Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 8 (1968)

Artikel: Thin-walled deep plate girders

Autor: Massonnet, Ch.

DOI: https://doi.org/10.5169/seals-8703

IIc

Thin-Walled Deep Plate Girders

CH. MASSONNET
Professeur à l'Université de Liège

1. Introduction

There has been a marked increase, during these last few years, in the use, in wide-span steel bridges, of plate girders which outclass lattice girders from the aesthetic point of view and in regard to ease of construction—by manual or automatic welding—and of maintenance, and consequently of resistance to corrosion.

The record span for these bridges is still held by the continuous girder bridge over the Save, in Belgrade, constructed in 1956 by the Société M.A.N., with a span of 260 m and a web depth of 9.6 m over the inner bearings, with a thickness of 14 mm (which gives a b/e ratio $= 685$). Considerably wider spans have been achieved in suspension bridges with stiffening plate girders, and designs with simple girders exceeding 300 m span were submitted in recent competitions for bridges over the Rhine (see various articles on this subject in the journal "Der Stahlbau").

The bridges hitherto constructed generally comprise single-web or box girders, seldom girders with double web plates. High-strength steel (A 52) has been employed in several instances, and this has necessitated the solution of new technological problems connected with the welding [30]. There is an increasing tendency towards the use of steels of different strengths, in order to reduce costs while at the same time complying with the essential requirements as regards stiffness [65]. There is nothing to prevent the utilisation, in these bridges, of very high strength steels of the American type T 1, nor to the simultaneous use of different grades of steel, in order to achieve the maximum economy.

2. Development of the Shape of the Cross-Section

The shape usually adopted, even today, is that of a double T, which is very easily constructed, by welding, from steel sheets and wide flats. Economy, however, necessitates the employment of very thin sheets, because the web is generally excessive for withstanding the shearing force. But sheet steel is not efficient as a member for withstanding buckling, and hence there arose, quite naturally, the idea of utilising sandwich sheets, with a honeycomb or light concrete core. No solution of this kind appears to have been found, so far, which is competitive with the plate type of web, but rapid advances in bonding and in the utilisation by folding of cold-rolled thin sheets allows us to hope that there will be developments of this kind in the near future.

For girders of small or medium span, the cutting out of the web into perforated shapes by means of oxy-acetylene cutting, makes it possible to construct castellated beams (with or without the insertion of additional web panels) from rolled sections, a subject that we shall not discuss in the present Report.

For wide-span girders, there have been successively advocated (Brotton) a girder with a web of variable thickness (fig. 2.1 a) and (Dörnen, Radojkovic) a girder with a tubular flange which can be constructed by means of thin-walled angles (fig. 2.1 b) or better still (MASSONNET [36]) from a trapezoidal section of folded sheet steel, in order to obviate welding over the zone of segregated metal of the angle, or lastly the conventional section strengthened by welded oblique flat bars (the section employed in Eastern European countries according to Professor Tesár).

Section a) possesses the advantages of a reduction in the height of that part of the web in danger of buckling and of a lessening of the equivalent tensile stress

$$\sigma_c = \sqrt{\sigma^2 + 3\tau^2}$$

at the flange junction.

Sections b), c) and d), in addition to the two foregoing advantages, also have that of a tubular flange with a considerable torsional stiffness which forms a practically perfect recess for the insertion of the horizontal edges of the web panels and furthermore possesses a marked flexural rigidity, so that—with the transverse stiffeners, which are generally tubular—it forms a rigid framework capable of withstanding the membrane stresses which develop in the web for stresses greater than the critical buckling stress.

The present author has demonstrated theoretically [34] and experimentally [36] the superiority of section c) over the conventional section. It is, in fact, possible to develop, in girders of this system, bending moments that are 8% greater than the *plastic moment* (corresponding to the *bi-rectangular* distribution of the stresses), provided that the girder is protected against any lateral warping or buckling of the compressed flange, with distortion of the cross-section, by adequate bracing members.

The main problem in this field seems to be the perfecting of economical methods of fabrication.

The above sections mark a definite improvement on the conventional double-T section, but constitute a partial palliative only for the fundamental problem of buckling of the web.

At the Symposium on the post-buckling behaviour of plates, which was held in Liège at the end of 1962 [13], SHANLEY showed that, from the purely constructional point of view—that is to say, leaving aside the question of economy—it is always advisable to give preference to a structure stressed below its critical load of instability over a structure stressed in the post-buckling range; he advocated webs made of very thin sheet steel folded in triangular or trapezoidal shapes (fig. 2.2a) and b)], which are fully resistant to buckling and to the shearing stresses due to the shear force. As drawbacks of this system, we may mention the enhanced cost of fabrication and the fact that the web is incapable of withstanding the flexural stresses, owing to the "concertina" effect.

In 1964, at the request of the S.A. Cockerill-Ougrée, the present author tested, in his laboratory, models of four different types of girders with thin webs (depth 400 mm, thickness 1 mm):

1) corrugated sheet with transverse corrugations;
2) corrugated sheet with longitudinal corrugations;
3) flat sheet and corrugated sheet with transverse corrugations, spot welded;
4) flat sheet and corrugated sheet with longitudinal corrugations, spot welded.

Certain of these models gave results that were to some extent unsatisfactory, because the spot welding had not been done properly and had given way prematurely. Nevertheless, it was possible, in well fabricated models, to reach the elastic limit throughout the web without buckling (fig. 23).

Girders with a web folded in a broken line (fig. 2.2) have also been employed recently in Belgium.

Rockey has informed the present author of the forthcoming publication of a report on girders with webs made of corrugated sheets.

3. Problems arising in Connection with the Optimum Design of Large Girders with thin Webs

The design of a plate-web girder, when the constructional arrangements that have been made obviate any risk of instability, is elementary; it is based on the well-known bending formula of Navier $\sigma = My/I$ and on the elementary shearing equation of Jourawski $\tau = TS/Ie$, combined with one of the criteria of plasticity of Tresca or of von Mises which give the respective equivalent stresses

$$\sigma_c = \begin{cases} \sqrt{\sigma^2 + 4\tau^2} & \text{(Tresca)} \\ \sqrt{\sigma^2 + 3\tau^2} & \text{(von Mises)} \end{cases}$$

The main problem is therefore to prevent any instability phenomena; these phenomena are of three types:

1) local buckling of the flange under compression,
2) warping of the girder;
3) buckling of the web[1]).

In actual fact, these three phenomena are interconnected and it is incorrect to consider them separately; in particular, the destruction of plate girders of the conventional type always occurs as the result of the collapse of the compressed flange (which is in the plastic state) combined with buckling of the adjacent web panel, the collapse of the flange being accelerated by the membrane stresses which the web panel in its post-buckling state exerts on its framework.

In the case of girders with webs made of flat sheets, an additional danger arises, the reality of which was demonstrated by the fatigue tests of HALL and STALLMEYER [21]; it is the rupture of the web by alternating flexural fatigue at its junction with its rigid framework. We shall make further reference to this phenomenon during the discussion of buckling tests (paragraph 5).

4. Design and Stiffening of Webs in Accordance with the Linear Theory of Buckling

The linear theory of buckling assumes, as does the conventional theory of the bending or plates, that:

1) the plate is, initially, perfectly flat;

2) its middle layer, when buckling occurs, undergoes transverse displacements w that are slight in relation to the thickness e (that is to say, practically $w \leq 0.3e$); it then exhibits a well-defined critical buckling stress, which can be calculated, either by means of the equation due to Saint-Venant (1886)

$$\nabla^2 \nabla^2 w = \frac{1}{D} (N_x w_{xx} + N_{yy} w_{yy} + 2N_{xy} w_{xy}) , \qquad (4.1)$$

or by means of the energy method of Raleigh-Ritz.

---

[1]) An additional problem arises in connection with girders over which moving loads travel, such as the girders of a travelling crane, namely the problem of local buckling due to the action of a considerable concentrated load applied to the upper flange. The most recent theoretical contribution to this problem is that due to ZETLIN [64]; it does not take into account, however, the distribution of the stresses brought about by the upper flange. This distribution was analysed by Girkmann in his book "Flächentragwerke".

Timoshenko has made the greatest contributions to the development of this theory, which is described in all the standard manuals on instability [61, 9, 26, 12]. He has, in particular, developed the theory of stiffened webs and introduced the concept of the *strictly rigid stiffener*. The relative stiffness γ^* of this stiffener is the smallest value of $\gamma = EI/bD$ for which the stiffener remains rectilinear when buckling takes place (fig. 4.1). During the last twenty years, additional results have been obtained by the research workers of the N.A.C.A. [60], P. DUBAS [18], KLÖPPEL [24, 25], MASSONNET [31, 34], ROCKEY [38 to 51], STÜSSI and DUBAS [19], etc.

Klöppel has made the greatest contribution to the practical application of this theory by writing a book [25] containing a large number of design nomographs (obtained by the energy method by means of an I.B.M. 704 computer) for rectangular plates supported at their four edges, provided with open-section stiffeners and subjected in their plane to bending or shearing stresses, or a combination of these stresses.

The webs of large plate-girder bridges constructed since 1945 by the Germans have generally been designed in accordance with this theory, with a safety coefficient of 1.35 with respect to buckling.

The book by Klöppel and Scheer contains the values of the buckling coefficients, both for panels with flexible stiffeners ($\gamma < \gamma^*$) and for those provided with rigid stiffeners ($\gamma > \gamma^*$).

According to the information he has given to the present writer, Klöppel has in preparation a book that will supplement the manual [25] by providing:

a) new tables relating to plates with one and with two transverse stiffeners;
b) new tables for plates stiffened in the longitudinal direction by a series of uniformly spaced stiffeners, and provided, in addition, with one or two transverse stiffeners;
c) a method making it possible, in the case of plates subjected to composite bending, to achieve practically continuous stiffening by a series of longitudinal stiffeners, by spacing them unevenly in a transverse direction in accordance with the intensity of the stresses;
d) details regarding the design of transverse stiffeners bounding a plate for various values of the longitudinal stiffening.

In Belgium, the present writer developed, from 1948 to 1954, a similar method of design [37]. On the basis of his tests, he recommended that the safety factor for buckling by bending should be reduced to 1.15, but that, on the other hand, the theoretical optimum rigidity γ^* of the stiffeners should be multiplied by a coefficient k varying, according to the mode of stress, from 3 to 6, in order to ensure that the stiffeners should remain practically rectilinear until the collapse of the girder was approached.

The present writer would now tend to agree to slightly exceeding the critical stresses in service by accepting a minimum safety coefficient of 0.8. ROCKEY [42] expressed similar views, as long ago as 1958, by accepting stresses equal to 1.5 times the critical tension. In bending plus shear the maximum permissible stresses σ and τ acting simultaneously would be limited by the relationship

$$\left(\frac{\tau}{\tau_{cr}}\right)^2 + \left(\frac{\sigma}{\sigma_{cr}}\right)^2 = 2.25 \tag{4.2}$$

The majority of the theoretical researches, including Klöppel's tables, are based on the simplifying hypothesis that the web plate is simply supported on its four edges. In a theoretical study, the present author and his co-workers [34] have shown how it was possible to increase the stability of the web by 100 to 200% by tailing it in at its edges. However, the numerical results obtained on an I.B.M. 650 computer are very incomplete and should be supplemented. The theoretical predictions are in good agreement with the tests to destruction carried out on two beams 1.20×18 m with tubular flanges and stiffeners [36]. It must be pointed out, however, that the estimation of the torsional rigidity by the formula due to Bredt

$$C = \frac{4GS^2}{\oint \frac{ds}{e}}$$

is optimistic because it neglects the distortion of the stiffener. DOOLEY has shown [66] that the true torsional stiffness may be 25% less than that calculated in this manner.

Some studies of girders stabilised by tubular stiffeners have been made by ROCKEY [46, 50, 52 and 53], who states that research is in progress to render it possible to determine the rational stiffening of plate webs subjected to shear and provided with transverse and longitudinal tubular stiffeners.

5. Lessons to be Drawn from Tests to Destruction on Models of Plate Girders with Thin Webs

The first important tests to destruction to be carried out on welded girders were those of WÄSTLUND and BERGMAN [63] and these were followed, in chronological order, by the experiments of MASSONNET [32, 33, 36], ROCKEY [38, 42, 49], etc., LONGBOTTOM and HEYMAN [29], BASLER and THÜRLIMANN [2, 3], COOPER et al. [15, 16, 20].

The main conclusions of all these tests are closely similar and may be summarised as follows:

1) the phenomenon of web buckling is a continuous phenomenon, gradually damped down by membrane stresses. It in no way resembles the buckling of bars subjected to compressive stresses. The web plates always exhibit unfore-

seeable initial buckles and it is frequently impossible to detect experimentally a critical load on the deflection-load curves.

2) In the post-buckling range, the web plate supports the loads elastically, while undergoing increasing transverse deformations, the distribution of which depends mainly on the initial deformations of the plate. The elastic range fixed by the ratio P/P_{cr} is the more extensive the thinner is the web plate (e/b lower) and the greater is the R_e/E ratio of the metal (where R_e is the yield stress). The present writer has observed, in steel girders, an elastic range up to $P/P_{cr} = 2.8$ [31] and ROCKEY [38], on aluminium alloy girders found values of the order of 4.

3) The load causing collapse has no correlation with the critical buckling load; if there are no longitudinal stiffeners, an incomplete diagonal tension field, which subjects the flanges to bending stresses, becomes established in the girder. The load resulting in destruction is consequently greatly influenced by the rigidity of the framework provided by the flanges and the stiffeners[1]). For thin webs ($b/e \approx 500$), the ratio P_{ult}/P_{cr} may reach 4 for mild steel girders [31] and 8 for light alloy girders [38].

4) The distribution of the bending stresses over the depth of the cross-section differs to a fairly marked extent (particularly in the region under compression) from the bitriangular distribution of Navier [42, 2, 3, 4].

5) The longitudinal stiffeners designed in accordance with the linear theory of buckling, to remain strictly rigid ($\gamma = \gamma^*$) bend as soon as the load is applied and particularly in the post-buckling range [33]. Let us term the load for which the ratio $w_{stiffener} : w^{plate}_{maximum}$ passes through a minimum value, the "limiting load of efficiency" of the stiffener. In order to obtain a limiting load of efficiency in the vicinity of the load causing collapse of the girder, it is necessary to employ stiffeners with a relative rigidity of $\gamma = k\gamma^*$, where k depends on the position of the stiffener and is at least equal to 3.

6) In addition, the experiments have also shown the danger of fastening the longitudinal stiffener by discontinuous weld beads (local buckling between two successive beads), of employing as stiffeners flat bars that are too thin (local plastic buckling of the stiffener) and of employing stiffeners made of a milder grade of steel than the steel of the girder (for example, stiffeners made of A 37 steel for a girder made of A 52 steel) because such stiffeners enter into the plastic state prematurely.

7) Finally, the experiments show that the behaviour of stiffeners placed on

---

[1]) ROCKEY [42] has shown that, if the flanges give way, fewer buckling blisters are observed and they are much less deep. He recommended, for the minimum moment of inertia of the flanges, the formula:

$$\frac{I}{b^3 e_{min}} = 0.00035 \left[\frac{P}{P_{cr}} - 1\right] \left(1 \leq \frac{P}{P_{cr}} \leq 4\right).$$

He has stated that he is engaged, with R. D. MARTIN, on a study to supplement his previous paper and with SKALOUD on an investigation to study more especially the effect of flexibility of the flange on behaviour at the point of collapse.

one edge is complicated. The width of the strip of web co-operating in the bending of the stiffener is not well understood, but seems, from unpublished tests of the present writer, to be of the order of 20 times the thickness of the web.

6. Application of the Linear Theory of Buckling

The majority of experts agree nowadays that, although the linear theory gives reliable structures, it does not provide economical solutions.

This theory may be justified from the point of view of the theory of permissible stresses, but gives us no information regarding the actual safety of the structure as regards collapse. It is based entirely on the necessity of obviating a phenomenon (buckling) which is not, in itself, a source of any danger, since it is immediately arrested by the development of membrane stresses.

In particular, the rules regarding the rigidity and the spacing of the stiffeners, which follow from the concept of the strictly rigid stiffener in view of the idea of achieving uniform safety in all the partial sub-panels, do not lead to an optimum girder as far as resistance to collapse is concerned.

Following the practice that has long been accepted in aircraft construction (Wagner, Kuhn) and in the field of structures made of cold-formed thin steel sheets (G. WINTER), attention is being directed towards finding more economical dimensions giving both a satisfactory behaviour in service (transverse deflections limited so as to be invisible to the naked eye, taking into account the possibility of fatigue) and an adequate collapse load.

Thus, after a century, there is a tendency to revert to the designs of the English constructors of 1850 who subjected the steel sheets of their tubular bridges to stresses in the post-buckling range, as is shown by the calculations made in France by the S.N.C.F. (French national railways) in connection with the repairs to the bridge over the Lot at Aiguillon which was designed by the famous English bridge builder R. STEPHENSON [27].

The present writer is consequently of the opinion that the linear theory of buckling, although preparing the way for the non-linear theory that we are about to discuss, has to some extent hindered technical progress by the inaccurate description that it gives of the phenomenon of buckling.

7. The Non-Linear Theory of Buckling and its Application to the Problem of the Design of the Webs of Large Girders with Thin Web Plates

The non-linear theory is, in reality, a theory of moderate deformations; it takes into account the extension of the middle fibre but assumes low values for the slopes $\partial w/\partial x$, $\partial w/\partial y$ in order to make their sine and tangent equal to the angle itself and their cosine equal to unity.

The basic equations for a perfectly flat isotropic plate are due to Kármán; they may be written:

$$\left.\begin{aligned}\nabla^2\nabla^2 w &= \frac{e}{D}\,[\varphi_{yy} W_{xx} + \varphi_{xx} W_{yy} - 2\varphi_{xy} W_{xy}]\\ \nabla^2\nabla^2\varphi &= E\;[W^2_{xy} - W_{xx} W_{yy}]\end{aligned}\right\} \tag{7.1}$$

by introducing for convenience the Laplace operator

$$\nabla^2 F = F_{xx} + F_{yy}\,. \tag{7.2}$$

The membrane stresses in the web are derived from the stress function by the well-known equations of Airy

$$\sigma_x = \varphi_{yy}\,; \quad \sigma_y = \varphi_{xx}\,; \quad \tau_{xy} = -\,\varphi_{xy}\,. \tag{7.3}$$

In 1934, Marguerre extended these equations to the case of plates having a slight initial curvature, then SOPER [14] extended them to the case of slightly curved orthotropic plates, SKALOUD and DONEA [56] to the case of plates exhibiting residual stresses, and finally LEPIC [28] to the case of plates in an elastoplastic state.

The equations that hold good for the elastic behaviour of isotropic plates having a slight initial curvature w_0, and residual stresses characterised by the stress function φ_0, which will be sufficient for the discussion of this report, are given below

$$\left\{\begin{aligned}&\nabla^2\nabla^2 w = (\varphi_0+\varphi)_{yy}(w_0+w)_{xx} + (\varphi_0+\varphi)_{xx}(w_0+w)_{yy} - 2(\varphi_0+\varphi)_{xy}(w_0+w)_{xy}\\ &\nabla^2\nabla^2\varphi = E\{(w_0+w)^2_{xy} - (w_0+w)_{xx}(w_0+w)_{yy} - (w_0)^2_{xy} + (w_0)_{xx}(w_0)_{yy}\}\end{aligned}\right. \tag{7.4}$$

The pioneer work in the application of the non-linear theory to the behaviour of web plates is that of BERGMAN [8]. Important results have been obtained by ALEXEEV [1], BROUDE [10, 11], VOLMIR [62], the Czech research worker SKALOUD ([13, 54 to 59]) and the Slovak investigator DJUBEK [17].

They show quite clearly that there is a considerable reserve of strength in thin web plates with a rigid framework. The chief criticism that can be made of the calculations carried out by the majority of the above-mentioned authors is that, although they take into account the extensional rigidity $E\Omega$ of the framework surrounding the plate, on the other hand, they all assume that the flexural rigidity EI is infinitely great, which gives over-optimistic results that are consequently not realistic. ROCKEY and SKALOUD state that they are engaged on a paper in which the flexural rigidity will be taken into account in the calculations.

The great problem is to overcome the almost unsurmountable mathematical difficulties that arise as soon as an attempt is made to study realistically the interaction between the membrane plate and its stiffening framework. In this connection, the present author would recommend methods by finite differences or by finite elements, particularly suitable for computers, rather than analytical methods utilising developments of Fourier series.

As the chief aim of the non-linear theory is to predict collapse, a collapse criterion must be selected. SKALOUD [13] has discussed this problem in detail; he comes to the conclusion that, in order to define the limiting state of the web, it may be assumed that the effect of the peak stresses is reduced to zero in the plastic range and the limiting state may be determined on the basis of the membrane stresses alone.

Tests show that the collapse of web plates is often preceded by considerable plastic deformations, so that the application of the *elastic* non-linear theory for predicting the stage of collapse may be contested.

However this may be, the principal results of applications of the non-linear theory made so far are as follows:

1) The reserve of post-buckling strength is the greater the more marked is the slenderness (b/e) of the web.

2) All other things being equal, it increases with the stiffness of the framework (flanges + stiffeners) bounding the web plate. This justifies the rule of minimum rigidity of the flanges deduced by Rockey from his experimental researches.

3) The theory [59] confirms the conclusions termed by the present writer from his experiments [33, 36], namely that:

a) the strictly rigid stiffener (γ^*) steadily undergoes deflection with the web in the post-buckling range;
b) in order to obtain a stiffener that remains rigid until the point of collapse of the entire girder is approached, its strict relative rigidity, γ^* must be multiplied by 3.

4) However, it has not been proved that stiffeners which remain rectilinear until collapse is approached are the best. SKALOUD suggests [57] as criterion of optimisation that the plate-stiffeners assembly should, for a given ultimate strength, have a minimum cost.

5) While awaiting the results of more far-reaching researches, SKALOUD and the present author have suggested [13] various simple but empirical design rules, resulting in considerable saving as compared with rules based on the linear theory.

6) The theoretical studies of BROUDE [10] relating to the non-linear behaviour of web plates exhibiting an initial deflection lead to the conclusion, firstly, that, in service, stresses σ_{max} equal to 1.1 times the buckling stresses given by the linear theory may be assumed and, secondly, a normal initial deflection (equal to threetenths of the thickness) gives rise to total stresses of the order of $\sigma_{max}/0.89$.

BROUDE therefore advises that for the web plates of industrial welded plate girders, stresses equal to $1.1\,\sigma_{cr} \cdot 1/0.89 \approx \sigma_{cr}$ should be accepted. Soviet standard specifications are based on these studies and assume a safety factor of one for girders subjected to static loads with transverse stiffeners located at the points of application.

For the girders of travelling cranes, traversed by moving loads, the Russian rules are more cautions.

Professor TESÁR has informed the present writer that horizontal fatigue cracks have been detected in Czechoslovakia in the web plates of welded plate girders of travelling cranes, a short distance from the web-flange weld bead.

The possibility of creating fatigue cracks in the webs of plate girders with thin web-plates has been demonstrated experimentally by HALL and STALLMEYER [21] and this phenomenon should be studied in detail if it is desired to take advantage in service of the post-buckling strength of the web plates.

8. The American Approach

Since 1957, Lehigh University has been engaged on further large-scale tests [2, 3] on plate girders with thin webs for the purpose of endeavouring to work out simple design rules ensuring a pre-determined resistance to collapse.

The research work was concerned initially with girders comprising only transverse stiffeners. It led to a semi-empirical theory suggested by Wagner's theory of a diagonal stress field, and due to BASLER and THÜRLIMANN [4, 5, 6]. As this theory has been widely disseminated, we shall confine ourselves to indicating its governing principles, while referring our readers, for its detailed application, to the specifications of the American Institute of Steel Construction (A.I.S.C.).

8.1. Bending strength

BASLER and THÜRLIMANN [4] observe, after ROCKEY [42], that the buckling of the compressed part of the web plate has the effect of markedly diminishing the capacity of this part for transmitting compressive stresses, and that is why they assume that, on collapse, everything happens as though a part of the compressed region was disappearing, as a result of which the neutral axis is lowered (fig. 8.1).

Moreover, the curvature of the girder gives rise to transverse forces of the flange on the web plate; by stating that the web plate should be just capable of withstanding these forces, they find, as the limiting slenderness of the web, the formula

$$\frac{b}{e} = \frac{0.48\,E}{\sqrt{R_e\,(R_e + R_r)}} \tag{8.1}$$

which gives, in the case of mild steel, $b/e = 360$. They also develop equations governing the resistance to torsional buckling and to warping of the section formed by the flange under compression and the effective portion of the web.

8.2. Shear strength

BASLER [5] assumes that, on collapse, the stress condition in the web plate results from the superposition of two fields:

a) a field of pure shear $\sigma_1 = -\sigma_2 = \tau_{cr}$ having the value given by the linear theory of buckling;

b) a "Wagner" type diagonal stress field, superimposed on this first field, in which there are tensile stresses inclined at an angle φ to the horizontal (fig. 8.2.).

The mathematical developments lead, for the ultimate value of the shear force, to the simple expression

$$T_{ult} = T_p \left[\frac{\tau_{cr}}{R''_e} + \frac{\sqrt{3}}{2} \frac{1 - \frac{\tau_{cr}}{R''_e}}{\sqrt{1 + \alpha^2}} \right] \tag{8.2}$$

where $R''_e = R_2/\sqrt{3}$ is the elastic limit in pure shear, $\alpha = a/b$ is the ratio of the sides of the plate and $T_p = R''_e\, b e$ is the shear force producing complete plasticisation of the web.

One of the merits of the American experiments is that they revealed the necessity for anchoring the diagonal stress field in the end panels (where the shear force is at a maximum) by the use of a reinforced stiffener.

8.3. Bending strength plus shear strength

In order to take into account the simultaneous effect of bending and shear, BASLER [6] suggests (fig. 8.3) the interaction equation

$$\frac{M}{M_e} = \frac{M_s}{M_e} + \frac{M_p - M_s}{M_e} \left[1 - \left(\frac{T}{\Omega_a} \frac{\Omega_a}{T_u} \right)^2 \right] \tag{8.3}$$

where Ω_a is the cross-section of the web

$M_p = R_{ez} = 1.10 M_e$, the plastic moment

$M_e = R_e I/v$, the maximum elastic moment

$M_s = R_e b \Omega_s$, the moment taken up by the plasticised flanges.

The foregoing equation finds expression in fig. 8.3 which shows that the value of the shear force T determined by equation 8.2 is not affected as long as the moment remains below the value M_e.

8.4. Extensions of the American method of design

The method of design developed by Basler and Thürlimann has been adopted in the American Standard Specifications of the A.I.S.C. It is simple and is in good agreement with the experiments carried out by these authors. In view of the fact that several parameters of the theory have had to be adjusted in order to bring about this agreement, it would be advisable to compare this theory with the results of the tests carried out by other authors.

The major defects of the American method of approach are:

1) that it is only applicable to girders without longitudinal stiffeners, whereas the large European bridges are all provided with several stiffeners of this type,

2) that it is closely connected with the conventional double-T shape of the cross-section, so that its application to the design of girders with tubular flanges and girders would result in structures of excessively heavy weight.

The first of the above-mentioned drawbacks has been palliated by a fresh series of researches by Lehigh University [15, 16, 20] carried out on girders possessing a longitudinal stiffener. Cooper [15] extends the method of Basler and Thürlimann by assuming (fig. 8.3) that separate incomplete diagonal tension fields arise in each of the sub-panels separated by the longitudinal stiffener.

Tests have shown that the longitudinal stiffener only remains effective until collapse provided that its relative stiffness γ is a multiple of the theoretical value γ^*. Although the reporter is fully in agreement with Cooper on this point, it does seem to him that additional investigations and, in particular, a comparison of the new method of design with the whole of the known experimental results, are necessary before it is possible to have complete confidence in a collapse model which appears to be somewhat inadequate for girders with one longitudinal stiffener and would be still more so for girders possessing several such stiffeners.

9. Conclusions

1. The linear theory of buckling has reached a high degree of development; it has made possible the reliable construction of wide-span plate-girder bridges. The numerical data require to be supplemented for plates built-in along their edges and strengthened by tubular stiffeners.

This theory has the advantage of being immediately applicable to all structures made from flat sheets, including for example, box-girder bridges and hydraulic constructions. It is suitable for structures where the risk of fatigue is a determinant factor, but it does not provide a structure that is optimised as regards elasto-plastic static collapse.

2. The present tendency is towards establishing a theory that is sufficiently simple and capable of estimating with safety the load that would cause collapse

of the girder. This theory must be supplemented by rules ensuring satisfactory operation in service and excluding any risk of fatigue.

This tendency is general, not only in steel construction, but also in reinforced concrete and prestressed concrete structures (work of the C.E.B. and the F.I.P.) and corresponds to recent progress in the fields of plastic calculation and the analysis of the conception of safety.

3. The equations of the non-linear theory of buckling are known, but the numerical data for actual cases of deformable framework are very difficult to obtain and are totally inadequate at the present time. The principal merit of the non-linear theory is, for the moment, to show that it is possible to adopt safety factors that are variable with the thinness of the plate and are, if necessary, less than unity. The extension of this theory to elasto-plastic buckling, which would lead to a thoroughly accurate prediction of the load causing collapse, is still in its early stages.

4. The American theory appears to be sufficiently verified experimentally for girders without longitudinal stiffeners. For those with one longitudinal stiffener, additional tests and theoretical researches are necessary. For girders with a large number of longitudinal stiffeners which are currently employed in Europe, no solution up to the point of collapse is yet available.

10. Recommendations for Future Researches

In the opinion of the reporter, it would be particularly advisable to investigate the following problems:

1. Study of plate girders with curved or folded webs, but devoid of stiffeners, except at points where heavy loads are concentrated, particularly with a view to developing economic methods of fabrication.

2. Inquiry into any fatigue cracks that may have been observed in the web in the vicinity of the stiffeners in large bridges with plate girders.

3. Extension of the fatigue tests of Hall and Stallmeyer to girders comprising horizontal stiffeners.

4. Study of the possibility of creating, by a judicious sequence of welding operations or by localised heating operations (with a welding torch, for example), residual stress fields stabilising the web plates.

5. Tests to destruction on girders with diagonal stiffeners.

6. Problems of buckling in the case of girders in which the web is made of steel having less strength than that of the steel used for the flanges and similarly in the case of webs formed of plates of steel with different strengths.

7. Researches into the possibility of increasing the ultimate strength of flat web plates by giving them an adequate initial deformation (for example, pressed bosses such as those advocated by JUNGBLUTH [22]).

8. Researches in order to develop a method of design for girders fitted with several longitudinal stiffeners.

9. New researches into the mode of action of longitudinal stiffeners located on one side only of the web. (A joint study on this point by Rockey and Skaloud is foreshadowed.)

10. Researches into plate girders with expanded metal webs.

11. Researches into the buckling of the web under the combined action of bending, shear and a concentrated force applied to the upper flange (girders of travelling cranes) taking into account the stiffening effect of the rails of the travelling cranes.

11. Acknowledgments

In concluding this report, the author wishes to thank his colleagues, Professors B. M. Broude, P. B. Cooper, Kireenko, K. Klöppel, W. H. Munse, K. C. Rockey, M. Skaloud, J. Stallmeyer, and R. N. Wright, who kindly informed him of the work they had in progress.

He would like to express his special thanks to Professor A. Tesár, of the Bratislava Institute of Technology, who assisted him in drafting the present report, particularly in regard to the contributions of research workers from Eastern Europe, during a visit he paid to the author's department in the University of Liège.