Oscillation of Cylindrical Structure in Wind

Oscillation de structures cylindriques sous l'effet du vent

Schwingungen zylindrischer Bauwerke unter Windeinfluss

NOBUHIRO UKEGUCHI   HIROSHI SAKATA   HIROKI TANAKA
Dr. Eng. Manager   Dr. Eng. Senior Research Engineer   Research Engineer

Aero-Hydraulic Research Laboratory
Mitsubishi Heavy Industries, Ltd.
Nagasaki Technical Institutes
1-1 Akunoura-machi, Nagasaki, Japan

1) Introduction

It is well-known that tall and slender structures like smokestacks, towers, etc. oscillate in wind. Many researches have been made to predict the oscillation of structures. The prediction, however, could not always be made successfully. There are many stacks in Japan for example and only a few percents of them were subjected to the oscillation.

From this fact, it can be said that the oscillation of a cylinder is easily effected by wind conditions and structural conditions. However, the effect of such conditions should be ascertained for safety and economical designs.

This paper deals with the oscillation of the cylindrical structures.

2) Review of researches conducted

Formerly, the oscillation of a circular cylinder was researched with an interest in an acoustic field known as Aeolian tone and was considered to be a forced-oscillation.

After that, a theory of random vibration was introduced and fluctuating lift forces on a cylinder were measured by Fung (1). The comparisons of experimental results based on the random vibration with the oscillations of actual stacks were made by Fujino and Nakagawa(2), Smith and McCarthy(3), and they had good agreements. In some special cases, however, the amplitude of oscillation of actual stack exceeds a result of experiment based on the random vibration.

Recently, the oscillation has been and is considered to be a self-excited or self-controlled oscillation. Lift forces on

a cylinder vary with the oscillation of cylinder and varied lift forces oscillate the cylinder, therefore the system of the oscillation is considered to have a feed back circuit.

Koopmann(4) showed that the so-called "locking in" region, in which frequencies of vortex shedding and cylinder oscillation coinside, is widened with an increase in amplitude of oscillation and vortex become less irregular. Toebes(5) ascertained fluctuating fluid flows behind a cylinder in stationary and oscillating conditions and also made sure of the correlations of flow along a cylinder axis. Bishop and Hassan(6), Ferguson and Parkinson (7) disclosed the phase difference of lift forces from cylinder oscillation. Keefe(8), Gerrard(9) showed pressures on a cylinder.

Reynolds number is an important factor in the oscillation of a structure. Roshko(10) revealed the features of aerodynamic forces at very high Reynolds number. Jones, Cincotta and Walker (11) showed the oscillating lift forces on an oscillating cylinder at high Reynolds number. Achenbach(12) made sure of the pressure distribution around a cylinder. As far as the oscillating lift force is concerned, the features of lift force are similar to that at sub-critical Reynolds number.

For a practical reason, Scruton and Rogers(13),(14) have widely investigated the oscillations of cylindrical structures and devices for suppressing the oscillation. Fiedler and Wille (15) showed flows in near wake of a cylinder standing on a surface and three-dimensionarities. Vickery(16) also ascertained three-dimensionarities and effects of shear flow which is boundary layer on a ground. Wootton(17) researched smokestack oscillations at high Reynolds number and showed a boundary of random vibration and regular vibration. Cooper and Wardlaw (18), and Vickery(19) researched on the oscillation of circular cylinder in the wake of other cylinder and revealed that the amplitude of oscillation is very large as compared with the oscillation of isolated cylinder.

According to the above mentioned researches, the oscillation of a cylinder may be a combined oscillation of self-excited and random oscillations.

3) Aerodynamic lift force on a cylinder

Lift forces have been measured by many researchers as mentioned in the previous section, and the literatures(6),(7) show lift forces on an oscillating cylinder. However, no research has been conducted to reveal how the lift forces change with each amplitude of cylinder oscillation.

The lift forces at each amplitude of oscillation have the most important role in the self-excited oscillation. Therefore, an experiment was made to get the features of lift force at each amplitde. For details of an experimental results, refer to the literature(20).

A wind tunnel was used in this experiment. The test section of wind tunnel is 1.3 meter in height and 1.0 meter in width. A cylinder of 0.15 meter in diameter and 1.0 in length was mounted on a mechanical oscillator and lift forces on the

oscillated cylinder were measured with strain gauges attached to supports. Inertia force of the model was electrically taken away by means of accelerometer.

Aerodynamic lift force has two different kinds of frequencies as shown by Toebes(5). One of which is the frequency of cylinder oscillation and the other is the frequency of Karman vortex, however, they coinside with each other in locking in region.

Absolute values of the lift forces are shown in figure 1. The lift force represents only a component having a same frequency as the mechanically oscillated cylinder has, because they were calculated through Fourier series expanding of the original lift forces.

The lift forces increase with the amplitude of an oscillated cylinder and they become maximum at Strouhal number about 0.2. The maximum points, however, move to the region of smaller Strouhal number. The locking in region was not clear, because wave forms of the lift forces were irregular, but it is considered to be between dotted lines in figure 1.

A calculation was made of standard deviations of the lift forces which mean fluctuations of lift force in each cycle, in other words, irregularities of lift force. Figure 2 shows the standard deviations. The irregularities increase with a decrease of lift force and decrease at the region where Strouhal number is near to 0.2.

Phase differences of the lift force from cylinder oscillation are shown in figure 3. There are abrupt changes in phase angles in neighbourhood of Strouhal number 0.2, and the points of abrupt changes seem to coinside with the maximum points of lift forces. The phase angles are qualitatively similar to those of literatures(6),(7), but the experiment made by Bishop and Hassan(6) showed more abrupt changes. Causes of the difference are not sure but less abrupt change might be caused by irregularities of lift forces.
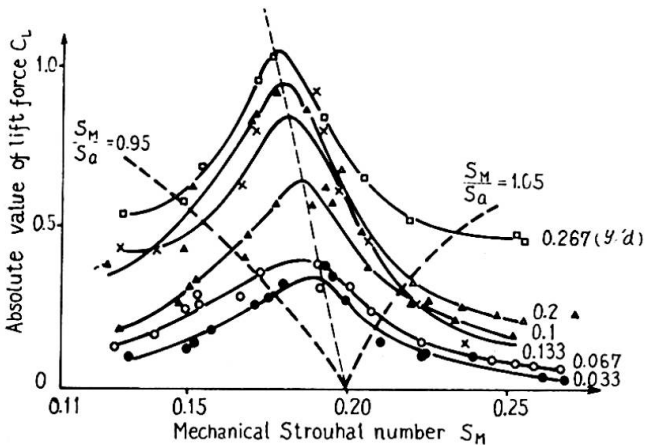

Fig.1 Absolute value of
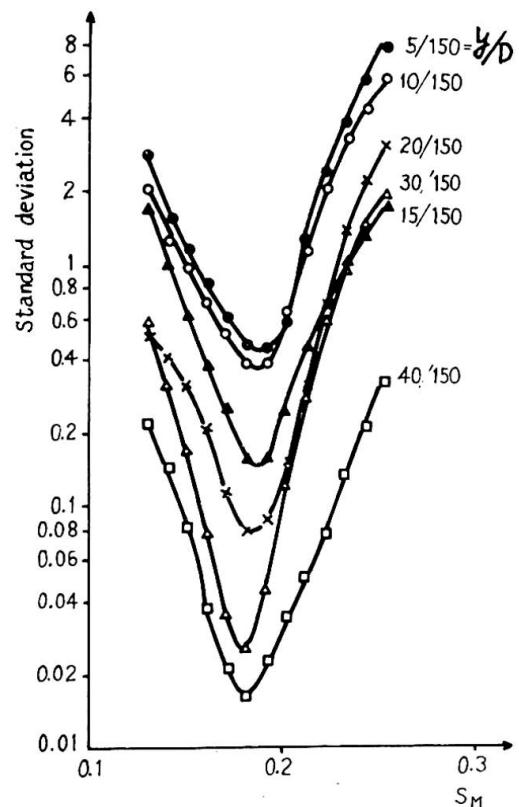   lift force

Fig.2 Standard deviation
       of lift force

Figure 4 shows imaginary part of lift forces which are out of phase components against the cylinder oscillation. Peak values of imaginary part versus each amplitude of the oscillated cylinder are shown in figure 5. The imaginary part does not always increase with the amplitude of oscillated cylinder, while the absolute values increase.

Lift forces obtained from free oscillation test with a cyliner on springs are also shown in figure 5. In the calculation of lift force from the oscillation test, the maximum amplitudes were used, while lift forces through the forced oscillation test were calculated as mean values. That is the reason why the result of free oscillation test showed larger forces especially at small amplitude region, and the difference might be caused by irregularities of lift forces.

4) Result of oscillation test in peculiar case

In a case of bluff cylinder, it is heard that amplitudes of oscillations versus wind velocity have hysteresis, which means that the amplitudes have different values in increasing and decreasing velocity processes. These phenomena some times occur especially in a circular cylinder with some accessories on it.

A circular cylinder with longitudinal strakes shown in figure 6 was supported on a spring system in wind tunnel. Mass of the model is 5.03 kilograms and logarithmic damping factor is 0.03. A result of the test is shown in figure 6. Solid line shows the maximum values of amplitude. When the oscillation suppressed, the amplitude does not increase
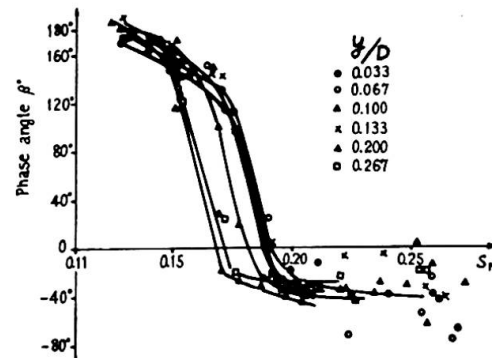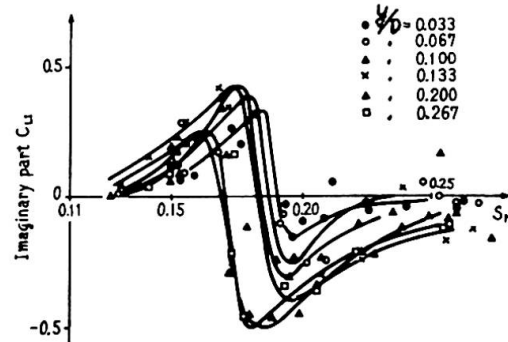
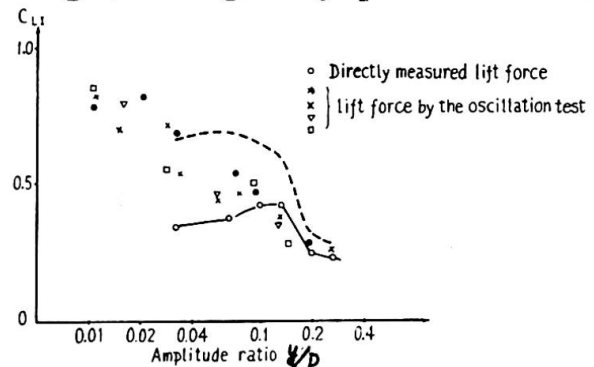Fig.3   Phase angle

Fig.4   Imaginary part of lift

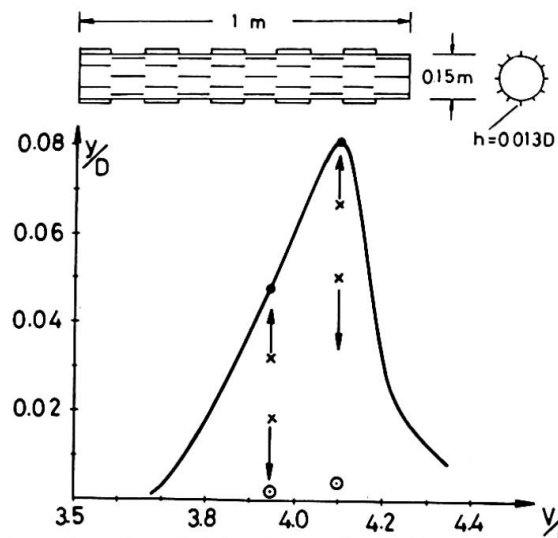Fig.5   Peak value of imaginary part in lift force

Fig.6   Amplitude of cylinder oscillation with strakes

after relieving the cylinder
from the suppression.
A combination of cross mark
( ) and arrow symbol indicates
the amplitude increases or
decreases from this initial
amplitude. This shows that
there are two stable points
on the amplitude of oscillation.

A cylinder with trip wires
of 0.06D in diameter winded
around it was also tested. The
model is shown in figure 7 and
mass of the model is 28 kilo-
grams and logarithmic damping
factor is 0.028. A result of
the test is shown in figure 7.

Solid line shows the ampli-
tude of oscillation without any
external disturbance. Once, an
external disturbance acts on the
cylinder, the amplitude becomes
great and after that, traces the
dotted line which is nearly
equal to an amplitude of a cyl-

Fig.7   Amplitude of cylinder
        oscillation with
        trip wires

inder with a smooth furnace. A cylinder with trip wires of
0.1D in diameter, however, did not oscillated even after a great
external disturbance.

In the former case, another stability point was found by
suppressing the oscillation, but in the other case, another sta-
bility point was detected by giving the external disturbance.

5) Equation of oscillation under lift forces

There are two different kinds of lift forces as mentioned
in section 2) and 3). One of which is lift forces induced with
a cylinder oscillation, and it causes a self-excited oscillation.
The other is the so-called fluctuating lift force, and it induces
a random vibration.

In a locking in region, a cylinder usually oscillates with
its natural frequency, and in that case, real part of lift
forces can be neglected as com-
pared with inertia force of the
cylinder.

Now, let us consider an
equilibrium of cylinder oscil-
lation as shown in figure 8.
The real part and imaginary
part of lift forces are os-
cilating with the natural fre-
quency and so they do not move
or circulate on the diagram in
figure 8, however, fluctuating
lift force is circulating free-
ly.

Fig.8   Diagram of cylinder
        oscillation

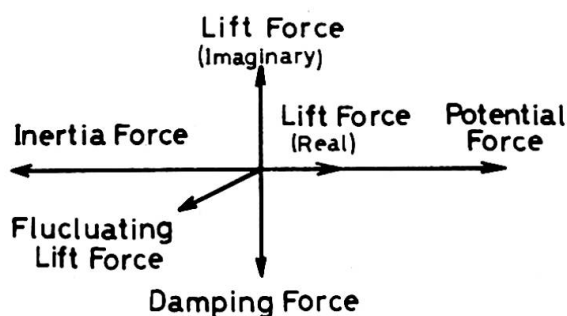$$= -L_o y_o \omega_o \int_0^{\frac{2\pi}{\omega_o}} e^{i\omega_o t} e^{i\omega_o t} dt \int f(x) g(x) dx$$

$$= -\pi L_o y_o \int f(x) g(x) dx \quad \text{—————— (4)}$$

4.4) Energy equilibrium

The oscillation of a cylinder is maintained by the above three energies, and flow of the energies per unit time must be balanced.

The energy variation is the following.

$$\frac{dT}{dt} - \frac{dW_D}{dt} + \frac{dW_a}{dt} = 0 \quad \text{—————— (5)}$$

From the equations(1),(2),(4)

$$\frac{dT}{dt} = \omega_o^2 M y_o \frac{dy_o}{dt} \qquad \frac{dW_D}{dt} = W_D \frac{\omega_o}{2\pi} = -\frac{1}{2} M \omega_o^3 g y_o^2$$

$$\frac{dW_a}{dt} = W_a \frac{\omega_o}{2\pi} = -\frac{1}{2} L_o y_o \omega_o \int f(x) g(x) dx$$

Putting the above equations into the equation(5) the following equation can be get

$$\omega_o^2 M y_o \frac{dy_o}{dt} + \frac{1}{2} \omega_o^3 M g y_o^2 - \frac{1}{2} L_o y_o \omega_o \int f(x) g(x) = 0 \quad \text{——— (6)}$$

Simplifying a little more, then

$$\frac{dy_o}{dt} + \frac{1}{2} \omega_o g y_o - A L_o = 0 \quad \text{—————— (7)}$$

where $\quad A = \dfrac{1}{2M\omega_o} \int f(x) g(x) dx$

4.5) Random vibration

If the lift force(L_o) is independent of cylinder oscillation, the equation(7) becomes a linear differential equation.

$$L = \sum L_n e^{i\omega_n t} \fallingdotseq \sum_j \frac{L_{N+J} + L_{N-J}}{2} e^{i\omega_j t} e^{i\omega_o t}$$

$$= \sum_j L_j e^{i\omega_j t} e^{i\omega_o t}$$

Then, the equation(7) becomes as follows.

$$\frac{dy_o}{dt} + \frac{1}{2} \omega_o g y_o - A \sum_j L_j e^{i\omega_j t} = 0$$

The solution is

$$y_o = -e^{-\int \frac{1}{2}\omega_o g dt} \int A \sum_j L_j e^{i\omega_j t} e^{\int \frac{1}{2}\omega_o g dt} dt$$

$$= -\sum \frac{2 A L_j e^{i\omega_j t}}{g\omega_o + 2 i g \omega_o} \quad \text{—————— (8)}$$

The mean value is

$$\frac{1}{T} \int_o^T y_o^2 dt = \sum \frac{2 A^2 L_j^2}{(g\omega_o)^2 + (2\omega_j)^2} \quad \text{—————— (9)}$$

This is the same equation as an approximate equation of random oscillation.

When the vector of fluctuating lift force directs upward, an amplitude of cylinder oscillation becomes larger and when it directs downward, the amplitude becomes smaller.

If a frequency of the fluctuating lift force is close to the natural frequency of cylinder oscillation, the vector of fluctuating lift force circulates slowly on the diagram. However, when the frequency is far from the natural frequency, the vector circulates quickly. In this case, the fluctuating lift force has no effect on the cylinder oscillation, because the inertia force is so great as compared with the lift force that the lift force can not increase the amplitude of oscillation quickly.

With the above assumption, the following equations are considered.

amplitude of oscillation $\quad y(t) = y_0(t)e^{i\omega_0 t}$

variation of amplitude with time $\qquad \dfrac{dy(t)}{dt} = i\omega_0 y_0(t)e^{i\omega_0 t}$

where $\qquad \dfrac{dy_0(t)}{dt} e^{i\omega_0 t} \ll i\omega_0 y_0 e^{i\omega_0 t}$

An energy of oscillation is stored in the system as a kinetic energy but the damping force of a structure always discharges the energy while the lift force is supplying the energy. Unfortunately however, the energy supply is not always constant with time.

5.1) Kinetic energy

Kinetic and potential energies are exchanging each other in each cycle of oscillation and so total energy is expressed using an amplitude of oscillation.

$$T = \frac{1}{2}\omega_0^2 y_0^2 \int mf^2(x)dx = \frac{1}{2}\omega_0^2 y_0^2 M \qquad\qquad (1)$$

5.2) Scattering energy due to damping force

$$\delta W_D = D\frac{dy}{dt}dy = iM\omega_0^2 gy_0 e^{i\omega_0 t}dy$$

Scattering energy in one cycle is

$$W_D = \oint \delta W_D = iM\omega_0^2 gy_0 \int_0^{\frac{2\pi}{\omega_0}} e^{i\omega_0 t}\frac{dy}{dt}\cdot dt = -M\omega_0^3 y_0 \int_0^{\frac{2\pi}{\omega_0}} e^{i\omega_0 t}e^{i\omega_0 t}dt$$

$$= -\pi M\omega_0^2 gy_0^2 \qquad\qquad (2)$$

5.3) Energy supply due to lift force

Real part of the lift force does not supply the energy, and so only imaginary part is considered.

$$L(t) = iL_0(t)e^{i\omega_0 t}g(x) \qquad\qquad (3)$$

$$\delta W_a = L(t)dy = iL_0 e^{i\omega_0 t}dy\int f(x)g(x)dx$$

Energy supply in one cycle is

$$W_a = \oint \delta W_a = iL_0 \int_0^{\frac{2\pi}{\omega_0}} e^{i\omega_0 t}\frac{dy}{dt}dt \int f(x)g(x)dx$$

5.6) Oscillation with linear lift force

The lift force linear to the amplitude of oscillation can be expressed as follows.

$$L_0(t, y_0) = C y_0 + \sum_j L_j e^{i\omega_j t}$$

Then, the equation(7) becomes as the following.

$$\frac{dy_0}{dt} + (\frac{1}{2}\omega_0 g - AC) y_0 - A \sum_j L_j e^{i\omega_j t} = 0 \qquad\qquad (10)$$

In this case, the oscillation changes as if the damping force of the system varied and if $2AC > \omega_0 g$, flutter occurs.

5.7) Oscillation with non-linear lift force

The lift force can be expressed as the following.

$$L_0(t, y_0) = L_0(y_0) + \sum_j F_j e^{i\omega_j t} = 0$$

The equation(7) becomes as the following.

$$\frac{dy_0}{dt} + \left\{ \frac{1}{2}\omega_0 g - A L_0(y_0) \right\} - A \sum_j L_j e^{i\omega_j t} = 0 \qquad\qquad (11)$$

This is the equation of oscillation combined with self-excited and random ones.

6) Discussion

The lift forces on a smooth cylinder shown in figure 5 is the aerodynamic forces expressed by $L_0(y_0)$ in equation(11). The difference between lift forces through free oscillation test and forced oscillation test may be caused by the fluctuating lift forces represented by $L_j e^{i\omega t}$. Unfortunately, we did not measure the frequency components of the fluctuating lift force, but when the frequency is close to the natural frequency of cylinder oscillation, a total lift force becomes expressed by dotted line in figure 5. However, it looks like over estimated.

The lift forces $L_0(y_0)$ on a cylinder with accessories were not measured, but may be represented by solid line shown in figure 9, because there are two amplitude of oscillation. The amplitude of oscillation increases in the region where a lift force is greater than a damping force and vice versa, and so the point(1) is a stable neutral point and point(2) is unstable neutral point. If there is no fluctuating lift force, the cylinder does not oscillate at all or oscillates steadily with the amplitude of point(1). If there is a little fluctuating lift force, the lift force is added to the solid line and becomes as dotted line(1) but the added value depends on an absolute value of the fluctuating lift force and frequency component. In this case, the point (A) is a stable point and this oscil-
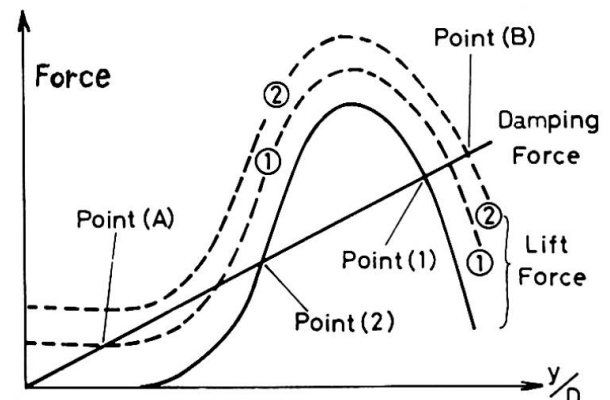

Fig.9   Oscillation mechanism of actual smokestack

lation can be considered a random vibration, because there is
not any regular oscillation. If there is a great fluctuating
lift force, the total lift force become like dotted line(2)
and in this case, the amplitude increases till point(B) and the
oscillation consists of random and regular oscillation.

Actual smokestacks or cylindrical towers have usually some
accessories e.g. ladders, platforms, etc. and so the lift force
against amplitude may be deformed even at very high Reynolds
number. That may be the reason why, on the actual structures,
some structures oscillate while the other similar structures
do not oscillate.

7) Notation

D : cylinder diameter
l : cylinder length
ρ : air density
v : wind velocity
L : lift force
C_L : lift coefficient $(L/2\rho v^2 Dl)$
y : amplitude of oscillation
f_0 : natural frequency
ω_0 : natural angular frequency
δ : Logarithmic damping factor

S_M : mechanical Strouhal number
I : imaginary part $(f_M D/V)$
K : kinetic energy
W_D : work due to damping force
W_q : work due to lift force
$f(x)$: mode of cylinder oscil-
lation
$g(x)$: mode of lift force
g : damping coefficient $(\delta/2\pi)$
f_M : frequency of mechanically
oscillated cylinder

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Summary

    Tall and slender structures have a trend to oscillate in
wind. The oscillation is usually in combinations of self-excited
and random oscillations, and the lift force varies with the amp-
litude of cylinder oscillation. The lift force on a cylinder
with accessories is so deformed that, some times, a trigger is
needed to oscillate the cylinder. Fluctuating lift force due to
turbulence in natural wind or in wake of other structures may
become the trigger.