Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 9 (1972)

Artikel: Double-layer space frame shells

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DOI: https://doi.org/10.5169/seals-9599

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Double-Layer Space Frame Shells

Coupole à deux nappes et à treillis

Zweischichtige schalenförmige Rahmen

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Introduction: The double-layer reticulated shells are being used with increasing frequency to span large areas because they are less sensitive than single-layer shells to the progressive instability. The interest in prefabbricated units for these double-layer shells is growing constantly. In the first section of this paper we present some types of prefabbricated units of standard size and shape for these structures. In the second section we suggest a simple procedure for evaluating the collapse-load taking into account plasticity and post-buckling behaviour for reticulated structures. This evaluation is the main requirement for a minimum-weight design.

I,1) Units of standard size for double-layer reticulated shells

Let us think a plate subdivided in elementar cubes whose edges are equal to the plate thickness t (fig. 1A). This elementar cube can be divided in five tetrahedra (this is possible in two different ways). One of these tetrahedra is regular its edges measuring $t\sqrt{2}$, (fig. 1B). Let us consider, now, as first step, instead of each cube its fundamental tetrahedron defined by the condition that two connected tetrahedra have the upper (or lower) edge perpendicular each other. As second step we substitute to the compact tetrahedron a reticulated one consisting of six bars laying along the tetrahedron edges. In this manner we have a reticulated plate structure formed by reticulated tetrahedra. Now we note that if we eliminate from the reticulated tetrahedron two bars (those corresponding to the diagonals of opposite faces in the elementar cube) we obtain four bars in the position shown in fig. 1B. This is the standard unit, with rigid joints, that we call: "standard farfalla". In the fig. 1C we show the assembling of four standard units, while in the foto 1 (top-left) it appears a specimen in which have been assembled several units.

This "standard farfalla", with some small variations, can be utilized for generating several double-curved reticulated shells. A first solution can be obtained assembling two types of farfallas

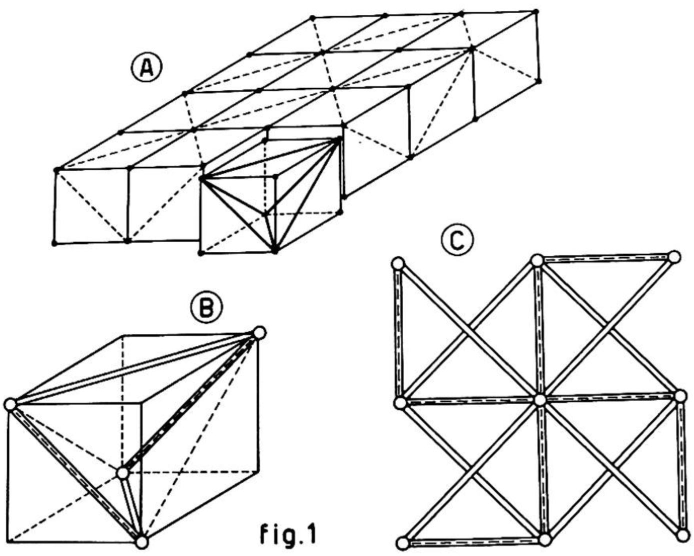


fig.1

one of which passess a bar shorter than the others, for generating double-layer cylindrical shells (foto 1 top-right).

The generation of a spherical double-curved shell starts from the truncated icosahedron inscript in a sphere (fig. 2a). This polihedron consists of 20 hexagons and 12 pentagons. Projecting the central points of these poligons on the spherical surface and connecting them with the vertexes of the corrispondent poligon, we obtain a triangulated net (fig. 2b) formed by 120+60 isosceles triangols of two types. Now we generate the double-layer spherical shell by four types of different farfallas (differents for side lenght), two of them having one side coincident with a side of two types of triangols. The other two are realized to complete the double-layer.

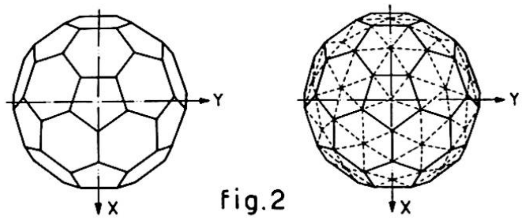
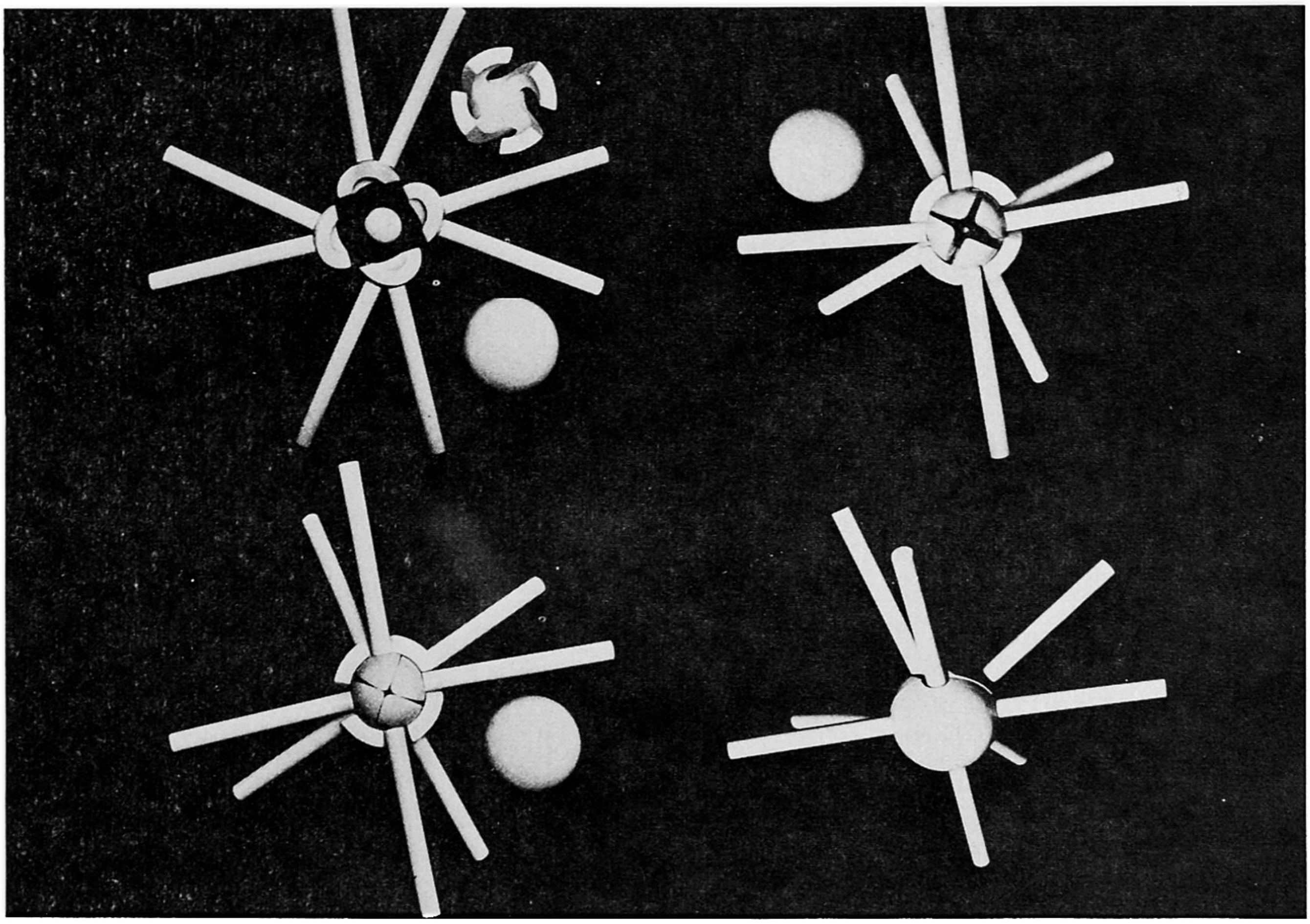


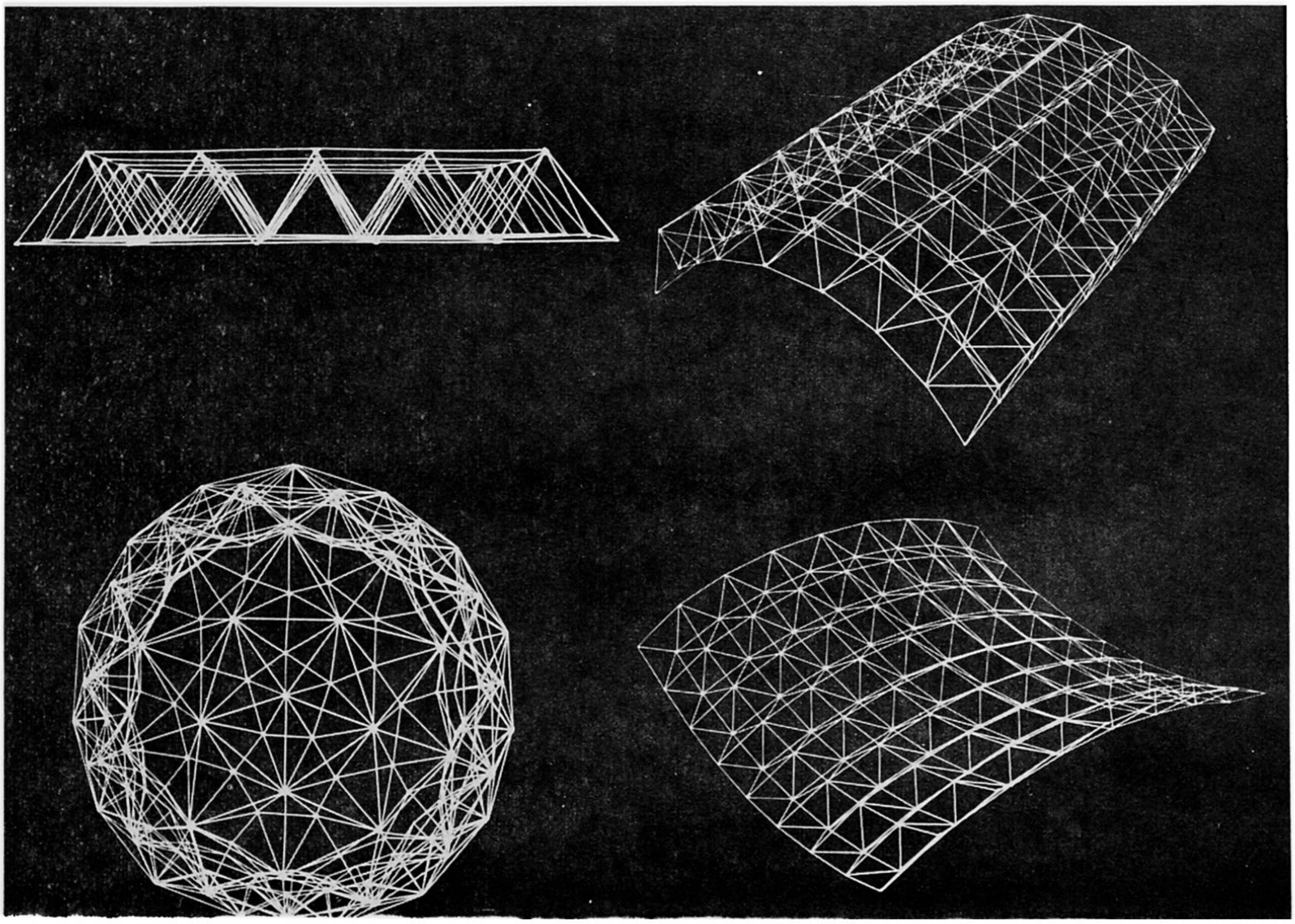
fig.2

A study for a connector of farfalla units is shown in foto 2.

II,1) Structural Analysis of reticulated structures

The main objective in the construction of reticulated structures is the employment of bars with equal section. The minimum-weight design requires the evaluation of the collapse load for a rational definition of the safety-factor. The procedures utilizing the limit-design teorems are unapplicable because these structures reach the collapse with bars in buckled configuration. Then the structural analysis must be conducted with "step by step" or "incremental" procedures. The relationship "axial force-elongation" N- (in tension and in compression) under the hypotesis of frictionless joints, represents tha basis for two approaches. The definition of this relationship is function of several parameters; this makes the practical application very arduous. For this reason it seems useful to define a simple $N-\varepsilon$ law taking into account the plastic and unstable behaviour of bars.





II,2) A simple N-ε relationship

Under the following hypotheses it is possible to define a simple N-ε relationship:

a) elastic-perfectly plastic behaviour of material,
b) bending moment-curvature with bilateral law,
c) yield function linearized,
d) deformability of bars localized in the middle section.

These hypotheses assure a prudent evaluation of the ultimate load. Indicating with N_o the limit axial force in tension and with M_o the limit bending moment, the linearized yield function is expressed by:

$$\Psi(M,N) = \pm \frac{N}{N_o} \pm \frac{M}{M_o} - 1 = 0 \qquad (1)$$

For the bars in compression the buckling axial load occurs for $N = N_{cr}$ in the elastic range that is before the bar reaches the limit axial force in compression $N = -N_o$. With the position $k = N/N_o$ and $k_{cr} = N_{cr}/N_o$ we have $-1 \leq k_{cr} \leq 0$, (fig. 3). For perfectly straight bars the lateral deflection of the middle section f will be zero before k reaches the condition $k/k_{cr} = 1$. According to the linear theory of stability and to elasto-plastic behaviour of material, the deflection f is indeterminate if $N = N_{cr}$ and $\Psi(M,N) \leq 0$. When $\Psi(M,N) = 0$ the deflection f is univocally determinate by (1) where we can consider $M = N\,f$. By putting $d = M_o/N_o$ the (1) can be written:

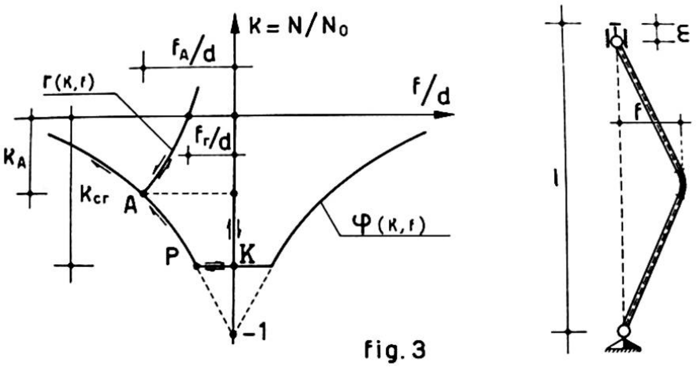


fig. 3

$$\varphi(k,f) = -k(1 \pm f/d) -1 = 0 \qquad (2)$$

that in the (k, f/d) plane represents two branches of hyperbola (fig. 3).

Following the yield line $\varphi(k,f) = 0$, an increasing deflection f requires a reduction of $|N|$. Starting from a point A on the descending k-f/d branch, for a guided deflection, it is possible a decrease of the deflection's magnitude f ; the bar behaves elastically according to the law:

$$r\,(k,f) = \frac{k}{k_{cr}} + \left(1 - \frac{k_A}{k_{cr}}\right) \frac{f_A}{f} -1 = 0 \qquad (3)$$

where f_A/d and k_A are the coordinates of the point A from which started the unloading process. Considering $N = 0$ that is $k = 0$ in (3), we obtain a "residual deflection" $f_r = f_A\,(1-k_A/k_{cr})$. The function r (k,f) represents a set of equilateral hyperbolas depending upon point A parameters, whose asymtotes are $k = k_{cr}$ and $f/d = 0$. For $A \equiv P$ (fig. 3) the hyperbola degenerates in its asymptotes. The "total elongation" is composed by two parts, the first depending on the elastic axial deformation, the second ($\Delta\ell/\ell$) depending on the lateral deflection. The relationship f $-\Delta\ell$, under the previous hypotheses, is $\Delta\ell = 2\,f^2/\ell^2$. In fig. 4 we show the diagrams and the equations of these relationships.

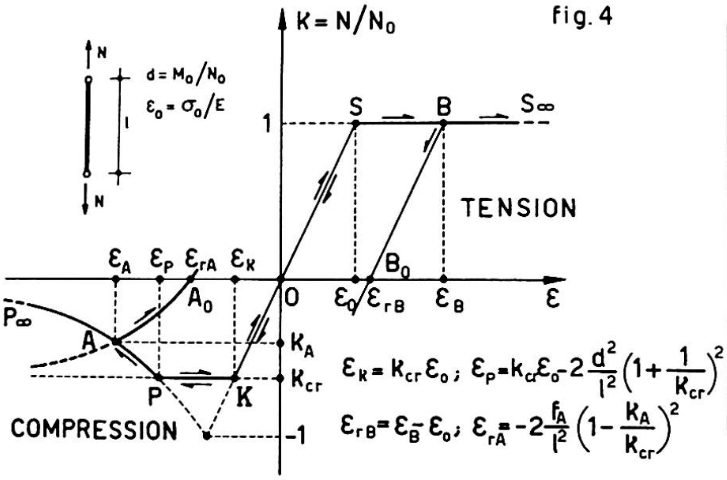


fig. 4

	BRANCH	K	ε	LAW	BEHAVIOUR
TENSION $K>0$	O-S	$0 \leq K < 1$	$0 \leq \varepsilon < \varepsilon_o$	$\varepsilon = K\varepsilon_o$	ELASTIC
	$S-S_\infty$	$K=1$	$\dot{\varepsilon} = \frac{\dot{\mu}}{N_o} > 0$	$\dot{K} = 0$	PLASTIC
			$\dot{\varepsilon} < 0$	$\dot{K} = \dot{\varepsilon}/\varepsilon_o$	ELASTIC
	$B-B_o$	$0 \leq K < 1$	$\varepsilon_{rB} \leq \varepsilon < \varepsilon_B$	$K = \frac{\varepsilon - \varepsilon_{rB}}{\varepsilon_o}$	ELASTIC
COMPRESSION $K<0$	O-K	$K_{cr} < K \leq 0$	$\varepsilon_K < \varepsilon \leq 0$	$\varepsilon = K\varepsilon_o$	ELASTIC
	K-P	$K = K_{cr}$	$\varepsilon_P \leq \varepsilon \leq \varepsilon_K$	$\dot{K} = 0$	ELASTIC (NEUTRAL EQ.)
	$P-P_\infty$	$0 \leq K \leq K_{cr}$	$\varepsilon < \varepsilon_P$ $\dot{\varepsilon} < 0$	$\varepsilon = K\varepsilon_o - 2\frac{d^2}{l^2}\left(1+\frac{1}{K}\right)^2$	PLASTIC (INSTABLE EQ)
			$\varepsilon < \varepsilon_P$ $\dot{\varepsilon} < 0$	$\varepsilon = K\varepsilon_o + \varepsilon_{rA}\left(1-\frac{K}{K_{cr}}\right)^{-2}$	ELASTIC
	$A-A_o$	$0 \leq K \leq K_{cr}$	$\varepsilon \geq \varepsilon_A$		ELASTIC (STABLE EQ)

Then the structural analysis can be performed, for each load increment, by the definition of the corresponding behaviour of every bar depending on the position of the representative point in the diagram of fig. 4. This procedure, repeated for successive finite load increments, leads to the evaluation of the collapse-load.

II,3) Numerical example

To clarify the method proposed for the evaluation of the collapse-load for reticulated structures we consider here the simple example shown in fig. 5.

The equilibrium and compatibility equations are:

$$N^A - N^B - F = 0 \qquad (4a)$$

$$\mathcal{E}^A + 2\,\mathcal{E}^B = 0 \qquad (4b)$$

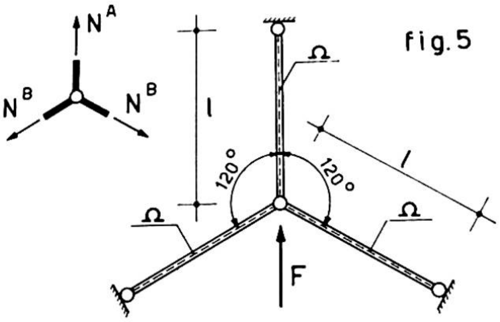


fig.5

In the elastic field the axial forces are: $N^A = -2F/3$; $N^B = F/3$. Being $|N^A| > |N^B|$ the load increment leads to point K (fig. 4) where is $\mathcal{E}^A = k_{cr}\ \mathcal{E}_o$. The corresponding value of the external force is $F_K = -1{,}5\ k_{cr}\ N_o$. Further external load increments will correspond to constant axial forces in the bar A (K P branch in fig. 4) while the representative point on the N- diagram will reach the point P for the bar A or the point S for the bar B.

In the case:

$$-2\,\mathcal{E}_o \geqslant k_{cr}\ \mathcal{E}_o\ -2\ \frac{d^2}{\ell^2}\ \left(1 + \frac{1}{k_{cr}}\right)^2 \qquad (5)$$

the point S will be reached before or simultaneously to the point P. Then the structure can't bear further load increments because beyond the point P and S the force in the tension bars remains constant while the force in the compressed bear must have a reduction (in absolute value). The limit load in this case is $F_\ell' = N_o\ (1-k_{cr})$.

In the other case, if the (5) is unsatisfied the compressed bar will reach the point P before the tension bars reach the point S. The internal force in the compressed bar must have a reduction in absolute value, while in the tension bars the force can have an increment as far as the point S will be reached. The compatibility equation (4b) allows to joint the deformation of the three bars, by putting $\varepsilon^A = -2\,\varepsilon_o$ we obtain the axial force in the bar A while in tension bars the force is equal to N_o. The limit load is:

$$F''_\ell = N_o - (-2\,\varepsilon_o)\,N^A \tag{6}$$

If the three bars have a tubular section whose diameter is D and tickness t, indicating with λ the slenderness, we have:

$$k_{cr} = -\frac{\pi^2}{\lambda^2}\,\frac{1}{\varepsilon_o} \quad ; \qquad \frac{\ell}{d} = \frac{\sqrt{2}}{4}\,\pi\lambda \tag{7}$$

being $\lambda = \ell/\varrho = 2\sqrt{2}\,\ell/D$. Assuming $\varepsilon_o = 2\times10^{-3}$, $\lambda = 100$ we have:

$$F_k = 0{,}74\,N_o \quad ; \quad F''_\ell = 1{,}17\,N_o \quad ; \quad F''_\ell/F_k \cong 1{,}59$$

where F_k is the external force corresponding to compressed member instability and F''_ℓ is the collapse-load.

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SUMMARY: In this paper we show a solution for the construction of double-layer space frame shells with simple prefabbricated units of standard size and shape. Then a procedure is proposed for the evaluation of the actual safety-factor by means of the calculation of the collapse-load of these structures taking into account plasticity and local buckling of simple bars.