Some Results in the Optimization of Tall Building Systems

Quelques résultats dans l'optimisation des systèmes pour bâtiments élevés

Einige Resultate in der Systemoptimalisierung von Hochhäusern

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INTRODUCTION

With the advent of high-speed digital computers, a significant amount of work has been accomplished in structural optimization [1] [2] [3] [4] [5] [6]. Presently available techniques enable the economical proportions or other parameters of structural systems with specified topology to be determined for buildings of moderate size and complexity under given loading schemes.

However, at the next higher level of building optimization, namely the selection of an overall optimal building system from among many candidate systems and building topologies, much remains to be done. The paper, which is an attempt in this direction, describes an approach to the optimization of topology and structural systems for tall buildings and focuses on obtaining realistic trends for possible use in design practice.

THE PROBLEM

This study is concerned with frameworks that are, or can be considered in groups of units that are, rectangular in plan. It is assumed that the usable space at each level is constant.

The structural system consists of the following component sub-systems: 1) floors, 2) framing for gravity loads, 3) framing for wind loads and 4) cladding.

1. *Floor Systems.* Candidate floor systems [7] in Fig. 1 are used in the study and are designed for a live load of 70 psf. An additional one way concrete slab derived from CRSI designs [8] is also included.
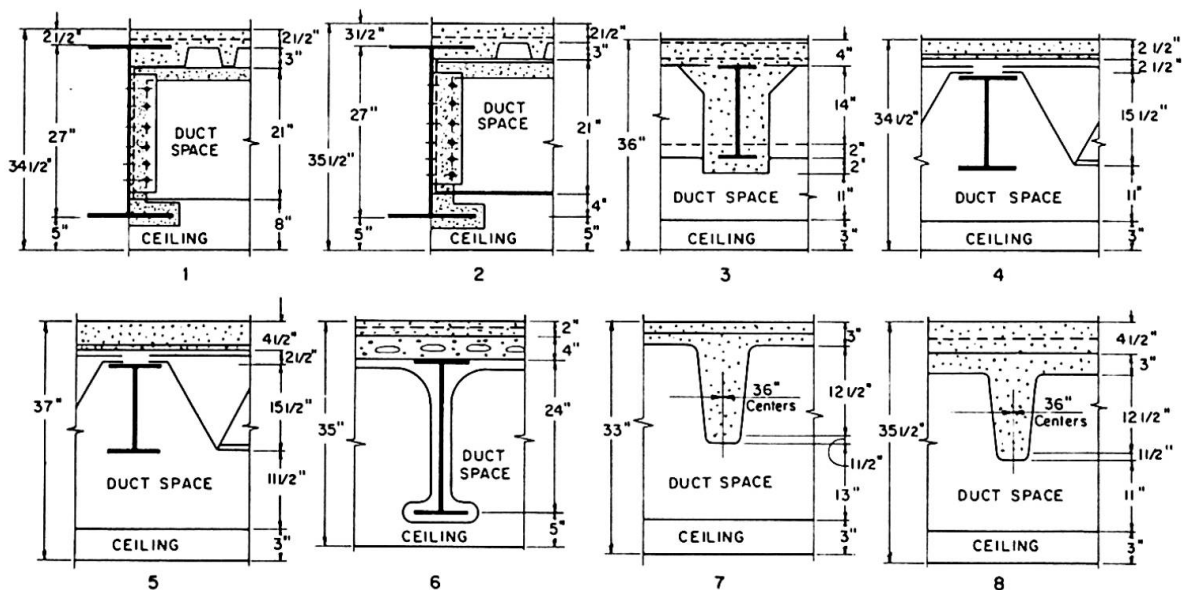

Fig. 1 Candidate floor systems (after [7])

2. *Framing for Gravity Loads* consists of beams and columns of either structural steel or reinforced concrete and their combinations. Their design conforms to current practice, i.e. the elastic design based on AISC 1963 [9] is used for structural steel and the ultimate strength design based on ACI Code 318-63 [10] is used for reinforced concrete. Pertinent design parameters include $f_s = 36$ ksi, $f_y = 60$ ksi, $f'_c = 4$ ksi, $E_s = 29,000$ ksi and $E_c = 3,000$ ksi.

3. *Framing for Wind Loads.* Three types of framing are considered: a) *conventional unbraced frames*, rigidly connected in both directions; b) *tube unbraced frames* with moment resisting exterior frames; and c) *braced frames* having two lines of cross-bracing in each direction. Drift limitation is taken as H/200, where H is the total height of the building.

4. *Cladding.* Two types of exterior walls are considered, i.e. metal curtain wall and masonry, for which average unit costs and weights are determined.

Regression analysis of the available floor data enables functional relations for the unit weight, W_i, and cost, C_i, of the typical floor systems to be evaluated in terms of the bay dimensions B and D. Thus, for a typical floor system:

$$W_i = f_i(B,D) \qquad C_i = g_i(W_i) \tag{1}$$

A regression analysis for the weight and cost of the *structural frame* is not possible and initial designs are necessary. Columns are assumed in a regular pattern, with spacings of B and D in the width, W, and length, L, directions, respectively. Columns and girders are designed at control sections of H/4 for dead and wind load combinations. The column properties are assumed to vary linearly between the quarter points.

For such a variety of systems and dimensions to choose from, a means of identification is required: systems that transmit vertical load to the frame (floor and wall systems) are identified by their unit weight, W_i, while structural frames, columns and girders are identified by their cross-sectional area, A_i.

With the above premises, the optimization problem can be stated as follows: *Given*: a required floor area of a building, a set of candidate structural components characterized by an appropriate parameter (W_i or A_i), a set of design rules and pertinent cost data, *find*: the structural topology (bay width and length and building height), the structural framing, type and material, and the system components, *such that*: the total cost of the structure is a minimum and *subject to*: all functional requirements of the current codes of practice and appropriate dimensional constraints.

THE MATHEMATICAL MODEL

The mathematical model used in the optimization process is illustrated by the flow diagram in Fig. 2. The program consists of four main operations: 1) a preliminary design of the column and girders on the basis of vertical loads is first performed; 2) this design is then revised with regard to the wind forces; 3) drift requirements are next satisfied and finally 4) a system optimization is performed to select systems from among the competing alternatives. Results of the optimization then become feedback input for the next iteration.
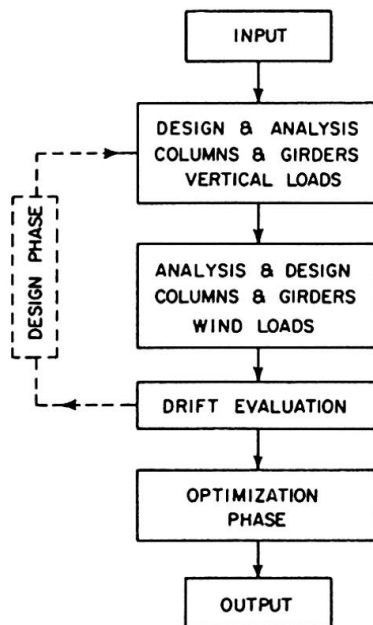

Fig. 2 - General Flow Diagram

Preliminary Design. Vertical loads are assembled from the data on the various individual floor systems, the wall system, and the spandrels. Live loads are uniformly applied in all bays. Axial forces in the columns for these loadings are determined at the quarter heights, neglecting all effects of continuity. The effect of the column and girder weights is added to these thrusts.

Typically, the weight of a composite floor system may be represented as

$$W_i^2 = \Sigma\ R_i W_i \tag{2}$$

where W_i, the weight of a composite floor system, consists of contributions from all the component floor systems, W_i, and their corresponding participation ratios, R_i.

The participation ratio, R_i, defines the relative contribution of a structural component to the total configuration. While $0 \leqslant R_i \leqslant 1$ is a continuous variable, during the process of optimization it converges to either 0 or to 1, indicating that component i does or does not participate in the optimal solution.

Wind Analysis. The following standard assumptions are adopted: a) points of inflection occur at mid point of columns and girders; b) at any elevation the vertical wind shear at all columns is identical and c) the increment of wind between mid story heights at any level is neglected. Wind thrusts in the columns are computed, assuming that the structure behaves as a cantilever beam. Column bending is computed assuming "portal action". Girders and columns are proportioned in accordance with the pertinent ACI and AISC codes, using the story height as an effective length.

Drift Analysis. The total drift is approximated by the sum of its components, the cantilever and portal drifts.

The cantilever drift is calculated by:

$$\Delta_c = SH^3/6(REI)_t \tag{3}$$

where S is the total wind force on the building, H is the building height and $(REI)_t$ is the transformed stiffness at the building base, allowing for the participating ratios of the component elements.

The portal drift in the direction B over the height, h, of the story n is derived from:

$$\Delta_n = [h^3/\Sigma(REI)_c + h^2B/\Sigma(REI)_g]\ S_n/12 \tag{4}$$

where S_n is the total shear at the level of the n^{-th} story, indices g and c refer to girders and columns, respectively, and the summations extend over elements of the n^{-th} story. A similar expression applies to the drift in the direction D by replacing B with D in eq. (4).

The value of Δ_n is computed at each quarter point of the building height. A linear variation of the relative drifts is assumed between quarter points and the drift at the top is assumed to be twice the value at the first quarter-point from the top. Then the portal drift, Δ_p, can be expressed in terms of the quarter-points drifts, Δ_1, Δ_2, Δ_3, and Δ_4, as:

$$\Delta_p = (2\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4/2)n/4 \tag{5}$$

The total drift ,Δ, becomes:

$$\Delta = \Delta_c + \Delta_p \tag{6}$$

If the drift limitation (H/200) is exceeded, a subroutine is provided to increment columns and girders in an optimal way. The sequence of column and girder design and drift analysis is repeated in a sufficient number of iterations.

Optimization. In this study, the total structural cost, C, is chosen as the merit function to be minimized. The merit function is expressed as:

$$C = \Sigma\ R_i\ W_i\ C_i \tag{7}$$

where W_i is the weight (area) parameter of the i-th component and R_i and C_i are its corresponding participation ratio and cost, respectively. All costs are Pittsburgh, U.S.A., costs converted to a 1970 base.

The formulation of the optimization problem involves 117 variables, (including the bay dimensions B and D, and all candidate structural component parameters) and 202 equality and inequality constraints. The behavioural constraints represent the column design, the assemblage of floor and spandrel weights and vertical columns loads. The side constraints limit the values of participation ratios, bay dimensions and column and girder sizes. The merit function and the constraints are highly nonlinear. Search and penalty function techniques did not prove successful for the nonlinear programming problem on hand, due to the difficulty of enforcing all of the constraints in the presence of a large number of variables. Instead, a cutting plane technique [11] was adopted wherein both the merit function and constraints were linearized by a Taylor's series expansion. The resulting linearized form was solved by a linear programming technique, "Optima" [12].

SOME OPTIMIZATION RESULTS

On the basis of the mathematical model described, a large number of optimal solutions have been investigated. These fall into two groups: a) certain ratios of the building dimensions are held constant and the volume of the building is allowed to vary: b) the total floor area is held constant while the pertinent building dimensions are allowed to vary. Buildings of 12, 20 and 32 stories and with height to least plan dimension ratios of 2 or 4 are selected in the first group; whereas buildings of 50,000, 500,000 and 5,000,000 sq. ft. floor area are chosen for the second group. In all optimization studies reported, the story height is taken as 11 ft.

In general, the program tends to select reinforced concrete frames supporting steel floor systems. The cost of connecting such systems imposes constraints to overrule this combination. Masonry walls are most often chosen. Whenever only floors in ref. [7] are available, the program chooses the steel joist floor system for steel frame designs and the waffle floor for concrete frame designs. The CRSI floor systems tends to be selected when available and for minimum 15 ft. bay dimensions. On larger bay dimensions, this conclusion does not hold.

Optimal bay dimensions range between 10 and 20 ft. However, since no value is assigned to the advantage of open space, this conclusion is only of relative validity. Generally, the total structural cost increases only slightly as the bay dimensions increase from 15 ft. to 40 ft.

Reinforced concrete frames prove advantageous for bay dimensions of 15 and 40 ft., for both conventional and tube unbraced frames. Apparent advantages decrease as the bay dimension increases. On the other hand, the total structural cost appears to be relatively insensitive to the frame material, as illustrated by Figs. 3,4. These show the total cost ratio of a steel frame structure to a reinforced concrete frame structure with various floor areas for unbraced and braced frames, respectively. In general, braced frame structures are slightly more economical, in steel than in concrete, whereas for unbraced frame structures, the economics of steel over concrete depends on the building aspect ratio, W/H. Moderate changes in unit costs would, probably alter these results. However, the trend of the cost ratio (structural steel to concrete) increasing as the building becomes more slender would remain valid.
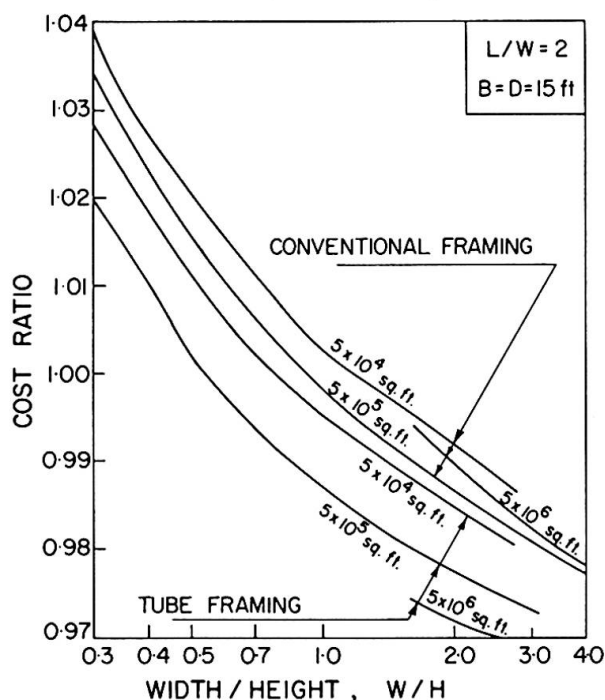

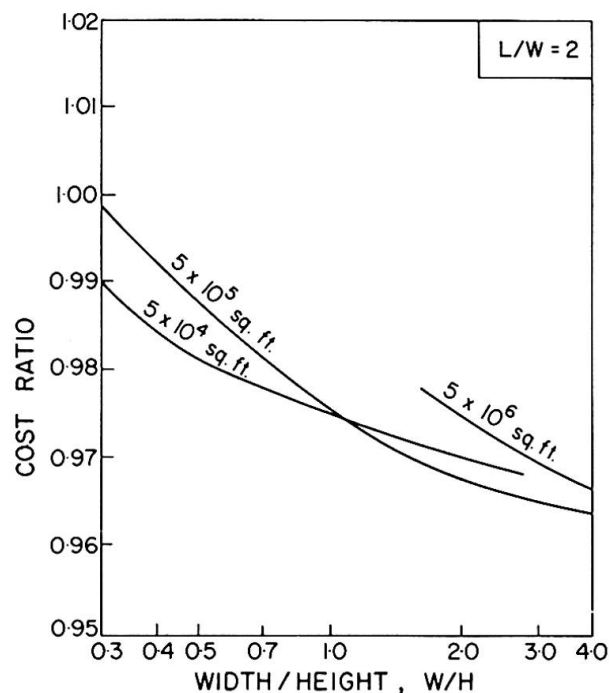Fig. 3 - *Total Cost Ratio of Steel to Reinforced Concrete Unbraced Frame Structures for Various Floor Areas*

Fig. 4 - *Total Cost Ratio of Steel to Reinforced Concrete Braced Frame Structures for Various Floor Area*

The type of frame plays a much more significant role in the overall optimization. Unbraced and braced frames are compared in Figs. 5,6,7, for various required floor areas for bays of at least 15 x 15 sq. ft. Consistently, braced frames prove to be more economical even for low rise structures. Also it is seen that a cost premium must generally be paid for more slender buildings. From these data, one might further infer that one large structure is more economical than a series of small buildings of equivalent total area.

Certain numerical solutions of this study related to the land cost effects could prove useful in an early decision-making process. Assuming first that land costs and constraints are not considered, Figs. 5,6,7, indicate that the optimal number of stories for buildings of 50,000, 500,000, and 5,000,000 sq. ft. of required floor space is approximately four to five, six to nine, and ten to thirteen, respectively. Thus, squatty buildings are more economical where there is no relation between structure cost and the land requirements of the building.
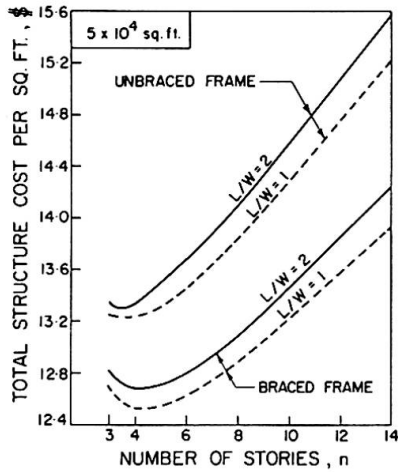
Fig. 5 - Total Structural Cost for
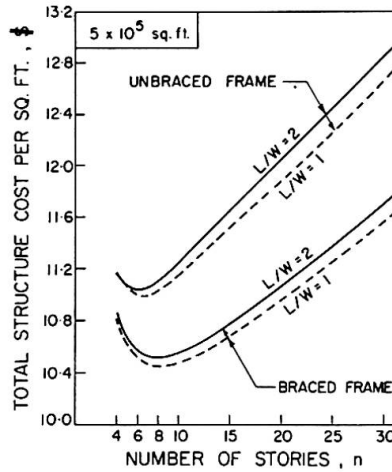*5 x 10⁴ sq.ft. Total Floor Area*

Fig. 6 - Total Structural Cost for
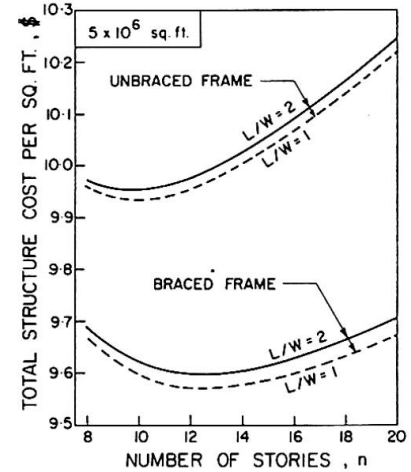*5 x 10⁵ sq.ft. Total Floor Area*

Fig. 7 - Total Structural Cost for
5 x 10⁶ sq.ft. Total Floor Area

Alternatively, if land becomes a constraint, then plan dimensions as close as possible to the plan dimensions at optimum height produce the most economical structure. Fig. 8 shows that this condition prevails since, for constant floor area, unit cost increases with height.

When land costs play a governing role, they can significantly modify the optimum heights. For example, the effect of land costs of two, twenty and $200/sq. ft. has been investigated. Fig. 9 shows the total costs of the structure including land cost for a 500,000 sq. ft. floor area requirement. Similar curves can be developed for other area needs. As a result, the most economical heights now increase over the no cost land heights, the increase becoming larger as land becomes more expensive. Table 1 illustrates these results.
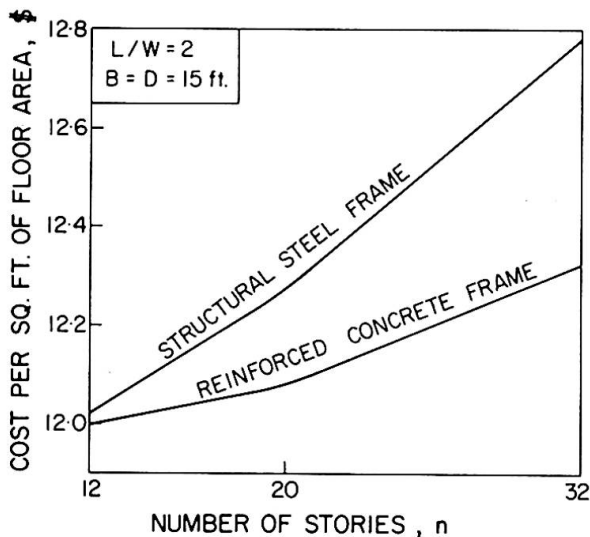

Fig. 8 - Variation of Structural Cost With The Number
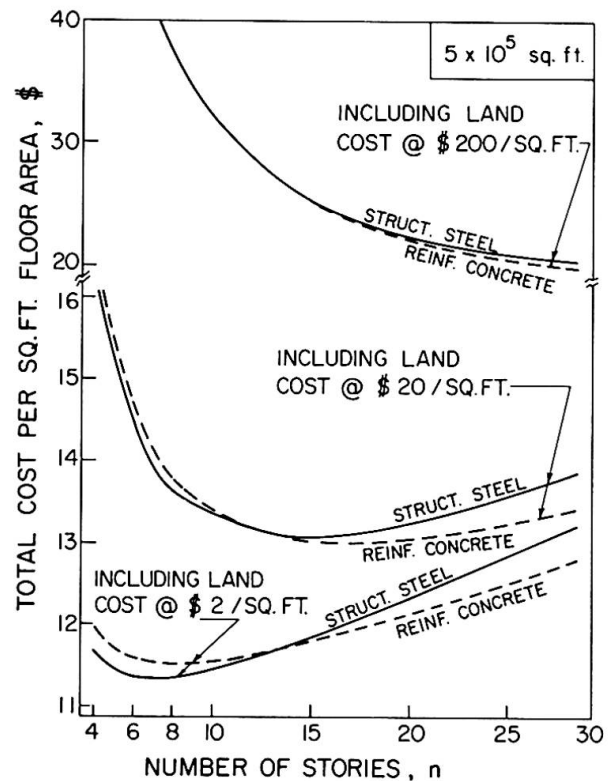*of Stories of Conventional Unbraced Frames*

Fig. 9 - Total Costs of Structures Including Land Costs
For 500,000 Sq. Ft. Total Floor Area

TABLE 1

OPTIMAL BUILDING HEIGHTS

Total Floor Area (sq.ft.)	Optimal number of storys for land cost of:			
	$ 0/sq.ft.	$ 2/sq.ft.	$ 20/sq.ft.	$ 200/sq.ft.
5×10^4	4–5	4–5	8–10	> 14
5×10^5	6–9	7–10	15–18	> 30
5×10^6	10–13	10–15	20	> 20

CONCLUSIONS

The present study is an attempt at a comprehensive building system optimization. Its efficiency is related to the successful development of a computer based cost-minimizing procedure for selecting a set of subsystems and topology parameters.

A number of factors such as the foundation, electrical, mechanical and architectural subsystems, have intentionally been excluded. While the economic trends will somewhat be altered by these factors, most of the present results and conclusions will remain essentially valid.

Some design trends are noted from a large number of studies based on the procedures developed. Use of this information may lead to designs that are much closer to the optimum than by intuitive judgement and experience, particularly in the conceptual or preliminary stage of planning. All trends must be tempered, of course, by the limitation of the model and the specific cost data adopted. Land cost may play a major role as a decision variable, can be included in the model and its effects can be evaluated.

Still more comprehensive optimization programs can be attempted, wherein all major building systems can be considered. The success of the present program establishes the precedent for such a bold approach to building systems optimization.

ACKNOWLEDGEMENT

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NOTATION

A_i	=	cross sectional area of component i.
B	=	bay width.
C	=	total cost of the building.
C_i	=	cost of the i-th system component.
D	=	bay length.
E_c, E_s	=	young moduli for concrete and steel, respectively.
EI	=	flexural stiffness of a structural component.
f_s	=	yield stress for structural steel.
f_y	=	yield stress for reinforcing steel.
f_c	=	cylinder strength of concrete.
H	=	building height.
h	=	story height.
L	=	building length.
n	=	number of stories of the building.
R_i	=	participation ratio of component i.
S	=	total wind force on the building.
S_n	=	total shear at story n level.
W	=	building width.
W_i	=	unit weight of the i-th system component.
Δ	=	total drift of the building.
Δ_c	=	cantilever drift of the building.
Δ_n	=	contribution of story n to the portal drift of the building.
Δ_p	=	portal drift of the building.

SUMMARY

The possibility of using programming techniques for the optimization of realistic structural building systems is explored. The object is to determine the bay dimensions the framing type and the system components, such that the total cost of the structure be minimized and that the functional requirements of the current codes of practice be satisfied. Some typical results of the optimization process and trend of optimal solutions are briefly discussed.

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