Dynamic Effects on Precast Bridge Structures

Effets dynamiques sur des ponts en préfabriqué

Der dynamische Einfluß auf vorfabrizierte Brückenteile

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In recent times, prestressed structures assembled of precast concrete elements are used also for railway bridges. There is not much experience about their dynamic properties and therefore research first theoretical and then experimental on actual bridges had to be undertaken.

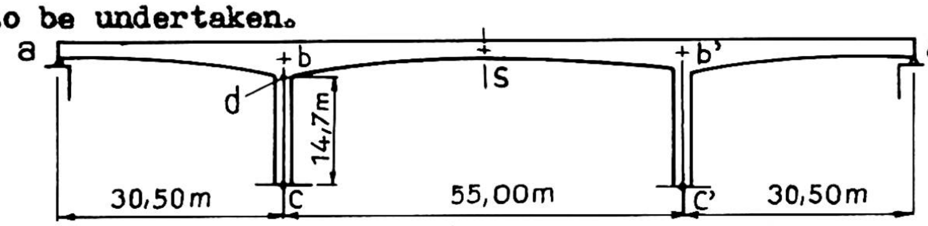

Fig. 1.

The statical system of the structure which we have used in the investigation is a three span rigid frame. (Fig.1) The cross-section in the middle of the central span is in fig.2. The elements of the superstructure which were manufactured in a central precasting plant were transported on a trailer, lifted, rectified and prestressed.

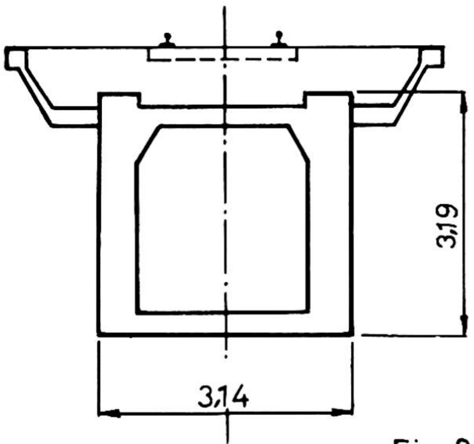

Fig. 2

The object of the research was to find theoretically the dynamic characteristics of the system i.e. the natural frequencies and modes, and to determine the forced vibrations produced by the load crossing the bridge. The results were then compared with the results of measurement on an actual structure.

The theoretical investigation.

The horizontal beams of the frame structure are of box-shaped cross-section with variable mass and moment of inertia. The theoretical analysis of such a system can be executed by various methods of different accuracy and laboriousness. The differential equation for the vertical motion in this case becomes

$$\mu(x)\,\frac{\partial^2 v(x,t)}{\partial t^2} + E\,\frac{\partial}{\partial x^2}\left[I(x)\,\frac{\partial^2 v(x,t)}{\partial x^2}\right] = 0 \qquad (1)$$

where the notation is as follows

$\mu(x)$ ..... is the variably distributed mass

.. .. is the abscissa of the point in question if the origin is at the left end of each span

$v(x,t)$..... is the vertical deflection of the point x at the time t

$I(x)$ ...... the variable moment of inertia

E ....modulus of elasticity

Solution of the equation (1) can be found in the explicit form for special cases only. The major part of solutions [2] start from the work by Kirchhoff[1] who investigated the vibrations of a conical cantilever. They are available e.g. for the beams with the distribution of $\mu(x)$ and $I(x)$ as follows

$$I(x) = I_b\left(\frac{x}{L}\right)^{n+2} \qquad (2)$$

$$\mu(x) = \mu_b\left(\frac{x}{L}\right)^{n} \qquad (3)$$

where μ_b and I_b are the mass and moment of inertia on the right end of the beam and L is the distance of the right end from the conveniently chosen origin (fig.3).There are only four arbitrary constants in expressions(2) and (3) and it is evident that not any distribution of $\mu(x)$ and $I(x)$ can be expressed. Consequently, for an actual structure this laborious solution represents, as a rule, an

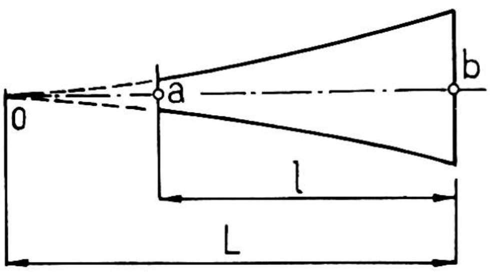

Fig. 3.

1) Kirchhoff G.: Vorlesungen über mat. Physik. Mechanik. Leipzig 1876
2 Коренев Б.Г.: Некоторые задачи тесрии упругости. Москва 1960.

approximation only.

Equally, some aproximative methods such as those of Rayleigh, Stodola, Ritz or Galerkin can be used, but if an adequate accuracy of calculus is to be attained, all these methods end in tedious computations.

Therefore, the author of this contribution has used his own procedure which enables us to determine the dynamical characteristics in a relatively simple way and with arbitrary required accuracy. This method, which can be called the simplified slope--deflection method, starts from the following considerations. Let us consider a beam which vibrates harmonically, and mark on it some points (fig.4). Between the points the deformed axis of the bar creates a curve, whose shape

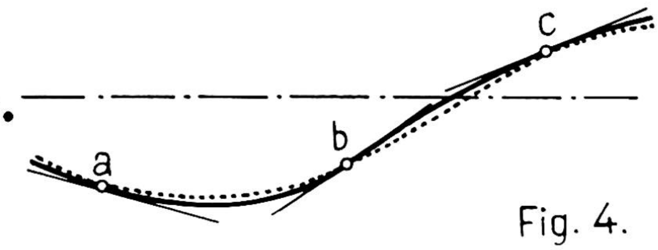

Fig. 4.

is determined not only by the position of the points a, b, c .... and their rotations, but also by the inertial forces which act on the distributed mass of the vibrating bar between these points. Let us imagine now the same beam which, however, does not vibrate but is statically deformed by some forces and moments acting in the points a, b, c ... so that the displacements and rotations of these points are the same as in the first case. It is evident that the deformed axis of the bar between the marked points will now be different, owing to the absence of the inertial forces (see the dotted line in fig.4). The difference between both shapes will decrease with decreasing both of the frequency of vibrations and the distance of the points a, b, c ..... .

Using the slope-deflection method, we divide the system by joints into singular bars. The displacements of joints can be determined by means of slope-deflection equations which are obtained from conditions of equilibrium of end forces and moments of all bars connected in singular joints. In our case, the joints are in the points a, b, s, b', a' and the bars a-b, b-c, b-s, s-b', b'-c', b'-a'.

The first task is the determination of moments and forces acting on the bar ends if they displace or rotate with an amplitude equal to unity. In fig.5 the bar a-b is represented in the case that the end b rotates harmonically.The amplitudes of end moments are[3]

$$M_{ab}(\xi_b=1)=M_{ab\ stat}(\xi_b=1)-\omega^2\int\mu(x)\,v_1(x)\,v_2x\,dx$$
$$M_{ba}(\xi_b=1)=M_{ba\ stat}(\xi_b=1)-\omega^2\int\mu(x)\,v_1^2(x)\,dx \qquad (4)$$

where ω is the angular frequency and $M_{ab\ stat}(\xi_b=1)$ is the end moment in the point b if this point is statically deformed with $\xi_b=1$. If the first natural frequency of the system is to be determined, the dynamical curves $v_1(x)$, $v_2(x)$ can be substituted by the statical ones $\bar{v}_1(x)$, $\bar{v}_2(x)$. The curves $\bar{v}_1(x)$, $\bar{v}_2(x)$ (statical influence lines) determined for the bar a-b of the system represented in fig. 1, are in fig. 6,

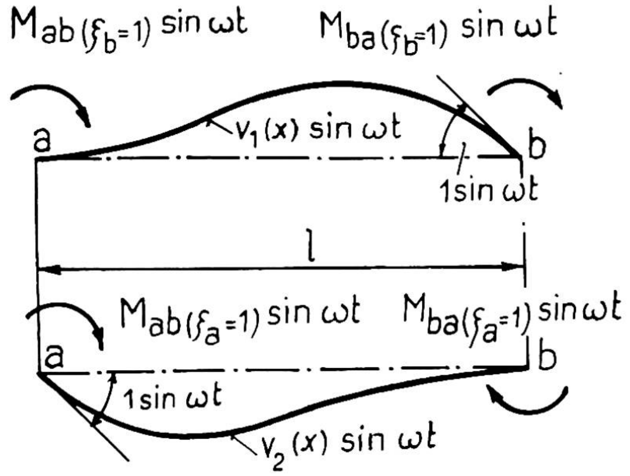

Fig. 5.

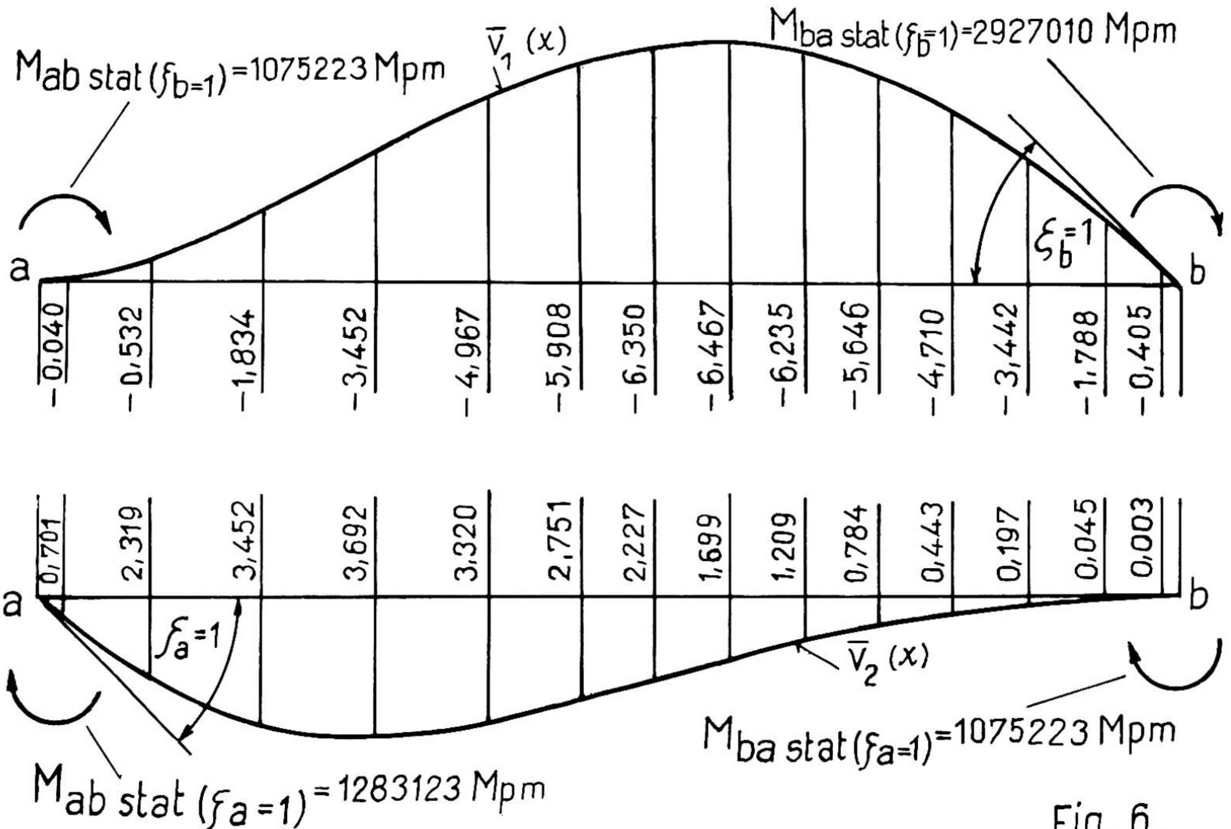

Fig. 6.

3) Kaloušek V.: Vibrations of Systems with Curved Members. Publications IABSE.VXXIII. Zurich 1963.P.219-232

Where also the end moments are indicated supposing E =3850 000 Mp/m^2. The integral in (4) can be evaluated with sufficient accuracy by numerical summation of finite differences dividing the bar into strips. Then it is

$$M_{ab}(\xi_b=1) = M_{ab\ stat}(\xi_b=1)-\omega^2 \sum_i m_i \bar{v}_{1i} \bar{v}_{2i} \qquad (5)$$

where m_i denotes the mass of the strip i and $\bar{v}_{1i}$, $\bar{v}_{2i}$ the vertical displacementsin the centre of gravity of the strip i. Another proceeding can be applied for the determination of the end-moments of piers which are of constant cross-section. The pier and the horizontal beams penetrate in the upper part of the pier d-b (fig.7) and the rigidity of the pier increases in this part substantially. It can be assumed that the moment of inertia is infinitely large there. The low end was assumed to be fixed rigidly, which holds only when the foundation reposes on solid rock. The end moment acting on the upper end can be expressed by frequency functions F (λ) which have been developed by the author [4)5)]. It is

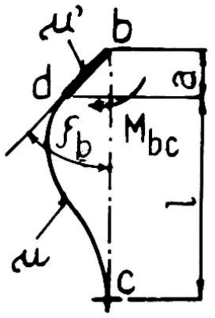

Fig. 7.

$$M_{bc}(\xi_b=1) = \frac{EI}{\ell}\left[F_2(\lambda) - \frac{2a}{\ell}F_4(\lambda) + \frac{a^2}{\ell^2}F_6(\lambda)\right] - \frac{1}{3}\mu' a^3\omega^2 \qquad (6)$$

The last term in exp. (6) expresses the moment of inertial forces of the rigid part d-b of the length a and μ' is its mass per unit of length. It was assumed

$$I = 2{,}286\ m^4$$

$$\mu = \mu' = 17{,}31 t/m\ (=1{,}76\ Mps^2\ m^{-2})$$

$$\lambda = \sqrt{\omega}\,\ell\sqrt[4]{\frac{\mu}{EI}} = 0{,}3354$$

$$E = 2850\ 000\ Mp/m^2$$

Further it can be assummed for small values of λ

$$F_2(\lambda) \doteq 4-\frac{1}{105}\lambda^4 \quad F_4(\lambda) \doteq -6+\frac{11}{210}\lambda^4 \quad F_6(\lambda) \doteq 12-\frac{13}{35}\lambda^4$$

Substituting into eq.(6) we obtain

$$M_{bc}(\xi_b=1) = 2143961 - 301{,}15\ \omega^2$$

The values of end moments and forces of all bars which were determined according to exp. (5) or (6) are given in Table I.

4) Kloušek V.: Baudynamik der Durchlaufträger und Rahmen. Fachbuchverlag, Leipzig 1953

5) Kloušek V.: Calcul des efforts dynamiques dans les ossatures rigides. Dunod, Paris 1959

Table I.

	$\xi_a = 1$	$\xi_b = 1$	$\xi_s = 1$	$v_s = 1$
M_{ab}	$1283123-286,20\omega^2$	$1075223+412,80\omega^2$		
M_{ba}	$1075223+412,80\omega^2$	$2927010-1124,46\omega^2$		
M_{bs}		$3421740-819,45\omega^2$		$-168850-57,35\omega^2$
M_{bc}		$2143961-301,15\omega^2$	$1221640+301,02\omega^2$	
$\sum$	$1075223+412,80\omega^2$	$8492711-2245,06\omega^2$	$1221640+301,02\omega^2$	$-168850-57,35\omega^2$
M_{sb}		$1221640+301,02\omega^2$	$1462100-207,56\omega^2$	
Y_{sb}		$-168850-57,35\omega^2$		$9701,97-15,111\omega^2$

F r e e s y m m e t r i c a l v i b r a t i o n s. The first natural mode is represented in fig. 8. The slope-deflection equations are

$$\begin{aligned} M_{ab} &= 0 \\ M_{ba} + M_{bs} + M_{bc} &= 0 \qquad (7) \\ Y_{sb} &= 0 \end{aligned}$$

where

$$\begin{aligned} M_{ab} &= M_{ab(\xi_a=1)}\cdot\xi_a + M_{ab(\xi_b=1)}\cdot\xi_b \\ M_{ba} &= M_{ba(\xi_a=1)}\cdot\xi_a + M_{ba(\xi_b=1)}\cdot\xi_b \\ M_{bs} &= M_{bs(\xi_b=1)}\cdot\xi_b + M_{bs(v_s=1)}\cdot v_s \qquad (8) \\ M_{bc} &= M_{bc(\xi_b=1)}\cdot\xi_b \\ Y_{sb} &= Y_{sb(\xi_b=1)}\cdot\xi_b + Y_{sb(v_s=1)}\cdot v_s \end{aligned}$$

ξ_a and ξ_b denote the amplitudes of rotation in joints a and b respectively, v_s is the amplitude of vertical displacement in the joint s, and Y_{sb} is the amplitude of end force.

After substituting numerical values into (8) and (7) we obtain the equations of the Table II. Setting the determinant of the equations equal to zero, we obtain

$$\omega^6 - 16\,078\omega^4 + 36\,175075\omega^2 - 9353\,402\,000 = 0$$

and thereof

$$\omega_{(1)} = 17,235\ s^{-1}, \quad \omega_{(2)} = 48,4\ s^{-1}, \quad \omega_{(3)} = 116\ s^{-1}$$

The last two values are of an informative character only. The first natural mode is given by ξ_a, ξ_b and v_s; one of them can be chosen and two other calculated from the equations of Table II.

Table II.

ξ_a	ξ_b	v_s
$1283123 - 286{,}20\,\omega^2$	$1075223 + 412{,}80\,\omega^2$	$= 0$
$1075223 + 412{,}18\,\omega^2$	$8492711 - 2245{,}06\,\omega^2$	$-168850 - 57{,}35\,\omega^2 \quad = 0$
	$-168850 - 57{,}35\,\omega^2$	$9701{,}97 - 15{,}111\,\omega^2 = 0$

Supposing that $v_s = 1\text{m}$, we have

$$\xi_a = -0{,}02003 \qquad \xi_b = 0{,}02804$$

The deformations of singular bars can then be obtained using the statical curves of deformations which for the bar a-b are in fig. 6. The first mode of the system is shown in fig. 8.

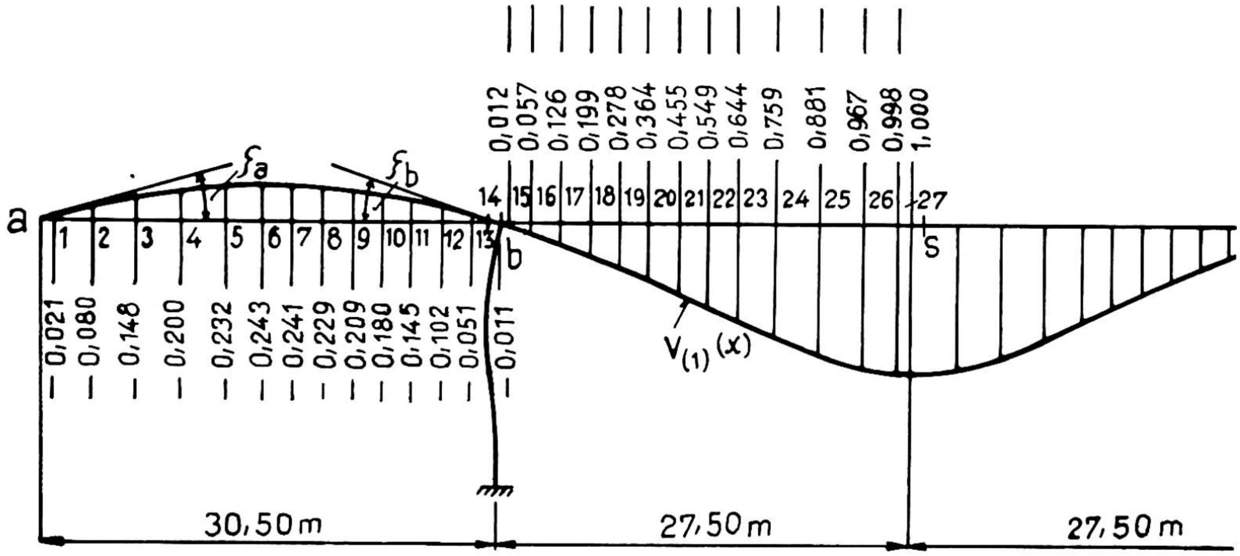

Fig. 8.

F o r c e d v i b r a t i o n s . The forced vibrations of the structure are produced by the movement of vehicles crossing the bridge. The computation was executed for the case of a two cylinder locomotive of the weight G = 97 t moving at a constant speed, the driving wheels of which produce the centrifugal force $P = 0{,}3\ N^2$ (in Mp, if N denotes the number of revolutions per second) with angular velocity $\Omega = 2\pi N$. The problem can be solved by expanding the vibrations into a series of natural modes by the same method which was described in detail in previous works of the author [5)6)7)].

6) Kolouŝek V.: Schwingungen der Brücken aus Stahl und Stahlbeton. Abhandlungen IVBH. B. XVI. Zürich 1956. S. 301-332.

7) Kolouŝek V.: Vibrations of Bridges with Continuous Main Girders. Publications IABSE. V. XIX. Zürich 1959. P. 111-132.

The critical speed by which N is equal to the first natural frequency is

$$c = \bar{n}_{(1)} \pi D$$

where $\bar{n}_{(1)} = \frac{\bar{\omega}_{(1)}}{2\pi}$ is the first natural frequency of the loaded bridge and D is the diameter of the driving wheels. It will be assumed that the alternating forces of axles act only when moving along the central span. The time variation of the deflection in the middle of the central span produced by the alternating force is given by the formula (45) in the paper [6] p.326

$$v\left(\frac{\ell}{2}, t\right) = \frac{A v_{(1)}(\ell/2) P \sin \bar{\omega}_{(1)} t}{2 \bar{\omega}_{(1)} (\omega^2 + \omega_b^2)} \left[(\cos \omega t - e^{-\omega_b t}) - \omega_b \sin \omega t\right] \quad (9)$$

where $\omega = \frac{\pi c}{\ell}$ and ω_b is a damping coefficient. A is given by

$$A = \frac{B_1}{\Sigma \int_0^\ell \mu(x)\, v_{(1)}^2 (x)\, dx} \quad (10)$$

where $v_{(1)}(x)$ denotes the first natural mode and the summation Σ is to be extended over all the spans of the system. B_1 is the coefficient of the first term in the Fourier series

$$v_{(1)}(x) = B_1 \sin \frac{\pi x}{\ell} + B_3 \sin \frac{3\pi x}{\ell} +$$

The angular frequency $\bar{\omega}_{(1)}$ of the bridge loaded by a locomotive is lower than of an unloaded one. If the centre of gravity of the locomotive is in x = d, it can be calculated according to [7] p.123

$$\bar{\omega}_{(1)} = \omega_{(1)} \sqrt{\frac{\mu}{\bar{\mu}}} \quad (11)$$

where

$$\bar{\mu} = \mu\left(1 + \frac{m_L\, v_{(1)}^2 (d)}{\Sigma \int_0^\ell \mu\, v_{(1)}^2 (x)\, dx}\right) \quad (12)$$

and m_L is the mass of the locomotive. In our case it is (see fig. 8) in the central span

$$v_{(1)}(x) = 0{,}8766 \sin \frac{\pi x}{\ell} - 0{,}0452 \sin \frac{3\pi x}{\ell} \quad (13)$$

so that

$$B_1 = 0{,}8766$$

We obtain further by numerical integration (see fig.8)

$$\Sigma \int_0^\ell \mu\, v_{(1)}^2 (x)\, dx = 42{,}103 \text{ Mp s}^2 \text{ m}^{-1}$$

In accordance with preceding experience, we can assume that $d = \frac{\ell}{3}$.
Then $v_{(1)}(d) = 0{,}75$ and

$$\frac{\bar{\mu}}{\mu} = 1 + \frac{97}{9{,}81} \quad \frac{0{,}75^2}{42{,}103} = 1{,}132$$

Further, it is $\bar{\omega}_{(1)} = \omega_{(1)}\sqrt{\frac{1}{1,132}} = 17,235.0,940 = 16,198\ s^{-1}$

$$\bar{n}_{(1)} = \frac{\bar{\omega}_{(1)}}{2\pi} = 2,578\ \mathrm{Hz}$$

$$A = \frac{0,8766}{1,132.42,103} = 0,01873\ \mathrm{Mp}^{-1}\mathrm{s}^{-2}\mathrm{m}$$

$$P = 0,3.2,578^2 = 1,994\ \mathrm{Mp}$$

With D = 1,26 m it is

$$c = 2,578.\pi\,.\,1,26 = 10,21\ \mathrm{m/s} = 36,75\ \mathrm{km\ p\,.\,h}$$

$$\omega = \frac{\pi.10,21}{55} = 0,583\ s^{-1}$$

We can put

$$\omega t = \frac{\pi a}{\ell}$$

where a = ct is the distance of the load of the beginning of the central span. Then we have

$$\bar{\omega}_{(1)}\, t = \frac{\bar{\omega}_{(1)}}{\omega}\,\frac{\pi a}{\ell} = 27,8\,\frac{\pi a}{\ell}$$

$$\omega_b\, t = \frac{\omega_b}{\omega}\,\frac{\pi a}{\ell} = 0,025253\ a$$

The logarithmic decrement was appreciated in the value $\delta = 0,1$ so that $\omega_b = \delta\bar{n}_{(1)} = 0,258\ s^{-1}$. Substituting into (10) we obtain

$$v(\ell/2,t) = \frac{0,01873.1,994\cos\frac{27,8\pi a}{\ell}}{2.16,198(0,583^2 + 0,258^2)}\left[0,583\left(\cos\frac{\pi a}{\ell} - e^{-0,02525a}\right) - 0,258\sin\frac{\pi a}{\ell}\right] \qquad (14)$$

The curve (14) is shown in fig.9a. According to our supposition that the resonance exists only when the locomotive passes the central span, we can determine the amplitudes A(t) of the curve v (ℓ/2,t) in the span b' - a', from the formula

$$A(t) = C\,e^{-\omega_b t}$$

where C denotes the amplitude of vibrations when the locomotive has left the central span. In fig. 9b the curve of fig.9a is superposed above the curve of statical deflection produced by the weight of the locomotive moving along the bridge. This curve represents the theoretical curve of dynamic deflection in the centre of the bridge. The dynamic augmentation of the static deflection is 20 % .

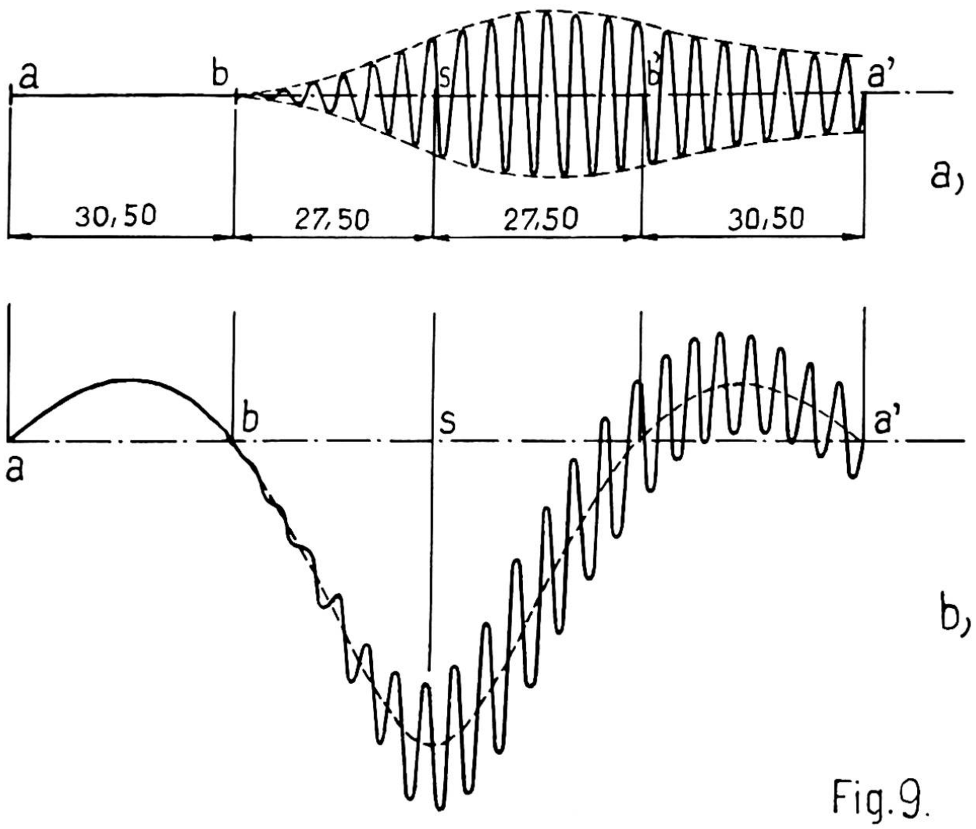

Fig.9.

Results of load-tests.

First of all, the bridge was tested by statical loads. The measured deflections attained only 70 % of theoretical values. The real rigidity was consequenty 1,42 times larger than the assumed one. The attained value E of concrete was larger than was supposed in the analysis and the moment of inertia was elevated by the monolithic execution of sidewalks. The natural frequency increases proportionally to the $\sqrt{EI}$. It can be expected that the actual first natural frequency and the critical speed will be $\sqrt{1,42}=1,19$ multiple of the theoretical value

$$c = 1,19.36,75 \doteq 44 \text{ km p.h}$$

A 97 ton two cylinder locomotive was used for the tests. The deflection in the middle of the central span was measured by means of a Stoppani deflection meter. At the same time the stresses in the lower part of the middle span girder were registered by means of strain-gauges and Brüell-Kjaer registration apparatus. The diagrams recorded at the speed of 44 km p.h. are shown in figs. 10 and 11. The first of them should be compared with the theoretical diagram of fig. 9. It is evident that the measured dynamic effects

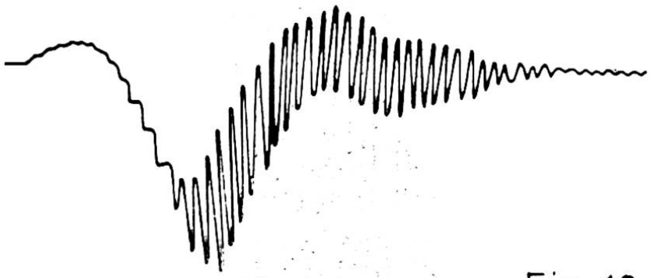

Fig. 10.

are larger than the calculated ones. Themeasured dynamic augmentation attains about 35 % of the statical deflection compared with the 20 % calculated. The difference is mainly the result of the higher rigidity of the actual structure. The centrifugal force P

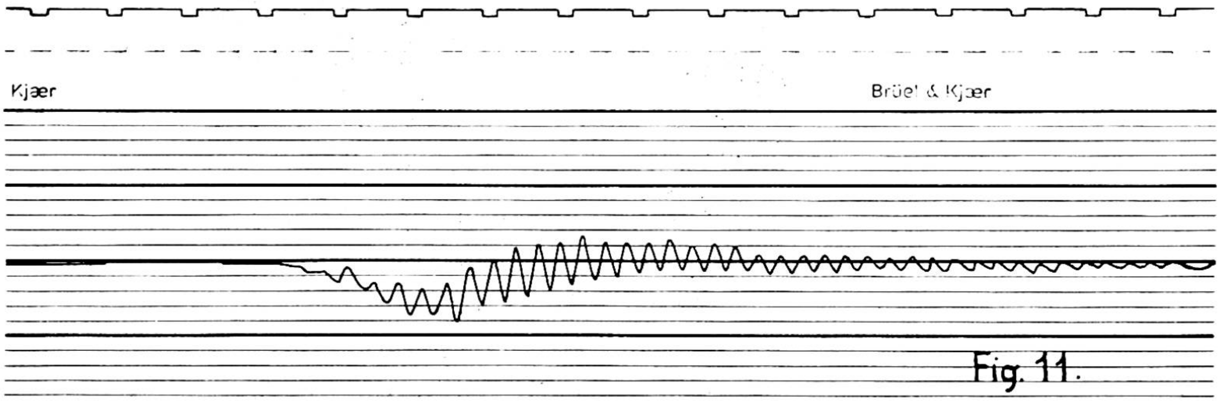

Fig. 11.

increases with the second power of the critical speed so that actually P will be 1,42 times larger than as calculated. In the same proportion, the dynamical augmentation of the deflection will increase and will theoretically attain the value of 28 %. The remaining difference between 28 % and 35 % is to be attributed to the inaccuracy of measurement, of presuppositions e.g. the value of damping and that of unbalanced masses of driving wheels and partly also to the inaccuracy of the theoretical analysis which neglects the movement of the masses along the bridge, etc.

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5) Kloušek V.: Calcul des efforts dynamiques dans les ossatures rigides. Dunod, Paris 1959
6) Koloušek V.: Schwingungen der Brücken aus Stahl und Stahlbeton. Abhandlungen IVBH. B. XVI, Zürich 1956. S 301-332
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SUMMARY

In the paper the dynamic effects of moving load on a railway bridge of prestressed concrete are discussed. The results of theoretical investigation are compared with the values obtained by the dynamic tests on the actual structure. The results are qualitatively in good accordance but the measured amplitudes of vibrations are somewhat larger than theoretically assumed.

RÉSUMÉ

Les effects dynamiques des charges mobiles sur les ponts rails en béton précontrainte sont analysés dans la contribution. Les valeurs de la solution théorique sont comparées avec les résultas experimentaux obténus sur la construction actuelle. Les amplitudes des vibrations mésurées surpassent un peu ceux de l'analyse théorique.

ZUSAMMENFASSUNG

In der Arbeit sind die dynamischen Einflüsse der beweglichen Belastung auf die Eisenbahnbrücken aus vorgespannten Beton untersucht. Die Ergebnisse der theoretischenUnter - suchung sind mit den experimentellen Werten verglichen, die bei dynamischen Messungen auf einer fertigen Brücke erhalten wurden. Beide Ergebnisse stimmen qualitativ gut überein, die gemessenen Amplituden sind jedoch ein wenig höher als die theoretisch gerechnetenWerte.