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**Earthquake Forces Acting on Tall Concrete Chimneys**

Charges sismiques sur des chemineées en béton de grande hauteur

Erdbebenkräfte auf hohe Betonschornsteine

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SCOPE OF THE STUDY

This investigation is based on studying the response of ten actual reinforced concrete chimneys varying in height from 352 ft. to 1200 ft. The physical properties of these ten chimneys are tabulated in Table 1. Accelerograms for the three actual earthquakes tabulated in Table 2 have been selected for the analytical study. All tabulated results are based on the average values obtained from the response due to these three accelerograms. It should be mentioned that the average response due to the three earthquakes has been found to be very close to the average response due to seven strong motion earthquakes which include the three used in the paper [1].

METHOD OF SOLUTION

The modal analysis techniques are used in finding the response of a chimney to the earthquake accelerations at the base of the chimney. The steps will be stated very briefly.

1. Determine the mode shapes and the shears and moments associated with each mode. The Stodola process combined with numerical integration is used [1], [2]. For practical purposes three or four modes of vibration will be enough.
2. The displacements,  $Y(x,t)$ , in the chimneys as well as the shears,  $V(x,t)$ , and bending moments  $M(x,t)$  at any section and at any time are then computed by the following equations [2], [3]:

$$Y(x, t) = \sum_{j=1}^n \phi_j(x) \cdot q_j(t) \quad (1)$$

$$V(x, t) = \sum_{j=1}^n V_j(x) \cdot q_j(t) \quad (2)$$

$$M(x, t) = \sum_{j=1}^n M_j(x) \cdot q_j(t) \quad (3)$$

Table 1 - Data for Chimneys Used in Study

Chimney No.	Height (ft.)	Outside Diameter (ft.)		Total Weight (kips)	Period (seconds per cycle)	E (kips/sq.in.)	Remarks
		Top	Bottom				
1	352	23.58	30.90	4532	1.74	3500	Corbel supported brick lining
2	450	16.33	35.79	6743	2.12	3500	Corbel supported brick lining
3	534	18.67	35.03	8374	2.26	3500	Independent liner
4	622	23.33	47.26	12526	2.33	4000	Independent liner
5	707	19.98	69.14	26236	2.91	3500	Corbel supported brick lining
6	797	31.33	62.50	23392	3.29	3625	Steel liner
7	825	25.00	63.96	22970	3.44	3625	Steel liner
8	840	41.66	74.42	40976	3.33	3630	Three steel liners
9	997	33.67	83.00	42440	3.64	3625	Steel liner
10	1200	37.00	95.29	65955	4.68	3820	Steel liner

Table 2 - List of Earthquakes

Designation	Location	Date	Direction
A	El-Centro, Cal.	May 18, 1940	West
B	Olympia, Wash.	April 13, 1949	N 10° W
C	Taft, Cal.	July 21, 1952	S 21° W

in which  $x$  is the distance along the chimney,  $t$  is the time,  $\phi_j(x)$  is the mode shape in the  $j^{\text{th}}$  mode and  $V_j(x)$  and  $M_j(x)$  are the shears and moments associated with the  $j^{\text{th}}$  mode.

The value of  $q_j(t)$ , which is a multiplier for the modal displacements, shears and moments, is obtained from the following equation:

$$\ddot{q}_j(t) + 2\beta\omega_j\dot{q}_j(t) + \omega_j^2q_j(t) = \frac{-a(t) \int_0^H m(x)\phi_j(x) dx}{\int_0^H m(x)\phi_j^2(x) dx} \quad (4)$$

in which  $\beta$  is the fraction of critical damping,  $\omega_j$  is the frequency in radians/sec.,  $a(t)$  is the acceleration of the earthquake and  $m(x)$  refers to the mass per unit length.

Equation (4) is solved numerically [1] using a third order Runge-Kutta process.

3. Although the solution of equations (1), (2), and (3) will give displacements, shears and moments at all intervals of time, yet the maximum values at any section are the only ones that are of interest. These maximum values are computed for each earthquake and the average is then obtained.

## RESULTS

### Base Shear

Many codes express the value of the maximum shear at the base of a chimney due to earthquakes as a function of the first mode period and of the total weight of the chimney. For this reason the maximum base shear has been computed for each chimney due to earthquakes A, B, and C and the average of these three maximums has been plotted as a ratio of base shear to total weight in Figure 1. It should be emphasized that the maximum base shear (Figure 1) is the maximum of the algebraic sum of four-mode responses.

### Base Moment

A dimensionless plot of the maximum base moment,  $M_b$ , is given in Figure 2. The ratio of  $M_b$  divided by the product of the base shear times the height,  $H$ , is plotted against the first mode period of the ten chimneys considered in this study.

### Shear Distribution from Accelerograms

The distribution of the maximum shears along the chimneys is presented in Figure 3 in normalized form for five of the ten chimneys considered. The numerical value of the maximum shear for any height above the base can be calculated from the value of the base shear recorded in the Figure.

### Maximum Bending Moment Curves

The maximum bending moments in five of the ten chimneys are presented graphically in normalized form in Figure 4. These bending moments are the average value of the maximum moment curves due to earthquakes A, B, and C.

## PROPOSED ACI EARTHQUAKE PROVISIONS [4]

1. Base shear. In the proposed chimney code of the American Concrete Institute, the base,  $V_b$ , is given by the empirical equation:

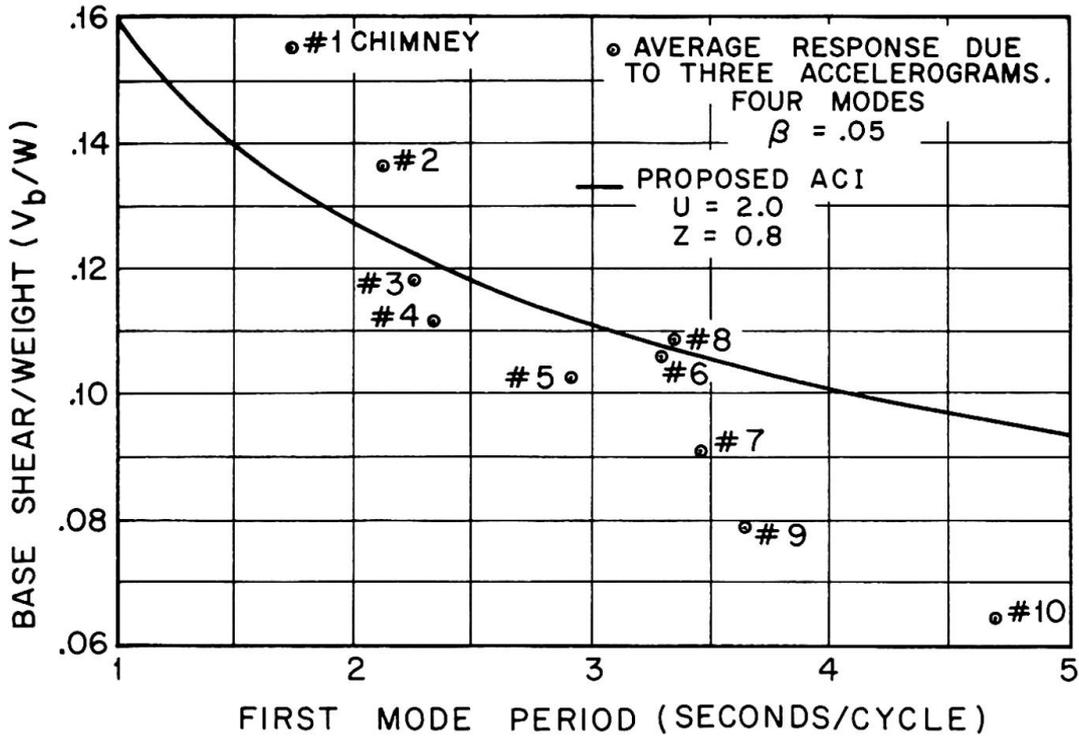


Figure 1 Base Shear/Weight Vs. First Mode Period

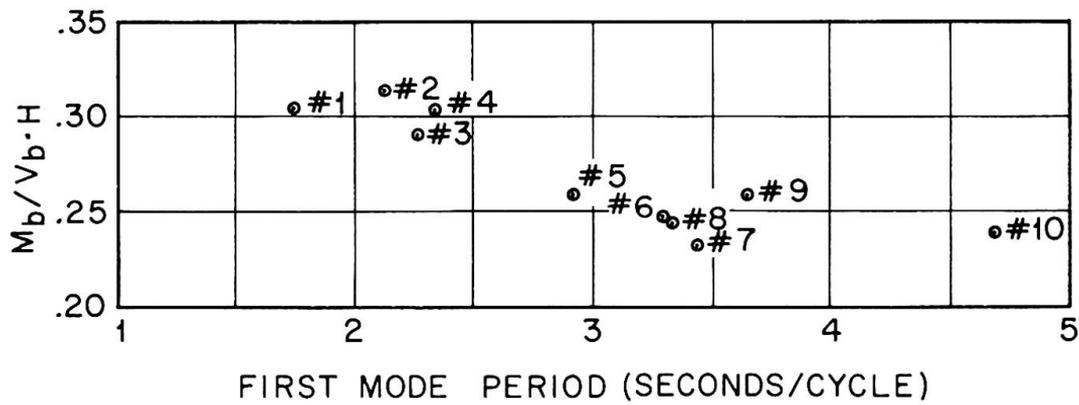


Figure 2 Base Moment/Base Shear Times Height Vs. First Mode Period

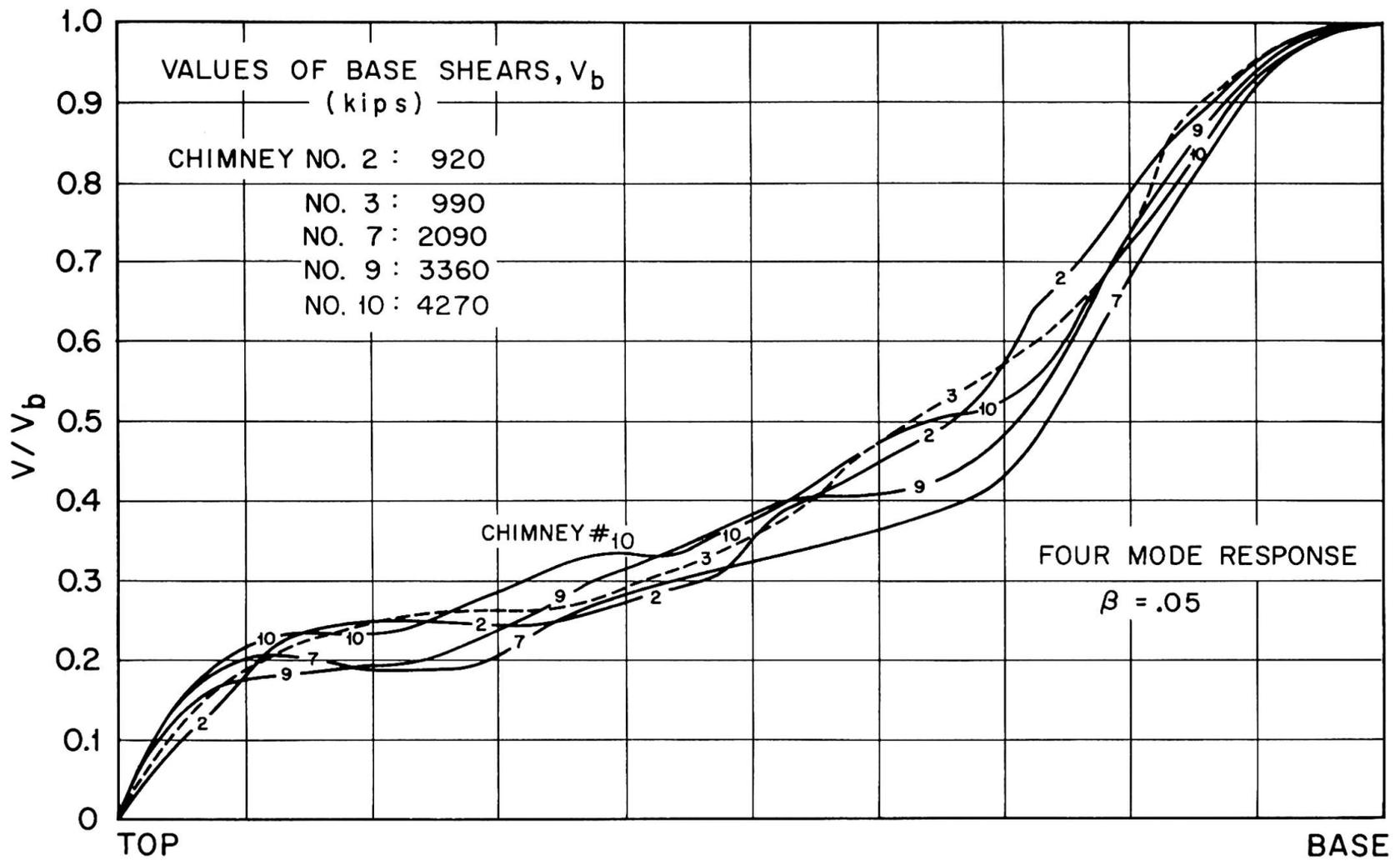


Figure 3 Normalized Maximum Shear Curves  
(Average Response Due to Three Earthquakes)

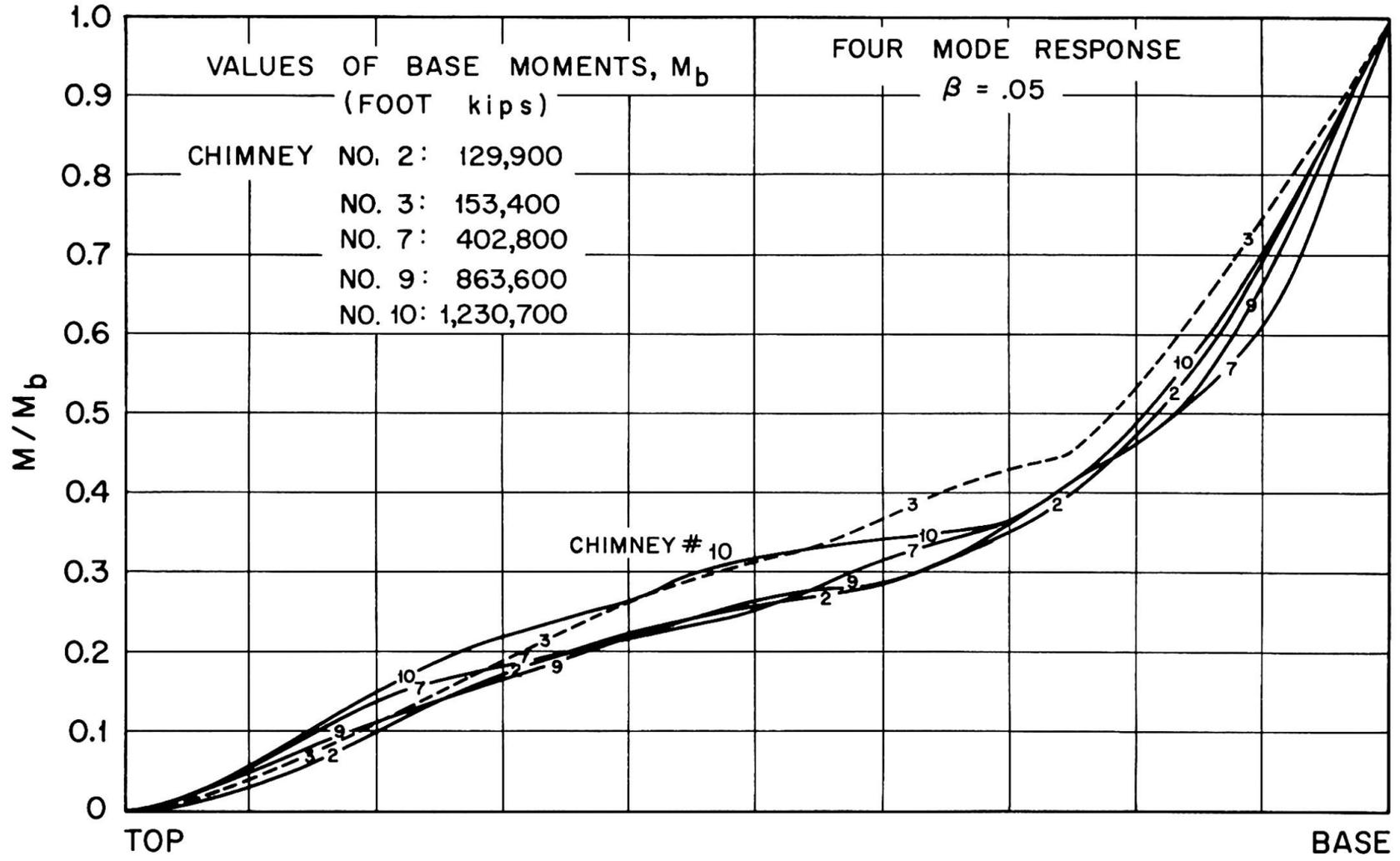


Figure 4 Normalized Maximum Moment Curves  
(Average Response to Three Earthquakes)

$$V_b = ZUCW \text{ or } V_b = ZUCW_1 \tag{5}$$

where

Z = a zone factor which shall not be less than 0.30 for Zone 1, 0.5 for Zone 2 and 1.0 for Zone 3. Zones are indicated on a map for the United States Uniform Building Code.

U = Use factor varying from 1.3 to 2.0.

W = Total weight of chimney without lining

W<sub>1</sub> = Total weight of chimney with lining

$$C = \frac{0.1}{3\sqrt{T}}$$

The period, T (secs. per cycle), may be approximated by:

$$T = \frac{1.8 H^2}{(3D_1 - D)\sqrt{E}} \cdot \sqrt{\frac{W_1}{W}} \tag{7}$$

H = Height of chimney in feet

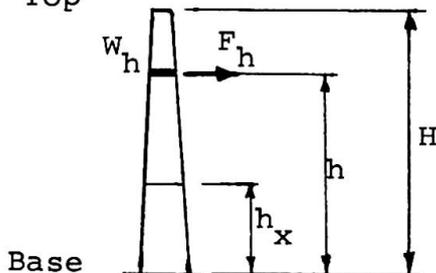
D<sub>1</sub> = Outside diameter of chimney shell at base (ft)

D = Outside diameter of chimney shell at top (ft)

E = Modulus of elasticity of concrete (lbs./sq./in.)

2. Distribution of Lateral Forces. Fifteen percent of the base shear, V<sub>b</sub>, is considered concentrated at the top of the chimney and the remainder is distributed in accordance with the following requirement:

Top



$$F_h = 0.85 V_b \frac{w_h h}{\sum w_h h} \tag{8}$$

3. Bending Moments. The bending moment at any level as provided by the proposed code is:

$$M_x = J_x \left[ 0.15 V_b (H - h_x) + \sum_{h=h_x}^H F_h (h - h_x) \right] \tag{9}$$

where

$$J_x = J + (1 - J) \left( \frac{h_x}{H} \right)^3 \tag{10}$$

$$J = 0.6 / \sqrt[3]{T} \quad (\text{But not less than } 0.45 \text{ nor more than } 1.0) \tag{11}$$

COMPARISON OF PROPOSED ACI PROVISIONS WITH THE ACTUAL RESPONSE DATA

The comparison with the proposed ACI Code will be presented under three parts:

(a) First Mode Period T

Both the computed values and those obtained from the ACI formula (equation 7) are tabulated in Table 3.

Table 3 - Comparison of the Computed First Mode Period, T (secs. per cycle), with the ACI T

Chimney No.	Height	Computed T	ACI T
1	352	1.74	1.98
2	450	2.12	2.34
3	534	2.26	3.18
4	622	2.33	2.94
5	707	2.91	3.03
6	797	3.29	3.92
7	825	3.44	4.02
8	840	3.33	3.86
9	997	3.64	4.43
10	1200	4.68	5.38

(b) The Base Shear or Total Lateral Force

The shape of the proposed ACI curve for base shear is plotted in Figure 1 to compare it with the data obtained from the mathematical analysis. This curve is a plot of Equation (5) for  $Z = 0.8$  and  $U = 2.0$ .

(c) The Bending Moment Curves

To compare the ACI bending moment curve with the computed curve it is necessary to use the same base shear. Therefore the maximum base shear that was obtained by the actual response is distributed according to the ACI provisions and the ACI bending moment curve is obtained from such distributions by using Equation (9). These ACI moments are compared with those obtained from the actual response in Figures 5, 6, and 7 for chimneys #4, #7, and #10 respectively.

IMPORTANCE OF MAXIMUM STRESS INVESTIGATION

The non-linear variation of the stress in the reinforcing steel of a typical reinforced concrete chimney with respect to the change in the bending moment is clearly shown in Figure 8. The values given have been calculated for a cross-section with a center line diameter,  $d$ , and thickness,  $t$ , of 51.97 ft. and .833 ft. respectively. The variation is affected considerably by the percentage of reinforcing steel. Procedures for design have been presented in a previous paper by the authors [3] and will not be repeated here. However, it is recommended that a load factor of at least 1.5 times the working load bending moments be used in the maximum stress design with upper stress limits of  $0.8 f'_c$  for concrete and  $F_y$  for steel. The value of  $f'_c$  is the specified compressive strength of the concrete and  $F_y$  is the yield strength of the steel. The above remarks do not apply to an ultimate strength design.

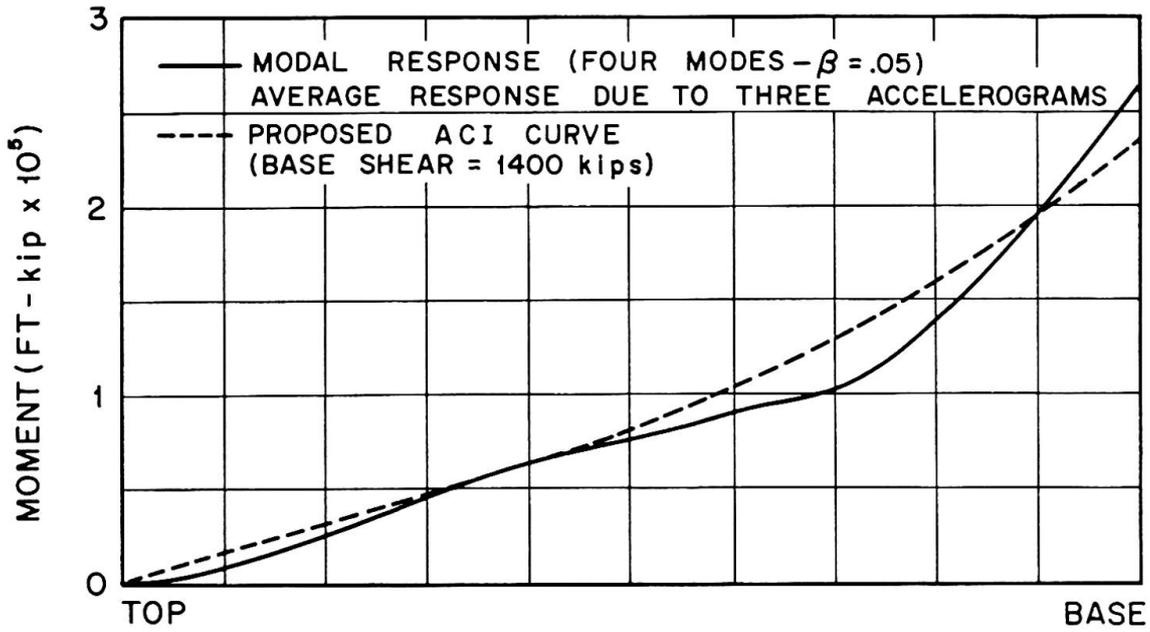


Figure 5 Comparison of Moment Curves - Chimney #4 (Actual Response Vs. Proposed ACI)

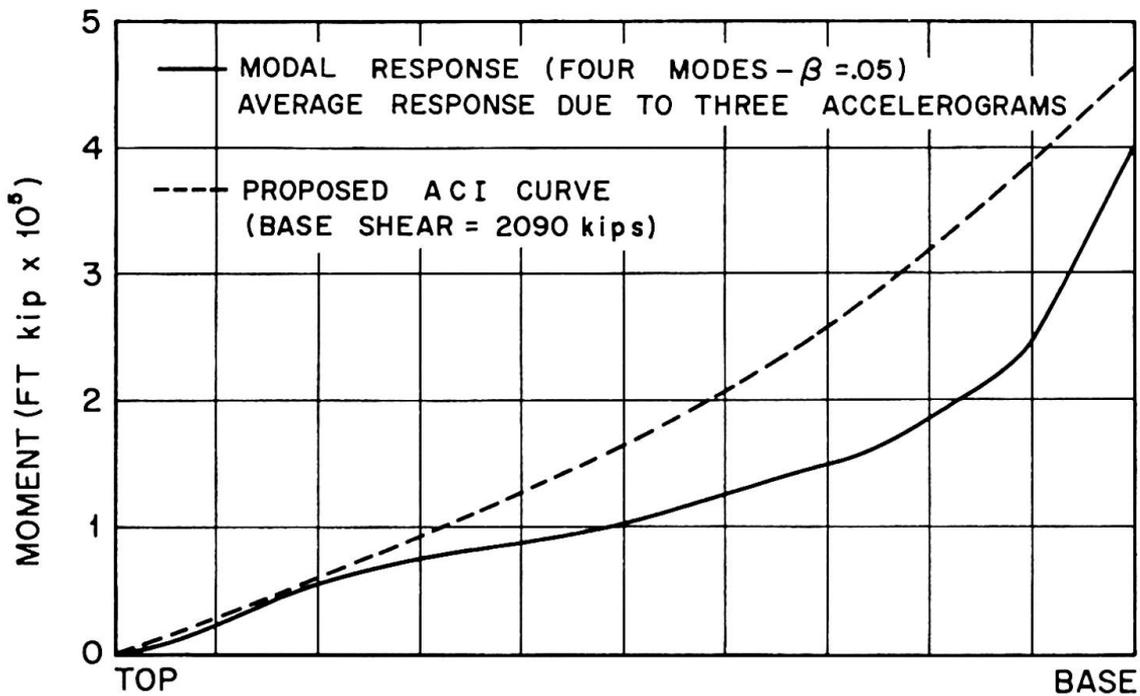


Figure 6 Comparison of Moment Curves - Chimney #7 (Actual Response Vs. Proposed ACI)

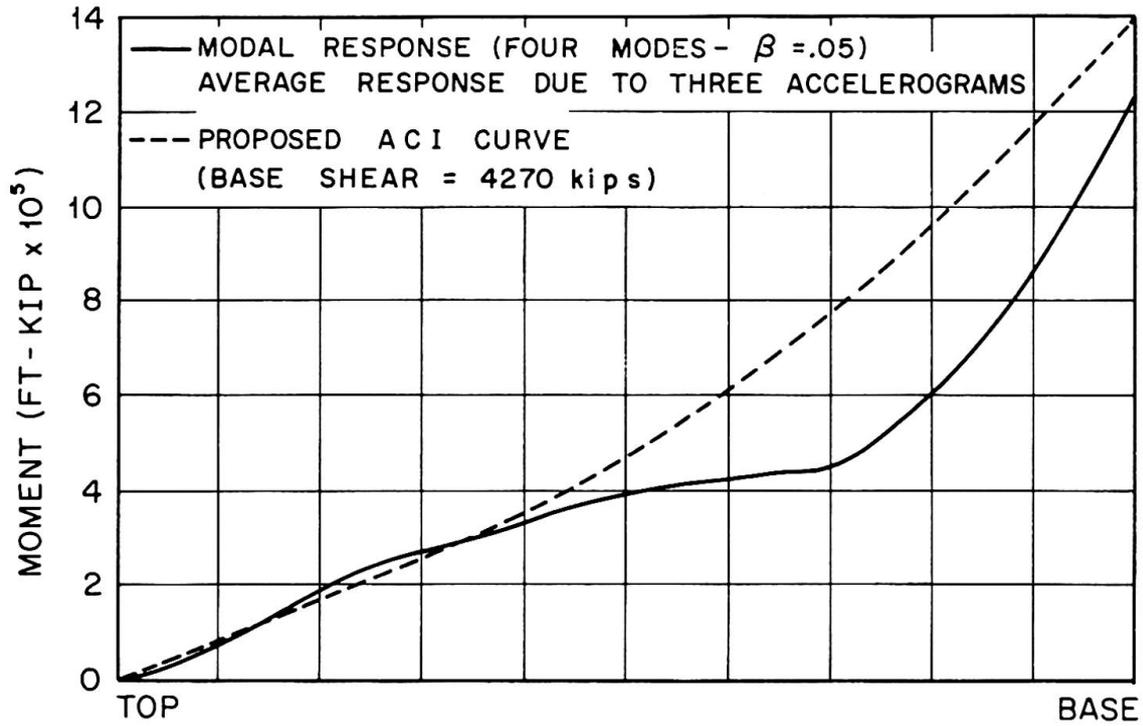


Figure 7 Comparison of Moment Curves-Chimney #10  
(Actual Response Vs. Proposed A C I)

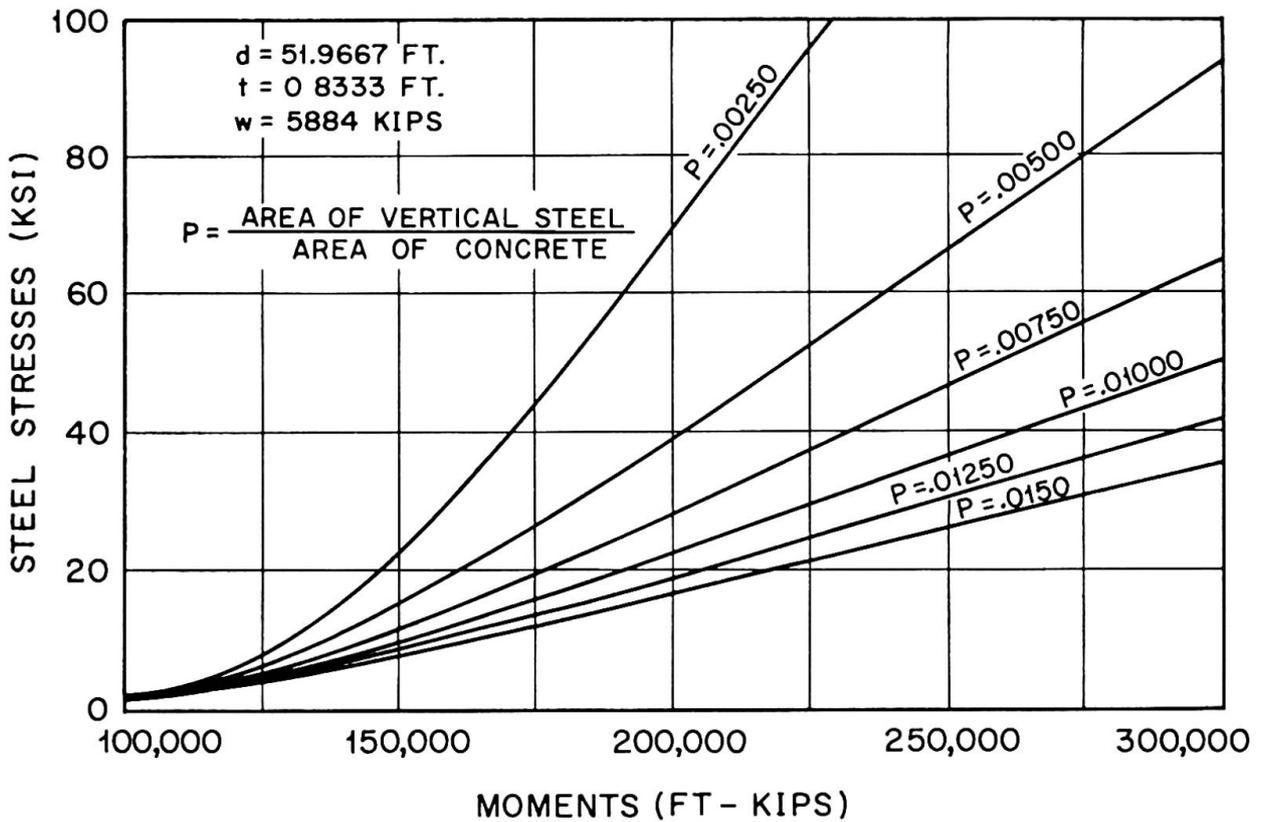


Figure 8 Steel Stresses Vs. Moments for Steel Ratios P

### CONCLUSIONS

Only certain pertinent facts of reinforced concrete chimney design for earthquakes have been presented in this paper. From the data given it seems reasonable to draw the following conclusions:

1. Although procedures that are presented in chimney codes are useful for preliminary designs they are not always sufficiently accurate for a final design.

2. A response analysis in which from three to seven carefully selected accelerograms are used is recommended for investigating the final design.

3. Both a working stress and a maximum stress investigation of the stresses should be made.

4. Although not discussed in this paper maximum shearing stresses should also be determined. These stresses may occur in the upper one-fifth of the chimney.

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## SUMMARY

This paper is primarily concerned with presenting some results of the response of actual reinforced concrete chimneys to recorded accelerograms of actual earthquakes. The results are based on an elastic response using the first four modes with a damping coefficient of .05 of critical.

The analytical results are then compared with those proposed by the American Concrete Institute Code (1968) for Earthquake Design of Chimneys. The provisions of this Code are summarized in the paper.

The last part of the paper emphasizes the necessity of designing reinforced concrete chimneys for both a working stress and a maximum stress condition as the stresses, especially in the steel, do not vary linearly with the bending moments.

## RÉSUMÉ

Cette rédaction présente quelques résultats de réactions de cheminées en béton précontraint sur les accélérations mesurées de plusieurs tremblements de terre.

Ces résultats analytiques sont comparés avec le Code de l'Institut Américain du Béton, dont les prescriptions sont résumées ici. Enfin, la rédaction démontre la nécessité de dimensionner les cheminées en béton précontraint et pour une charge de service et pour des conditions de charge maximales, vu que les tensions ne varient pas linéairement avec le moment, surtout dans l'acier.

## ZUSAMMENFASSUNG

Dieser Beitrag ist hauptsächlich bemüht, einige Ergebnisse zu zeigen, die man aus der Aufzeichnung der Beschleunigungen von Erdbeben als Wirkung auf Stahlbeton-Schornsteine erhält.

Die Ergebnisse stützen sich auf elastische Bestimmung, die ersten vier Fälle benützend, mit einem Dämpfungsbeiwert von 0.05 des kritischen. Die analytischen Ergebnisse sind dann mit denjenigen verglichen worden, die durch die Normen des amerikanischen Betoninstitutes für den Entwurf von Schornsteinen bei Erdbeben vorgeschlagen wurden. Der letzte Teil des Beitrags betont ausdrücklich die Notwendigkeit, Stahlbetonkamine für Gebrauchs- und maximale Spannungsbedingungen zu entwerfen, da die Spannungen, insbesondere jene des Stahles, nicht linear mit dem Biegemoment ändern.