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Load Distribution in Multi-Storey Shear Wall Structures

Répartition des charges dans des constructions à étages multiples avec murs de cisaillement

Lastverteilung in mehrstöckigen Scheibentragwerken

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1. Introduction

Recent developments in multi-storey buildings for residential purposes have led to the extensive use of shear-walls, or cross-walls, for the basic structural system. Whilst the walls are designed primarily to resist both vertical and horizontal loads, they can in addition, by careful planning, be utilised fully for the non-structural functions of dividing and enclosing space, whilst simultaneously providing fire resistance and acoustic insulation between dwellings. By this means, efficient structural designs can be achieved. The floor slabs, which keep storey and overall heights to a minimum, act as deep beams transferring the wind loads to the vertical wall elements. The regular system of walls and slabs lends itself to industrialised building techniques, using either in-situ or precast construction.

A typical example of such slab-type structures is shown in Fig.1. Planning requirements tend to evolve parallel assemblies of walls, coupled by floor slabs or lintel beams, in conjunction with box-type assemblies surrounding lift shafts and stair wells.

The majority of studies of coupled shear-wall structures has been devoted to the problem of plane walls subjected to standard systems of loads in their own plane. In the case of a complete building, the results are strictly accurate only if the structure consists of parallel systems of identical wall assemblies, so that any lateral load is divided equally amongst them, it being assumed that all walls deflect equally due to the very high in-plane stiffness of the floor slabs.

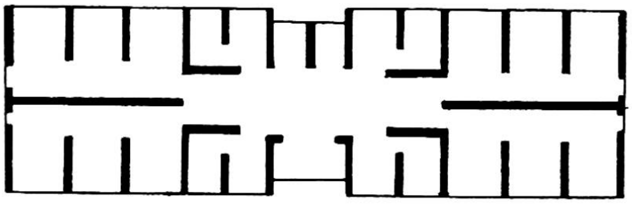

FIG. 1

If the structure consists of different forms of load bearing elements, or if, as is often the case, walls are curtailed at first floor level to provide an open concourse, considerable redistribution of load may take place. It may then become important to examine in some detail the distribution of load between the walls throughout the height of the building.

For shear-wall structures, suggestions have been made that lateral loads should be distributed amongst the walls in proportion to their stiffnesses[1]. If any coupling occurs, this process will give rise to significant errors, as may readily be seen by comparing the modes of deformation of a coupled wall and a cantilever box structure subjected to uniformly distributed loads[2]. The former bends with a reversal of curvature in the upper levels, whilst the latter bends in single curvature. In order to constrain the two to deflect equally, tensile linking forces are required in the upper levels, and compressive forces in the lower regions.

Two different methods have been developed for the analysis of coupled shear walls, the frame analogy and the continuous connection techniques. The first replaces the deep wall by a line column at the centroid, the finite depth being incorporated by the use of rigid arms to link the ends of the connecting beams to the column. The analysis is then carried out using standard frame-analysis techniques. In this case, an increase in the number of storeys leads to an increase in the degree of statical indeterminacy, and a corresponding increase in the number of equations to be solved. The second method replaces the discrete system of connecting beams by a continuous medium of the same stiffness. By assuming that the connecting beams deform with a fixed point of contraflexure, but do not deform axially, the behaviour of the system may be expressed by a single second-order differential equation, enabling a general closed solution to the problem to the obtained. This method has the advantage that the accuracy of the solution increases with an increasing number of storeys, with no extra labour involved. Because of its analytical nature, the method is less versatile in that it is more difficult to deal with variations in geometrical and stiffness parameters, although, because of the essential uniformity of the shear wall structure, this is not a serious limitation. The effects of variable thickness can easily be incorporated[3], and changes in width may be included in the general solution[4]. The accuracy of the technique has been demonstrated by numerous experimental investigations[3,4]. The discontinuities which may be present at ground floor level, where shear walls may be discontinued, may be treated by incorporating their influence in the lower boundary condition[5].

The aim of this paper is to present a method of analysis of complete multi-storey apartment-style concrete buildings, whose load-bearing structure consists essentially of parallel systems of shear wall and box elements, subjected to any system of lateral loads. The method is based on the continuous connection technique, and numerical results are presented for a typical representative structure.

2. Method of Analysis

Consider the coupled shear wall shown in Fig.2 subjected to a point load Pi applied at any height xi. The individual connecting beams of stiffness EI_p are replaced by an equivalent continuous medium or set of laminas of stiffness EI_p/h per unit height. It is assumed that the connecting beams do not deform axially so that both walls deflect equally, and that the point of contraflexure occurs at the mid-span position in all beams. If the laminas are assumed 'cut' at their mid-points, the only force acting at the cut section is a shear force of intensity q per unit height. On considering the deformations of the cut laminas, the compatibility condition may be set up to give no resultant relative deformation at the cut, and, when used in conjunction with the moment-curvature relationships for the walls, this leads to the establishment of the following governing differential equation,

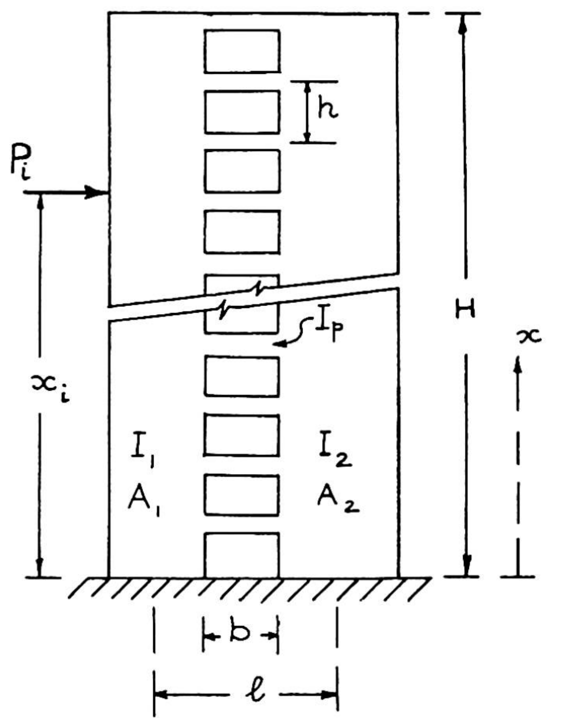

FIG. 2

$$\frac{d^2q}{dx^2} - \mu^2 q = -\alpha^2 Pi \langle x - xi \rangle^0 \qquad (1)$$

where $\alpha^2 = \dfrac{12\ell I_p}{b^3 hI}$

and $\mu^2 = \dfrac{\alpha^2}{\ell}\left(\ell^2 + \dfrac{IA}{A_1A_2}\right)$

For convenience in writing a single solution for the entire structure, a system of Macaulay's brackets has been employed in equation (1). These are defined in the usual manner as, When $x < xi$, $\langle x - xi \rangle^n = 0$, $\langle x - xi \rangle^o = 0$

When $x > xi$, $\langle x - xi \rangle^n = (x - xi)^n$, $\langle x - xi \rangle^o = 1$

The detailed derivation of equation (1) is not given, since similar equations have been derived in detail in earlier papers for different load conditions[3,4]. The appropriate boundary conditions for the fixed and free ends, expressed in terms of the shear force intensity 'q', may be shown to be, respectively,

$$\text{At } x = 0, \qquad q = 0$$

$$\text{At } x = H, \qquad \frac{dq}{dx} = 0 \tag{2}$$

The solution of equation (1), subject to the boundary conditions (2), may be shown to be,

$$q = \frac{\alpha^2 p_i}{\mu^2} \cdot \frac{1}{\cosh \mu H} \left\{ \cosh \mu H \, . \, \cosh \langle \mu (x - xi) \rangle^1 - \cosh \mu (H - x) - \sinh \mu (H - xi) \, . \, \sinh \mu x \right\} \tag{3}$$

At any level, the axial force in the wall, tensile or compressive, is given by,

$$\begin{aligned} N &= \int_x^H q \, dx \\ &= \frac{\alpha^2 p_i}{\mu^3} \, \frac{1}{\cosh \mu H} \left\{ \sinh \mu (H - xi) \, . \, \cosh \mu x - \sinh \mu (H - x) \right. \\ &\quad - \cosh \mu H \, . \, \sinh \langle \mu (x - xi) \rangle^1 \\ &\quad \left. - \langle \mu (xi - x) \rangle^1 \right\} \end{aligned} \tag{4}$$

The corresponding bending moments in the walls are,

$$\begin{aligned} M_1 &= \left\{ p_i \langle x - xi \rangle^1 - T\ell \right\} \frac{I_1}{I} \\ M_2 &= \left\{ p_i \langle x - xi \rangle^1 - T\ell \right\} \frac{I_2}{I} \end{aligned} \tag{5}$$

since the moment carried by each wall is proportional to its stiffness, by virtue of the assumption that both walls deflect equally.

The moment-curvature relationship is,

$$EI \frac{d^2 y}{dx^2} = p_i \langle x - xi \rangle^1 - T\ell \tag{6}$$

Integration of equation (6), and inclusion of the appropriate boundary conditions at top and bottom, yields the deflection at any level,

$$y = \frac{Pi}{EI}\left\{ \frac{1}{6}\left(1 - \frac{\alpha^2 \ell}{\mu^2}\right)\left(3xi \,.\, x^2 - x^3 + \langle x - xi\rangle^3\right)\right.$$
$$+ \frac{\alpha^2 \ell}{\mu^4}\left[\left(x - \langle x - xi\rangle^1\right) + \frac{1}{\mu \cosh \mu H}\left(\sinh \mu (H - x)\right.\right.$$
$$- \sinh \mu H + \sinh \mu (H - xi)\ (1 - \cosh \mu x)$$
$$\left.\left. + \cosh \mu H \ \sinh \langle \mu (x - xi)\rangle^1 \right]\right\} \qquad (7)$$

The single equation (7) yields the complete relationship between a unit load at any height xi and the deflection at any height x, enabling a complete set of influence coefficients fij (deflection at xi due to a unit load at xj) to be evaluated readily.

It may readily be shown that the same solution holds for a symmetrical shear wall containing two bands of openings, provided the parameters α and μ are defined slightly differently[3,4].

Analysis of Complete Structure

Suppose that the structure consists of a number of parallel wall assemblies. For a coupled-wall element, the load-deflection relationship is given by equation (7). For a box-type element, the corresponding load-deflection relationship may be shown to be,

$$y = \frac{Pi}{6EI}\left\{3xi^2 x - xi^3 + \langle xi - x\rangle^3\right\} \qquad (8)$$

In either case, for the k^{th} wall element, the load-deflection relationship may be expressed in matrix form as,

$$\underset{\sim}{y}_{ik} = \underset{\sim}{fij}_k \ \underset{\sim}{Pi}_k \qquad (9)$$

where $\underset{\sim}{y}_i$ and $\underset{\sim}{Pi}$ are column vectors of deflections and applied loads at any chosen set of levels xi, and $\underset{\sim}{f}_{ij}$ is a square matrix of influence coefficients evaluated from equations (7) or (8). A similar relationship can be derived for each wall assembly. Fortunately, in buildings of this nature, the number of different forms of wall elements is quite small, so that only a very limited number of matrices $\underset{\sim}{fij}$ will have to be evaluated.

For any applied load system whose resultant passes through the centre of rotation, all deflections will be the same at any given level, if it is assumed that the floor slabs are infinitely stiff in their own plane. Hence,

$$\underset{\sim}{fij}_k \ \underset{\sim}{Pi}_k = \underset{\sim}{y}_i = \text{constant for all 'k'} \qquad (10)$$

For equilibrium, the total applied load Pi at each storey level must be equal to the sum of the loads on the wall assemblies at that level. That is,

$$Pi = \sum_k \underset{\sim}{Pi}_k \qquad (11)$$

summing over all wall assemblies.

Hence, from equations (10) and (11), the complete load distribution throughout the structure may be obtained. From equation (10) the applied loads on any wall assembly 'k' may be expressed in terms of the load Pi_1 on any particular wall '1', say,

$$\underset{\sim}{Pi}_k = \underset{\sim}{fij}_k^{-1} \; \underset{\sim}{fij}_1 \; \underset{\sim}{Pi}_1 \qquad (12)$$

Hence, from equation (11), the loads on wall '1' become,

$$\underset{\sim}{Pi}_1 = \left\{ \underset{\sim}{I} + \sum_{k = 2,3,\ldots m} \underset{\sim}{fij}_k^{-1} \; \underset{\sim}{fij}_1 \right\}^{-1} \underset{\sim}{Pi} \qquad (13)$$

and the loads on all other walls follow from equation (12).

The deflections and stresses in individual wall assemblies may then be evaluated by the continuous connection solution, using equations (3), (4), (5) and (7).

Suppose, for example, that the structure consists of four, six and one element of types 'a', 'b' and 'c' respectively. Then, from equation (9),

$$\underset{\sim}{yi} = \underset{\sim}{fij}_a \; \underset{\sim}{Pi}_a = \underset{\sim}{fij}_b \; \underset{\sim}{Pi}_b = \underset{\sim}{fij}_c \; \underset{\sim}{Pi}_c \qquad (14)$$

For equilibrium, $$\underset{\sim}{Pi} = 4\underset{\sim}{Pi}_a + 6\underset{\sim}{Pi}_b + \underset{\sim}{Pi}_c \qquad (15)$$

or using equation (14)

$$\underset{\sim}{Pi} = \left\{ 4\underset{\sim}{I} + 6\underset{\sim}{fij}_b^{-1} \; \underset{\sim}{fij}_a + \underset{\sim}{fij}_c^{-1} \; \underset{\sim}{fij}_a \right\} Pi_a \qquad (16)$$

The loads on the wall assemblies of type 'a' become,

$$\underset{\sim}{Pi}_a = \left\{ 4\underset{\sim}{I} + 6\underset{\sim}{fij}_b^{-1} \; \underset{\sim}{fij}_a + \underset{\sim}{fij}_c^{-1} \; \underset{\sim}{fij}_a \right\}^{-1} \underset{\sim}{Pi} \qquad (17)$$

and the loads on the other elements follow from equation (14).

If the resultant of the applied loads does not pass through the centre of rotation, but acts at a distance z' from it, a torsional moment Ti of magnitude $Pi\, z'$ is applied to the building. Owing to the high in-plane stiffness of the floor slabs, they will undergo a rigid body rotation through an angle θi, say, such that the total displacement of the k^{th} wall assembly is equal to $yi + \theta i z_k$, where z_k is the distance of the wall element from the centre of rotation.

If the twisting moments on the wall elements are neglected, the second condition of rotational equilibrium becomes, for the entire system,

$$\underset{\sim}{Ti} = \sum_k \left(\underset{\sim}{Pi}_k \, z_k \right) \qquad (18)$$

The second condition of equilibrium enables the additional unknown displacement $\underset{\sim}{\theta}_i$ to be evaluated and a complete solution obtained.

By virtue of the assumption of a rigid body rotation of all floor slabs, the rotations of all wall elements may be obtained and the twisting moments evaluated in terms of the rotations Θi from standard strength of materials relationships. If the twisting movements on individual wall elements are included, therefore, the second condition of rotational equilibrium (18) must be amended to,

$$\underset{\sim}{Ti} = \sum_k (\underset{\sim}{pi}_k \, 3_k) + \sum_k \underset{\sim}{Ti}_k \qquad (19)$$

where Ti_k is the twisting movement on the k^{th} wall element at level xi. These twisting movements may then be expressed in terms of the rotations, and the complete solution obtained as before.

Lack of space does not permit the torsional condition to be treated in greater detail.

3. Numerical Example

In order to illustrate the results, a representative shear-wall structure of the form shown in Fig.3 (in plan) is considered. The structure consists of a central core (wall 3), heavy flank walls (walls 1), and four pairs of interior cross-walls (walls 2). It is assumed that the building is 20 storeys high, with a storey height of 8 ft.9in., the floor slabs being assumed to be 6 in. thick. For the purpose of this example, it is assumed that the building is subjected to a wind loading of intensity 10 lb/ft^2 in a direction parallel to the cross-walls.

The effective width, or stiffness, of the floor slabs may be determined by a subsidiary calculation, using such numerical techniques as the finite element or finite difference method; in the present instance, the latter was employed, using curves prepared by Qadeer and Stafford Smith[6]. In this case, the effective widths of slab connecting walls 1 and 2 are taken to be 10 ft. and 13.2 ft. respectively.

A computer program has been written to perform the entire calculations necessary for a complete analysis of the building. Because of the regularity of such structures, the program requires a relatively small amount of input data. The output consists of the influence coefficients for each type of wall unit, the loads on the various walls at each level, the bending moments, axial forces, and shear forces on all walls at each level, the stresses at the extreme fibres of each wall, and the deflection. In order to check the computation, the deflection of each wall is calculated separately.

In the present case, the influence coefficients and forces were computed at each storey level. The computed deflections for the different wall units agreed with each other to six significant figures. The results obtained are shown in Figs.4, which show (a) the percentage of the total applied load carried by each type of wall unit throughout the height of the building, and (b) the deflected form.

The results indicate that for structures of this nature, considerable variations in the load distribution can occur throughout the height of the building.

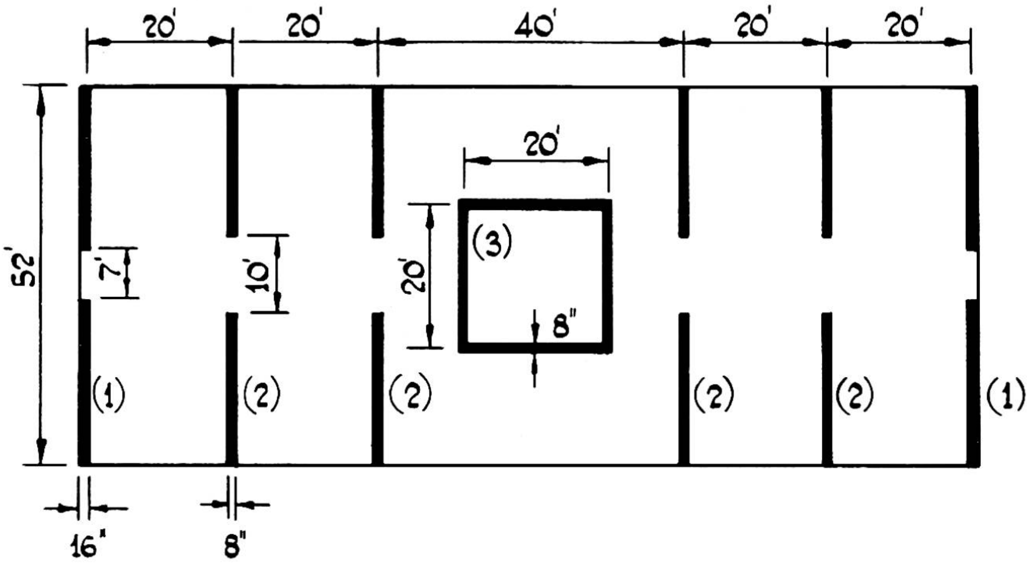
20'
20'
40'
20'
20'
20'
52'
7'
10'
20'
(3)
8"
(1)
(2)
(2)
(2)
(2)
(1)
16"
8"

Fig. 3.

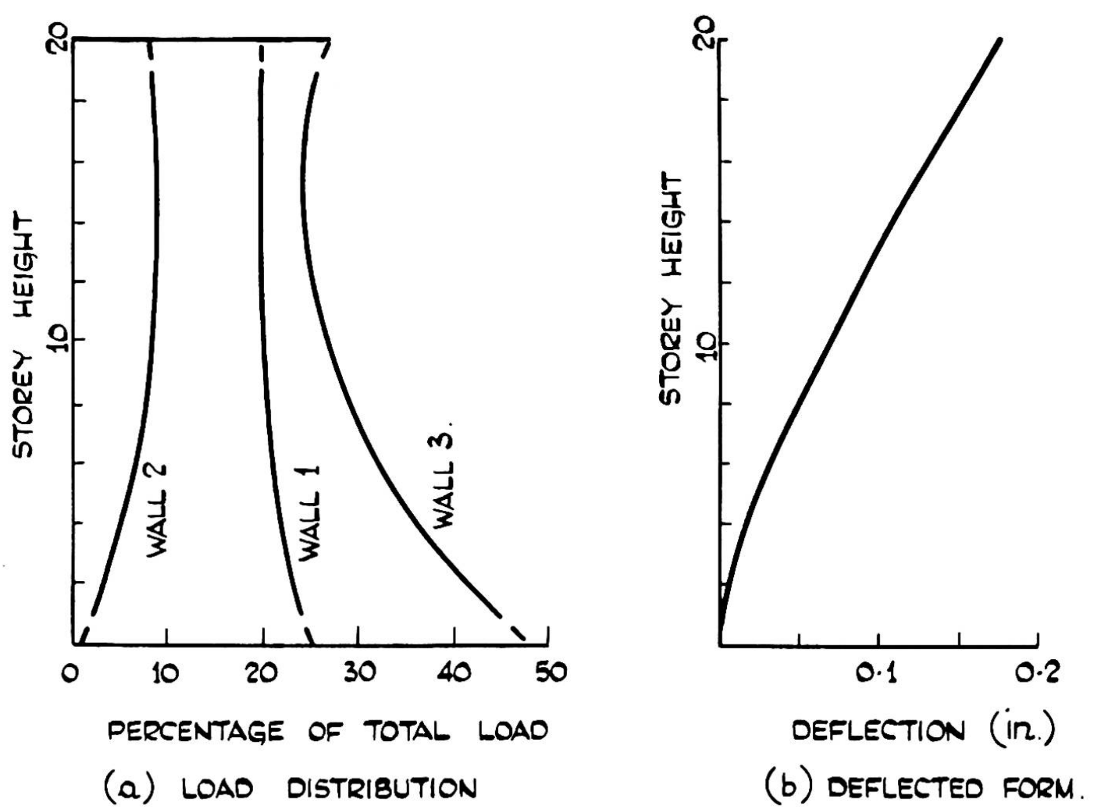
STOREY HEIGHT
20
10
WALL 2
WALL 1
WALL 3
0
10
20
30
40
50
PERCENTAGE OF TOTAL LOAD
(a) LOAD DISTRIBUTION
STOREY HEIGHT
20
10
0·1
0·2
DEFLECTION (in.)
(b) DEFLECTED FORM.

Fig. 4.

4. Conclusions

A method has been presented for the analysis of multi-storey structures which consist essentially of parallel assemblies of coupled shear-walls. The continuous connection technique is used to determine the load-deformation characteristics of individual wall assemblies, and the complete structure is analysed using compatibility and equilibrium conditions. Any applied load system can be dealt with, and the technique can be extended to include torsion of the structure.

The analysis deals with matrices of small order, and can readily be programmed for digital computation. In order to reduce the amount of computation, the influence coefficients need not be evaluated for every storey level, since rapid variations in load distribution do not occur in regular structures of the form considered. Having regard to the form of structure and applied load, the designer can assess the number of reference levels which may be required in the analysis to give a solution which is sufficiently accurate for practical purposes. A closer spacing in the reference levels may be adopted for regions where the load distribution is changing most rapidly.

In the paper, the general solution for a coupled shear wall structure subjected to a joint load at any height is, as far as the authors are aware, presented for the first time. By simple integration, the general solution for any load form may be derived from equation (3).

References

1. Albiges, M. and Goulet, J. "Contreventement des Batiments" Annales de L'Institut Technique du Batiment et des Travaux Publics. No.149, May, 1960, pp.473 - 500

2. Khan, F.R. "Current Trends in Concrete High-Rise Buildings" Proceedings of Symposium on Tall Buildings, Ed. Coull and Stafford Smith, Pergamon Press, 1967, p.575

3. Coull, A. and Puri, R.D. "Analysis of Coupled Shear Walls of Variable Thickness" Building Science, Vol.2, 1967, pp 181-188

4. Coull, A. and Puri, R.D. "Analysis of Coupled Shear Walls of Variable Cross-section" Building Science, Vol.2, 1968, pp 313-320

5. Rosman, R. "Approximate Analysis of Shear Walls Subject to Lateral Loads" Jnl. A.C.I., Vol.61, 1964, pp 717 - 732.

6. Qadeer, A. and Stafford Smith, B. "The Bending Stiffness of Slabs Connecting Shear Walls" (submitted for publication to A.S.C.E.)

Notation The following symbols are used in this paper:

x	Height
y	Horizontal deflection
Θ	Rotation
ζ	Distance from centre of rotation
H	Total Height
h	Storey Height
b	Clear distance between walls
ℓ	Distance between centroidal axis of two walls
I_p	Moment of inertia of connecting beam
I_1, I_2	Moment of inertia of walls 1 and 2
I	$I_1 + I_2$
$A_1 A_2$	Cross-sectional areas of walls 1 and 2
A	$A_1 + A_2$
Pi	Applied load at level xi
Ti	Applied twisting moment at level xi
q	Shear force intensity in substitute connecting medium
$M_1 M_2$	Bending moments in walls 1 and 2
N	Axial force in wall
fij	Deflection at level xi due to unit load at xj
k	Suffix denoting k^{th} wall assembly

SUMMARY

A method is presented for the analysis of the distribution of load amongst the shear walls of a multi-storey apartment-style building. The method is based on the continuous connection technique. Numerical results are presented for a typical twenty-storey structure.

RÉSUMÉ

L'article présente une méthode pour l'analyse de la distribution des charges parmi les murs de cisaillement dans une maison d'appartement de beaucoup d'étages. La méthode est basée sur la technique de liaison continue. Des résultats numériques sont donnés pour un bâtiment typique de 20 étages.

ZUSAMMENFASSUNG

Dargestellt ist ein Verfahren zur Berechnung der Lastverteilung in Scheiben eines mehrstöckigen Gebäudes. Das Verfahren baut auf der durchlaufenden Gewebetechnik auf. Numerische Ergebnisse werden für ein typisches zwanzigstöckiges Haus gegeben.