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Evaluation of Gusts on Flexible Structures

Estimation des efforts dynamiques aux structures souples

Abschätzung der Einwirkung von Stössen auf flexible Bauwerke

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Introduction

The contribution described herein forms part of a study of the gust responses of flexible structures such as long-spanned bridges, tall-slender buildings and towers. Dynamic excitation of structures by wind action can usually referred to the following causes: (1) instability of the galloping, stall flutter and (classical) flutter types, (2) buffetting by vortices and turbulence shed in the wake of the structure, (3) buffetting by gusts and (4) buffetting by vortices shed by the other surrounding structures. These causes are possible either together or separately.

The indication of criteria for the determination of the effects of gusts have been so far considered, in the form of gust factor *1*), *2*), with regard to the oscillation of mean wind direction (drag component), and a number of other factor have not been so much investigated owing to the difficulty of generalization. They could, however, be indispensable, for instance, when there is a possibility of negative aerodynamic damping. Many common structural shapes have usually a potential of instability at certain critical wind velocities. Such structural shapes as square, rectangular and H-shaped sections, plate girder bridges, truss bridges and streamlined sections are those with which aerodynamic instability or vortex excitation has been known to occur.

In this paper it is indicated that the dynamic effects of gusts on items except (4) described above are generally expressed by using unsteady aerodynamic forces obtained experimentally or theoretically, and a couple of illustrative examples are presented in the calculation of gust responses of particular structural cases.

Formulation of Gust Responses

The proper recognition of the dynamic effects of wind in conjunction with design wind velocity estimates depends on the prediction of the statistical distribution of the responses of the structure--stress, deflection or local pressure. To derive these distributions it is well known that two types of information are required with the distribution of reference wind velocities observed at the site of the structure and the aerodynamic responses based on a certain reference wind velocity. The reference wind velocity and the structure of the wind are assumed to be already known in the following discussion.

In smooth flow the unsteady aerodynamic forces acting on a two-dimensional body oscillating with displacements of lift motion $Z(t)$ and pitching moment motion $\phi(t)$ at circular frequency ω are expressed as:
for the lift force,

$$L = \pi\rho(\frac{B}{2})^3\omega^2 \; [(L_{ZR} + iL_{ZI})\frac{Z}{B/2} + (L_{\phi R} + iL_{\phi I})\phi]$$

and for the pitching moment, (1)

$$M = \pi\rho(\frac{B}{2})^4\omega^2 \; [(M_{ZR} + iM_{ZI})\frac{Z}{B/2} + (M_{\phi R} + iM_{\phi I})\phi]$$

where $i = \sqrt{-1}$, ρ is air density, B is the width of the body in the wind direction and the unsteady aerodynamic coefficients L_{ZR}, L_{ZI}, ... and $M_{\phi I}$ are real functions of reduced frequency

$$\xi = \frac{n\ B}{U} = \frac{1}{\pi}\ (\frac{\omega}{2}\frac{B}{U}) \tag{2}$$

at wind velocity U and frequency n. The only theoretical analysis for the unsteady aerodynamic coefficients have been derived for a flat plate section as follows:

$$L_Z = L_{ZR} + iL_{ZI} = -\frac{2i}{\pi\xi}C(\pi\xi),\ L_\phi = L_{\phi R} + iL_{\phi I} = -\frac{2}{(\pi\xi)^2}C(\pi\xi) - \frac{i}{\pi\xi}[1 + C(\pi\xi)]$$
$$M_Z = M_{ZR} + iM_{ZI} = \frac{i}{\pi\xi}C(\pi\xi),\ M_\phi = M_{\phi R} + iM_{\phi I} = \frac{1}{(\pi\xi)^2}C(\pi\xi) - \frac{i}{2\pi\xi}[1 - C(\pi\xi)] \tag{3}$$

in which $C(\pi\xi) = F(\pi\xi) + iG(\pi\xi)$ is a complex Theodorsen function *3*). As far as structural shapes described above are concerned, generally speaking, it is difficult to derive these unsteady aerodynamic coefficients analytically, and there is no way to derive other than experimental measurements. The measured coefficients are given in Fig. 1 for a couple of sections *4*) together with a flat plate section. The measuring experiments were carried out by a so-called forced oscillation method, in which the unsteady coefficients are obtained by giving a body forced oscillations with constant amplitude and frequency as a parameter of ξ.

In turbulent flow with mean wind velocity $\overline{U}$ and fluctuating velocity components of along wind u and cross wind w (vertical) or v (lateral), the aerodynamic forces that act on a body are partly due to the fluctuating components of turbulent flow and partly due to the motion of the body itself, that is, part of unsteady aerodynamic forces. The latter part must, strictly speaking, be different from that in smooth flow. Thus, the aerodynamic forces can be expressed in turbulent flow as:

$$L^* = \pi\rho(\frac{B}{2})^3\omega^2 \; [(L_{ZR}^* + iL_{ZI}^*)\frac{Z}{B/2} + (L_{\phi R}^* + iL_{\phi I}^*)\phi] + L_f(t)$$
$$M^* = \pi\rho(\frac{B}{2})^4\omega^2 \; [(M_{ZR}^* + iM_{ZI}^*)\frac{Z}{B/2} + (M_{\phi R}^* + iM_{\phi I}^*)\phi] + M_f(t) \tag{4}$$

in which the reduced frequency $\xi = \frac{1}{\pi}(\frac{\omega B}{2\overline{U}})$. Whether or not the forces induced by above two parts are linearly superposable will be a question requiring further attention in the understanding of the identification of each part and the complex

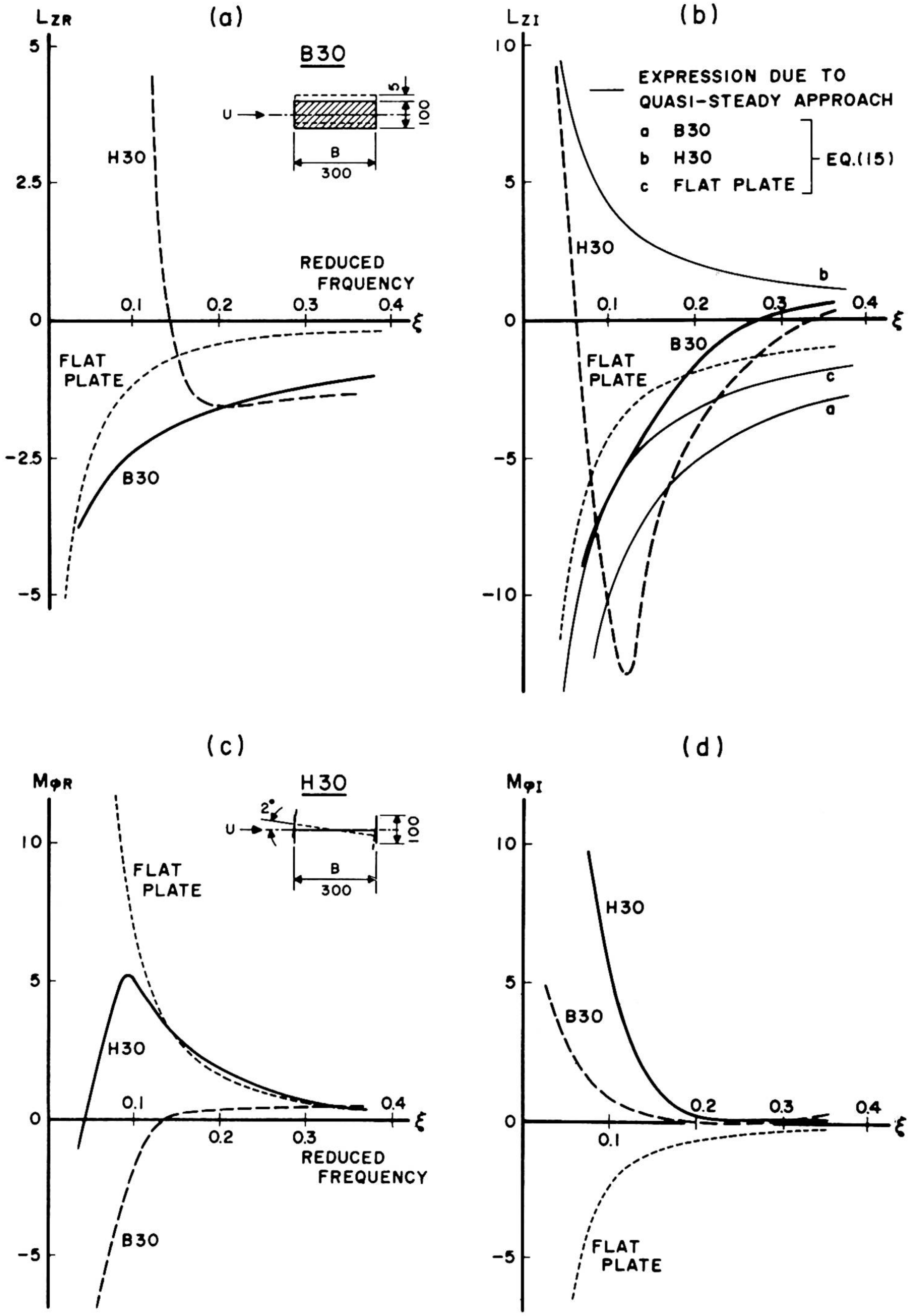

Fig. 1 EXAMPLES OF MEASURED AERODYNAMIC COEFFICIENTS FOR LIFT AND MOMENT RESPONSES OF RECTANGULAR AND H-SHAPED SECTIONS TOGETHER WITH THEORETICAL ONES OF FLAT PLATE SECTION

interaction between the turbulence in the approaching flow and the wake generated by the oscillatory body. Each coefficients of eq. (4) is available in principle by the same forced oscillation method as in smooth flow, if the turbulent flow in wind tunnel is well simulated to the natural wind.

It can be seen that the justification of the separation into two independent parts and the superposition of them in eq. (4) depends on the assumption of small disturbances. Then, as the first approximation, the expression of eq. (4) may be valid and the replacement of coefficients $L_{ZR}{}^*$, $L_{ZI}{}^*$, ... and $M_{\phi I}{}^*$ by L_{ZR}, L_{ZI}, ... and $M_{\phi I}$ in smooth flow can be assumed for turbulence of small intensity.

Including several kinds of oscillations such as vertical one of bridges due to vertical components of turbulence, lateral one of tall-slender structures transverse to the mean wind direction caused by lateral components of turbulence as well as oscillations of the mean wind direction caused by longitudinal components, the gust responses of flexible structures to atmospheric turbulence can be expressed as in the following discussion by means of such the aerodynamic forces as given in eq. (4). All the aerodynamic characteristics of structural shapes, whether the section is stable or there is a possibility of negative aerodynamic damping, should be represented in the forms of unsteady aerodynamic forces.

Let us consider an illustrative analysis of vertical responses to vertical components w of turbulent flows of a system that has coupled oscillation characteristics with vertical and torsional motions. The spectrum $S_Z(x;\, n)$ of the displacement $Z(x;\, t)$ at a point x of the span in the rth mode can be expressed by

$$S_Z(x;\, n) = S_w(n)\ |X_L(n)|^2\ |J_w(n)|^2\ |X_Z(x;\, n)|^2 \tag{5}$$

in which $S_w(n)$ is the spectrum of vertical component w of turbulence, $|X_L(n)|$ the frequency response (admittance) of the lift to a sinusoidal gust w, $|X_Z(x;\, n)|$ the frequency response of the displacement to a sinusoidal lift force, and $|J_w(n)|^2$ named the joint acceptance.

As to the frequency response of the displacement to a sinusoidal lift force, it can be obtained by examining the dynamic solution of the system subjected to a harmonic exciting lift $L_f(t) = L_o e^{iwt}$ at $x = x_L$. Using the expressions of eqs. (1) and (4) and letting $M_f(t) = 0$, the equations of motion are:

$$\begin{aligned} m\ [\ddot{Z} + i \cdot 2\zeta_1\omega_1\dot{Z} + \omega_1{}^2 Z] - L &= L_f \\ \theta\ [\ddot{\phi} + i \cdot 2\zeta_2\omega_2\dot{\phi} + \omega_2{}^2\phi] - M &= 0 \end{aligned} \tag{6}$$

where m and θ are the mass and the polar moment of inertia per unit length, ζ_1 and ζ_2 the mechanical damping ratio, and ω_1 and ω_2 the circular natural frequency in vertical and torsional motions, respectively. The solution is

$$Z(x;\, t) = \frac{L_o e^{iwt}}{\pi\rho(B/2)^2\omega^2}\ \frac{A_4}{\begin{vmatrix} A_1 & A_2 \\ A_3 & A_4 \end{vmatrix}}\ \frac{\phi(x)\phi(x_L)}{\int_0^l \phi^2(x)dx} \tag{6a}$$

in which $\phi(x)$ is the rth mode shape, l the span length and

$$A_1 = \mu[-1 + i\cdot 2\zeta_1\frac{\omega_1}{\omega} + (\frac{\omega_1}{\omega})^2] - (L_{ZR} + iL_{ZI}), \quad A_2 = -(L_{\phi R} + iL_{\phi I})$$

$$A_3 = -(M_{ZR} + iM_{ZI}), \quad A_4 = \nu[-1 + i\cdot 2\zeta_2\frac{\omega_2}{\omega} + (\frac{\omega_2}{\omega})^2] - (M_{\phi R} + iM_{\phi I}) \tag{6b}$$

$$\mu = \frac{m}{\pi\rho(B/2)^2}, \qquad \nu = \frac{\theta}{\pi\rho(B/2)^4} \tag{6c}$$

The quantity $|X_Z(x;\ n)|$ is, therefore, derived as

$$|X_Z(x;\ n)| = \frac{\phi(x)}{\pi\rho(B/2)^2\omega^2}\ \frac{A_4}{\begin{vmatrix} A_1 & A_2 \\ A_3 & A_4 \end{vmatrix}} \tag{7}$$

which differs from the familiar resonance curve of the mechanical admittance, and the joint acceptance is expressed by

$$|J_w(n)|^2 = \int_0^l\int_0^l \phi(x_1)\phi(x_2)R_w(x_1,\ x_2;\ n)dx_1dx_2\ /\ [\int_0^l \phi^2(x)dx]^2 \tag{8}$$

in which $R_w(x_1,\ x_2;\ n)$ is the spanwise cross correlation of w at points x_1 and x_2 and at frequency n.

The lift force caused by vertical component w of turbulence, that is, $L_f(t)$ in eq. (4) may be approximately given in the form:

$$L_f(t) = \frac{1}{2}\rho B(\frac{dC_L}{d\alpha})\overline{U}^2 \cdot \frac{w(t)}{\overline{U}}\ X_w(\xi) \tag{9}$$

in which $X_w(\xi)$ is the term corresponding to the frequency response (aerodynamic admittance) of the lift to gust w, and $dC_L/d\alpha$ the rate of change of steady lift coefficient with flow inclination α. Vickery *5*) has investigated the drag force/velocity relationship for bluff prismatic structures of low aspect ratio, being in a reasonable agreement with theoretical estimates based on a lattice structure. Bearman *6*) has also examined the relationship between the approaching turbulent flow and the mean and fluctuating forces on a series of flat plates set normal to the flow. It was concluded that at values of $\xi = nB/U$ less than 0.1 the drag/velocity relationship helped to justify the concept of aerodynamic admittance, but the measurements suggested, at high values of ξ, a further contribution to drag fluctuations, uncorrelated with upstream velocity, perhaps resulting from wake-induced fluctuations on the rear face, although the level of the spectra in turbulent flow was three orders greater than in smooth flow at the same value of ξ. This may suggest that it is significant to use unsteady aerodynamic forces in evaluating the gust responses.

Sears *3*) has shown the expression, for a flat plate section, in the form

$$L_f(t) = \frac{1}{2}\rho B(2\pi)\overline{U}^2 \cdot \frac{w_o e^{iwt}}{\overline{U}}\ X_w(\xi) \tag{9a}$$

and the approximation of $X_w(\xi)$ has been given by Liepmann *3*) as follows:

$$|X_w(\xi)|^2 = \frac{1}{1 + 2\pi^2\xi} \tag{10}$$

The frequency response (admittance), in eq. (5), therefore, is presented as

$$|X_L(n)| = \pi\rho B\overline{U}\ |X_w(\xi)| \tag{11}$$

If the response is limited to one degree of freedom, the expressions of the frequency response of the displacement to a sinusoidal exciting force are rewritten in simpler forms. For the combination of translational (vertical or lateral) displacement and lift force,

$$|X_Z(x;\ n)|^2 \text{ or } |X_x(Z;\ n)|^2 = \frac{\phi^2(x)}{16\pi^4 m^2 n_o^4} \frac{1}{[1-(\frac{n}{n_o})^2-(\frac{n}{n_o})^2\frac{L_R}{\mu}]^2+[2\zeta_o\frac{n}{n_o}-(\frac{n}{n_o})^2\frac{L_I}{\mu}]^2} \tag{12a}$$

in which L_R and L_I are real and imaginary unsteady aerodynamic force components due to translational displacement respectively, n_o the natural frequency and ζ_o the mechanical damping ratio, and for torsional displacement,

$$|X_\phi(x;\ n)|^2 = \frac{\phi^2(x)}{16\pi^4\theta^2 n_2^4} \frac{1}{[1-(\frac{n}{n_2})^2-(\frac{n}{n_2})^2\frac{M_{\phi R}}{\nu}]^2+[2\zeta_2\frac{n}{n_2}-(\frac{n}{n_2})^2\frac{M_{\phi I}}{\nu}]^2} \tag{12b}$$

The same expression as in eq. (12a) is also possible with the drag response of the mean wind direction, provided the aerodynamic coefficients are available by measuring each component of unsteady drag force giving the body an along-wind oscillation.

The coefficients such as L_{ZI} and $M_{\phi I}$ indicate the effects of aerodynamic damping (or exciting), because the imaginary terms are correlated with phase lag between displacement and force acting on the body. The aerodynamic damping (or exciting) ratio can be derived from the quasi-steady approach as well. Davenport 7) has shown the logarithmic decrements for drag direction responses

$$\delta_P = \frac{\overline{P}}{n_o\overline{U}m} \tag{13a}$$

and for translational responses

$$\delta_L = \frac{d\overline{L}/d\alpha}{2n_o\overline{U}m} \tag{13b}$$

in which $\overline{P} = \frac{1}{2}\rho B\overline{U}^2C_D$, $\overline{L} = \frac{1}{2}\rho B\overline{U}^2C_L$ and C_D and C_L are steady drag and lift coefficients. The term in eq. (12a) associated with damping is rewritten as

$$2\zeta_o\frac{n}{n_o} - (\frac{n}{n_o})^2\frac{L_I}{\mu} = 2\frac{n}{n_o}(\zeta_o - \frac{n}{n_o}\frac{L_I}{2\mu})$$

and, thus, the aerodynamic damping (or exciting) term is

$$\zeta_{aero} = -\frac{n}{n_o}\frac{L_I}{2\mu} \tag{14}$$

Combining eq. (13) and eq. (14), the expressions of L_1 due to quasi-steady approach are derived for translational responses and along-wind responses respectively:

$$P_I = -\frac{2C_D}{\pi^2}\frac{1}{\xi}, \qquad L_I = -\frac{dC_L/d\alpha}{\pi^2}\frac{1}{\xi} \tag{15}$$

Numerical Examples

Vertical Gust Responses of Flat Plate Section To find the vertical responses of a taut strip model of flat plate section to boundary layer turbulent flows, derive the root mean square of response from the results of $S_Z(x; n)$ in eq. (5).

$$\sigma_Z(x) = \sqrt{\int_0^\infty S_Z(x; n)dn} \tag{16}$$

The factor $X_Z(x; n)$, the frequency response of the displacement to a sinusoidal lift force can be computed according to eq. (7), using the aerodynamic coefficients in eq. (3) with regard to frequency n as a parameter of wind velocity. The results are shown in Fig. 2, which indicates that peak responses shift from a frequency close to the natural frequency to that close to the critical frequency with increase of wind velocity. Every dimension used in calculation is due to the work by Davenport, Isyumov and Miyata *8*) as follows:

l = 228.6 cm, B = 9.36 cm

μ = 11.1, ν = 2.26 from eq. (6c)

n_1/n_2 = 7.6/21.2 = 0.36

ζ_1 = 0.0024, ζ_2 = 0.01

The root-mean-square responses are, finally, obtained as indicated in Fig. 3 together with experimental results *8*). In the case of 9% of intensity of turbulence, the agreement of both is reasonably good.

Comparison of Aerodynamic Coefficients with Those due to Quasi-steady Approach As to the sections shown in Fig. 1, compute aerodynamic coefficients L_I due to eq. (15).

	$\{dC_L/d\alpha\}_{\alpha = 0°}$		L_I
B 30	9.98	}ref. *4*)	$-\frac{1.01}{\xi}$
H 30	-4.02		$\frac{0.407}{\xi}$
FLAT PLATE	2π		$-\frac{0.638}{\xi}$

The results are shown in Fig. 1(b) together with experimental values measured by the forced oscillation method. The agreement of both is poor with a slight exception of small values of ξ. On the other hand, in the case of drag responses of a truss bridge, it is likely that the expression of aerodynamic damping due to quasi-steady approach is comparatively reasonable. Fig. 4 shows a result of aerodynamic coefficients for lateral (drag) motions of a suspension bridge model. Dimensions of the truss bridge model are l = 16 m, B = 35.5 cm, $m = 4.66 \times 10^{-2}$ g sec^2/cm^2, n_o = 0.46 c/s and C_D = 0.29. For this case,

$$P_I = -\frac{2 \times 0.29}{\pi^2}\frac{1}{\xi} = -\frac{0.059}{\xi} \qquad \text{from eq. (15)}$$

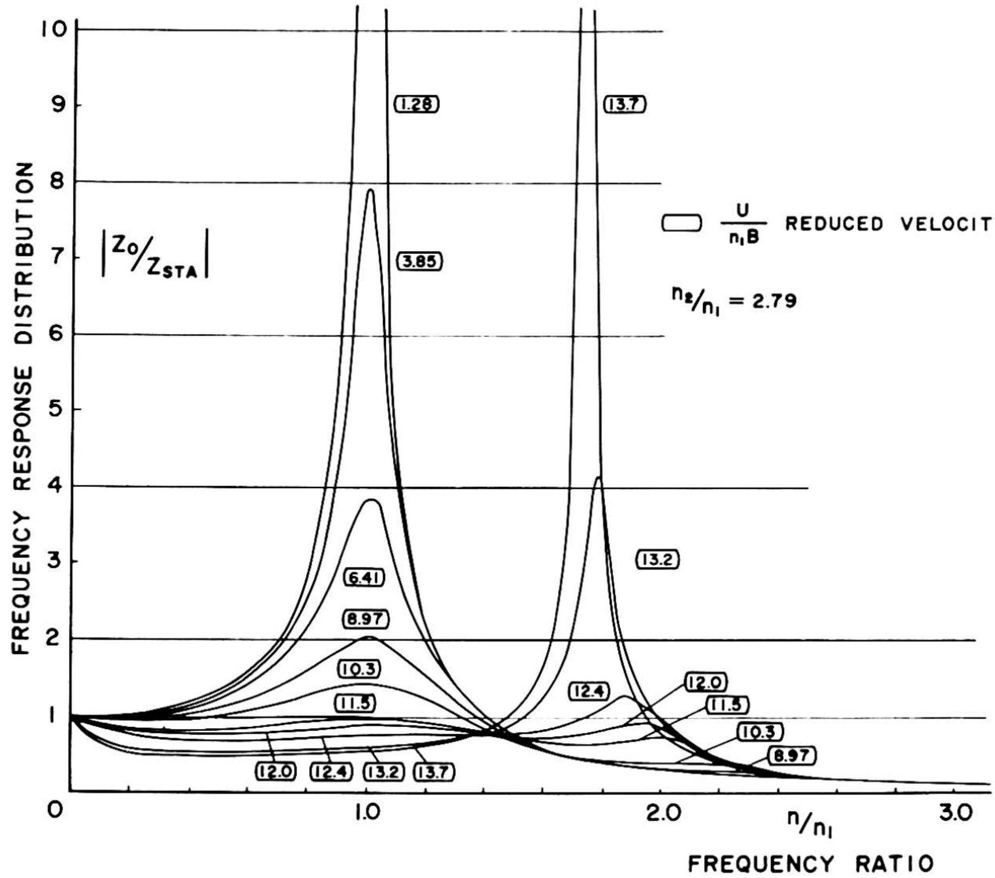

Fig. 2 FREQUENCY RESPONSE OF VERTICAL DISPLACEMENT $Z = Z_o e^{i\omega t}$ OF FLAT PLATE SUBJECTED TO SINUSOIDAL EXCITING LIFT FORCE $L_f = L_o e^{i\omega t}$

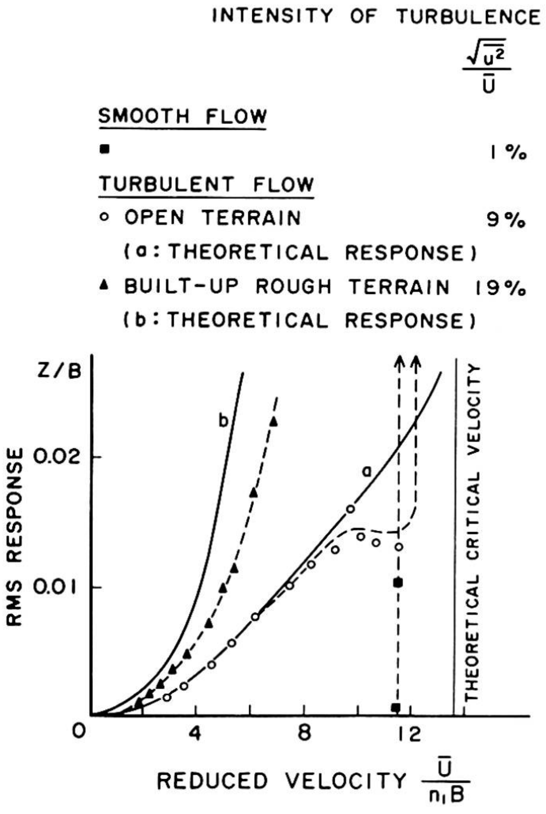

Fig. 3 VERTICAL GUST RESPONSES OF FLAT PLATE TAUT STRIP MODEL [ORIGINAL EXPERIMENTAL DATA DUE TO REFERENCE *8*)]

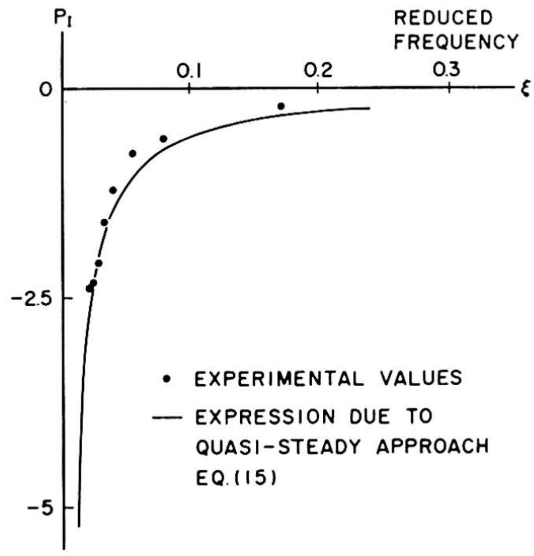

Fig. 4 AERODYNAMIC COEFFICIENT FOR ALONG-WIND (DRAG) RESPONSE OF TRUSS BRIDGE

Application to Along-Wind Gust Responses of Suspension Bridges It is well known that long-spanned suspension bridges are determined in designing due to the along-wind action of wind. The evaluation of gust responses of suspension bridges has been examined by Davenport *7*), and the statistical approach has been used in Japan *9*) to derive a factor to give the design wind velocity for the static action of gusts. According to Davenport's treatment, find the gust factor G for suspension bridges with center span length of 500-1500 m and truss stiffening girders. As parameters of calculation, assumed the height Z of stiffening girders and the reference wind velocity $\overline{U}_{10}$ (mean wind velocity averaged over 10 min. at Z = 10 m) appropriately. The wind conditions over open sea are chosen as follows:

wind velocity at Z; $\overline{U}_Z = (\frac{Z}{10})^{1/7}\overline{U}_{10}$, roughness coefficient K; 0.003

spanwise cross correlation of u; $R_u(x_1, x_2; n) = \exp(-\frac{7n}{\overline{U}_Z}|x_1 - x_2|)$

The mechanical damping δ_o = 0.03 is assumed, and the aerodynamic damping effect is considered due to $P_I = -\frac{2C_D}{\frac{\pi}{2}}\frac{1}{\xi}$. The frequency response (aerodynamic admittance) of the drag to gust u is, due to Vickery's expression *2*),

$$|X_u(n)|^2 = \frac{1}{1 + 2(nD/\overline{U}_Z)^{4/3}} \tag{17}$$

Finally, the gust factor $G(x)$ to be computed is in the form of

$$G(x) = \sqrt{1 + g(x)\frac{\sigma_M(x)}{\overline{M}(x)}} \tag{18}$$

in which $g(x) = \sqrt{2ln[600\nu(x)]} + \frac{0.5772}{\sqrt{2ln[600\nu(x)]}}$; and $\overline{M}(x)$ is mean bending moment by mean wind load, $\sigma_M(x)$ the variance of bending moment by gusts and $\nu(x)$ the effective frequency. The results for a series of suspension bridges, of which the properties *10*) are chosen as follows, are given in Fig. 5.

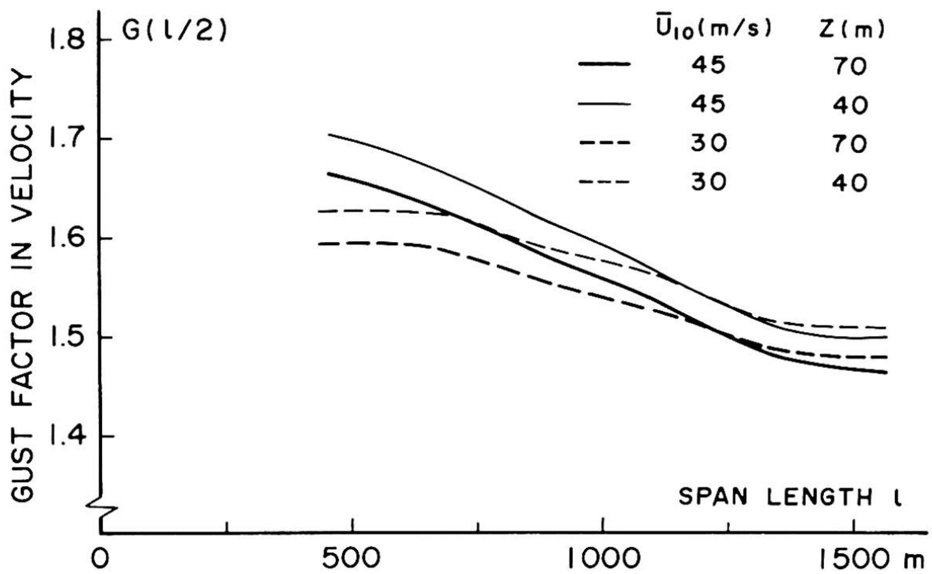

Fig. 5 GUST FACTOR IN WIND VELOCITY FOR ALONG-WIND (DRAG) RESPONSE - BENDING MOMENT - OF SUSPENSION BRIDGES

span length l	(m)	1500	1300	1000	800	650	500
width of truss B	(m)	36	36	33	33	33	33
height of girder D	(m)	14	14	11	11	8	8
sag dip	(m)	150	130	100	73	59	45
lateral rigidity $EI_h \times 10^{-9}$	(tm^2)	1.266	1.249	0.923	1.012	0.731	0.781
weight of truss w_t	(t/m)	20.28	20.05	19.32	19.66	19.01	19.01
weight of cables w_c	(t/m)	11.40	9.35	7.95	5.57	4.51	3.47
drag coefficient C_D		0.26	0.26	0.23	0.23	0.21	0.21
ratio of load (cables/truss)		0.261	0.233	0.214	0.184	0.167	0.148

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Summary

The gust responses of flexible structures are evaluated by taking into account unsteady aerodynamic forces due to the motion of the body itself. That is, under assumption of small intensity of turbulence, the aerodynamic forces are assumed to consist of the part due to the fluctuating components of turbulence and the unsteady part in smooth flow. As the unsteady aerodynamic force expression could include the effect of instability due to negative aerodynamic damping or vortex excitation phenomena as well, the general treatment of gust responses is probable. A couple of numerical examples show the validity of this concept.