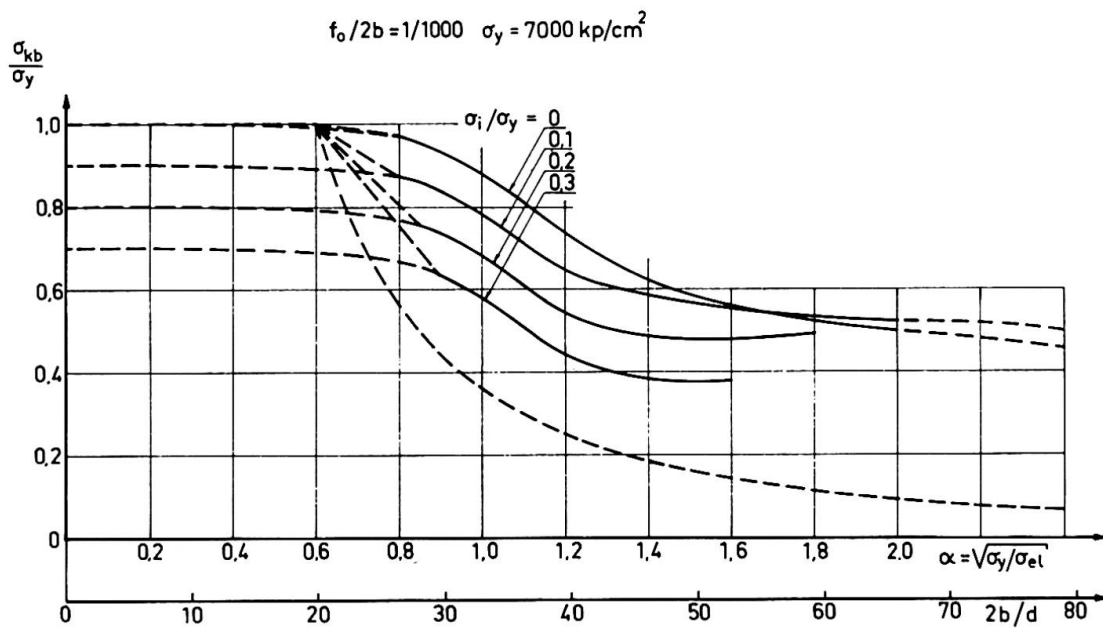
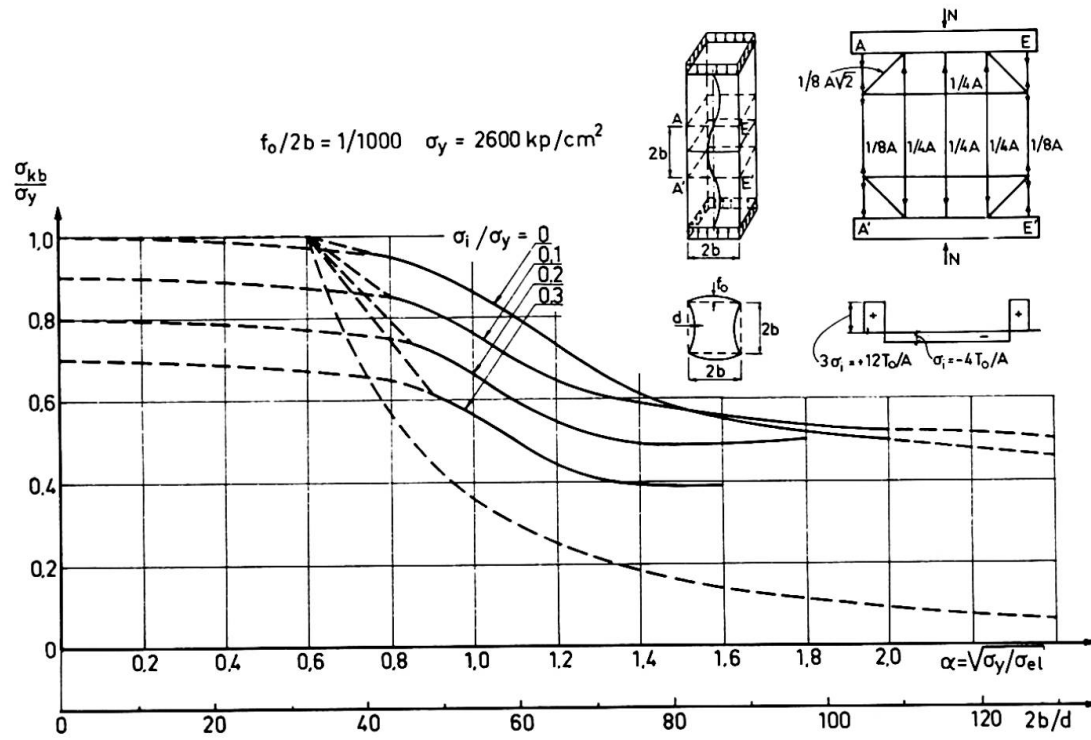


Fig. 2 a and 2 b. Values of $\sigma_{kb} = N/2bd$ at buckling failure divided with σ_y for different σ_i/σ_y as function of $\alpha = \sqrt{\sigma_y/\sigma_{el}}$

a) $\sigma_y = 2600 \text{ kp/cm}^2$

b) $\sigma_y = 7000 \text{ kp/cm}^2$

and the normal forces. Then that load has been determined at which total yielding (yield stress over the whole cross section) will occur at the point considered, if the bending and torsional moments as well as the normal forces have the values calculated from the theory of elasticity. At the judgement of the failure load the following points have been considered:

1. The midpoint of strip (A-A', Fig. 1 a). Yielding due to normal force (compression) in the direction of the load N.
2. The midpoint of the strip 2 (B-B', Fig. 1 a). Yielding due to bending moment and normal force in the direction of the compressive load N.
3. The centre of the plate (midpoint of strip 3). Yielding due to bending moment and normal force in the direction of the compressive load N.
4. The corner points. Yielding due to torsional moment and normal force in the direction of the load N.

The results are given in Fig. 2 for two values of the yield stress: 2 600 kp/cm² and 7 000 kp/cm² and for the ratio initial deflection over plate width $f_0/2b = 1/1\,000$. For most of the calculated points of the diagrams the alternatives 2) and 3) above were most dangerous and the failure loads were for these points calculated as the average values of the failure loads for the alternatives 2) and 3).

For $\sigma_i/\sigma_y = 0$ and $\alpha \geq 1,2 \leq 2,0$ and for $\sigma_i/\sigma_y = 0,1$ and $\alpha \geq 1,8$ alternative 4) was most dangerous. For $\sigma_i/\sigma_y = 0$ and $\alpha > 2,0$ alternative 1) was most dangerous.

The effective width b_e (see Fig. 3) is of importance for the column buckling. Calculated values at failure load are given in Fig. 3 for different σ_i/σ_y ($f_0/2b = 1/1\,000$; $\sigma_y = 2\,600$ kp/cm²). It is seen from the Figure that the initial stresses highly affect the values of b_e/b .

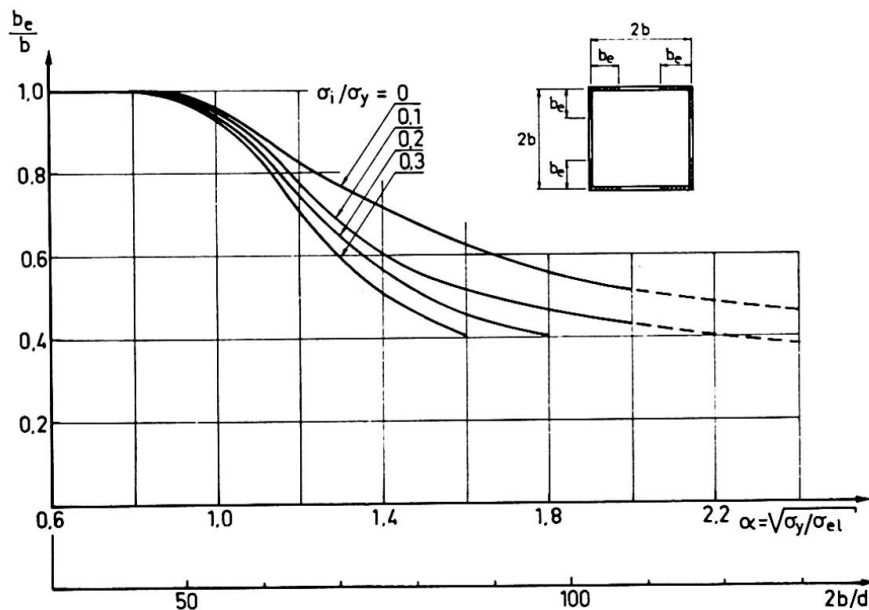


Fig. 3 Ratio b_e/b for the failure load as a function of α ($\sigma_y = 2\,600$ kp/cm²)

It is seen from Fig. 2 that the initial stresses have a very important negative influence on the critical buckling stresses especially for $0,8 < \alpha < 1,6$. The initial stresses have a negative effect on the effective width b_e (see Fig. 3). Both these effects reduce the column buckling load. The applied distribution of the initial stresses is unfavourable. Calculations of a case where $\sigma_i = 0$ in the strip 3 have given higher failure loads. It is therefore a need of studying the influence of the fabrication methods on the distribution of initial stresses. Finally the author among investigations will remind of those by Nishino, Ueda, Tall [2]; Dwight, Moxham [3] and Dwight, Ractcliffe [4] of buckling of welded columns of hollow sections, where it was pointed out that the initial stresses have a large unfavourable effect on the failure load.

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SUMMARY

The behaviour of compressed steel plates in the overcritical range is studied. A simplified model of calculation, Fig. 1, which enables to consider initial stresses and deflections is used. The results are intended to serve as a basis for design rules. It is shown that the initial stresses reduce the failure load especially for the dimensions corresponding to $0,8 < \alpha < 1,6$, Fig. 2. Furthermore the effective width is reduced by the initial stresses, Fig. 3. Here omitted results for other distributions of the initial stresses are more favourable.

RESUME

L'auteur étudie ici le comportement de plaques en acier comprimées, dans le domaine post-critique. Il emploie un modèle mathématique simplifié, fig. 1, qui permet de tenir compte des tensions et des déviations initiales. Les résultats sont destinés à servir de base pour déterminer des règles de dimensionnement. On remarque que les tensions initiales réduisent la charge ultime, spécialement pour les dimensions correspondant à $0,8 < \alpha < 1,6$, fig. 2. De plus la largeur effective est réduite par les tensions initiales, fig. 3. Les résultats pour d'autres distributions des tensions initiales, qui ne sont pas mentionnées ici, sont plus favorables.

ZUSAMMENFASSUNG

Es wird das Verhalten gedrückter Stahlplatten im überkritischen Bereich untersucht, dabei wird ein vereinfachtes Berechnungsmodell (Fig. 1) verwendet, das die Berücksichtigung der Eigenspannungen und Anfangsdurchbiegungen erlaubt. Die Ergebnisse sind als Basis für die Bemessung gedacht. Es wird gezeigt, dass die Eigenspannungen die Bruchlast reduzieren, insbesondere für Dimensionen entsprechend $0,8 < a < 1,6$ (Fig. 2). Zudem wird die tatsächliche Breite infolge der Eigenspannungen vermindert (Fig. 3). Die hier ausgelassenen Ergebnisse für andere Eigenspannungsverteilungen sind günstiger als die gezeigten.

Contribution to the Free Discussion Regarding the Paper "Interaction of Postcritical Plate Buckling with Overall Column Buckling of Thin-Walled Members" by J. De Wolf, T. Pekoz and G. Winter

Contribution à la discussion libre de l'article "Interaction of Postcritical Plate Buckling with Overall Column Buckling of Thin-Walled Members" par J. De Wolf, T. Pekoz et G. Winter

Diskussion bezüglich des Beitrages "Interaction of Postcritical Plate Buckling with Overall Column Buckling of Thin-Walled Members" von J. De Wolf, T. Pekoz und G. Winter

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To begin with, I would like to congratulate Professor Winter and his co-workers on obtaining very interesting and valuable results regarding the limiting state of thin-walled columns.

I would also like to take this opportunity to mention that our team at the Institute of Theoretical and Applied Mechanics in Prague has been concerned with the interaction of overall column buckling with plate buckling for some 14 years. Several investigations, both theoretical and experimental, have been carried out. For example, a few years ago, twenty eight thin-walled columns were tested, with the slenderness ratio of the column and the width-to-thickness ratio of its plate elements being varied in a way that both the column and plate buckling could be studied.

It is beyond the scope of this contribution to the Free Discussion to describe all our results and observations; therefore, I have to limit myself to a few conclusions, which may be of some interest in connection with Professor Winter's

paper.

To start with, it is, perhaps, worth mentioning that in all our tests a pronounced interaction of column buckling with the buckling of its plate elements was observed. It follows from this observation that it is not possible to separate (as is frequently done when following the currently held design concept) the behaviour of the column as a whole from that of its plate elements. A steel column is always a system of plates, the overall deformation of which and the local one (i.e. buckling of plate elements) are interconnected.

Further, I would like to draw attention to the fact that the performance of thin-walled columns is considerably affected by unavoidable initial irregularities (like an initial curvature, residual stresses, etc.).

For example, in the case of a centrally loaded column, the initial irregularities make the column deflect from the very beginning of loading. As a result of the flexure of the column as a whole, the loads of the plates on the concave side of the deflected column are increased, whereas those acting on the plates on the convex side are reduced. That is why, in the most loaded section of the bar, the load acting on the concave side plate can be substantially (in the case of very slender bars even several times) larger than that which acts on the plate element on the convex side of the deflected column (Fig. 1). Furthermore, it is of importance that this increased load of the plate on the concave side is, at the most stressed section of the column, frequently substantially greater than the average value $\sigma = P/A$, which is considered in the design if an "ideal" column without initial deviations is assumed.

The influence of the initial irregularities upon the

loading of the plate elements is reflected in the character of the waving of these elements (Fig. 2). That is why plate buckling is more pronounced on the concave side of the deflected bar than it is on the convex one. Moreover, in view of the fact that in the case of the plate on the concave side the load acting on it is larger in the central section than at the boundaries of the bar, the buckled pattern is more pronounced in the middle of the column than at its ends. On the other hand, the plate on the convex side is less loaded in the central part of the bar than at the boundary sections; therefore, the wave pattern tends to be less pronounced in the middle of the column than in the boundary zones. The aforesaid analysis shows that the currently held model of behaviour, according to which the plate elements of a centrally compressed bar are uniformly loaded and, consequently, uniformly waved, is not compatible with the behaviour of ordinary thin-walled steel columns.

The effect of initial deviations upon the limiting state and the ultimate load is shown in Fig. 3, where the experimental load-carrying capacities σ_{cr}^{exp} of one test series are plotted in comparison with a/ the critical load σ_{cr} of the bar as a whole, evaluated regardless of plate buckling, b/ the critical load σ_{cr}^p for the buckling of the weakest plate element, and c/ the critical load σ_{cr}^e determined for the stability of the column as a whole with due regard to plate buckling and, therefore, for an effective cross section. In this case, the effective widths of the plate elements were determined by using Winters' formula.

An inspection of the figure indicates that there is no definite relation between the experimental ultimate load on one side and a/ the critical load σ_{cr}^p of the weakest plate

element and b/ the critical load Q'_{cr} of the column as a whole, on the other. The curve Q'_{cr}^e calculated for the effective cross section is closer to the experimental results than the two aforementioned quantities; however, it does not seem to follow the experimental results either. As it disregards the effect of initial irregularities, it frequently gives values higher than is the actual limiting state of the thin-walled column.

I would like to conclude by making a suggestion to the Working Commission II. I think that the above discussed problems of interaction belong to the most important lines of the present research on the behaviour of steel structures. This problem is being dealt with at several places; for ex. at Cornell, in Cambridge, in Liège, in Darmstadt and in Prague. It may, perhaps, be time to give thought whether it would not be profitable to organize a colloquium with this line of work. This colloquium, which could be organized in a way similar to last year's London colloquium on plate girders, and in which all researchers concerned would take part, could significantly contribute to further progress in the aforementioned field.

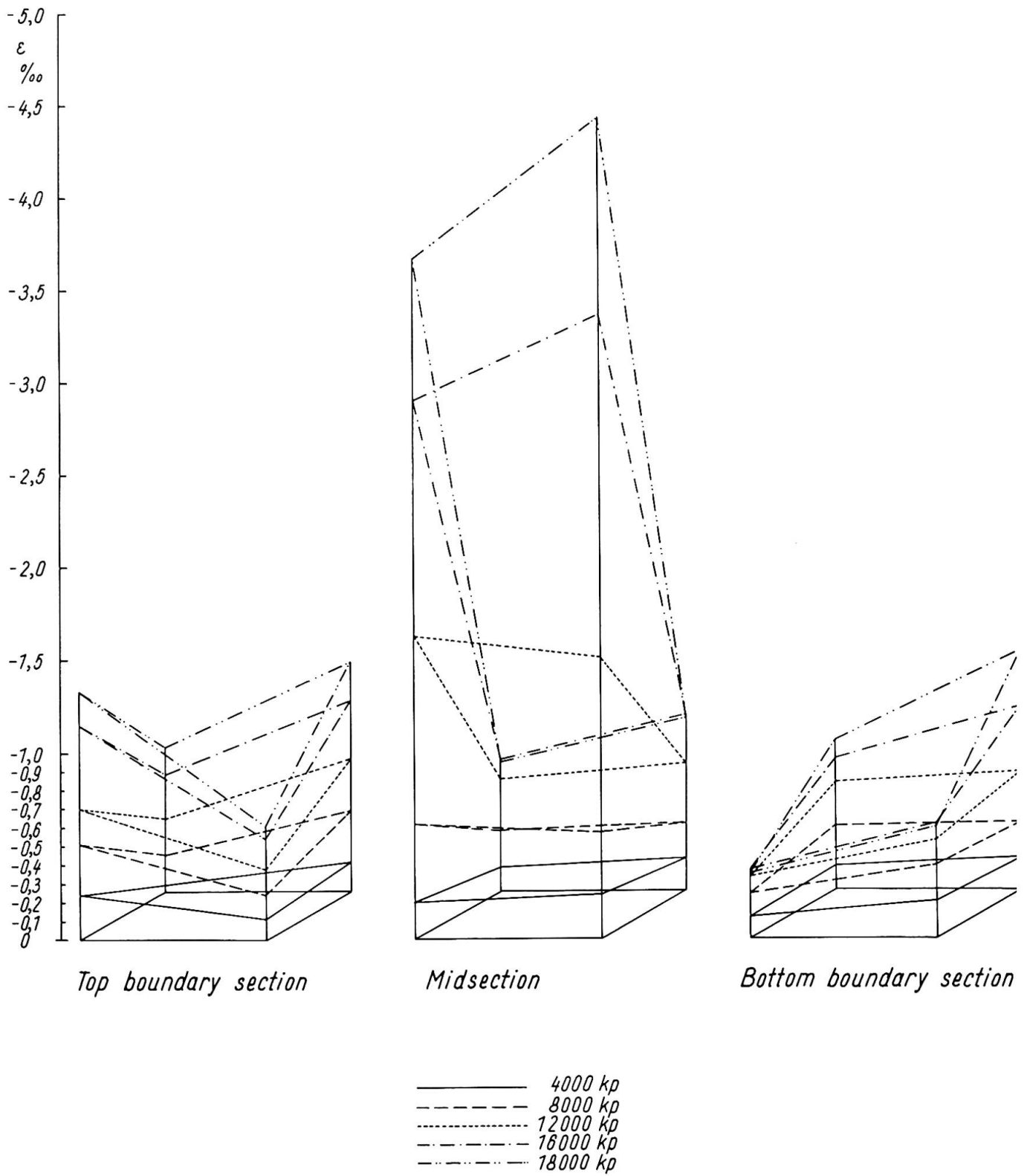


Fig. 1

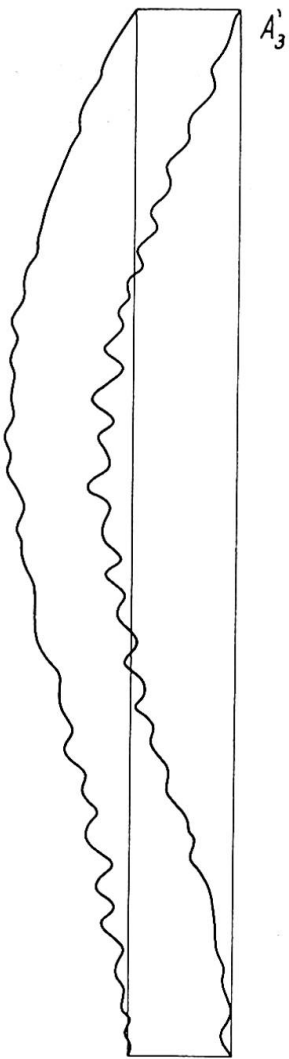


Fig. 2

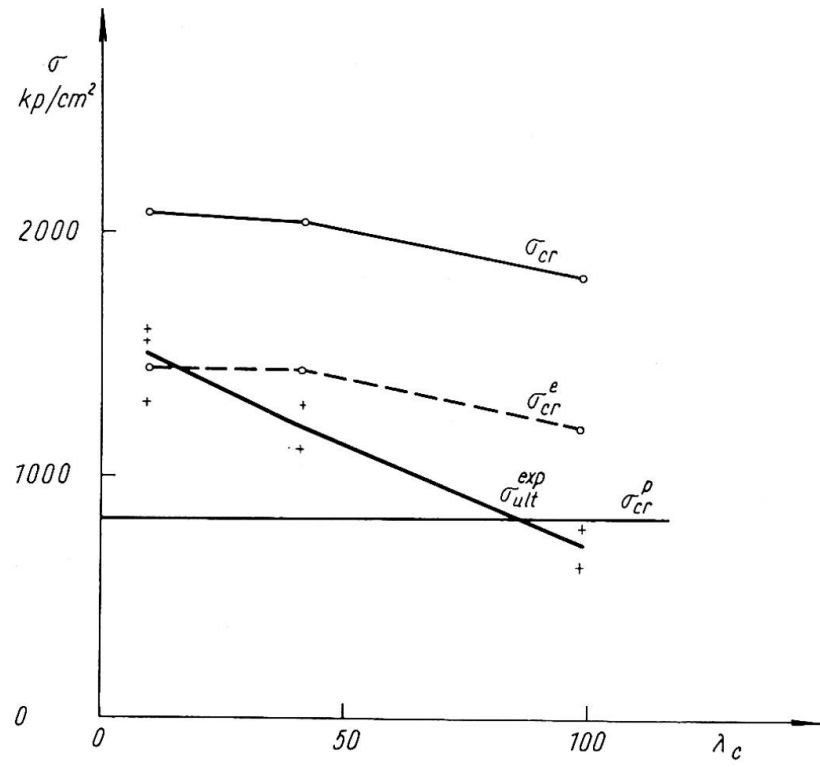


Fig. 3

Application d'une méthode de calcul par éléments finis à l'étude du comportement des plaques minces raidies dans le domaine des grands déplacements

Anwendung einer Berechnungsmethode mittels endlicher Elemente zur Untersuchung des Verhaltens dünner ausgesteifter Platten im Bereich grosser Verschiebungen

Application of a Method of Calculation by Finite Elements for the Study of the Behaviour of Thin-Walled Stiffened Plates in the Range of Large Displacements

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Ce mémoire expose une méthode d'analyse approchée, par éléments finis, du champ de déplacements des plaques minces raidies très déformables et présente une étude comparée du comportement d'une structure par la simulation sur ordinateur et par l'expérimentation sur modèle.

Cette méthode trouve, en particulier, son application dans l'étude du comportement non linéaire, correspondant au domaine dit "post-critique", des panneaux plans ou à faible courbure constituant les parois des structures légères ou des poutres de grande portée.

Le modèle mathématique étudié est défini à partir d'un maillage principal à éléments finis rectangulaires et d'un maillage secondaire à éléments triangulaires (triangles rectangles) utilisable aux frontières du domaine.

Les hypothèses sont exprimées sur le champ des déplacements en introduisant un champ cinématiquement admissible. L'application du principe des travaux virtuels ou du principe de variation des déplacements conduit à satisfaire aux exigences de la compatibilité non seulement aux noeuds du maillage, mais encore le long de l'interface des éléments rectangulaires contigus. La continuité des déformations est assurée dans le plan moyen de la plaque.

Les raidisseurs associés à la plaque mince peuvent être orientés suivant des directions parallèles, orthogonales ou obliques (cas des membrures obliques des poutres de hauteur variable) et disposés symétriquement ou non par rapport au plan moyen de la plaque. Ils sont à parois minces et à section droite ouverte ou fermée. Ils peuvent être traités dans le domaine des déplacements finis.

Nous avons admis pour le matériau une loi de comportement élastique et des déformations infinitésimales.

L'introduction d'une déformée initiale de faible courbure aussi bien en ce qui concerne la plaque que les raidisseurs et d'un état de contraintes propres dans la plaque permet de compléter la prise en compte des données initiales dans le but d'une bonne simulation du comportement réel de la structure.

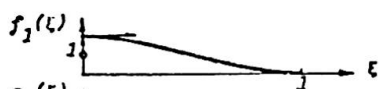
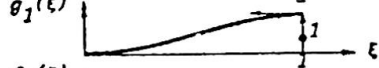
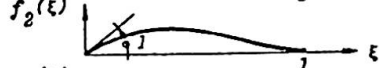
Le champ de déplacement s'établit sous forme d'une combinaison linéaire de fonctions dont les paramètres correspondent respectivement aux degrés de liberté du modèle mathématique adopté et définissent les coordonnées du vecteur déplacement. La détermination du vecteur déplacement s'effectue par accroissements tangents ou paraboliques à partir des différents états successifs du modèle en donnant aux charges des petits accroissements. Chaque état du modèle est caractérisé par une matrice de raideur tangente.

Les études de simulation du comportement des structures dans le domaine des déplacements finis portent actuellement sur différents modèles expérimentaux traités au préalable en laboratoires :

1. - une série de panneaux rectangulaires simples, raidis sur leur contour, appuyés suivant les raidisseurs d'extrémité et chargés ponctuellement à mi-portée de la membrure supérieure.
2. - une poutre en I, à âme raidie transversalement, de 8 m de portée.
3. - deux poutres en I, à âme mince raidie transversalement et longitudinalement, de 20 m de portée.

Nous présentons, ci-après, une étude comparée du comportement d'un panneau de poutre en I chargé localement dans son plan, par la simulation sur le modèle mathématique ainsi défini et par la mesure sur un modèle expérimental.

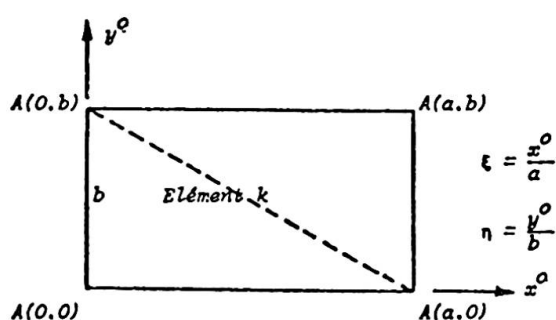
1. - SCHEMA DE DISCRETISATION

$f_1(\xi) = 1 - 3\xi^2 + 2\xi^3$	$(f_1^{IV}(\xi) = 0)$	
$\theta_1(\xi) = 3\xi^2 - 2\xi^3$	$(\theta_1^{IV}(\xi) = 0)$	
$f_2(\xi) = \xi - 2\xi^2 + \xi^3$	$(f_2^{IV}(\xi) = 0)$	
$\theta_2(\xi) = -\xi^2 + \xi^3$	$(\theta_2^{IV}(\xi) = 0)$	

2. - CHAMP DE DEPLACEMENT DANS UN ELEMENT DE PLAQUE

$$u_i^k(x^0, y^0) = \left\{ \phi(x^0, y^0) \right\} \left\{ D_i \right\}^k$$

$(i = 1, 2, 3)$



$\xi = \frac{x^0}{a}$
 $\eta = \frac{y^0}{b}$

$$u_i^k(x^0, y^0) = \left\{ \begin{array}{l} f_1(\xi) \cdot f_1(\eta) \\ a \cdot f_2(\xi) \cdot f_1(\eta) \\ b \cdot f_1(\xi) \cdot f_2(\eta) \\ a \cdot b \cdot f_2(\xi) \cdot f_2(\eta) \\ g_1(\xi) \cdot f_1(\eta) \\ a \cdot g_2(\xi) \cdot f_1(\eta) \\ b \cdot g_1(\xi) \cdot f_2(\eta) \\ a \cdot b \cdot g_2(\xi) \cdot f_2(\eta) \\ g_1(\xi) \cdot g_1(\eta) \\ a \cdot g_2(\xi) \cdot g_1(\eta) \\ b \cdot g_1(\xi) \cdot g_2(\eta) \\ a \cdot b \cdot g_2(\xi) \cdot g_2(\eta) \\ f_1(\xi) \cdot g_1(\eta) \\ a \cdot f_2(\xi) \cdot g_1(\eta) \\ b \cdot f_1(\xi) \cdot g_2(\eta) \\ a \cdot b \cdot f_2(\xi) \cdot g_2(\eta) \end{array} \right\} \left\{ \begin{array}{l} u_i \\ u_i, x^0 \\ u_i, y^0 \\ u_i, x_{ij}^0 \end{array} \right\}_{0,0} \left\{ \begin{array}{l} u_i \\ u_i, x^0 \\ u_i, y^0 \\ u_i, x_{ij}^0 \end{array} \right\}_{a,0} \left\{ \begin{array}{l} u_i \\ u_i, x^0 \\ u_i, y^0 \\ u_i, x_{ij}^0 \end{array} \right\}_{a,b} \left\{ \begin{array}{l} u_i \\ u_i, x^0 \\ u_i, y^0 \\ u_i, x_{ij}^0 \end{array} \right\}_{0,b}$$

(i = 1, 2, 3)

3. - CHAMP DE DEPLACEMENT DANS UN RAIDISSEUR

$$\begin{aligned} \left\{ \Delta_1 \right\}_O &= \begin{bmatrix} u_r(O) \\ u_r, r(O) \end{bmatrix} & \left\{ \Delta_1 \right\}_C &= \begin{bmatrix} u_r(C) \\ u_r, r(C) \end{bmatrix} \\ \left\{ \Delta_2 \right\}_O &= \begin{bmatrix} u_t(O) \\ u_t, r(O) \end{bmatrix} & \left\{ \Delta_2 \right\}_C &= \begin{bmatrix} u_t(C) \\ u_t, r(C) \end{bmatrix} \\ \left\{ \Delta_3 \right\}_O &= \begin{bmatrix} w(O) \\ w, r(O) \end{bmatrix} & \left\{ \Delta_3 \right\}_C &= \begin{bmatrix} w(C) \\ w, r(C) \end{bmatrix} \\ \left\{ \Delta_4 \right\}_O &= \begin{bmatrix} \theta(O) \\ \theta, r(O) \end{bmatrix} & \left\{ \Delta_4 \right\}_C &= \begin{bmatrix} \theta(C) \\ \theta, r(C) \end{bmatrix} \end{aligned}$$

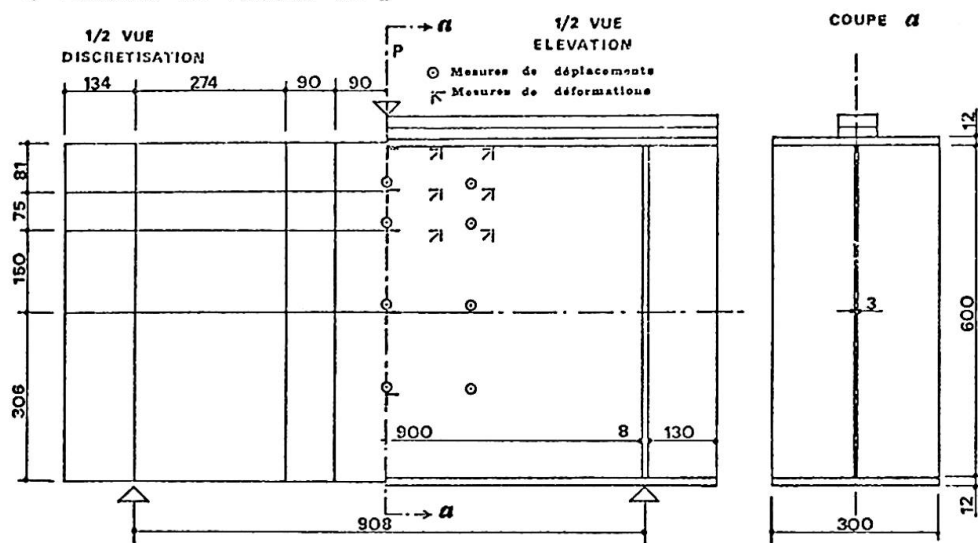
$$\text{Posons : } \left\{ \Delta_i \right\}^r = \begin{bmatrix} \Delta_i O \\ \Delta_i C \end{bmatrix} \quad \text{composantes de déplacement de } A(O) \text{ et } A(C) \text{ dans } (r, t, z)$$

- Champ de déplacement dans le système (r, t, z)

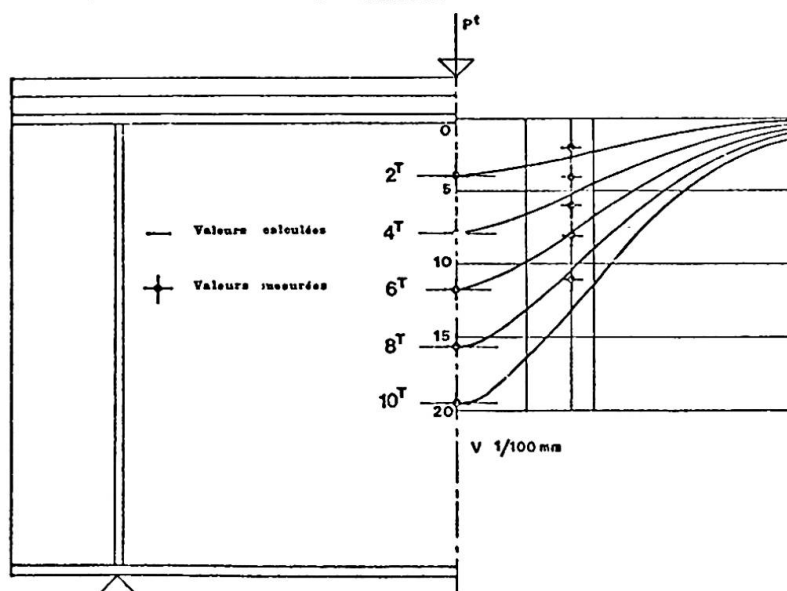
$$u_i^r(r) = \left\{ \rho(r) \right\} \cdot \left\{ \Delta_i \right\}^r \quad (u_i = u_r, u_t, w, \theta)$$

$$\left\{ \rho(r) \right\} = \left[f_1(\xi), c \cdot f_2(\xi), g_1(\xi), c \cdot g_2(\xi) \right] \quad \left(\xi = \frac{r}{\alpha} \right)$$

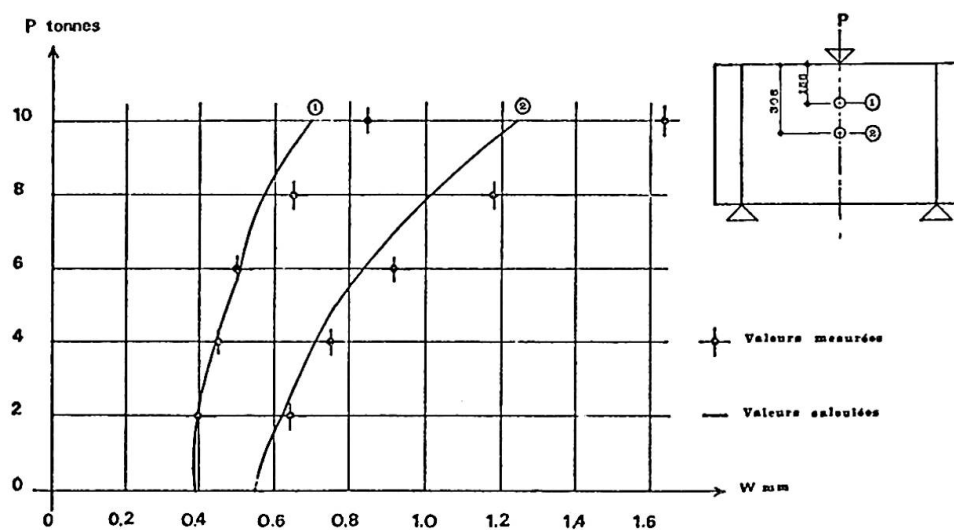
4. PANNEAU DE POUTRE EN I

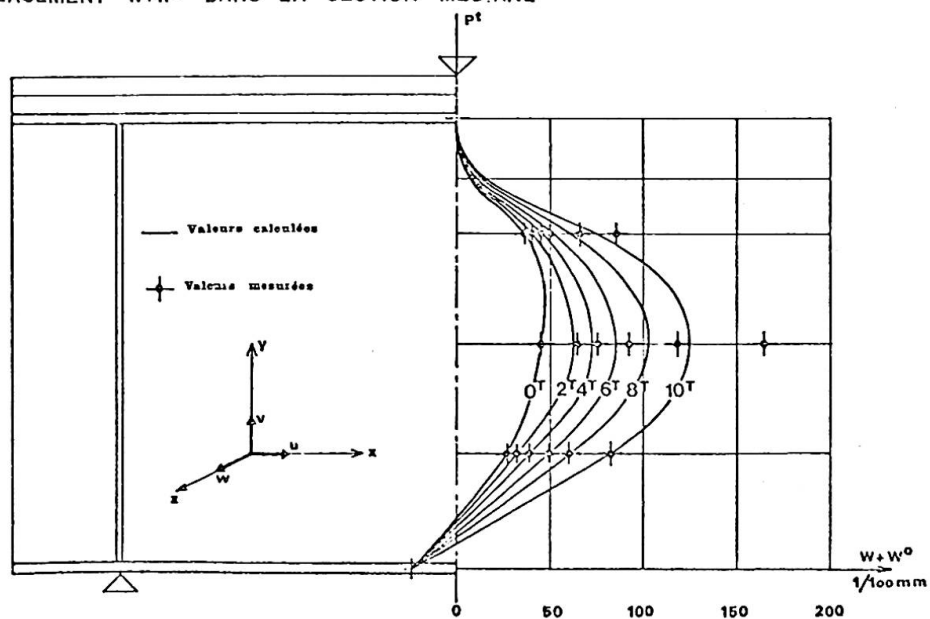
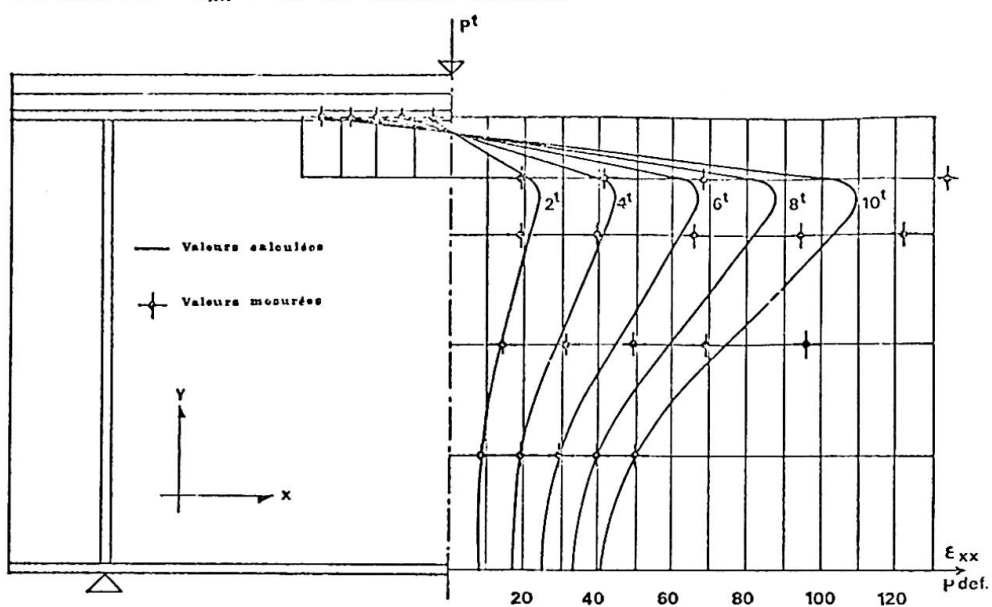
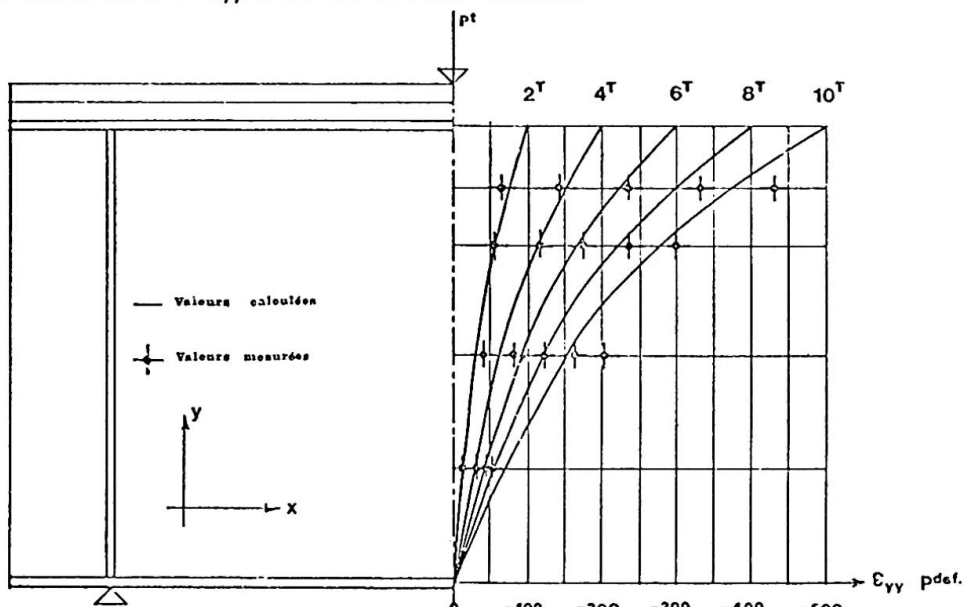


5. DEFORMÉE DE LA MEMBRURE SUPERIEURE



6. DEFORMATION DE L'AME HORS DE SON PLAN



7-DEPLACEMENT $w+w^0$ DANS LA SECTION MEDIANE8-DEFORMATION ϵ_{xx} DANS LA SECTION MEDIANE9-DEFORMATION ϵ_{yy} DANS LA SECTION MEDIANE

RESUME

Ce mémoire expose une méthode d'analyse, par éléments finis, du champ de déplacement de plaques minces raidies présentant un champ de contraintes propres et une courbure initiale. Il présente une étude comparée, d'un panneau de poutre en I, chargé localement dans son plan, par la simulation sur modèle mathématique et par la mesure sur modèle expérimental.

ZUSAMMENFASSUNG

Diese Arbeit zeigt eine mittels endlicher Elemente durchgeführte Analyse-methode des Verschiebungsfeldes dünner ausgesteifter Platten mit einem Feld von Eigenspannungen und anfänglicher Krümmung. Durch Simulation an einem mathematischen Modell und durch Messung am experimentellen Modell legt sie eine vergleichende Studie über ein in seiner Ebene örtlich begrenztes T-Balkenfeld vor.

SUMMARY

This paper exposes an analysis by the finite element method of the displacement field of thin stiffened plates presenting a field of residual stresses and an initial curvature. It shows a comparing study of a I-girder panel loaded locally in its plain, by simulation on a mathematical model and by measuring on an experimental model.

Post-Buckling Behaviour of Webs under Concentrated Loads

Comportement post-critique de voilement des âmes soumises à des charges concentrées

Überkritisches Beulverhalten von Stegblechen infolge Einzellasten

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Steel and Timber Structures
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Some comments are here given on the paper [1] by M. Škaloud and P. Novák : Post-buckled Behaviour and Incremental Collapse of Webs Subjected to Concentrated Loads. This paper gives results from tests performed in order to make it possible to build up a complete theory.

It is of course interesting to compare their results with already existing preliminary formulas. In a paper [2] to the London colloquium 1972 on Design of Plate and Box Girders for Ultimate Strength, I gave a preliminary formula and a diagram illustrating the influence of flange stiffness on the ultimate bearing capacity under a concentrated load (A. Bergfelt : Studies and Tests on Slender Plate Girders without Intermediate Stiffeners. I, Shear strength and II, Local web crippling).

Putting the test results of Škaloud and Novák into my formula, fig. 1 illustrates that their results confirm at least its tendency. The curve in the figure is from fig. 14 of my London paper, completed with the influence (from fig. 11 of the paper) of a somewhat distributed load, as in the tests of Škaloud and Novák. The curve is valid for point load action combined with small bending stresses. As my investigation considered girders without intermediate stiffeners and the girders of their tests had stiffeners with the same distance as the height of the girder it is of course a slight difference in behaviour, especially for extremely thick flanges. In that case the web is not so dominating, but the girder to some extent also acts as a Vierendeel-beam consisting of the flanges and the stiffeners. Owing to this fact their result for flanges with a thickness of about 10

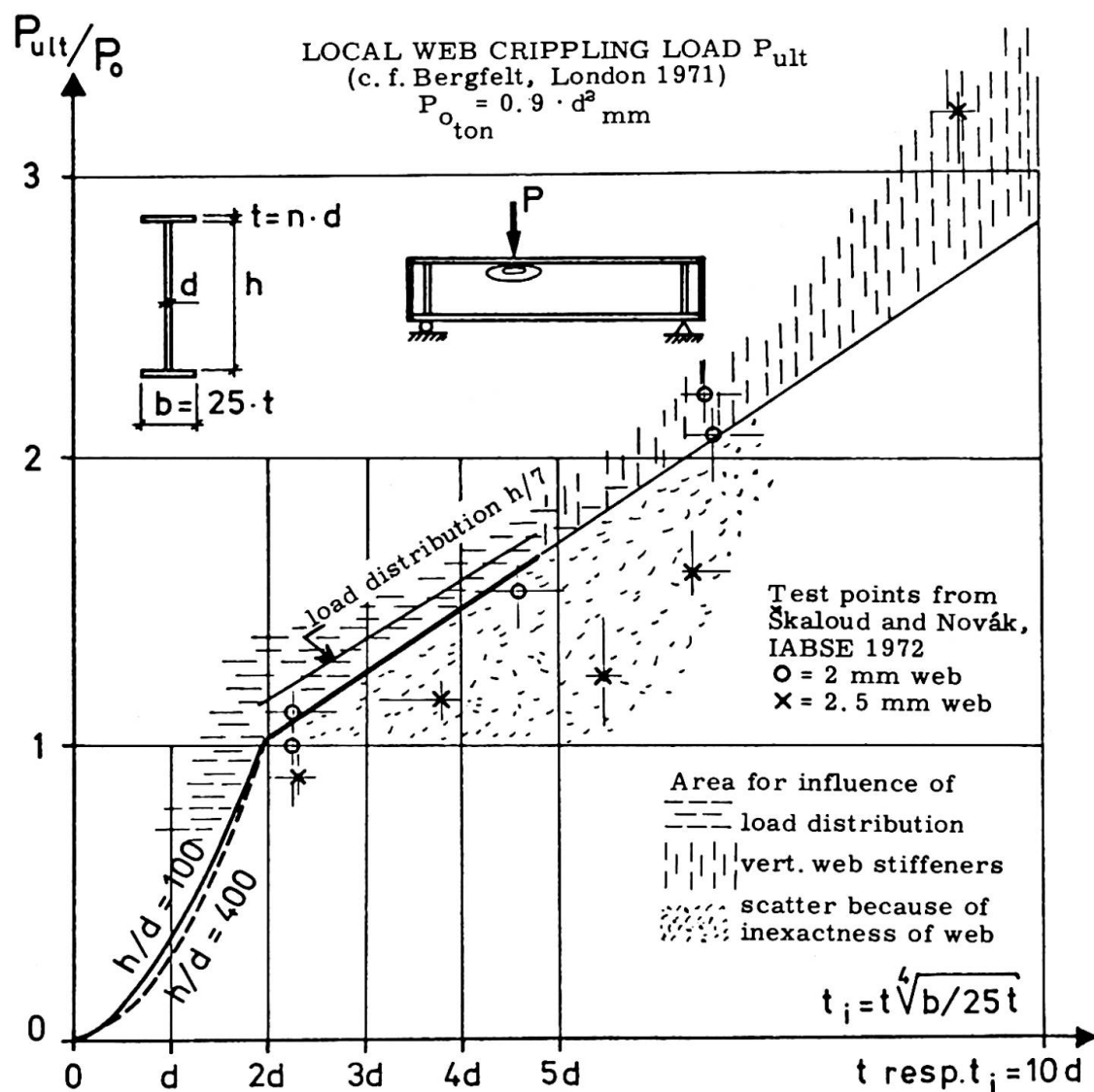


Fig.1 Test values according to Škaloud and Novák [1] marked in the diagram from [2].

times the thickness of the web is as seen from the only test in that region, a little higher than predicted from my curve. The scatter of the test values due to fabrication and material deficiencies is within the predicted limits.

The curve in the original figure was based on tests on girders with flanges about $b = 25t$, which is commonly used. When the ratio of flange dimensions differs very much from that mentioned it is of course necessary to adjust t — as single variable for stiffness — to $t_i = t \sqrt[4]{b/25t}$ as in fig. 1.

The test points of Škaloud and Novák are marked in fig. 1 as if the web thickness was really the nominal values 2 and 2.5 mm given in their paper. When their careful measurements are published and used some adjustments can of course be motivated. The steel quality is not reported but seems to have some influence, which is for the moment the scope of tests at my laboratory.

In order to illustrate the influence of stiffeners when the flanges are very thick the bearing capacity of the flange can be computed as if there was no web at all. Considering the flange 250 x 30.88 mm, which corresponds to t_i near 10, its bearing capacity is 12.5 ton (plastic design and $\sigma_Y = 2200 \text{ kg/cm}^2$).

As the test load was 18.0 ton, the influence of the web is only 5.5 ton (which in this case happens to be just about the 5.6 ton taken by the web with a very weak flange only).

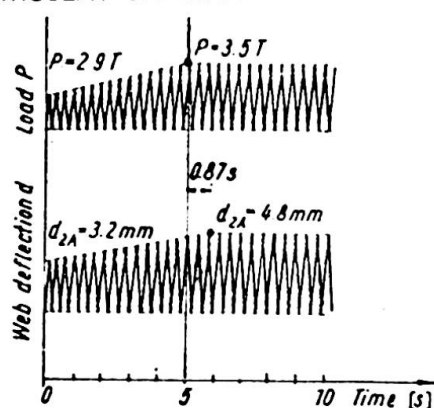
As pointed out in the paper much work remains to be done and it is considered favourable that several laboratories are interested in the problem.

*

The second part of the paper by Škaloud and Novák deals with cyclic load tests. They state that the maximum loads in several cases were higher than in the static tests and that the cyclic loads did not cause any reduction in ultimate strength.

The reason for this result is, as I think, the short loading times which result from 3 loading cycles per second. There is no time for yielding.

MIROSLAV ŠKALOUD – PAVEL NOVÁK



ALLAN BERGFELT

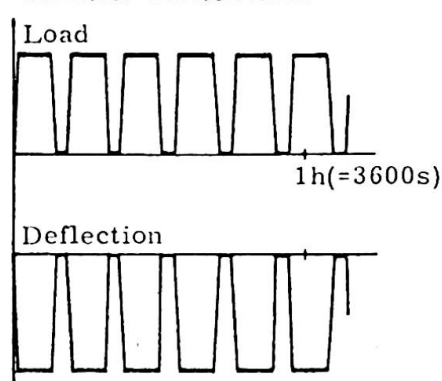


Fig. 2 Load-time dependency as used in the tests of [1] and [2]

As mentioned in my London paper 1971 I have made loadings with longer periods, for instance with so long periods that there are only 5 cycles per hour. Now test girders have been loaded up to about 1000 such cycles, which could be called repeated statical loads, and there are of course no increase in ultimate strength. It ought to be a reduction which was, however, in our tests very small. I have performed tests with 1000 cycles to more than 90% of the ultimate statical load without collapse. It is called attention to the fact that ultimate loads are meant, which as also observed in the paper by Škaloud and Novák could be 2-6 times higher than some elastic critical load for the dimensions of our test girders.

The comparison with statical loading tests holds true even in tests with combined action of bending stresses and stresses caused by the point load. Tests have been performed with bending stresses in the girder up to about half the yield stresses.

In order to obtain a well-founded general formula for cyclic loads it is necessary to test both slow and rapid cycles. So extremely slow loadings as corresponding to only 5 cycles per hours as mentioned above is probably not necessary. Perhaps for instance one cycle per minute is satisfactory. The rapid tests of Škaloud and Novák are necessary for machine foundations and bridges, while my slow tests are needed for roof structures. Anyhow it is very positive that testings at different laboratories complete each other.

References

- [1] ŠKALOUD, M. , and NOVÁK, P. : Post-buckled Behaviour and Incremental Collapse of Webs Subjected to Concentrated Loads. IABSE, Ninth Congress Amsterdam 1972, p.101.
- [2] BERGFELT, A. : Studies and Tests on Slender Plate Girders Without Intermediate Stiffeners. I, Shear strength and II, Local web crippling. IABSE Colloquium on Design of Plate and Box Girders for Ultimate Strength, London 1971.

SUMMARY

The test values according to Škaloud and Novák (1) for local web crippling under a concentrated static load, are compared with a preliminary formula given in (2). Comments are given on the influence of the rates of loading on the collapse under cyclic loadings.

RESUME

Dans ce travail, les valeurs obtenues lors d'essais par Škaloud et Novák (1) pour la ruine locale de l'âme soumise à une force statique concentrée, sont comparées à celles déterminées théoriquement dans (2). Ensuite l'auteur commente l'influence de la fréquence de charge sur l'état de ruine dans le cas de charges périodiques.

ZUSAMMENFASSUNG

Die Versuchsergebnisse nach Škaloud und Novák (1) für lokales Stegblechbeulen unter statischer Einzellast werden mit einer vorläufigen Formel aus (2) verglichen. Ueber den Einfluss der Laständerungsrate auf den Kollaps unter Wechsel-lasten wird berichtet.

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**Criteria of Column Strength in Paper by De Wolf, Pekoz and Winter
– the Significance of Imperfections**

Critères de résistance des colonnes dans les travaux de De Wolf, Pekoz
et Winter – l'importance des imperfections

Kriterien der Festigkeit von Stützen im Beitrag von De Wolf, Pekoz
und Winter – die Bedeutung von Unvollkommenheiten

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The authors, De Wolf, Pekos and Winter, have considered an important question - design curves for columns with local plate buckling. They are modest about progress to date, but I am compelled to ask whether their basic philosophy will yield the desired results.

The authors report that three column tests were discounted presumably because eccentricities of load could not be eliminated. In structural design we are not so much concerned with the average margin of safety between load and strength as we are concerned with minimising the number of failures. We are not interested in the average, or even maximum attainable, column strength, but in the minimum strength below which the risk of failure is acceptably low.

In this class of structure, so sensitive to imperfections, it is necessary to base our philosophy on probable imperfections. The load-shortening characteristics of plates are very sensitive to initial plate curvature. An initially curved plate is less stiff, axially, than a flat plate when the load is less than the initial buckling load. It is more stiff after buckling.

A thinwalled column has both local plate initial curvature and overall initial curvature. Unsymmetrical initial plate curvature can cause an overall curvature to develop from the start. Overall initial curvature will cause plate buckling to initiate unsymmetrically in the cross-section with subsequent magnification of initial deflection with further load. Dr. Skaloud's report of experiments earlier in this session supports this observation.

I am therefore compelled to ask whether the effective width concept will lead to reliable design curves for thin-walled columns. Without considerable manipulation, the effective width concept does not give the current axial stiffness of plate elements which must be known for a stability analysis at any given load. The effective width changes with load and it is also sensitive to initial imperfections. I believe a more fruitful line of attack lies in assessment of column stiffness at the current load level, using reasonable

imperfections. In this way, the three tests discarded by the authors might turn out to be useful data in the overall estimation of column strength on a statistical basis.

Finally, I would like to observe that there seems to be a pressure on the designer of columns built up from separate plates to seek an apparent optimum proportion in which plate buckling stress, column buckling stress and yield stress are about the same value. It seems that predictions of strength are most subject to error in this region; and test results are most variable, due to the extreme sensitivity of the system to imperfections. This variability is enough to wipe out any benefits of this apparent optimum proportioning of column cross-section.