Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 8 (1968)

Artikel: Response of structures subjected to sonic booms

Autor: Herrmann, George / Krajcinovic, Dusan

DOI: https://doi.org/10.5169/seals-8869

VI

Response of Structures Subjected to Sonic Booms

Influence des détonations supersoniques sur les constructions

Wirkung des Überschallknalls auf Bauwerke

GEORGE HERRMANN
Professor of Civil Engineering
Northwestern University
Evanston, Illinois

DUSAN KRAJCINOVIC
Solid Mechanics Research
Ingersoll-Rand Research Center
Princeton, New Jersey

1. Introduction

The advent of supersonic commercial air-transport operations brings with it a host of new and different problems, such as the transient pressure generated by the sonic boom that is associated with the shock waves stemming from the aircraft. The response problem will involve not only people, but also structures on the ground, and thus transient response of buildings and other structures to the supersonic shock has to be studied.

As measurements of the history of the far-field atmospheric pressure (signature) induced by a sonic boom indicate, the loading on a structure consists of a sudden overpressure followed immediately by a sharp underpressure. The total duration of this applied disturbance has been measured to be of the order of a fraction of a second. Because of the shape of this signature and the relatively short duration time, the authors are proposing to represent the applied load as a dipole in time. A dipole has been defined and used extensively as a generalized derivative of a Dirac delta function only if the independent variable is a spatial coordinate. Even though some work has already been carried out on the effects of the sonic boom on structures, the proposed representation of the loading as a dipole in time (called here bipulse) has the advantage that the structural response may be treated conveniently as a homogeneous initial value problem.

In Section 2 the proposed representation of the sonic boom loading is discussed, while the response of some simple structures is analyzed in Section 3.

A more extensive treatment of the response of structures to sonic booms, including problems of continuous structures, will be presented by the authors in a later study.

2. The Sonic Boom Loading

A large number of measurements of the pressure on the ground generated by sonic booms has been recorded and published in the last few years [1]. The measured diagram showing the variation of the pressure with respect to time (the

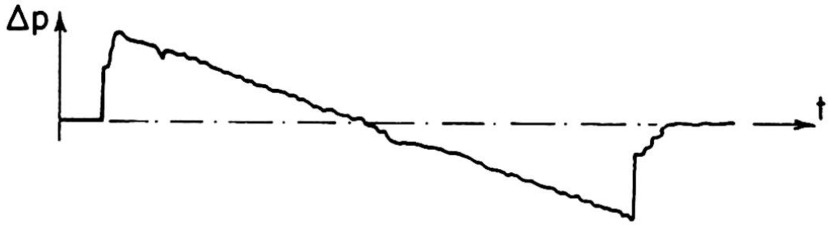

Fig. 1. Pressure signature due to the sonic boom
$\Delta p = 0$ corresponds to atmospheric pressure.

signature), Fig. 1, may be closely approximated by two triangles of identical area, Fig. 2.

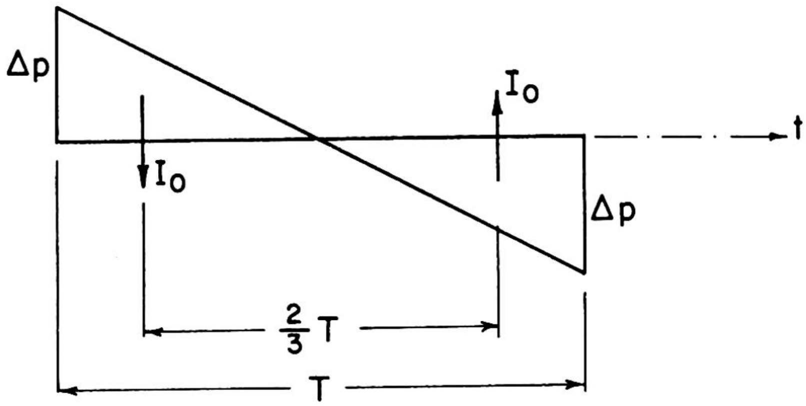

Fig. 2. Idealized pressure signature due to the sonic boom.

The two most significant parameters for the structural engineer are the peak overpressure Δp and the time duration T.

According to the measurements reported in Ref. [1], the duration T is 0.04 sec for the sonic boom generated by the present-day fighters, 0.1 sec for the largest present-day operational aircrafts, and is expected to be 0.4 sec for the supersonic transport (SST). It has been established also that Δp is not considerably different for the three cases mentioned.

In analyzing a structure subjected to sonic boom loading, it is necessary to represent the pressure signature in a mathematically convenient manner. For loads which have a large intensity and short duration (impulsive loads) it is a standard procedure to make use of the Dirac delta function defined by

$$\delta(t-\tau) = \begin{cases} 0 & t \neq \tau \\ \infty & t = \tau \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t-\tau)dt = 1 .$$

It can be shown readily that such impulsive loading may be regarded as the difference between two step loads of the same intensity P(x) applied immediately one after another, i.e.,

$$\lim_{\Delta t \to 0} P(x)\Delta t \, \frac{H(t-\tau) - H(t-\tau+\Delta t)}{\Delta t} = I(x)\delta(t-\tau), \tag{1}$$

where $H(t-\tau)$ is the Heaviside step function defined by

$$H(t-\tau) = \begin{cases} 0 & t < \tau \\ 1 & t > \tau \end{cases}$$

and

$$I(x) = \lim_{\substack{\Delta t \to 0 \\ P(x) \to \infty}} [P(x)\Delta t]. \tag{2}$$

It occurred to the authors that the sonic boom signature may be conveniently represented by two impulsive loads, opposite in sign, and acting in rapid succession one after another. This implies mathematically that the signature may be idealized by a time derivative of a Dirac delta function, i.e.,

$$\lim I(x)\Delta t \, \frac{\delta(t-\tau) - \delta(t-\tau+\Delta t)}{\Delta t} = - B(x)\dot{\delta}(t-\tau), \tag{3}$$

where $\dot{\delta}(t-\tau)$ is a dipole of positive unit moment applied at $t = \tau$ on the time axis and is defined by

$$- \dot{\delta}(t-\tau) = \lim_{\Delta t \to 0} \frac{\delta(t-\tau+\Delta t) - \delta(t-\tau)}{\Delta t} = \frac{d\delta(t-\tau)}{dt} .$$

The function B(x), having the dimension of the force multiplied by the square of the time unit, is defined as

$$B(x) = \lim_{\substack{\Delta t \to 0 \\ I(x) \to \infty}} [I(x)\Delta t]. \tag{4}$$

Spatial derivatives of the Dirac delta function have been used extensively in various branches of physics and engineering (e.g., acoustics) under the name dipole of doublet. To distinguish these from the time derivative defined by Eq. (3), we propose to use the term "bipulse" for the latter.

Based on these definitions, let us now calculate the bipulse idealizing a sonic boom signature. The impulse I(x), see Fig. 2, is

$$I(x) = \frac{\Delta p(x)T}{6} ,$$

while the bipulse is the "moment" of the two impulses, i.e.,

$$B(x) = \frac{2}{3} TI(x) = \frac{1}{9} \Delta p(x)T^2.$$

3. Response of a Lumped-Parameter Structure

In this Section we wish to calculate the response of a simple system subjected to a sonic boom loading and compare the results with the responses to other types of dynamic loads, such as step load and an impulse. The structure shall be considered to possess only a single mass m and a single stiffness k. The differential equation of such a system with one degree of freedom is, in terms of a displacement w,

$$\frac{d^2w}{dt^2} + \omega^2 w = \frac{F(t)}{m} , \tag{5}$$

where

$$\omega^2 = \frac{k}{m}$$

is the natural frequency of free vibrations, t is the time, and F(t) is the applied load. The static displacement w_{st} due to the static force $F(t) = F_0$ is

$$w_{st} = \frac{F_0}{k} = \frac{F_0}{m\omega^2} . \tag{6}$$

The general solution to Eq. (5), as combined from the transient and steady-state part, reads

$$w(t) = w(0)\cos\omega t + \dot{w}(0)\,\frac{\sin\omega t}{\omega} + \frac{1}{\omega}\int_0^t \frac{F(t)}{m}\,\sin\omega(t-\tau)dt, \tag{7}$$

where w(0) and $\dot{w}(0)$ are the initial displacement and the initial velocity, respectively. We will confine our attention to the solution of an initial value problem with

$$w(0) = \dot{w}(0) = 0.$$

It is well known that the response to a step load results in a maximum displacement

$$w_{max} = 2w_{st}$$

such that the so-called displacement amplification factor $A \equiv |w_{dyn\ max}|/|w_{st}|$ is in this case

$$A_H = 2. \tag{8}$$

Next we calculate the response to a single impulse applied at time t = 0. The forcing term in Eq. (5) now reads

$$\frac{F(t)}{m} = \frac{I_0}{m}\,\delta(t) \tag{9}$$

resulting in a displacement

$$w_I = \frac{I_o}{m\omega} \sin\omega t = \frac{I_o\omega}{F_o} w_{st} \sin\omega t. \tag{10}$$

The amplification factor is now a function of the structural parameters and reads

$$A_I = \frac{I_o\omega}{F_o} . \tag{11}$$

Finally, for the bipulse loading, the forcing term in Eq. (5) is

$$\frac{F(t)}{m} = - \frac{B_o}{m} \dot{\delta}(t) \tag{12}$$

such that the displacement is

$$w_B = \frac{B_o}{m} \cos\omega t = \frac{B_o\omega^2}{F_o} w_{st} \cos\omega t \tag{13}$$

and the displacement amplification factor is

$$A_B = \frac{B_o\omega^2}{m} , \tag{14}$$

which again depends on the properties of the structure.

It is useful, now, to define the ratio μ between the maximum displacement due to the bipulse and due to the impulse, i.e.,

$$\mu = \frac{|w_{B\ max}|}{|w_{I\ max}|} = \frac{B_o\omega}{I_o} . \tag{18}$$

Since $$B_o = \frac{2}{3} T I_o$$

μ can also be written as

$$\mu = \frac{2}{3} T \omega. \tag{19}$$

The limiting value ω^* of the natural frequency of the free vibrations of the system, above which the bipulse induces larger displacements than the impulse is, according to Eq. (19), obtained for $\mu = 1$. Using the values of T for the three different aircrafts mentioned in Section 2, we obtain

$$\omega^* = \begin{cases} \dfrac{3}{2(0.04)} = 37.5 \text{ sec}^{-1} & \text{(fighters)} \\[2ex] \dfrac{3}{2(0.1)} = 15 \text{ sec}^{-1} & \text{(present-day aircrafts)} \\[2ex] \dfrac{3}{2(0.4)} = 3.75 \text{ sec}^{-1} & \text{(SST)} \end{cases} \tag{20}$$

It is obvious from the numerical values given above that the representation of the sonic boom by a simple impulse, as this is sometimes assumed, may lead to results which greatly underestimate the structural response.

As an illustration let us consider a cantilevered beam of length ℓ and mass per unit of length m. If only the lowest mode is assumed to contribute to the response, the corresponding natural frequency of free vibrations is known to be

$$\omega^2 = 3\,\frac{20\,EI}{9\,m\ell^4},$$

where EI is the flexural rigidity. If ω^* is taken as 15 sec^{-1} (largest present-day aircrafts), then for

$$\frac{EI}{m\ell^4} > 33.7\ sec^{-2}$$

the response to a bipulse is larger than that to an impulse.

Note that for $\mu = 1$ the duration of the bipulse is almost 1/4 of the natural vibration period. This means that the proposed idealization leads to an over-estimation of displacements. Consequently $\mu = 1$ may be regarded only as a lower bound under which sonic boom results in smaller displacements than predicted by an impulsive load.

4. Conclusions

Having proposed that the effect of the sonic boom on a simple structure may be represented as a dipole in time ("bipulse"), the authors show that this idealization leads to convenient mathematical analysis. The commonly used displacement amplification factor as a measure of the severity of dynamic response is introduced. It is shown that the amplification factor depends on the free vibration frequency of the structure and that, consequently, some structures undergo larger displacements due to the bipulse load than due to other types of dynamic loads (such as impact load). The procedure is exemplified by a simple discrete system.

In a later paper the authors intend to treat the problem in greater detail, proving that it reduces to an initial value problem and extending it to continuous systems.

5. Acknowledgment

The authors are very grateful to Professor C. Dyrbye of the Technical University of Denmark for a most constructive discussion of this work. They also wish to acknowledge the support of the Air Force Office of Scientific Research under grant AF-AFOSR-100-67.

Reference

1. "Proceedings of the Sonic Boom Symposium," J. Acoust. Soc. Am. 39, No. 5, Part 2, S1-S80 (1966).

SUMMARY

It is suggested in this paper that the loading on structures induced by the sonic boom generated by supersonic aircraft can be represented by a dipole in time. The term "bipulse" is introduced for this type of transient loading. It is shown that simple structures subjected to such bipulse loading may be conveniently analyzed and the response readily compared with that due to other types of dynamic effects such as, for example, step loading and impulsive loading.

RÉSUMÉ

La rédaction suggère de représenter la charge appliquée à une construction due à une détonation supersonique par un dipôle du temps. On·introduit le terme "bipulse" pour désigner ce type de charge variable. Cette charge "bipulse" peut être facilement analysée et son effet comparé à celui d'autres types d'effets dynamiques comme charge progressive ou charge impulsive, dans le cas de structures simples.

ZUSAMMENFASSUNG

In diesem Beitrag wird vorgeschlagen, dass die aus Ueberschallknall der Ueberschallflugzeuge entstandene Belastung durch ein Dipol der Zeit dargestellt werden kann. Der Ausdruck "Bipuls" ist für diese Art veränderlicher Belastung eingeführt worden. Es wird gezeigt, dass einfache Bauwerke unter solcher Bipulslast bequem gelöst werden können, und dass die Antwort leicht fällt verglichen mit jenen, die anderen dynamischen Wirkungen unterworfen sind, zum Beispiel Stufenlast und impulsiver Last.

Leere Seite
Blank page
Page vide