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Free Vibrations of Cable Networks Utilizing Analogous Membranes

Oscillisations libres de constructions en câbles par utilisation de membranes analogues

Freie Schwingungen von Kabelnetzwerken unter Anwendung analoger Membranen

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1. Introduction

The application of membrane theory for the static response of cable roof systems has been demonstrated by Shore and Bathish (1) and Schleyer (2). This paper considers an analogous membrane technique to study the free vibration of a certain class of cable systems. The major objective of this study is to present a simplified and accurate technique for predicting the natural frequencies of flat cable networks by utilizing an appropriate analogous membrane to mathematically model the discrete or cable system.

2. Assumptions and Limitations

The following assumptions are made: (1) the cables and the membrane obey Hooke's Law, (2) the cables and membrane have only extensional stiffness, (3) linear strain-displacement relationships only will be considered, (4) Poisson's effect in the membrane is neglected, (5) the cable and membrane tension everywhere is always greater than zero, (6) the mass of the cable system is concentrated at the nodes, (7) damping is negligible.

The following limitations apply to this study: (1) the cable system at $t = 0$ is flat and an orthogonal network with the cable intersections connected; (2) the boundary planform is rectangular; (3) only linear, free vibrations normal to the network are considered.

3. Governing Equations of Motion

A. Membrane:

The equations of motion for a flat, prestressed, homogeneous membrane of thickness, h, are (3)

$$Eh\, u_{xx} + F_x(t) = \rho h \ddot{u} \tag{1}$$

$$Eh\, v_{yy} + F_y(t) = \rho h \ddot{v} \tag{2}$$

$$Eh\, [u_{xx} w_x + v_{yy} w_y + (u_x + u_x^0)\, w_{xx} + (v_y + v_y^0)\, w_{yy}] + F_z(t) = \rho h \ddot{w} \tag{3}$$

where all symbols are defined in Section 7. Since the major interest is the free transverse vibrations of the system, then for the assumptions and limitations noted in Section 2, it is permissible to neglect in-plane displacements and inertia terms. Thus, equations (1),(2), and (3) reduce to the following single equation of free vibration:

$$u_x^0 w_{xx} + v_y^0 w_{yy} = \frac{\rho}{E}\ddot{w} \tag{4}$$

The following displacement function will be chosen to describe the free vibration of a rectangular membrane (see Figure 1):

$$w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} f(t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{5}$$

Note that the displacement function satisfies the boundary conditions at the edges of a rectangular membrane, that is,

$$w(o,y,t) = w(a,y,t) = w(x,o,t) = w(x,b,t) = 0 \tag{6}$$

The initial conditions are taken as

$$w(x,y,o) = \bar{w}(x,y), \ \dot{w}(x,y,o) = 0 \tag{7}$$

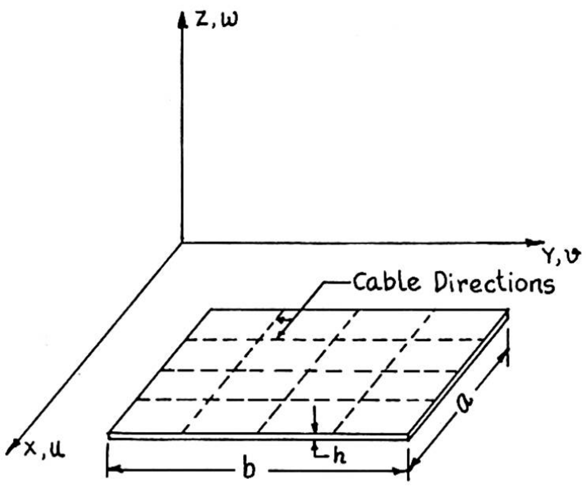

Figure 1

Substituting equation (5) into equation (4) leads to the well known equation of simple harmonic motion

$$\ddot{f}(t) + \omega_{mn}^2 f(t) = 0 \tag{8}$$

where ω_{mn} represents the frequency of free vibration in the mn-th mode written explicitly as

$$\omega_{mn}^2 = \frac{\pi^2}{\rho h}\left[U\left(\frac{m}{a}\right)^2 + V\left(\frac{n}{b}\right)^2\right] \tag{9}$$

where $U = Ehu_x^0$ and $V = Ehv_y^0$, the initial membrane tensions per unit length.

B. Discrete Cable Network

For a prestressed cable network, the equations of motion of a typical joint based on the assumptions of Section 2 are (3)

$$T_{jq}\frac{\xi_q-\xi_j}{x_q-x_j} + T_{jr}\frac{\xi_r-\xi_j}{x_r-x_j} + T_{js}\frac{\xi_s-\xi_j}{y_s-y_j} + T_{jt}\frac{\xi_t-\xi_j}{y_t-y_j} + F_\xi^j(t) = m_j\ddot{\xi} \tag{10}$$

$$T_{jq}\frac{\eta_q-\eta_j}{x_q-x_j} + T_{jr}\frac{\eta_r-\eta_j}{x_r-x_j} + T_{js}\frac{\eta_s-\eta_j}{y_s-y_j} + T_{jt}\frac{\eta_t-\eta_j}{y_t-y_j} + F_\eta^j(t) = m_j\ddot{\eta} \tag{11}$$

$$T_{jq}\frac{\zeta_q-\zeta_j}{x_q-x_j} + T_{jr}\frac{\zeta_r-\zeta_j}{x_r-x_j} + T_{js}\frac{\zeta_s-\zeta_j}{y_s-y_j} + T_{jt}\frac{\zeta_t-\zeta_j}{y_t-y_j} + F_\zeta^j(t) = m_j\ddot{\zeta} \tag{12}$$

If the same assumptions and limitations are imposed on the cable network as for the membrane, then the equations of free vibration reduce to one per node. Therefore, if there are N nodes in the network the governing equations of free transverse vibrations are (See Figure 2)

$$\sum_{j=1}^{N} T_{jq}\frac{\zeta_q-\zeta_j}{x_q-x_j} + T_{jr}\frac{\zeta_r-\zeta_j}{x_r-x_j} + T_{js}\frac{\zeta_s-\zeta_j}{y_s-y_j} + T_{jt}\frac{\zeta_t-\zeta_j}{y_t-y_s} = m_j\ddot{\zeta} \tag{13}$$

If the network is flat, $T_{jq} = T_{jr} = X$, $T_{js} = T_{jt} = Y$; for equally spaced cables in the x-direction $(x_q - x_j) = (x_r - x_j) = d_x$; for equally spaced cables in the y-direction, $(x_s - x_j) = (x_t - x_j) = d_y$. Now equation (8) can be written in the following simplified matrix form

$$[M]\{\ddot{\zeta}\} + [K]\{\zeta\} = 0 \tag{14}$$

where

$$[M] = \begin{bmatrix} m_1 & & & & 0 \\ & m_2 & & & \\ & & \ddots m_j & & \\ & & & \ddots & \\ 0 & & & & m_n \end{bmatrix} \quad [K] = \begin{bmatrix} \left(\frac{2X}{d_x} + \frac{2Y}{d_y}\right) & \left(-\frac{X}{d_x}\right) & --- \\ \left(-\frac{X}{d_x}\right) & \left(\frac{2X}{d_x} + \frac{2Y}{d_y}\right) & ---- \\ | & | & \\ | & | & \\ | & | & \end{bmatrix} \tag{15}$$

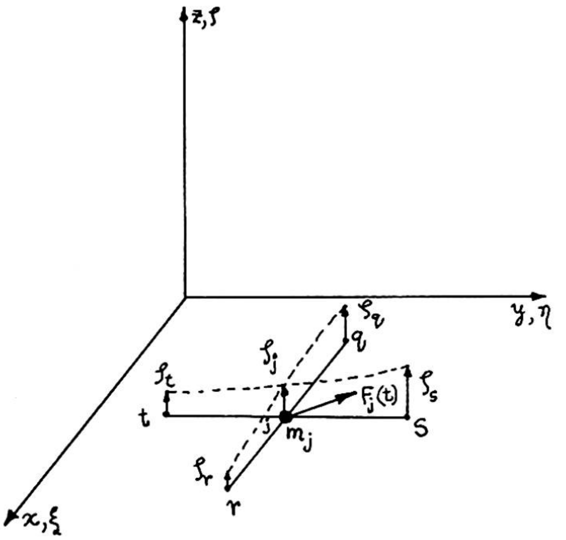

Figure 2

If simple harmonic vibration of the cable network is assumed, the k-th mode response is $\zeta_{jk} = Z_{jk} \sin(\Omega_k t + \alpha)$. Placing this function into equation (14) leads to the characteristic value problem of determining the eigenvalues or frequencies Ω_k^2, and the corresponding eigenvectors or mode shapes $\{\zeta_k\}$ of the following matrix

$$\left[[M]^{-1}[K] - \Omega_k^2\,[I]\right] = 0 \tag{16}$$

4. Membrane Analogy

Equation (9) represents the frequency equation of free vibration of a flat rectangular membrane of thickness h, uniform mass distribution, and initial pretensions U and V; equation (16) represents the matrix whose eigenvalues are the frequencies of free vibration of a flat cable network with cable spacings of d_x and d_y and initial cable tensions X and Y, and concentrated masses at the network nodes. Thus, for this study the discrete cable network is completely defined once the appropriate nodal masses are determined. Although it is recongized that the nodal mass can be frequently dependent, it is assumed that they are determined on the basis of tributary lengths or areas of cables and/or network coverings (with extensional stiffness only).

To determine the membrane parameters to replace the discrete network, the following equivalences are made:

$$a_C = a_M = a; \; b_C = b_M = b \tag{17}$$

$$\rho_C = \rho_M = \rho \tag{18}$$

$$\rho_C\,(A_x a + A_y b) = \rho_M abh; \; h = \frac{A_x}{b} + \frac{A_y}{a} \tag{19}$$

$$U = \frac{X}{a}\,;\; V = \frac{Y}{b} \tag{20}$$

5. Example

To demonstrate the usefulness of an analogous membrane to predict the frequencies of free vibration of discrete cable networks, consider a 120" x 240" rectangular cable system. The data pertaining to the cable system, as well as the analogous membrane parameters, are shown in Fig. 3. Note that the parameters relating to the membrane do not change and equation (9) is used to predict as many frequencies as is desired. The total cable areas and weight, and total pretensions in the x and y directions remain constant but the span to cable spacing ratio, R, is varied as the number of cables is varied. Thus, the magnitude of each nodal mass, each cable area, and each cable tension vary with R. For each R value, the appropriate [M] and [K] matrix is calculated and the eigenvalues, Ω, obtained on the basis of equation (16) using an IBM 360/65 computer. Table 1 summarizes the results of these calculations for R = 3,4,5,6, that is, two, three, four and five cables in both the x and y directions.

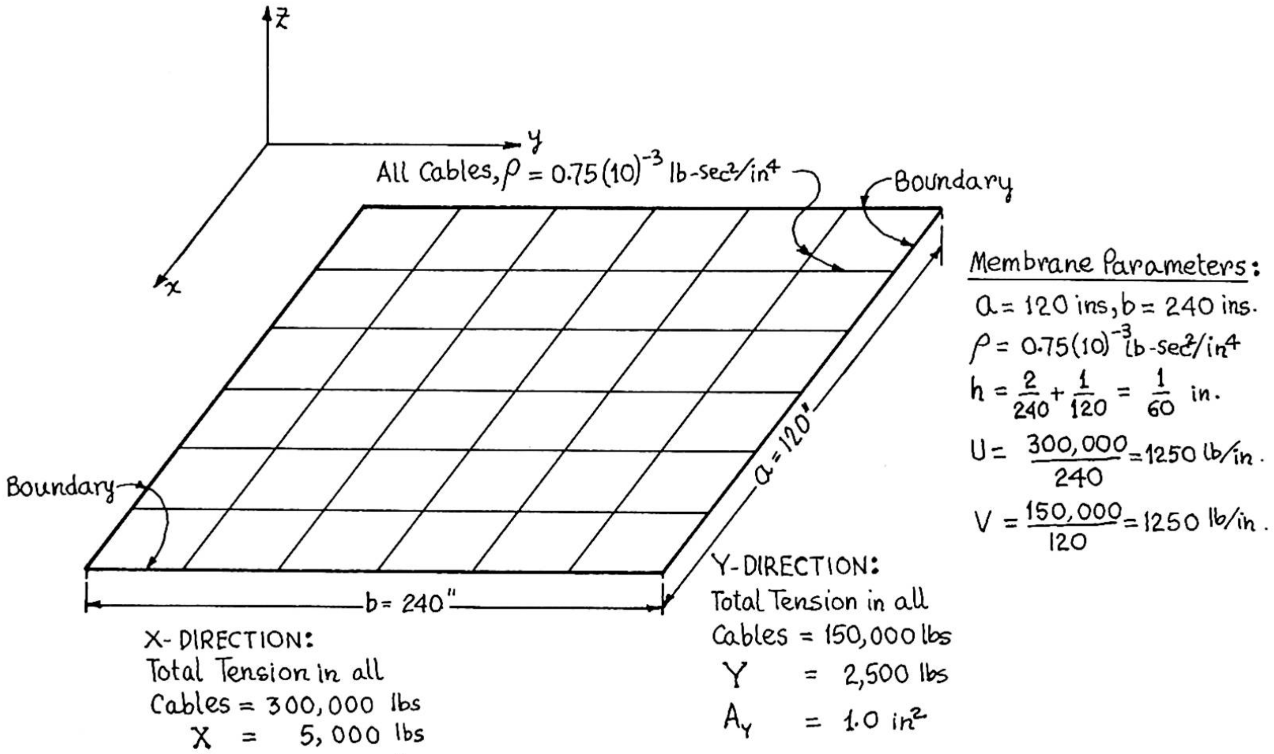

Figure 3

From the results shown in Table 1, errors in the form $(\omega - \Omega)/\omega$ as a function of R are plotted in Fig. 4. Note that the extrapolations in this figure used a least square polynomial approximation.

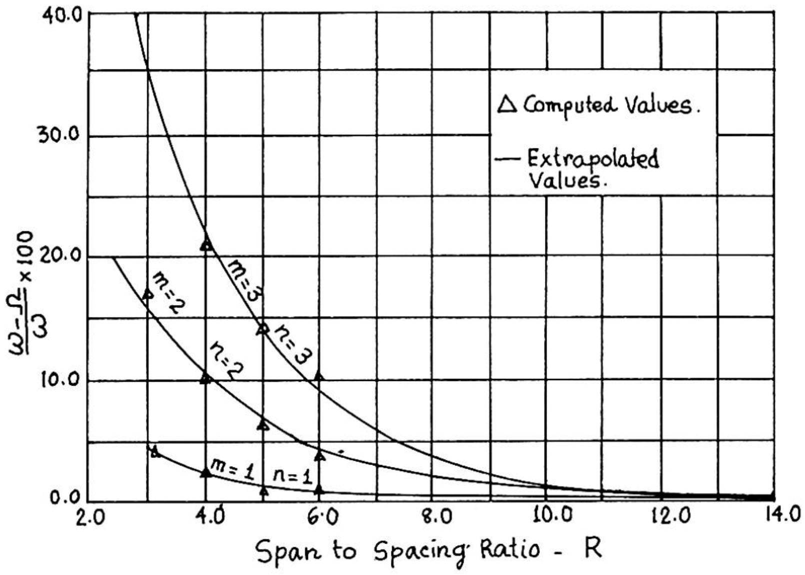

Figure 4

Since the percentage error for a particular span to spacing ratio increases for higher modes or natural frequencies, and the cable networks used in practice are expected to have span to spacing ratios of more than 10, a plot of percentage error against the frequency numbers in ascending order for a span to spacing ratio of 10 is shown in Fig. 5.

Since the errors in the natural frequencies, using a membrane obtained by uniformly distributing the mass of the cables over the area of the network, are known, it is now

TABLE 1

Frequency No.	Mode Shape m	Mode Shape n	Frequency CPS Membrane	Discrete System - Frequency CPS 2 Cables	3 Cables	4 Cables	5 Cables
ω_1	1	1	290.0	278.0	284.0	287.0	288.0
ω_2	1	2	369.5	329.5	345.5	354.0	358.0
ω_3	2	1	537.5	448.0	486.5	505.0	513.5
ω_4	2	2	584.5	482.5	524.5	546.0	556.0
ω_5	1	3	470.0	---	398.0	422.5	436.0
ω_6	3	1	794.0	---	626.0	685.0	715.0
ω_7	2	3	653.0	---	560.0	593.5	609.0
ω_8	3	2	826.0	---	656.0	715.0	745.0
ω_9	3	3	875.0	---	686.0	751.5	785.0
ω_{10}	1	4	584.0	---	---	471.0	501.0
ω_{11}	4	1	1050.0	---	---	800.5	870.0
ω_{12}	2	4	739.0	---	---	627.0	658.0
ω_{13}	4	2	1075.0	---	---	827.0	897.5
ω_{14}	3	4	941.0	---	---	779.0	825.0
ω_{15}	4	3	1115.0	---	---	858.0	930.0
ω_{16}	4	4	1068.0	---	---	882.5	962.5
ω_{17}	1	5	702.5	---	---	---	513.5
ω_{18}	5	1	1310.0	---	---	---	970.0
ω_{19}	2	5	835.0	---	---	---	691.0
ω_{20}	5	2	1330.0	---	---	---	986.0
ω_{21}	3	5	1018.0	---	---	---	851.0
ω_{22}	5	3	1361.0	---	---	---	993.0
ω_{23}	4	5	1228.0	---	---	---	1022.0
ω_{24}	5	4	1405.0	---	---	---	1052.0
ω_{25}	5	5	1460.0	---	---	---	1072.0

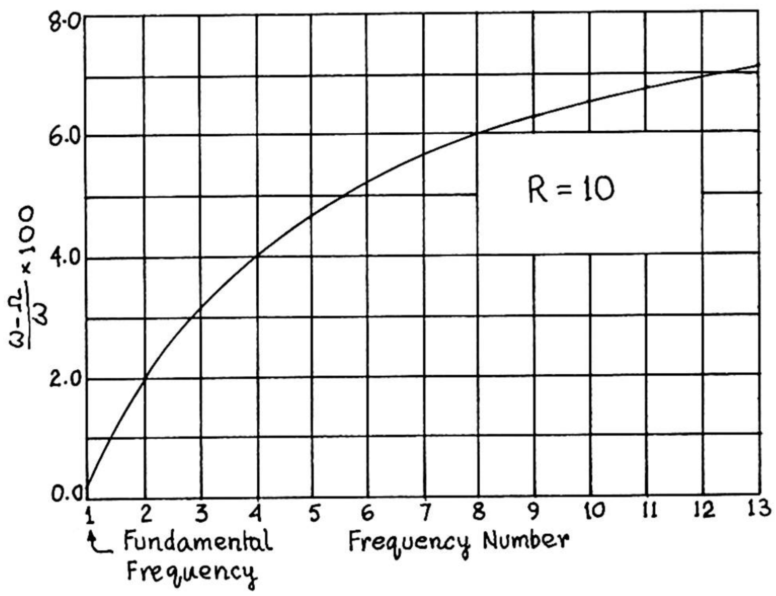

Figure 5

possible to adjust the mass distribution such that the error is minimized. This is accomplished by multiplying the natural frequency of the membrane in which the mass of cables is uniformly distributed, by a factor called the mass ratio, μ. For cable networks with span to spacing ratio greater than 10, the mass ratio for various frequencies is shown in Fig. 6.

6. Bibliography

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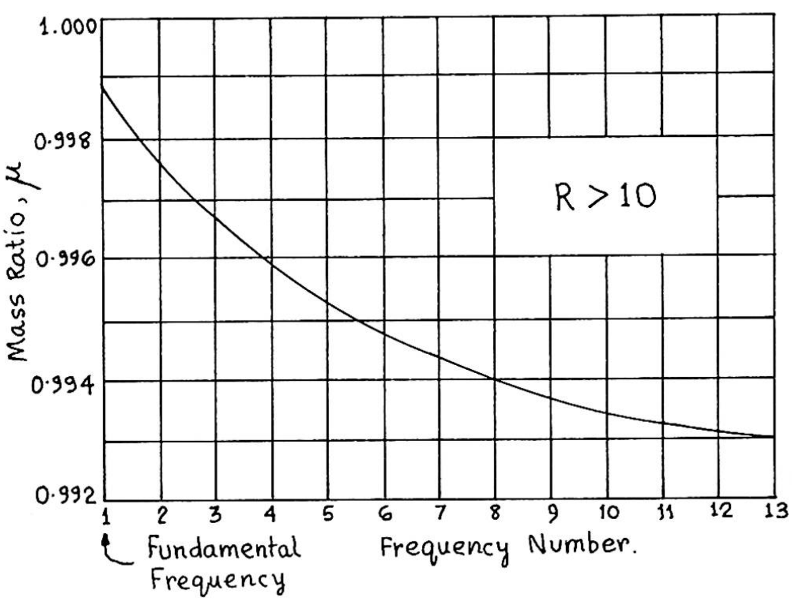

Figure 6

2. Schleyer, F., "Die Berechnung Von Seilnetzen", Hanging Roofs, North Holland Publishing Co., Amsterdam, 1963, pp. 48-55.
3. Shore, S., and Chaudhari, B., "Dynamic Response of Cable Systems", Graduate Division of Civil Engineering Report, University of Pennsylvania, May 1970.

7. Symbols

- a Length of rectangular boundary parallel to x axis, in.
- A_x Total area of cables parallel to x axis, $in.^2$
- A_y Total area of cables parallel to y axis, $in.^2$
- b Length of rectangular boundary parallel to y axis, in.
- d_x Cable spacing parallel to x axis, in.
- d_y Cable spacing parallel to y axis, in.
- E Modulus of elasticity of membrane, $lbs./in.^2$
- $f(t)$ Time dependent function
- $F_i(t)$ Time dependent forcing function ($i = x, y, z$)
- h Membrane thickness, in.
- $[I]$ Identity matrix
- $[K]$ Stiffness matrix of cable network defined in equation (15)
- m_j Concentrated mass at node j, $lb.sec.^2/in.$
- $[M]$ Mass matrix of cable network defined in equation (15)
- R Ratio of network span to cable spacing (a/d_x or b/d_y)
- t Independent time variable
- T_{jk} Tension in cable segment jk, lbs.
- u Displacement of membrane parallel to x axis
- u_x^0 Initial strain in membrane parallel to x axis
- U Membrane tension per unit of length parallel to x axis, lbs./in.
- v Displacement of membrane parallel to y axis
- v_y^0 Initial strain in membrane parallel to y axis
- V Membrane tension per unit length parallel to y axis, lbs./in.
- w Displacement of membrane parallel to z axis
- $\bar{w}(x,y)$ Initial shape of membrane at $t = 0$
- x, y, z Orthogonal cartesian coordinates
- X Tension in cable parallel to x axis, lbs.
- Y Tension in cable parallel to y axis, lbs.
- ω_{mn} Frequency of free vibration of the membrane in the mn-th mode, cps
- Ω_k Frequency of free vibration of cable network in the k-th mode, cps
- ρ Mass density, $lbs.sec.^2/in.^4$
- ξ, η, ζ Components of nodal displacements in the cable network parallel to the x, y, and z axes respectively.
- μ Ratio of mass of membrane obtained by uniform distribution of mass of cable system to modified mass of membrane to be used.
- $\cdot$ Derivative with respect to time variable t.

Subscripts:

x, y Derivatives with respect to space variables x and y

C Cable network

M Membrane

8. Summary

It has been shown that an appropriate flat membrane can be used to predict the frequencies of free vibrations of a flat cable network. Thus, the much simpler frequency equation of a membrane permits the accurate determination of the natural frequencies for the cable network with span to spacing ratios greater than 10. This simplified procedure eliminates the determination of eigenvalues of large order matrices by relatively complex numerical methods or computer computations.

9. Acknowledgment

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