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## Experimental Verification of the Dynamic Characteristics of a Free-Standing Tower

Vérification expérimentale des caractéristiques dynamiques d'une tour isolée

Experimentelle Nachprüfung der dynamischen Eigenschaften eines freistehenden Turmes

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### 1. INTRODUCTION

A free-standing, three-legged, latticed steel tower has been instrumented for studying its dynamic response to wind forces. This paper describes the location of the structure, the structural details, the mathematical representation of the structure, and the computed and experimentally determined dynamic characteristics of the tower. The tower is 150 feet high, and is located at the U.S. Marine Corps Air Station, Kaneohe Bay, on the island of Oahu, Hawaii. Permission for its use in this investigation was granted through the courtesy of the Station because this tower is part of a transmitting system that will be replaced and the tower is no longer needed in the present communication system.

A mathematical model of the tower was formulated from structural detail drawings of the structure. The first three translational frequencies and mode shapes were calculated for two orthogonal horizontal directions. The tower was then subjected to man-excited oscillations, and the fundamental frequency and critical damping ratio for the first mode were determined, for the same directions, from accelerograms. Finally, a Fourier Analysis was performed on the accelerograms for the tower under ambient conditions, and the first two periods were obtained for both directions. There was good agreement among the predicted and experimentally determined natural frequencies.

### 2. DESCRIPTION OF THE TOWER

Figure 1 shows the tower as seen from the north. The horizontal coordinate directions in Table 1 show that the x direction points to the west and the y direction to the south. Each leg of the tower is rigidly attached to a concrete footing at the base. The three concrete footings extend approximately 10 feet below the ground and are approximately 11 feet square at the bottom. Details concerning wind conditions at the tower site are given in Ref. [1].

The tower is fabricated from steel angles and is triangular in cross-section with 14 panel points. It is 25 feet wide at the base and tapers at a constant slope from the base to the third panel point which is 143 feet above the base. From the third panel point to the top, the tower width is constant at 4 feet. Riveted connections are used throughout the structure. All connections and structural members are relatively rust-free because all components were galvanized prior to construction. Figure 2 shows the structural details of the individual members of the structure.

Table 1 lists the projected areas and masses that were assumed to be concentrated at panel points for the mathematical model. Additional steel plates were welded on the horizontal struts at the two levels of the existing platforms to minimize the effect of torsional movements that could be induced because of eccentric mass distributions caused by the platforms.

### 3. INSTRUMENTATION

Open grating platforms were constructed in the center of the tower for placing accelerometers and anemometers at the five levels shown in Figure 2. A total of 10 Systron-Donner, Model 420, force balance type accelerometers were placed at each of the five levels of the tower as shown in Figure 3. Two accelerometers were mounted in each casing parallel to the x and y axes shown in Figure 2. Two R.M. Young, Model 27101X, Gill Anemometers were also placed at each of the five levels to measure the x and y components of the wind velocity as shown in Figure 4.

Analog signals from these accelerometers and anemometers were relayed via shielded cables to a Multiplexer-Analog/Digital Converter Unit (Redcor Corp., Model 720), and the resulting digital information was recorded on a 9-track, 800 BPI, magnetic tape by a Magnetic Tape Recorder (Cipher Data Products Model 70H). The Multiplexer-Analog/Digital Converter and Magnetic Tape Recorder System is housed in a building approximately 200 feet from the tower, and the front panel of the system is shown in Figure 5.

### 4. MATHEMATICAL MODEL AND COMPUTED RESULTS

The tower was idealized as a space truss, and the structural members were assumed to resist axial loads only. Horizontal loads were assumed to be applied only at panel points, and secondary stresses were assumed to be negligible. Under these assumptions, the column elements of the flexibility matrix were computed by successively applying a unit load horizontally at each panel point in the x or y direction.

For the dynamic analysis, the tower was modeled mathematically as a discrete system of fourteen masses lumped at the panel points as listed in Table 1. The mass between midheights above and below a panel point was concentrated at that point and assumed to be located at the centroid of the horizontal cross section. The mass may move in the horizontal plane designated by the x and y axes of Figure 2, but vertical motions were considered negligible. Motions in the horizontal directions were assumed to be uncoupled. The mass matrix was diagonal and identical for motion in both x and y directions. The base of the tower was assumed to be rigid; therefore, the entire tower was assumed to behave as a lumped mass cantilever beam.

The first three translational natural periods and corresponding mode shapes in each direction for this tower were computed on the IBM 360, Model 65, digital computer at the University of Hawaii. These periods were 0.537 sec, 0.169 sec, and 0.084 sec in the x direction, and 0.496 sec, 0.158 sec, and 0.080 sec in the y direction. The corresponding mode shapes are given in Table 2.

### 5. EXPERIMENTAL RESULTS

The tower was subjected to man-excited oscillation in the x and y directions according to the method discussed by Hudson, *et. al.* [2], to obtain natural periods of vibration of the tower. This experiment was repeated several times on different days of fairly calm wind conditions, so that damping effects could also be estimated, and the results were averaged. Figure 6 shows portions of accelerograms obtained at the top instrumentation

level (150 ft) for motions in the x and y directions. These records substantiate the assumption that vibration in the two orthogonal horizontal directions are uncoupled. Vertical and horizontal motions of the supports were not discernable from response records taken at the bases and hence the assumption of a rigid foundation is valid.

It was fairly easy to force the tower into vibrating in a purely fundamental mode as shown in Figure 6. From records such as these, the fundamental periods of vibration were determined for the two directions. It was difficult to force the tower into vibrating purely in the second or third modes. Figure 7 shows the responses caused by wind. The fundamental mode predominates although the second and third modes are discernable occasionally at relatively much smaller amplitudes of vibration. Thus, close inspection of the chart records for small ambient amplitudes of vibration permits estimates of these higher modes.

Table 3 shows good agreement among the predicted and experimentally determined natural periods of vibration for the first three translational modes. The periods determined from field data were 0.54 sec, 0.18 sec, and 0.08 sec for the x direction, and 0.50 sec, 0.16 sec, and 0.09 sec for the y direction.

Table 4 compares the experimental and predicted first mode shape. The experimental points were determined by taking ratios of the simultaneous amplitude of the accelerogram records for the various levels. All mode shapes are normalized with respect to the top level.

Critical damping ratios for the fundamental modes of vibration from the man-excited records were estimated using the logarithmic decrement method that is applicable when damping is small. Damping for the fundamental mode was estimated from several sections of the field data. The averaged critical damping ratios were approximately 0.3 and 0.5 percent of critical in the x and y directions, respectively.

A ladder which was attached to the leg of the tower along the y direction probably contributed to the difference in natural periods and critical damping ratios between the two horizontal directions. This ladder probably slightly increased the stiffness and energy absorption capacity along the y direction.

## 6. FOURIER ANALYSIS OF AMBIENT VIBRATION RECORDS

Digitized accelerograms on magnetic tape were subjected to Fourier analysis to determine the natural periods of vibration of the tower. Each continuous accelerogram consisted of 1023 data points taken at intervals of 0.025 seconds, for a time length of approximately 25.55 seconds. The resulting Nyquist frequency of 20 Hz is thus well above the first three frequencies of the tower in either direction. Similar studies have been reported recently by Murota and Ishizaki for a tower [3], and by Van Koten [4], and Trifunac [5] for buildings.

Although the natural periods could be obtained from the analysis of only one record in each direction, the frequency resolution of approximately 0.04 Hz was not considered satisfactory for this analysis. Thus longer records consisting of four sequential records connected together were used for both x and y horizontal directions. Because two data points between each individual record were lost due to a time gap between each record, these values were assumed to be equal to the mean for this analysis. While this procedure may introduce slight errors [6], its effect on the estimate of the lower frequencies in the Fourier Amplitudes was considered small enough to yield satisfactory values in this range.

The total time of each lengthened record corresponded approximately to 100 seconds, thus giving a frequency resolution of 0.01 Hz. Such lengthened accelerograms for the accelerometers at the top of the tower were subjected to computer analysis. A Fast Fourier Transform Program developed by Sande [7] was used to obtain the Fourier coefficients of the accelerograms. The coefficients were then modified by the formula:  $(F_i)_M = 0.25 F_{i-1} + 0.50 F_i + 0.25 F_{i+1}$ , where  $(F_i)_M$  and  $F_i$  are respectively the modified and original coefficients of point  $i$ . Figure 8 shows the squared modified Fourier coefficients plotted as a function of frequency.

From this figure, the two lowest translational frequencies in the x direction are 1.88 and 5.48 Hz, corresponding to natural periods of 0.53 and 0.18 seconds. For the y direction they are 2.03 and 5.85 Hz, or natural periods of 0.49 and 0.17 seconds. These are in agreement with values determined by other methods described previously (Table 3). The third translational frequencies were barely discernable in Figure 8; a torsional frequency of 7.39 Hz is also evident.

## 7. CONCLUDING REMARKS

The mathematical model of the tower, with the flexibility coefficients derived by assuming an idealized space truss and with lumped masses at the panel point, is adequate. Experimental data substantiated the assumptions of a rigid foundation and of essentially uncoupled motions in the horizontal orthogonal directions. Previous studies showed that three translational modes are adequate for dynamic analysis of free-standing towers that do not have eccentric distributions of masses.

The experimental data showed that the fundamental mode of vibration predominates in free-standing structures such as the tower used for this study. There was good agreement among the predicted and experimentally determined natural translational periods. Results from the Fourier analysis of ambient vibration records corroborated these periods. The tower has a fairly low damping ratio for the fundamental mode of vibration.

## 8. ACKNOWLEDGMENTS

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## SUMMARY

The dynamic characteristics of a free-standing latticed tower, obtained from analysis and field experiment, are discussed. Results from a Fourier analysis of ambient response records are also presented. There was good agreement among the three sets of answers for the translational frequencies.



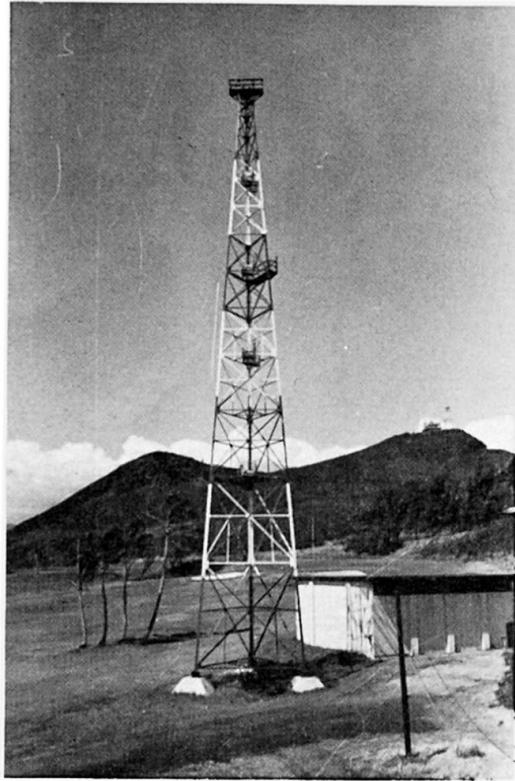


FIG. 1-NORTH VIEW OF TOWER

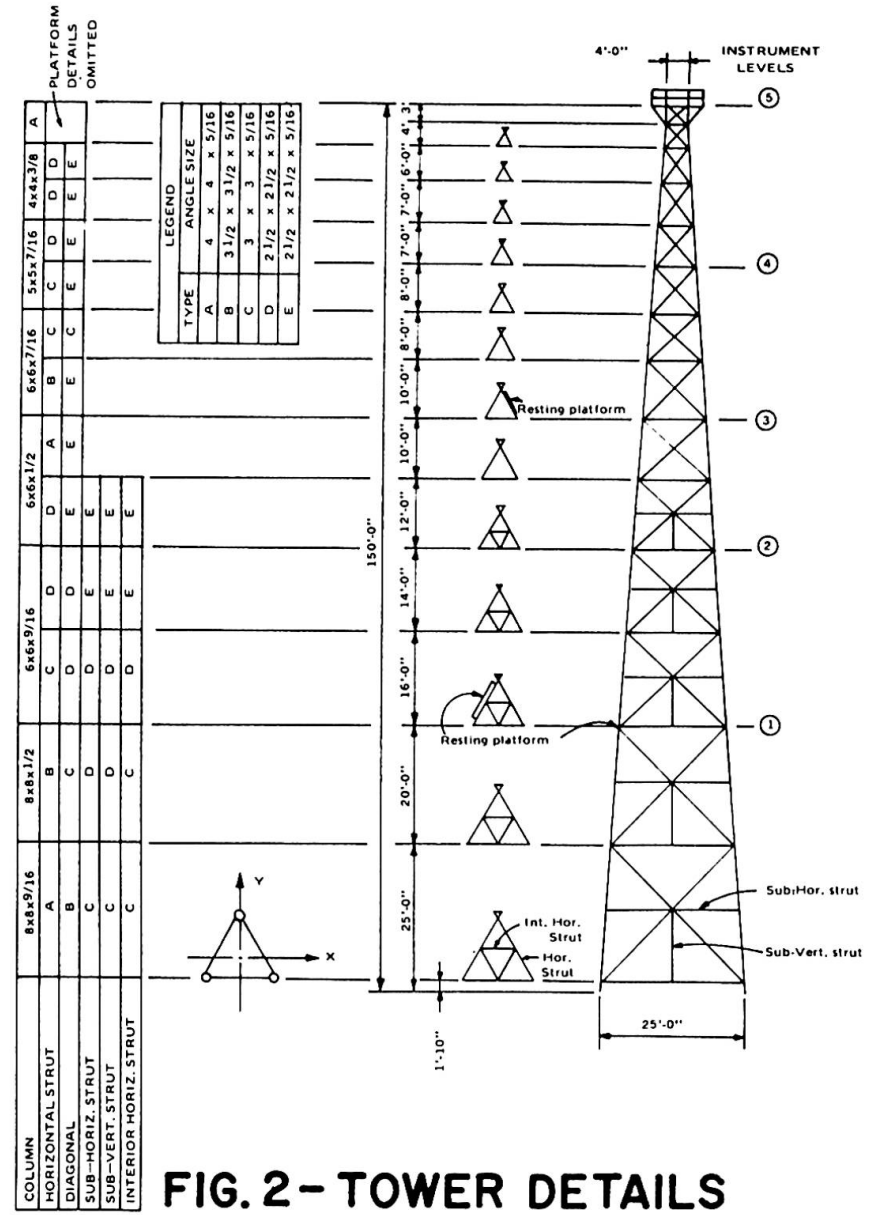
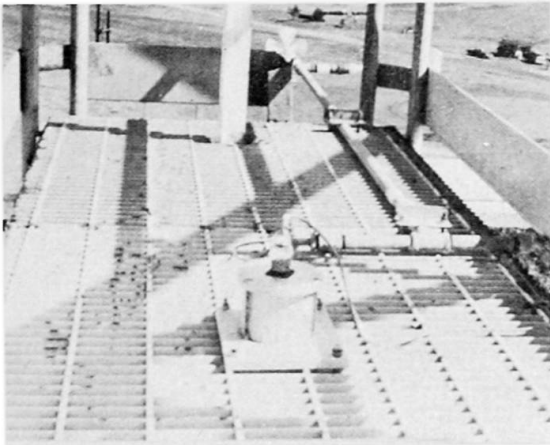
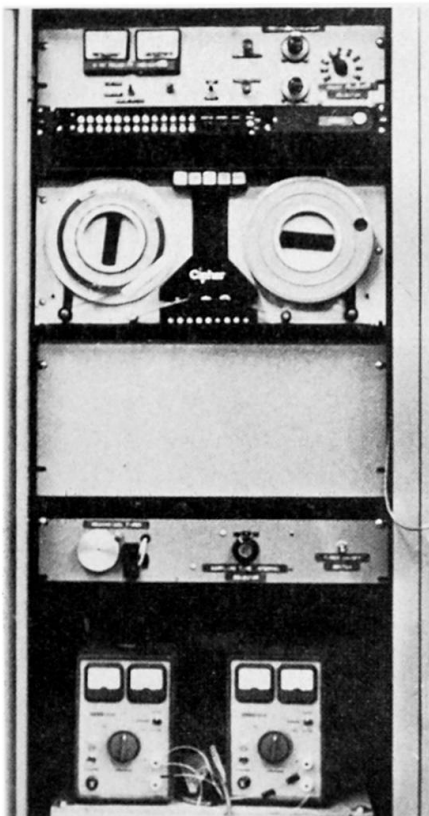


FIG. 2-TOWER DETAILS



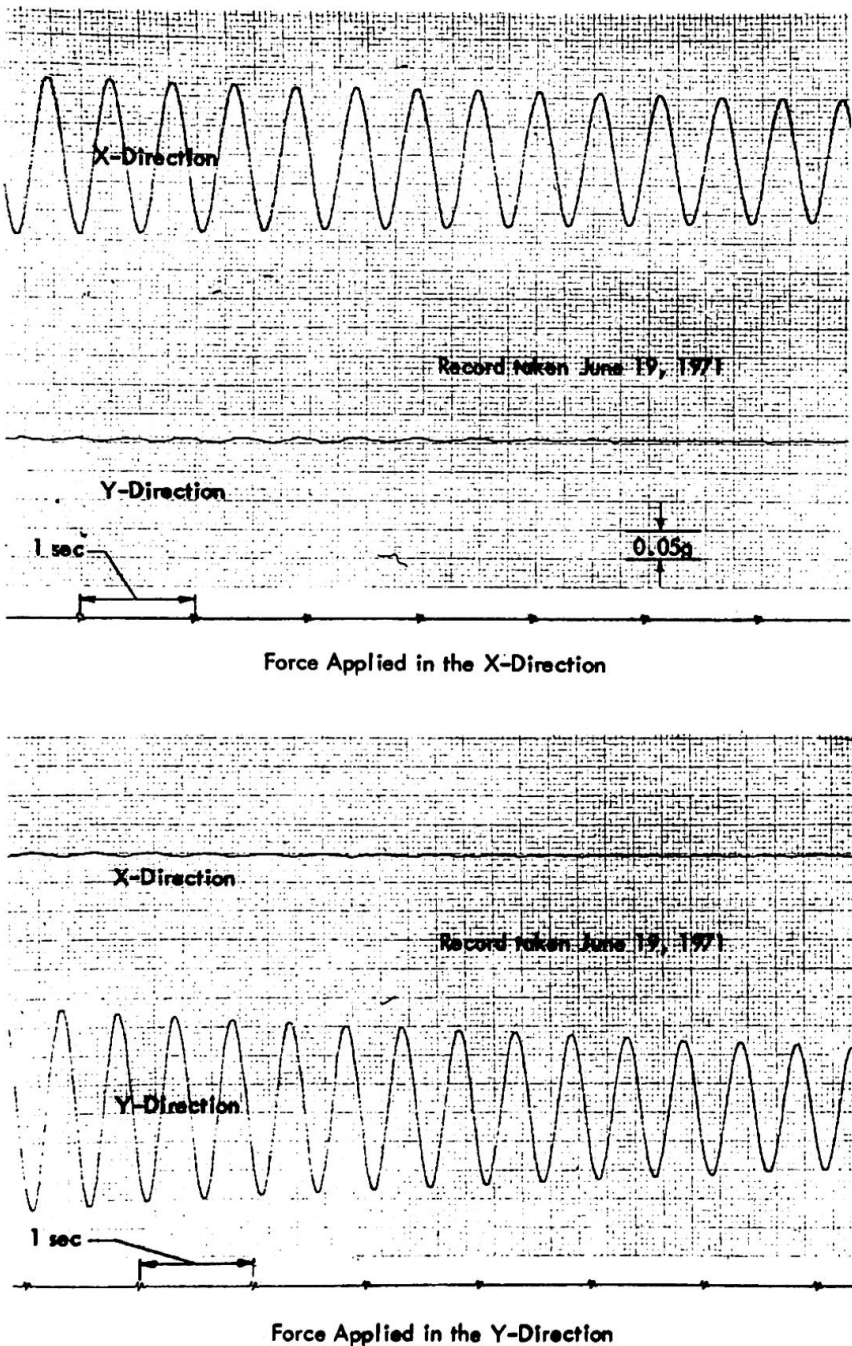
**FIG.3 – LOCATION OF  
ACCELEROMETER**

**FIG.4 – ANEMOMETER  
IN PLACE**

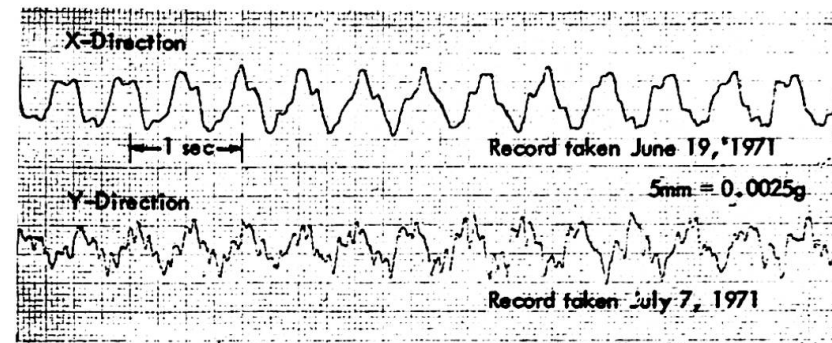


**FIG.5 – FRONT VIEW OF INSTRUMENT  
PANEL**

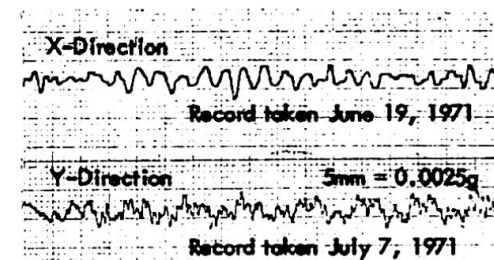




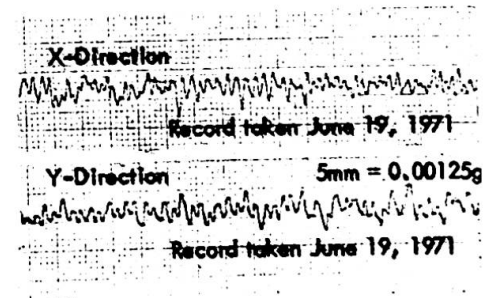
**FIG. 6 - MAN-EXCITED RESPONSE RECORDS**



**FIRST MODE**



**SECOND MODE**



**THIRD MODE**

**FIG. 7 - AMBIENT RESPONSE RECORDS**

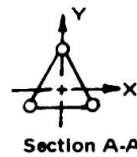
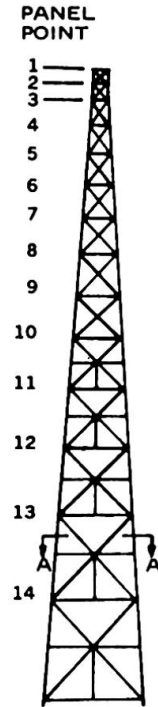
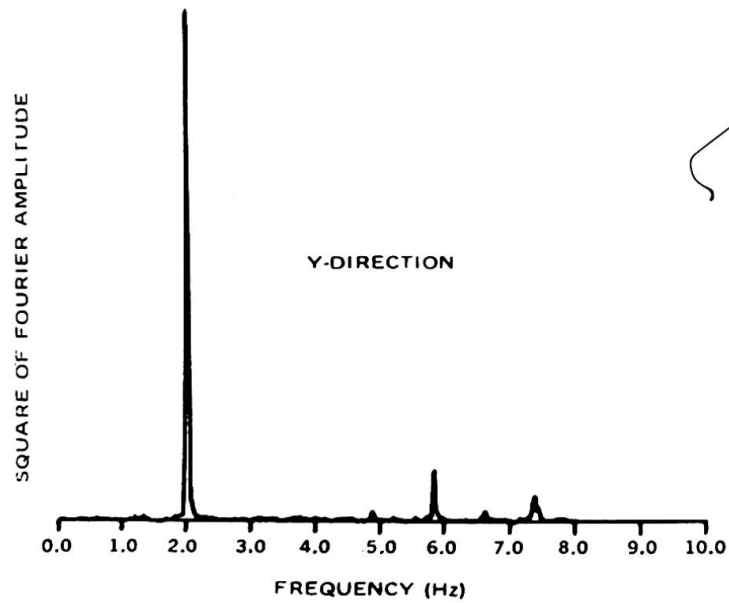
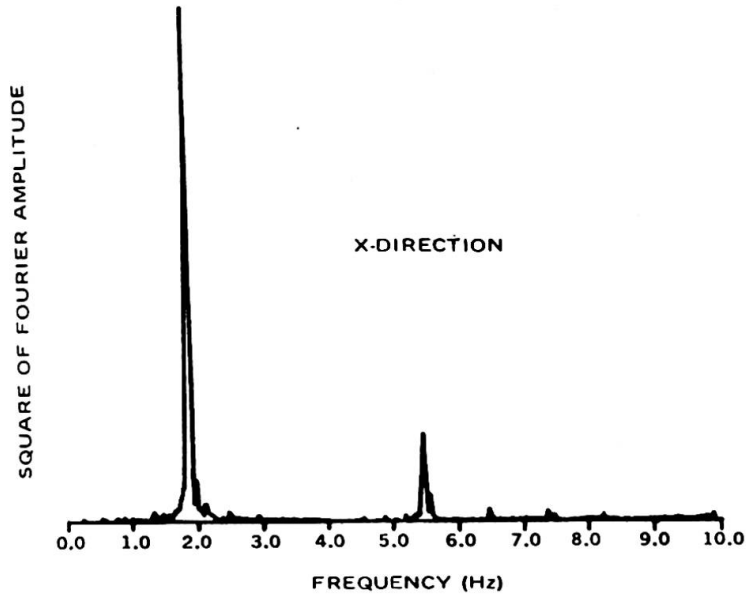


TABLE 1  
CONCENTRATED MASSES AND PROJECTED AREAS

Panel Point	Elev. (ft)	Projected Area (sq.ft.)	D.L. (LB.) at Panel Point
1	150	2.87	1810
2	147	5.22	510
3	143	8.06	750
4	137	8.75	810
5	130	10.32	920
6	123	11.97	1790
7	115	14.12	1190
8	107	16.76	1360
9	97	19.30	2940
10	87	23.32	1900
11	75	28.08	3580
12	61	32.35	2940
13	45	43.45	5670
14	25	60.37	5240

NOTE: 1. Values for projected area are for one face only.  
2. Values for D.L. are for entire tower.

FIG. 8 - FOURIER AMPLITUDE SPECTRA

TABLE 2

COMPUTED NORMALIZED MODE SHAPES  
(Normalized with respect to bottom panel)

Panel Point	X DIRECTION			Y DIRECTION		
	First Mode	Second Mode	Third Mode	First Mode	Second Mode	Third Mode
1	44.05	-6.16	1.93	39.93	-5.60	1.78
2	42.19	-5.15	1.30	38.31	-4.73	1.23
3	39.69	-3.77	0.42	36.15	-3.59	0.52
4	36.11	-1.98	-0.56	33.00	-2.00	-0.41
5	32.15	-0.20	-1.41	29.50	-0.38	-1.14
6	28.46	1.25	-1.79	26.22	0.93	-1.56
7	24.55	2.49	-1.76	22.71	2.10	-1.63
8	20.98	3.35	-1.43	19.47	2.95	-1.44
9	16.94	3.99	-0.87	15.82	3.61	-0.90
10	13.39	4.14	-0.06	12.56	3.83	-0.21
11	9.71	3.94	0.80	9.18	3.72	0.63
12	6.25	3.26	1.46	5.96	3.13	1.29
13	3.32	2.23	1.66	3.19	2.20	1.56
14	1.00	1.00	1.00	1.00	1.00	1.00

TABLE 3

COMPUTED AND EXPERIMENTALLY DETERMINED NATURAL PERIODS OF VIBRATION

Mode	Period (Sec.)					
	Computed	X DIRECTION		Computed	Y DIRECTION	
		Experimental			Experimental	
		(a)	(b)		(a)	(b)
1	0.537	0.54	0.53	0.496	0.50	0.49
2	0.169	0.18	0.18	0.158	0.16	0.17
3	0.084	0.08	--	0.080	0.09	--

a - Chart records; b - Fourier Analysis

TABLE 4

COMPARISON OF COMPUTED AND EXPERIMENTALLY DETERMINED FIRST MODE SHAPE

Instrument Level	Elevation (Ft.)	X DIRECTION		Y DIRECTION	
		Computed	Experimental <sup>a</sup>	Computed	Experimental <sup>a</sup>
5	150	1.000	1.00	1.000	1.00
4	123	0.646	0.67	0.656	0.71
3	97	0.385	0.40	0.396	0.45
2	75	0.220	0.25	0.230	0.28
1	45	0.075	0.10	0.080	0.11

a - Chart records