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## VI

### The Treatment of Damping Coefficient on the Dynamic Problem

Sur le coefficient d'amortissement dans les problèmes dynamiques

Die Behandlung des Dämpfungskoeffizienten bei dynamischen Problemen

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### INTRODUCTION

Recently the use of digital computer having become very popular, a number of papers dealing with the response analysis of complex structures is published. Very few of them set apart, however, they do not give detailed explanations about damping force. The writer having also developed a program for response analysis of framed structures, computed the responses of various types of them, and is in every time troubled by how the damping coefficients are selected. As response displacements depend largely upon them, even it is possible that we insist on the propriety of the certain damping coefficients obtained inversely from the required response displacements.

Damping force is a force that suppresses vibrations and comes from various origins. Though it is quite natural that efforts to catch the causes dominating the damping forces and to include them in the equation, such a frontal attack would not be so expected under existing circumstances. In case of complex structures, it is also very hard to determine the ratios to the critical damping coefficient, as in a one-mass-system, because of its complexity.

Then the writer, referring to the results of vibrational experiments about one-mass-system, and noticing that damping constant is of three terms (first inversely proportional, second unrelated and third proportional, to the frequency), has tried to extend the idea to multi-mass-system. There are such four forces, further saying, as inertia force ( $M\ddot{x}$ ), damping force ( $C\dot{x}$ ), restoring force ( $Kx$ ) and external force ( $-M\ddot{x}_e$  in case of earthquakes) which determine a vibration, the theme of this paper is then the second force. Restoring force is determined from the static relation between external force and deformations of the structure. This subject is dealt with in other papers of which one is published by the writer<sup>1)2)</sup>. In this paper is shown in another form extended thereafter. It is inevitable to encounter what type of seismic waves is selected, but such problem should belong to the field of seismology. Finally, as for inertia force, it is usual to concentrate the mass to some points, but as actually the mass is distributed along structural members, this effect must be introduced. The discussion about this problem is left for another chance.

### ONE-MASS-SYSTEM

The kinetic equation of one-mass-system is

$$Kx = -M(\ddot{x} + \ddot{x}_e) - C\dot{x}$$

Dividing by M and replacing

$$p = \sqrt{\frac{K}{M}} \quad , \quad h = \frac{C}{C_{cr}} \quad , \quad C_{cr} = 2\sqrt{K \cdot M}$$

Eq.(1) is reduced to

$$\ddot{\chi} + 2h\rho\dot{\chi} + P^2\chi = -\ddot{\chi}_e \quad (3)$$

in which  $p$  is circular natural frequency and  $h$  is called damping constant, the ratio of actual damping coefficient( $C$ ) to critical value of that ( $C_{cr}$ ). The relation between damping constant( $h$ ) and logarithmic decrement ( $\Delta$ ) is

$$\Delta = \frac{2\pi h}{\sqrt{1-h^2}} = 2\pi h$$

The solution of Eq.(3) is

$$\chi = P^{-1} S_v(t) \quad (4)$$

$$S_v(t) = -\int_0^t \frac{\ddot{\chi}_e}{\sqrt{1-h^2}} e^{-P(t-\lambda)} \cdot \sin P\sqrt{1-h^2}(t-\lambda) d\lambda$$

Substituting in Eq.(4) actual seismic waves, and calculating maximum values of  $S_v(t)$  for various values of  $p$ , we can get a response velocity spectrum by plotting  $S_v(t)$  against  $p$ . To average the values of  $S_v(t)$  for a number of cases of actually occurred earthquakes makes so-called average response velocity spectrum ( $S_v$ ) proposed by Hausner.

After our simple experiment,  $h$  is constant or proportional to  $p$  (see Fig. 1, 2). Making a reference<sup>3)</sup>,  $h$  is in inverse proportion with  $p$ . Then, we shall be able to put

$$h = h_0 P^{-1} + h_1 + h_2 P \quad (5)$$

Substituting this in Eq.(3) and using Eq.(2), Eq.(1) becomes

$$M\ddot{\chi} + (2h_0M + 2h_1\sqrt{KM} + 2h_2K)\dot{\chi} + K\chi = -M\ddot{\chi}_e \quad (6)$$

Damping coefficient is then expressed in such a form as

$$C = 2h_0M + 2h_1\sqrt{KM} + 2h_2K \quad (7)$$

Using Eq.(5) as damping constant under such condition that  $h_0$  and  $h_2$  have constant values, average response velocity spectrum of Hausner is calculated as shown in Figs. 3(a),(b).

### MULTI-MASS-SYSTEM (MODAL ANALYSIS)

The kinetic equation of a multi-mass-system is, by using matrices, expressed as follows.

$$IK\mathbf{x} = -M(\ddot{\mathbf{x}} + \mathbf{F}\ddot{\mathbf{x}}_e) - C\dot{\mathbf{x}} \quad (8)$$

Now, introducing a linear equation

$$(M\lambda^2 - K)\mathbf{x} = 0$$

let  $U_i$  be the root other than zero, and  $P_i^2$  be the value of  $\lambda^2$  (the number is as much as the rank of the matrices), that is to say, the eigenvector and eigenvalue. If  $V$  denotes the matrix arranging  $U_i$  in a column, and  $P^2$  the matrix arranging  $P_i^2$  diagonally, the relation between them is

$$V^T K V = V^T M V P^2 \quad (9)$$

Each element of  $P$  is circular natural frequency, and each column of  $V$  shows proper mode of vibration. Further, changing the independent variables  $\mathbf{x}_i$  of Eq.(8) to  $\mathbf{q}$  by the relation

$$\mathcal{X} = \mathbb{W} \mathbf{q} \quad (10)$$

and multiplying  $\mathbb{W}^T$  from the left side, Eq.(11) is obtained.

$$\mathbb{W}^T \mathbb{M} \mathbb{W} \ddot{\mathbf{q}} + \mathbb{W}^T \mathbb{C} \mathbb{W} \dot{\mathbf{q}} + \mathbb{W}^T \mathbb{M} \mathbb{W} \mathbb{P}^2 \mathbf{q} = - \mathbb{W}^T \mathbb{M} \mathbb{F} \ddot{\mathbf{x}}_e \quad (11)$$

Because the critical damping coefficient matrix of the kinetic equation (8) for a multi-mass-system is  $2 \mathbb{M} \mathbb{W} \mathbb{P} \mathbb{W}^{-1}$  (see APPENDIX I), defining, on an analogy of Eq.(7), the damping coefficient matrix of multi-mass-system as

$$\mathbb{C} = 2\hbar_0 \mathbb{M} + 2\hbar_1 \mathbb{M} \mathbb{W} \mathbb{P} \mathbb{W}^{-1} + 2\hbar_2 \mathbb{K} \quad (12)$$

and modifying the second term of Eq.(11) and considering Eq.(9), we find

$$\mathbb{W}^T \mathbb{C} \mathbb{W} = 2\hbar_0 \mathbb{W}^T \mathbb{M} \mathbb{W} + 2\hbar_1 \mathbb{W}^T \mathbb{M} \mathbb{W} \mathbb{P} + 2\hbar_2 \mathbb{W}^T \mathbb{M} \mathbb{W} \mathbb{P}^2$$

Eq.(11) is therefore transformed into

$$\ddot{\mathbf{q}} + 2(\hbar_0 \mathbb{P}^{-1} + \hbar_1 \mathbb{U} + \hbar_2 \mathbb{P}) \mathbb{P} \dot{\mathbf{q}} + \mathbb{P}^2 \mathbf{q} = - (\mathbb{W}^T \mathbb{M} \mathbb{W})^{-1} \mathbb{W}^T \mathbb{M} \mathbb{F} \ddot{\mathbf{x}}_e \quad (13)$$

When  $\mathbb{P}^{-1} \mathbb{S} \mathbf{v}_i(t)$  is the solution of Eq.(3) in which Eq.(5) and circular natural frequency  $\mathbb{P}_i$  of multi-mass-system are substituted,  $\mathbb{S} \mathbf{v}_i(t)$  being the matrix of diagonal arrangement of  $\mathbf{S} \mathbf{v}_i(t)$ , the solution of Eq.(13) is

$$\mathbf{q} = \mathbb{P}^{-1} \mathbb{S} \mathbf{v}(t) (\mathbb{W}^T \mathbb{M} \mathbb{W})^{-1} \mathbb{W}^T \mathbb{M} \mathbb{F}$$

and the relative displacement is obtained by substituting in Eq.(10), as follows:

$$\mathcal{X} = \mathbb{W} \mathbb{P}^{-1} \mathbb{S} \mathbf{v}(t) (\mathbb{W}^T \mathbb{M} \mathbb{W})^{-1} \mathbb{W}^T \mathbb{M} \mathbb{F} \quad (14)$$

Sectional forces would be then calculated from the displacement method of statics.

#### MULTI-MASS-SYSTEM (DIRECT METHOD)

Damping coefficient of Eq.(8) being substituted by equation (12), and replacing

$$\dot{\mathcal{X}} = \dot{\mathcal{Y}}, \quad \dot{\mathcal{Y}} = - \mathbb{F} \ddot{\mathbf{x}}_e - (\mathbb{C} \mathcal{Y} + \mathbb{K} \mathcal{X}) \quad (15)$$

Eq.(8) would be solved by the numerical integral method<sup>2)</sup> such as the Runge-Kutta-Gill or Milne's Method, under the initial condition,  $\mathcal{X} = \mathcal{Y} = 0$  at  $t=0$ . As described at the head, there are so few papers dealing with damping force that the writer has proposed the equation (12). But, when using direct method, the second term of equation (12) seems troublesome. So it would be better to compute Eq.(15) after normalizing the eigenvector by using the relation

$$\mathbb{W}^T \mathbb{M} \mathbb{W} = \mathbb{E}$$

into the form

$$\mathbb{C} = 2\hbar_0 \mathbb{M} + 2\hbar_1 \mathbb{M} \mathbb{W} \mathbb{P} \mathbb{W}^T \mathbb{M} + 2\hbar_2 \mathbb{K} \quad (16)$$

or letting include the influence of the second term to the first and the third term

$$\mathbb{C} = 2\hbar_0' \mathbb{M} + 2\hbar_2' \mathbb{K} \quad (16')$$

## STIFFNESS MATRIX

For calculation of the responses of multi-mass-system using Eq.(14) or (15), it is necessary to make up mass matrix ( $M$ ) and stiffness matrix ( $K$ ) in addition to damping coefficient matrix ( $C$ ). If the mass is concentrated to the structural nodes, mass matrix is to be diagonal matrix, but actually the mass is distributed. Though the writer is researching to take into consideration the influence of distribution, but it is not yet the time to publish.

Stiffness matrix is obtained from the static relation between loads ( $P$ ) and displacements ( $X$ )

$$KX = P \quad (17)$$

Many studies in this field being published, their results should be used. The writer has also published a method<sup>1)2)</sup>. Afterwards the writer modified to be able to use for a member with one hinged end. Here is a simple explanation.

The linear equation by which the framed structure is solved statically is written as follows

$$DKD^T X = P - AC^T F_{fa} - BC^T F_{fb} \quad (18)$$

$DKD^T$  is stiffness matrix,  $X$  is displacement vector and the first term of the right side is force vector composed of external forces acting on the nodes. The second and third terms of the right side are vectors composed of external forces acting on the intermediate members connecting the nodes,  $F_{fa}$  and  $F_{fb}$  are end reactions of fixed beam (or modified end reactions when hinged),  $C^T$  is transformation matrix of coordinates (local to global), and,  $A$  and  $B$  are also transformation matrices from sectional forces at the member's end to nodal forces. The contents of  $D$ ,  $k$  are shown in APPENDIX II.

Solving Eq.(18) with performing an operation to the supports, sectional forces of the both ends  $F_a$  and  $F_b$  would be obtained.

$$F_a = T_a k D^T X + F_{fa}, \quad F_b = T_b k D^T X + F_{fb} \quad (19)$$

The operation to the supports is, for example, to sweep out the corresponding row and column of the stiffness matrix, if the node  $i$  is fixed in one direction, and/or to add a spring constant to the corresponding diagonal element of the stiffness matrix, if the node  $j$  is supported elastically in one direction.

## NUMERICAL EXAMPLE

The suspension bridge shown in Fig.(4) is modelled and shown in Fig.(5). By substituted various values of  $h_0, h_1, h_2$  into equation (12), the numerical calculations are carried out. If the suspension bridge and the seismic wave acting at the both tower bases are symmetric, the response of displacements and/or member forces of the center span are reduced to extremely small. In order that we increased the masses of the right tower ten per cent more than that of the left for this numerical example.

Results of the calculations are tabulated in the table 1. The figures in this table are obtained from eq.(14) using a seismic wave of the reduced El Centro NS component (the maximum acceleration is 200 gals). Here we also calculated the response of displacements using other types of above seismic waves, but we can not show the results in this paper because of space limitations.

## ACKNOWLEDGEMENTS

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1. Ohchi, Y., "Matrix Solution of Framed Structures" Transactions of the Japan Society of Civil Engineers, No.87 (1962) or Quarterly Report of the Railway Technical Research Institute Vol.3, No.4 (1962).
2. Ohchi, Y., "Response Analysis of Framed Structures" Proc. of 3rd World Conference of Earthquake Engineering (1965).
3. Itō, H., and Katayama, T., "Vibrational Damping of Bridge Structure", Trans. of the Japan Society of Civil Engineers, No.117 (1965).

## APPENDIX I CRITICAL DAMPING COEFFICIENT MATRIX

Supposing  $q = Qe^{-\omega t}$  in the expression (11), let the right side equals zero, it becomes

$$(V^T M V \omega^2 - V^T C V \omega + V^T M V P^2) Q e^{-\omega t} = 0 \quad (a)$$

The above equation represents the system of free vibration accompanying with damping, if  $\omega$  is real, the system does not vibrate. In order that  $\omega$  be of a value at the border between being real and imaginary, that is to say  $\omega$  be identical roots, the next expression should stand.

$$V^T C_{cr} V = 2 V^T M V P \quad (b)$$

This would be confirmed by substituting (b) into the expression (a), which makes

$$V^T M V (F \omega^2 - 2 P \omega + P^2) Q e^{-\omega t} = 0$$

or 
$$V^T M V (F \omega - P)^2 Q e^{-\omega t} = 0$$

From the expression (b),  $C_{cr}$  is obtained.

$$C_{cr} = 2 M V P V^{-1} \quad (c)$$

## APPENDIX II EXPLANATION OF Eqs. (18) AND (19)

If the structure is constructed in the  $xy$  plane of the global co-ordinate  $xyz$ , the elements of matrices which are included in eqs. (18) and (19) are as follows.

- (1) For plane framed structure (Loads and deflections are restricted to the inside of the  $xy$  plane)

$$D = \begin{bmatrix} \delta X L^{-1} & \delta Y L^{-1} & 0 \\ \delta Y L^{-1} & \delta X L^{-1} & 0 \\ 0 & \frac{1}{2} \mu L & \frac{1}{2} \delta L \end{bmatrix} \quad R = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$T_a = \begin{bmatrix} U & 0 & 0 \\ 0 & U & 0 \\ 0 & \frac{1}{2} \epsilon_a (2U - \epsilon_b) L & \frac{1}{2} L \end{bmatrix} \quad T_b = \begin{bmatrix} -U & 0 & 0 \\ 0 & -U & 0 \\ 0 & \frac{1}{2} \epsilon_b (2U - \epsilon_a) L & \frac{1}{2} L \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ \theta_z \end{bmatrix} \quad P = \begin{bmatrix} P_x \\ P_y \\ m_z \end{bmatrix} \quad F_a = \begin{bmatrix} N_{ua} \\ S_{va} \\ M_{wa} \end{bmatrix} \quad F_b = \begin{bmatrix} N_{ub} \\ S_{vb} \\ M_{wb} \end{bmatrix}$$

- (2) For grid-type structure (Loads and deflections point to the outside of the  $xy$  plane)

$$D = \begin{bmatrix} \delta X L^{-1} & -\mu Y L^{-1} & -\delta Y L^{-1} \\ \delta Y L^{-1} & \mu X L^{-1} & \delta X L^{-1} \\ 0 & -(\epsilon_a + \epsilon_b) L^{-1} & 0 \end{bmatrix} \quad R = \begin{bmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{bmatrix}$$

$$T_a = \begin{bmatrix} U & 0 & 0 \\ 0 & \epsilon_a & U \\ 0 & -(\epsilon_a + \epsilon_b) L^{-1} & 0 \end{bmatrix} \quad T_b = \begin{bmatrix} -U & 0 & 0 \\ 0 & \epsilon_b & -U \\ 0 & -(\epsilon_a + \epsilon_b) L^{-1} & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} \theta_x \\ \theta_y \\ z \end{bmatrix} \quad P = \begin{bmatrix} m_x \\ m_y \\ P_z \end{bmatrix} \quad F_a = \begin{bmatrix} T_{ua} \\ M_{va} \\ S_{wa} \end{bmatrix} \quad F_b = \begin{bmatrix} T_{ub} \\ M_{vb} \\ S_{wb} \end{bmatrix}$$

Where

$$a = E A L^{-1} \quad b = 3(\epsilon_a + \epsilon_b)^2 \{U + (\epsilon_a + \epsilon_b)^2 \Omega\}^{-1} E I_w L^{-1} \quad c = 4 \epsilon_a \epsilon_b E I_w L^{-3}$$

$$d = G J L^{-1} \quad e = 3(\epsilon_a + \epsilon_b - \epsilon_a \epsilon_b) \{U + (\epsilon_a + \epsilon_b)^2 \Omega\}^{-1} E I_u L^{-1} \quad f = \epsilon_a \epsilon_b E I_v L^{-1}$$

$$\delta = \alpha - \beta$$

$$\mu = \begin{cases} \alpha \epsilon_a (2U - \epsilon_b) + \beta \epsilon_b (2U - \epsilon_a) \\ \alpha \epsilon_a + \beta \epsilon_b \end{cases}$$

for plane framed S.

for grid-type S.

$$\Omega = \begin{cases} 3\kappa EI_w (GAL^2)^{-1} & \text{for plane framed S.} \\ 3\kappa EI_v (GAL^2)^{-1} & \text{for grid-type S.} \end{cases}$$

$U$  = unit matrix

$E, G, \kappa$  = diagonal matrices, the  $(i, i)$  element in diagonal matrix shows the Young's modulus, the shear modulus and the shear coefficient of member  $i$ .

$A, I_v, I_w, J, L, X, Y$  = diagonal matrices in which the  $(i, i)$  element represent the cross sectional area, the moment of inertia of the section around the local  $v, w$  axis, the torsional moment of inertia of the section, the length and the projection of the length on the global  $x, y$  axis respectively.

$\alpha, \beta$  = matrices indicating with which member is connected at member's node. For example,  $\alpha_{ij} = 1$  or  $\beta_{ij} = 1$ , it shows that the end  $a$  or  $b$  of member  $j$  is connected with the node  $i$ ; otherwise  $\alpha_{ij} = 0$  or  $\beta_{ij} = 0$ .

$\epsilon_a, \epsilon_b$  = diagonal matrices, in which the  $(i, i)$  element equal zero, if a hinge is located at the end  $a$  or  $b$  of member  $i$ ; otherwise equals 1.

$\chi, \psi, \theta$  ( $\theta_x, \theta_y, \theta_z$ ) = column vectors, the  $i$ th element shows the deflection (deflection angle) of node  $i$ .

$N_{ua}, S_{va}, S_{wa}$  ( $T_{ua}, M_{va}, M_{wa}$ ) = column vectors, the  $i$ th element shows the  $U, V, W$  component of the sectional forces (moments) at the end  $a$  of member  $i$ .

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### APPENDIX III NOTATION

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$C, C$  = damping coefficient and damping coefficient matrix

$C_{cr}, C_{cr}$  = critical damping coefficient and critical damping coefficient matrix

$\Delta$  = logarithmic decrement

$F$  = This vector represents the difference of absolute and relative displacement vector dividing by  $\chi_{ei}$ ; while the displacement is the same direction as seismic acceleration, the values of elements in this vector are 1, otherwise equal zero.

$\zeta$  = damping constant ( $C/C_{cr}$ )

$h_0, h_1, h_2$  = constants defining  $h$  (see Eq.(7) or (12))

$K, K$  = spring constant and stiffness matrix

$M, M$  = mass and mass matrix

$P, P$  = circular natural frequency and circular natural frequency matrix

$\bar{S}_v$  = average response velocity spectrum

$S_v(t), S_v(t)$  = see Eqs.(4) and (14)

$U$  = unit matrix

$w_i, W$  = mode vector and mode matrix

$\chi, \chi$  = relative displacement and relative displacement vector

$\ddot{\chi}_e$  = seismic acceleration

$A, B, C^T, D, F_a, F_b, F_{fa}, F_{fb}, f_b, P, T_a, T_b$  = see APPENDIX II



FIG. 1

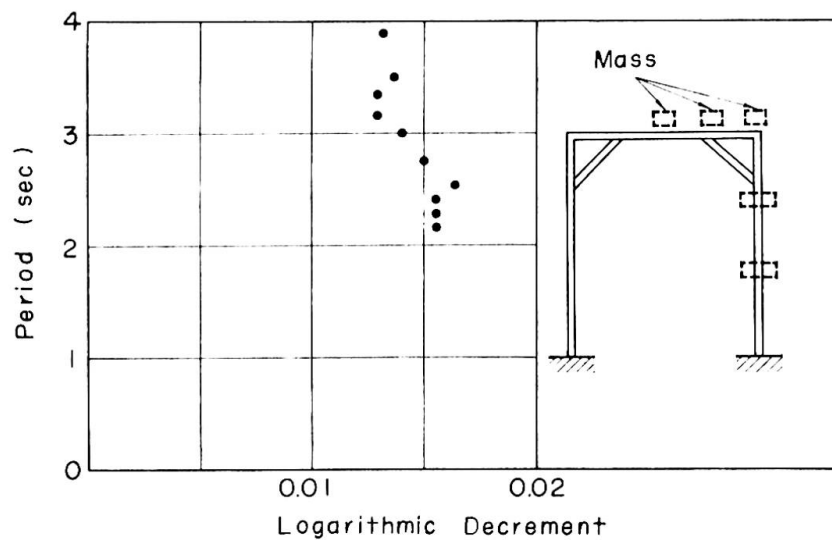


FIG. 2

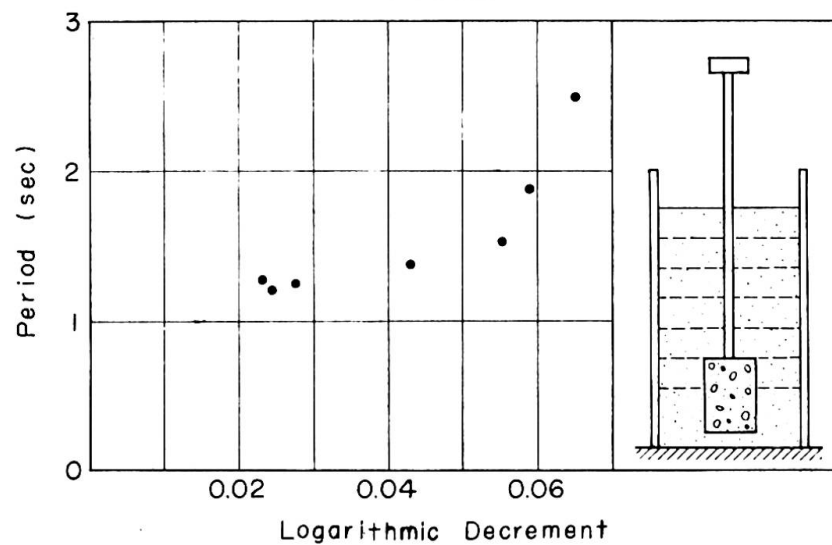


FIG. 3 (a)

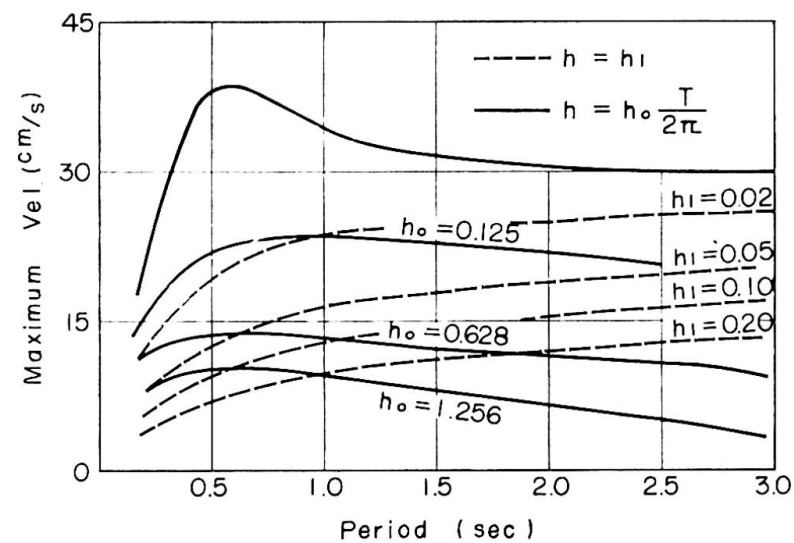
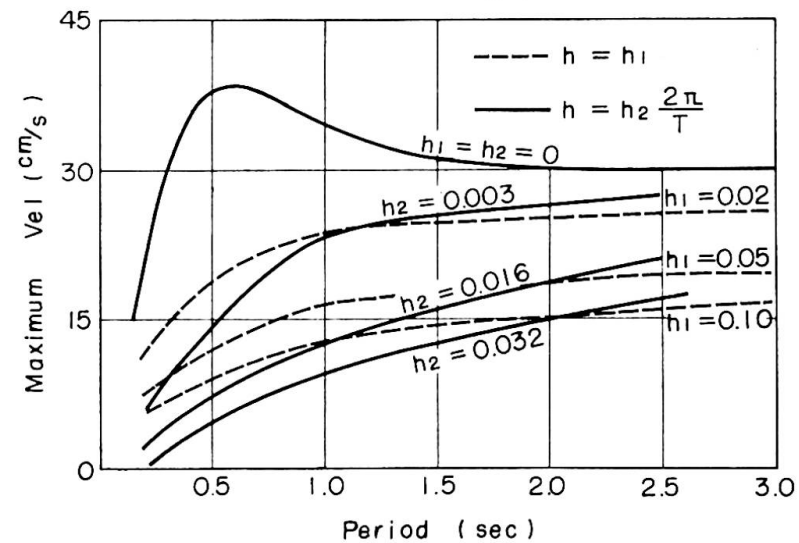


FIG. 3 (b)



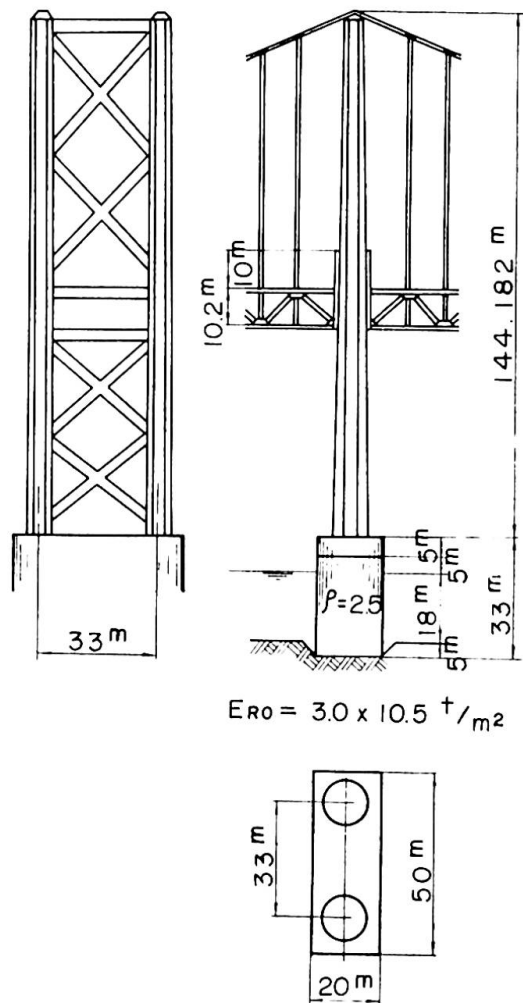


FIG. 4

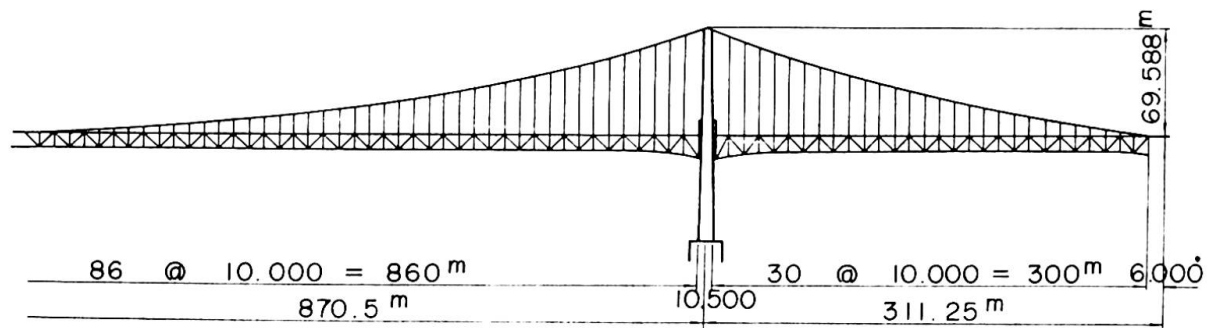


FIG. 5

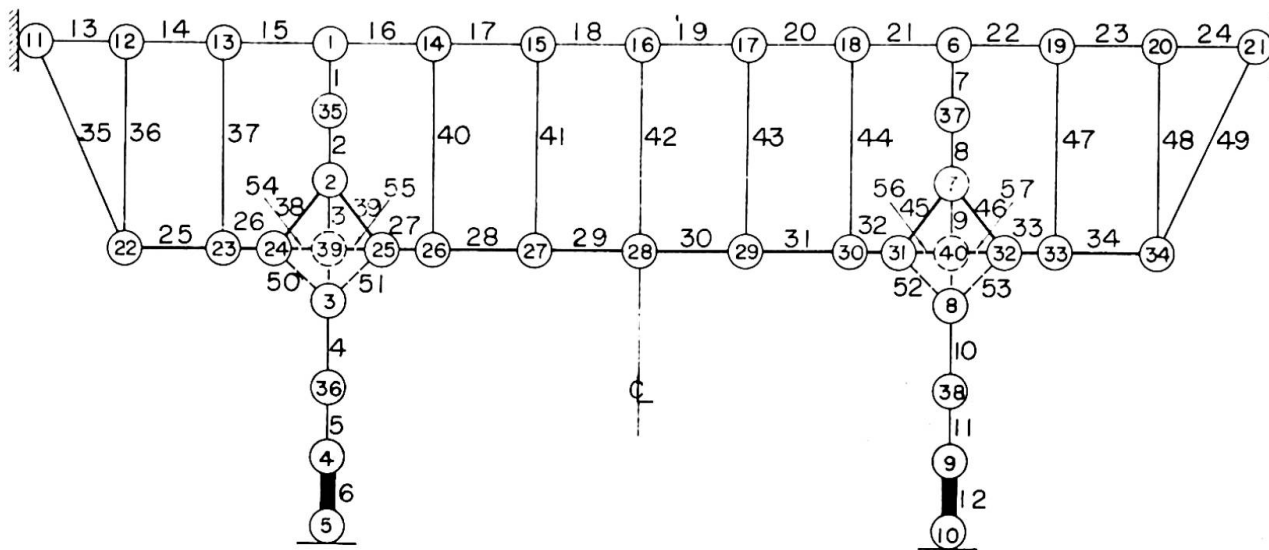


Table 1

damping Const.	h <sub>0</sub>	—	0.628	—	0.314	damping Const.	h <sub>0</sub>	—	0.628	—	0.314
	h <sub>1</sub>	0.100	—	—	—		h <sub>1</sub>	0.100	—	—	—
	h <sub>2</sub>	—	—	0.0159	0.008		h <sub>2</sub>	—	—	0.0159	0.008
NORMAL FORCE (t)						SHEARING FORCE (t)					
side cable	13	1 070	1 090	1 030	1 050	tower	1	575	566	611	570
	14	1 080	1 100	1 040	1 060		2	126	136	125	116
	15	1 100	1 120	1 060	1 080		3	549	588	523	544
center cable	16	57.6	70.3	56.8	55.7		4	870	878	854	863
	17	56.5	69.3	55.7	54.6		5	1 030	1 020	1 050	1 030
	18	55.7	68.0	55.0	53.9		6	14 800	14 800	14 800	14 800
DISPLACEMENT (cm)											
side hanger	35	201	228	174	187	left tower	1	16.9	3.84	19.0	4.88
	36	88.7	95.4	85.6	86.7		35	16.2	14.8	17.4	13.6
	37	55.6	57.1	52.5	53.7		2	16.6	16.2	16.8	14.7
	38	42.9	487	372	400		3	14.0	13.7	14.2	12.3
center hanger	39	44.4	54.4	41.4	40.4		37	8.37	13.5	8.44	7.24
	40	2.96	4.06	2.88	2.80		4	3.35	3.38	3.29	2.88
	41	5.50	6.67	5.46	5.30		5	—	—	—	—
	42	4.69	5.77	4.61	4.50						
BENDING MOMENT (t, m)											
tower	1	21 400	21 000	22 700	21 200	side span	22	132	23.8	154	39.1
	2	24 700	23 500	26 000	24 200		23	47.4	8.61	54.8	14.0
	3	16 600	16 400	17 100	16 600		24	41.5	11.2	46.8	15.4
	4	12 500	12 300	12 800	11 000	center span	25	3.31	3.30	3.42	3.26
	5	40 400	41 600	39 100	39 800		26	1.67	1.87	1.58	1.77
	6	522 000	522 000	524 000	522 000		27	2.60	0.66	3.05	0.51
stiff girder	26	1 150	844	1 250	987		28	3.39	0.55	4.04	0.93
	28	151	154	162	152		29	3.15	1.32	3.61	1.56
	29	79.8	58.7	96.1	62.7		30	2.68	2.22	2.70	2.29
	30	88.6	28.5	122	42.7		31	3.42	3.32	3.50	3.34

From our simple experiments about this field, we propose the equation (12) or (16') for the damping coefficient matrix of the multi-mass-system. Results obtained from usual method were compared with some series of our numerical calculations, we find that  $\dot{h}_0$  in eq.(12) is more important and influential than that of  $\dot{h}_2$  on conforming the result obtained from usual method. We consider that some questions still exist in adapting damping coefficient matrix to be used in usual method.

In order to obtain more adequate value of  $\dot{h}_0 \sim \dot{h}_2$ , we conclude that more field test or more detail of experiment for determining the damping coefficient matrix is necessary.

## RÉSUMÉ

De nos expériences dans ce domaine nous arrivons à proposer l'équation (12) ou (16') pour la matrice de coefficient d'amortissement du système à masses multiples. Les résultats reçus par la méthode habituelle ont été comparés avec quelques séries de nos calculs numériques. Nous trouvons le facteur  $\dot{h}_0$  dans l'équation (12) plus grand et influent que  $\dot{h}_2$ , en adaptant le résultat obtenu par la méthode habituelle. Nous pensons que tous les problèmes ne sont pas résolus dans l'adaptation de la matrice du coefficient d'amortissement à la méthode de calcul normale.

Nous concluons qu'il est nécessaire de faire plus de tests sur nature ou de détailler d'avantage les expériences pour obtenir des valeurs  $\dot{h}_0 \sim \dot{h}_2$  plus adéquates à la détermination de la matrice de coefficient d'amortissement.

## ZUSAMMENFASSUNG

Aufgrund unserer einfachen Versuche auf diesem Gebiet empfehlen wir die Gleichung (12) oder (16') für die Dämpfungskoeffizienten-Matrix des Viel-Massen-Systems. Ergebnisse der üblichen Verfahren sind mit einigen Sätzen unserer numerischen Berechnung verglichen worden, und wir finden, dass  $\dot{h}_0$  in Gleichung (12) wichtiger und einflussreicher denn  $\dot{h}_2$  bei Anpassung an die Ergebnisse der üblichen Verfahren ist. Wir berücksichtigen, dass einige Fragen bei der Anwendung der Dämpfungskoeffizienten-Matrix im üblichen Verfahren noch offen bleiben.

Um mehr hinreichende Werte  $\dot{h}_0 \sim \dot{h}_2$  zu erhalten, folgern wir, dass mehr Felduntersuchungen oder mehr Prüfungsdetails zur Bestimmung der Dämpfungskoeffizienten-Matrix notwendig sind.

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