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## Structural Dynamic Considerations in Horizontally Curved Bridges

Quelques considérations sur le comportement dynamique de ponts en courbe

Dynamische Betrachtungen an waagrechtgekrümmten Brücken

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### 1. INTRODUCTION

As the engineer is turning his attention more and more from aerospace to "geospace" and his environment, a more sophisticated understanding of the response of earthbound structures subjected to natural and other forces assumes greater importance. In particular with the development of high speed transportation systems, for example in the United States, Japan, and France, it is essential to consider the dynamic loads caused by the present and future vehicles, and the dynamic response of bridge or elevated structures.

A number of analytical studies have been reported in the past fifteen years of the dynamic response of bridges on straight alignments subjected to simulated highway or railway loading. However, little or nothing of substance has been reported for horizontally curved bridges, and as is seen from the results presented herein, this increasingly used geometry gives rise to substantially higher dynamic amplification factors for displacements and stress resultants (shear, flexural and torsional moments).

This contribution discusses some analytical results obtained for either a concentrated force or a simulated vehicle traversing a horizontally curved bridge at constant velocity. The significance of this type of study assumes greater importance when it is realized that in the next ten years the world will witness new and more efficient and probably automated transportation systems in which vehicle speeds will approach 500 miles per hour.

### 2. CURRENT SPECIFICATIONS

In the United States and in many foreign countries the American Railway Engineering Association Specifications [1] are used to determine the dynamic effects of all types of moving trains by a Cooper's E-72 loading. In applying the AREA Specifications to obtain dynamic effects, an impact factor, expressed as a percentage of the static live load, is calculated on the basis of only one independent variable, a characteristic length  $L$  in feet, which in general is taken as the loaded length of the member being examined. For example, for the direct vertical impact of moving trains for beam spans, stringers, girders, ...:

$$\text{Impact Percentage} = \begin{cases} 60 - \frac{L^2}{500} & L < 100 \text{ ft.} \\ \frac{1800}{L^2 + 40} + 10 & L \geq 100 \text{ ft.} \end{cases} \quad (1a)$$

and for truss spans

$$\text{Impact Percentage} = \frac{4000}{L+25} + 15 \quad (1b)$$

A simple calculation shows that the greatest impact percentage can never exceed 40% of the static live load.

The standard live loading for highway bridges in the United States is the HS 20 - 44 representing a highway truck-trailer of 72,000 pounds, or alternately a uniformly distributed lane loading of 640 pounds per linear foot of lane with either a concentrated force of 18000 pound (for moment) or 26000 pounds (for shear). Again, the dynamic effects are accounted for by utilizing only one independent variable  $L$ , which represents the length in feet of the portion of the bridge span that is loaded to produce the maximum stress in the member being investigated. The AASHO formula is:

$$\text{Impact Factor} = \frac{50}{L+25} \leq 0.30 \quad (2)$$

It must be noted that neither of these specifications consider other important parameters such as the velocity of the vehicle, the unevenness of the deck of the bridge, the initial conditions of the vehicle upon entering the span (pitching motion for example), or the geometry of the span, that is, a straight alignment, a vertical curve, or a horizontal curve.

### 3. STRAIGHT BRIDGES

Comprehensive analytical studies of the dynamic behavior of simple and multi-span bridges on a straight alignment have been reported in the literature [e.g. 3,4,5]. Some of the parameters considered in these studies involve: the speed of the vehicle; the ratio of the total weight of the vehicle to the total weight of the bridge; the ratio of the natural frequency of the  $j^{\text{th}}$  axle to the fundamental frequency of the bridge; rotatory inertia of the vehicle in pitching motion; axle spacing; shape of the roadway profile; initial condition of the vehicle (vertical and angular displacements) upon entering the span; initial condition of the bridge (dynamic deflection and velocity) when the vehicle enters the span. When these parameters are varied through the ranges of values that describe the vehicle-bridge system of today's dynamic increments as high as 1.0 are obtained; however, for the more basic parameters ratios involving vehicle velocity, weights of vehicle and bridge, and natural frequencies of vehicle and span, the maximum dynamic increments are of the order of magnitude of 0.30. The term "dynamic increment" is defined as the difference between the dynamic value of a function (e.g. deflection, shear, moment) at a specified section and the value of the function for the same force or load statically applied at the same specified section, this difference being divided by the absolute maximum static value of the function at the specified section.

Thus, it can be concluded that even though all the parameters upon which the dynamic response of a bridge depend are not included in the AREA and AASHO Specifications, the impact values specified by these organizations appear feasible and reasonable for current design procedures.

#### 4. HORIZONTALLY CURVED BRIDGES

As horizontally curved bridges (many times approximated by a series of short straight segments) were being utilized more frequently in highway design, the University of Pennsylvania initiated, a few years ago, a study to determine the dynamic response characteristics of such structures. The major objective of this study was to ascertain the dynamic increments for realistic bridge-vehicle systems and thus determine whether the specifications in current use were adequate.

A simply supported, single span, horizontally curved bridge was chosen (see Figure 1) and two types of input were used: (1) A single force traversing the bridge along its centerline at constant velocity, and (2) A rigid mass (sprung mass) connected by a linear spring and a viscous damper to a rigid mass (unsprung mass) which was always in contact with the bridge deck, traversing the bridge along its centerline at constant velocity. See Figure 2. The parameters considered and their corresponding ranges were:

1. Central Angle,  $\theta_1$   
 $0.125 \text{ radian} \leq \theta_1 \leq 1.0 \text{ radian}$
2. Radius of Horizontal Curvature,  $r$   
 $200 \text{ ft.} \leq r \leq 800 \text{ ft.}$
3. Rigidity Ratio of the Bridge Cross-section,  $A$   

$$A = \frac{\text{torsional rigidity} + \text{warping rigidity function}}{\text{Flexural rigidity}}$$
 $0.05 \leq A \leq 1.00$
4. Speed Parameter,  $\alpha_v$   
 (velocity of vehicle) (fundamental period of equivalent straight bridge\*)  

$$\alpha_x = \frac{2(\text{length of equivalent straight bridge*})}{\text{fundamental period of equivalent straight bridge*}}$$
 $0.06 \leq \alpha_v \leq 0.18 (20 \text{ mph} \leq v \leq 60 \text{ mph})$
5. Weight Ratio,  $R_v$   

$$R_v = \frac{\text{total weight of vehicle}}{\text{weight of bridge}}$$
 $0.08 \leq R_v \leq 1.00$
6. Frequency Ratio,  $\phi_v$   

$$\phi_v = \frac{\text{natural frequency of vehicle}}{\text{natural frequency of equivalent straight bridge*}}$$

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\*The equivalent straight bridge is defined as having the same length as the curved bridge.

The displacement equations of motion representing this system were coupled, non homogeneous partial differential equations which were solved by techniques described previously in detail by Tan and Shore [6,7] . The major conclusions drawn from this study of horizontally curved bridges were: (1) The dynamic increments (as defined in Section 3) for deflections and stress resultants for the moving constant force were generally higher by at least 10% than for an equivalent straight beam; (2) The dynamic increments for the moving vehicle for deflections and stress resultants were significantly higher than for an equivalent straight beam; (3) When the frequency ratio and the weight ratio are 0.30 or less the response of the bridge due to the constant force can be used; (4) For a rigidity ratio greater than 0.5 and a central angle less than 0.50 radians, the curved bridge response can be predicted by an equivalent straight bridge; (5) Preliminary results indicate that the dynamic increments for vertical deflection,  $w$ , rotation,  $\beta$ , and stress resultants are essentially the same for a given set of parameters. Two typical response curves for curved bridges are shown in Figures 3 and 4. In these Figures the following notation is used:

DIWSB = dynamic increment for vertical deflection of an equivalent straight bridge of length  $L_c$  ; DIWCB = dynamic increment for vertical deflection of the horizontally curved bridge.

On the basis of the results obtained in this study of the response of horizontally curved bridge the following recommendations appear in order:

- (1) An appraisal of the current specified impact and dynamic factors to determine whether other variables should be incorporated in addition to only a characteristic length.
- (2) Since for curved bridges the dynamic increment is extremely sensitive to the rigidity ratio parameter, attention should be given to methods for accurately calculating the torsional, warping, and flexural rigidities of complex bridge structures. It appears necessary and feasible that work on analytical methods by finite element techniques verified by model tests should be initiated for predicting these rigidity ratios.
- (3) Dynamic response tests on laboratory models of curved beams appears advisable. These models should simulate as closely as possible the mathematical model used in References 6 and 7, to verify the analytical results.
- (4) Field tests of actual curved bridge structures subjected to dynamic loads should be initiated to correlate both the analytical results and model tests.
- (5) Further analytical work should be initiated for curved bridges to include other effects such as superelevation which introduces an initial twist in the bridge, vehicle speeds up to possibly 500 mph, longitudinal forces due to braking, accelerations, and

(5) Continued:

decelerations at these high speeds, as well as the other parameters listed in Section 3 which were reported for straight bridges, but which were not included in the study reported in References 6 and 7.

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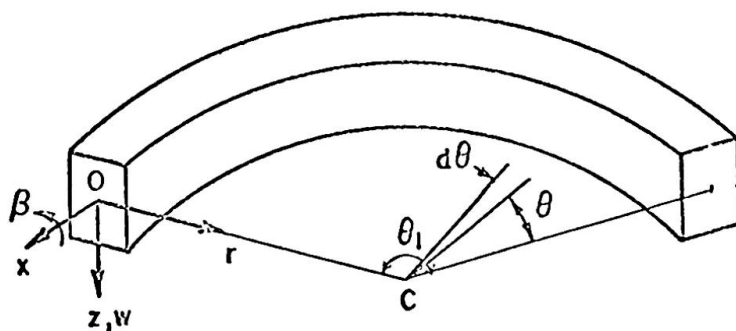


FIG. 1. CURVED BEAM GEOMETRY

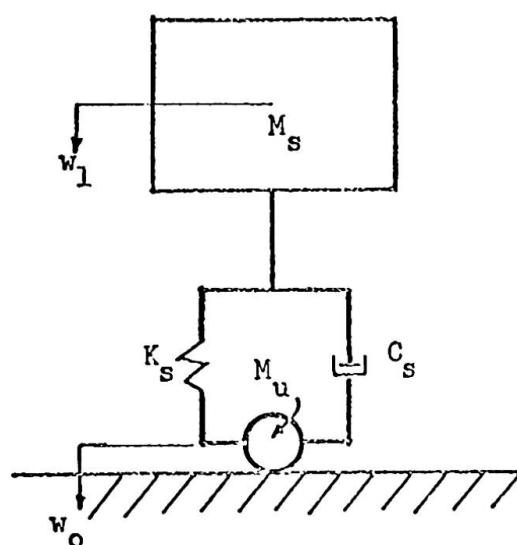


FIG. 2. IDEALIZED VEHICLE

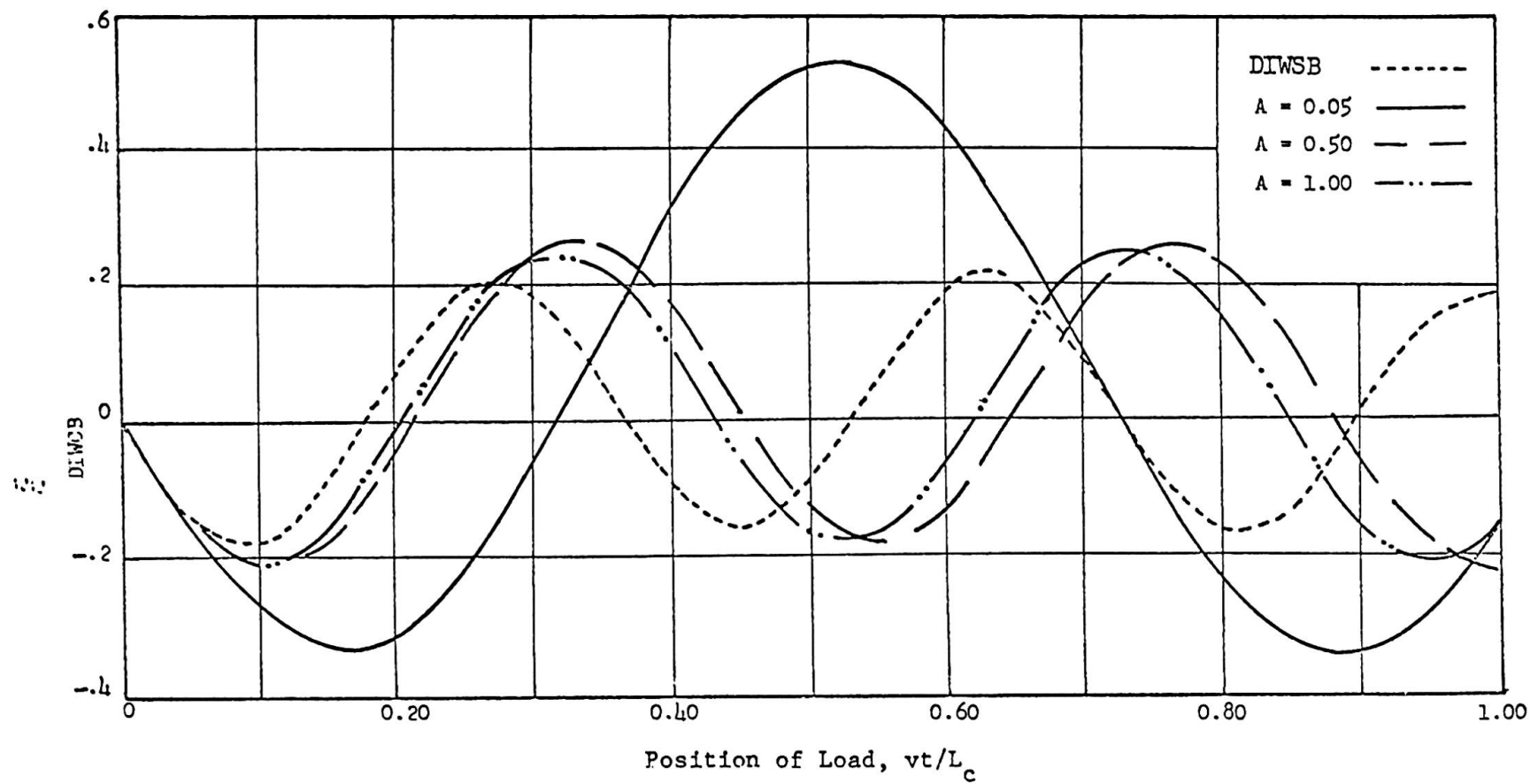


FIG. 3. EFFECT OF RIGIDITY RATIO ON DYNAMIC INCREMENT FOR DEFLECTION AT MIDSPAN  
 CONSTANT MOVING FORCE,  $L_c = r = 200'$ ,  $\alpha_v = 0.18$

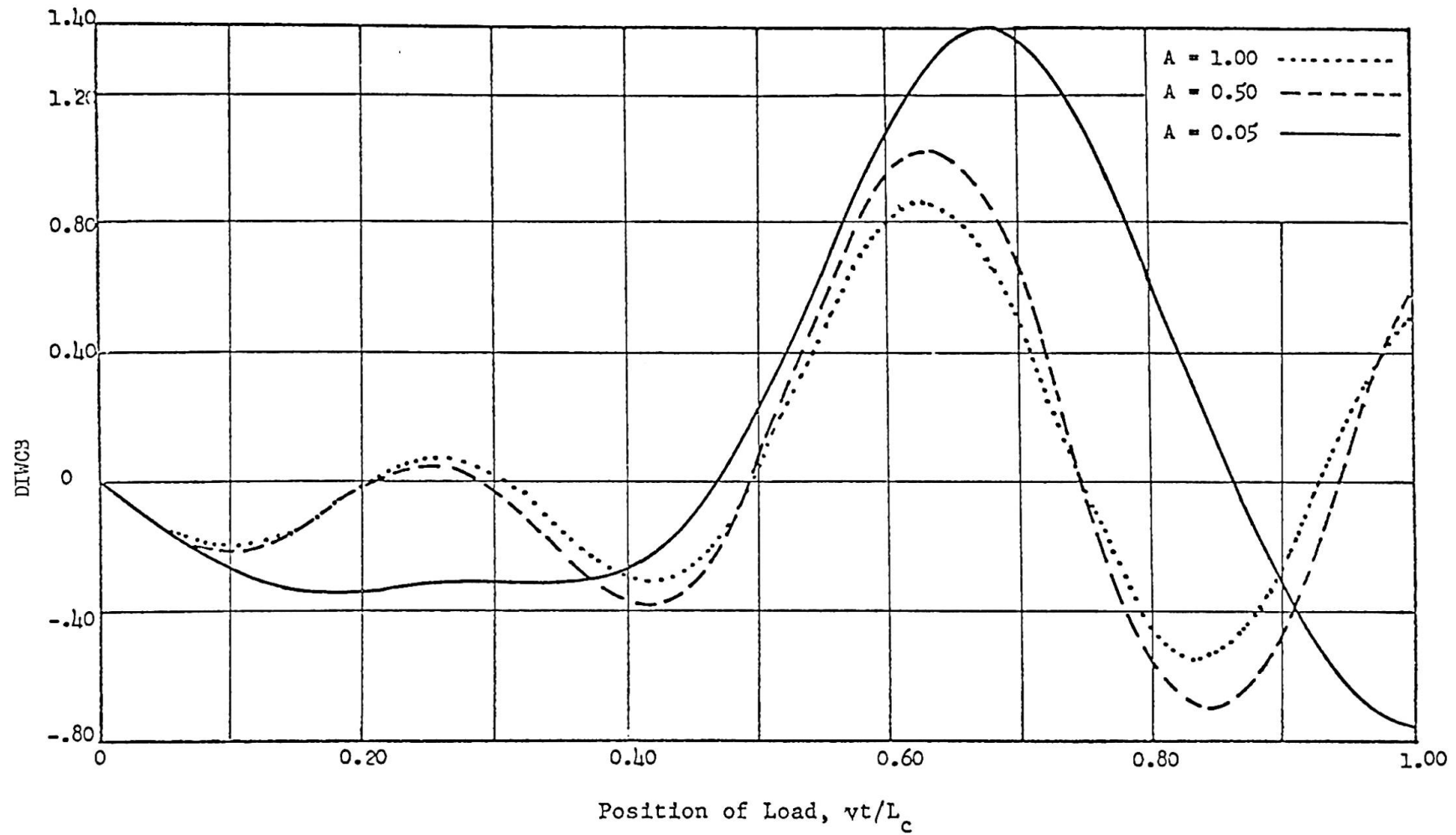


FIG. 4. EFFECT OF RIGIDITY RATIO ON DYNAMIC INCREMENT FOR DEFLECTION AT MIDSPAN  
SMOOTHLY MOVING VEHICLE,  $L_c = r = 200'$ ,  $\alpha_v = 0.18$ ,  $R_v = 0.50$ ,  $\varphi_v = 0.50$



## SUMMARY

Many studies have been reported in the past fifteen years concerning the dynamic response of bridges on straight alignments subjected to simulated highway loading. However, little has been reported for horizontally curved bridges and this rather common alignment on highway and railway systems gives rise to substantially higher dynamic amplification factors for displacements and stress resultants. Such a study has been made for a simulated highway vehicle traversing a curved bridge considering such parameters as radius of curvature, flexural to torsional rigidity ratio, velocity of the vehicle, and vehicle mass to bridge mass ratio. Some overall results will be reported and recommendations made in light of current specifications.

## RÉSUMÉ

De nombreuses études ont été faites ces dernières 15 années sur le comportement dynamique de ponts droits soumis à une charge d'autoroute simulée. Cependant, on a presque totalement négligé les ponts en courbe, beaucoup employés pour routes et chemins de fers. Pourtant, on a ici des facteurs d'amplification dynamique considérablement plus grands pour les déplacements et pour des tensions résultantes. Une telle étude a été faite pour un véhicule de route simulé traversant un pont courbe, considérant des paramètres tels rayon de courbure, rapport des rigidités à la flexion et à la torsion, vitesse du véhicule, et rapport des masses du véhicule et du pont. Quelques résultats universels et des recommandations concernant les exécutions courantes seront publiés.

## ZUSAMMENFASSUNG

Viele Untersuchungen sind in den letzten fünfzehn Jahren betreffend das dynamische Verhalten von geradlinigen Brücken unter Verkehrslast angestellt worden. Wie auch immer, wenig ist über waagrechtgekrümmte Brücken gesagt worden; diese weniger gebräuchliche Ausführung der Strassen- und Eisenbahnbrücken zeitigt erheblich höhere Schwingungsamplituden für die Verschiebungen und Spannungen. Eine solche Untersuchung wurde für ein simuliertes Fahrzeug bei folgenden Parametern angestellt: Halbmesser, Drillsteifigkeit, Geschwindigkeit des Fahrzeugs sowie Ausmass desselben im Verhältnis zu dem der Brücke. Einige Gesamtspannungen und Empfehlungen aus gebräuchlichen Ausführungen sind aufgeführt.