DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Specifications on Multi-Story Buildings and in Particular on Steel Structures in Seismic

Recommandations pour le calcul des bâtiments à plusieurs étages en zone sismique avec référence spéciale aux constructions en acier

Empfehlungen für mehrstöckige Gebäude, insbesondere für Stahltragwerke in Erdbeben-gebieten

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The Convention of European Constructional Steelwork Associations has appointed a special Commission for compiling the "Recommenda - tions" for the design of steel structures in seismic area. Such Commission, known as Commission XIII, has begun his works in 1965 and in a first stage has accurately studied all international standards on the argument by making comparison and analysing those parts that the different recommendations have in cpmmon.

On the basic of the aquired knowledges and taking into account the most recent standards, the Commission has outlined a text of re commendations with comments where the general principles of the stan dards have been explained and some particular cases have been trea- ted. Numerical coefficients have also been given. The Commission has decided that buildings in seismic area can be designed only if the equivalent static forces proportional to the masses in motion through a suitable seismic coefficient are introduced. Such coefficient has been fixed on the basis of the most recent studies of seismic engi- neering based on the spectral analysis of earthquakes and has been considered as the product of four elementary factors: the intensity seismic factor, the foundation factor, the response factor and the masses distribution factor.

The influence of each factor has been pointed out both from a qualitative and a quantitative point of view by introducing suita- ble schemes and simplified formulations.

In the text have also been treated the problems of torsional ac tions due to seismic motions as well as the stresses due to verti- cal actions.

Studies for preparing a further chapter on the efficiency of connections between members both of the up structure and of the foundation structure are going on. In examining the factors which define the seismic factor the Commission has paid a particular at

tention to the evaluation of the response coefficient of the struc
tures. As it is known this coefficient depends on the natural pe-
riod of vibration of the structure as it is shown by the spectral
analysis of the earthquakes.

In the following it has been reported on some researches carried
out for the evaluation of the natural period of vibration of buil-
dings.

- Period of vibration of framed structures

Because of the difficulties of evaluating the natural period of
vibration, simplified formulations have been suggested by some Au-
thors which relate the period T to the number of floors of a buil-
ding, and sometimes to its geometric and elastic characteristics.

Among the most widely used formulae, the following ones may be
listed:

$$T = 0,1\ n,$$

and:

$$T = \frac{0,05}{\sqrt{D}}\ H$$

which are respectively used by S.E.A.O.C. standards for framed
structures and structures with stiffening walls.

In the formulae above, n represents the number of floors,and H
and D respectively the height and the transversal dimension of the
building in feet.

It is worth writing also the formula suggested by Ifrim for pla-
ne frames:

$$T = \frac{2\pi}{\xi} \sqrt{\frac{M_o\ l_o^3}{E\ J_o}} \qquad (1)$$

where M_o, l_o and J_o are spectively the base mass lenght and moment
of inertia, E the elastic modulus of the material and ξ a parame-
ter which depends on the number of floors and columns as well as
on the masses and stiffnesses distribution on the height.

In order to check the validity of the proposed formulations so-
me studies on the argument have been carried out in the "Istituto
di Tecnica delle Costruzioni" of the University of Naples.

Firstly plane structures with or without windbracing have been
considered; secondly a dynamic analysis of three dimensional frames
has been undertaken.

1) - Plane structures

For plane structures a sistematic theoretic study has been car-
ried out by releasing all simplifying hypothesis formulated in the
papers dealing with the argument. In fact either the possibility
of joints to rotate and the inertial forces acting on each beam ha-
ve been taken into account. The latter circumstance requires an i-
terative process if the elastic characteristics vary with the pe -
riod of vibration.

If a value of the lowest frequency is fixed (in particular it
has been assumed the value corresponding to infinitely rigid beams
and weightless columns hypothesis) it is possible to calculate the
functions rectifying the elastic characteristics of all rods. Next

a unit displacement is given to all the joints of the structure, one per time, and the structure is solved each time. Then the matrix of the reactions of fictitious supports is obtained.

As it is known the n eigenvalues are proportional to the n squares of natural vibration of the structure. Comparing the lowest of the eigen values with the one fixed, a new iteration process can be carried out assuming the new frequency as starting value.

The investigation has been extended to frames with a number of floors ranging from 2 and 24, and with a number of spans m between 1 and 14.

In a first stage frames with fixed joints and without windbracing have been considered; then the analysis for braced structures has been formulated. With regard to common frames the fundamental period of vibration depends obviously on the number of floors and spans as well as on the variation law of moments of inertia on the height of the frame, on the inertia of beams and on the mass of all rods. The analysis of design characteristics of several steel buildings has suggested some variation laws of the cross section of columns with the height and consequently of their masses. With regard to beams the variation of their moment of inertia with the height does not seem to seriously influence the value of the period of vibration.

Using the Computer it has been possible to calculate the frequencies of vibration of a large number of frames whose geometric characteristics range as stated before. Besides it has been tried to find a variation law of the unknown parameter as function of the data.

With regard to unbraced frames the following expression has been obtained for the period of vibration:

$$T = 2 \pi l_o^2 \sqrt{\frac{\mu_o}{E J_n}} \; T' \qquad (2)$$

where l_0 is average height of columns, J_n the moment of inertia of the columns of the last floor, μ_o the distributed mass of the beam.

The quantity T' given by graphs on fig. 1 and 2 for 1 and 14 spanned frames respectively, can be expressed by:

$$T' = 0,31 + f(\alpha) n - \frac{1,3 \; m^3}{10^m}$$

where n is the number of floors, m the number of spans and $f(\alpha)$ is related to the variation law of moments of inertia of columns with the height of the frame through the expression:

$$\frac{J_i}{J_n} = \alpha (n-i) + 1 \qquad (3)$$

Obviously if the behaviour of the J_i/J_n is not linear a suitable value for α has to be chosen which is able to better approximate the exact curve. Function $f(\alpha)$ is shown in fig.3.

With regard to the variation laws of moments of inertia 3 and 4 in fig. 1 and 2 some comparison have been made with the formula sug
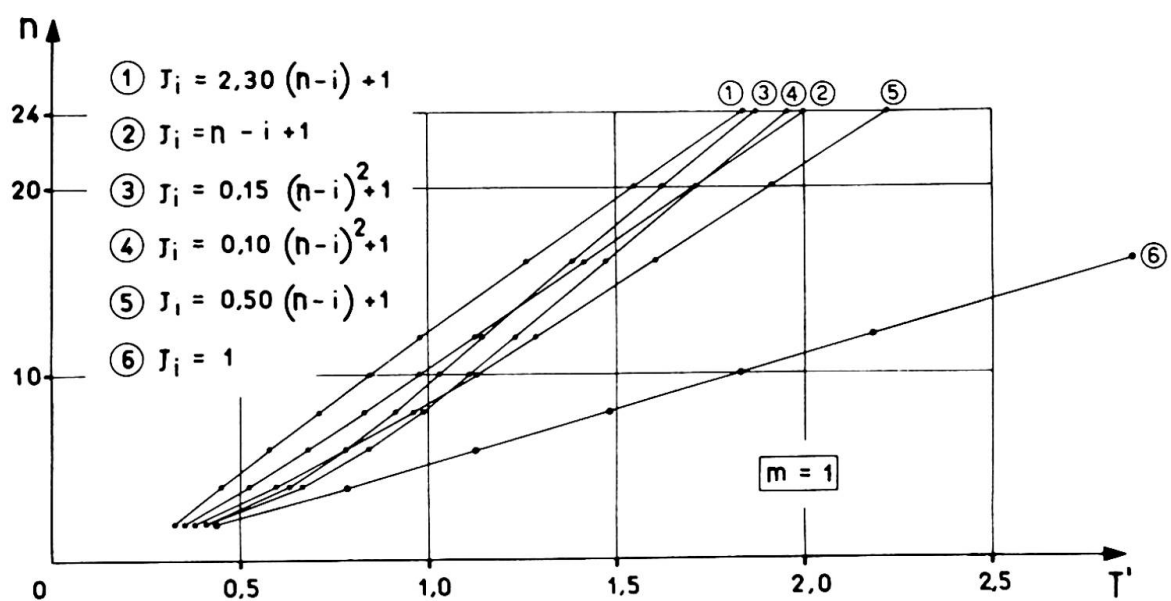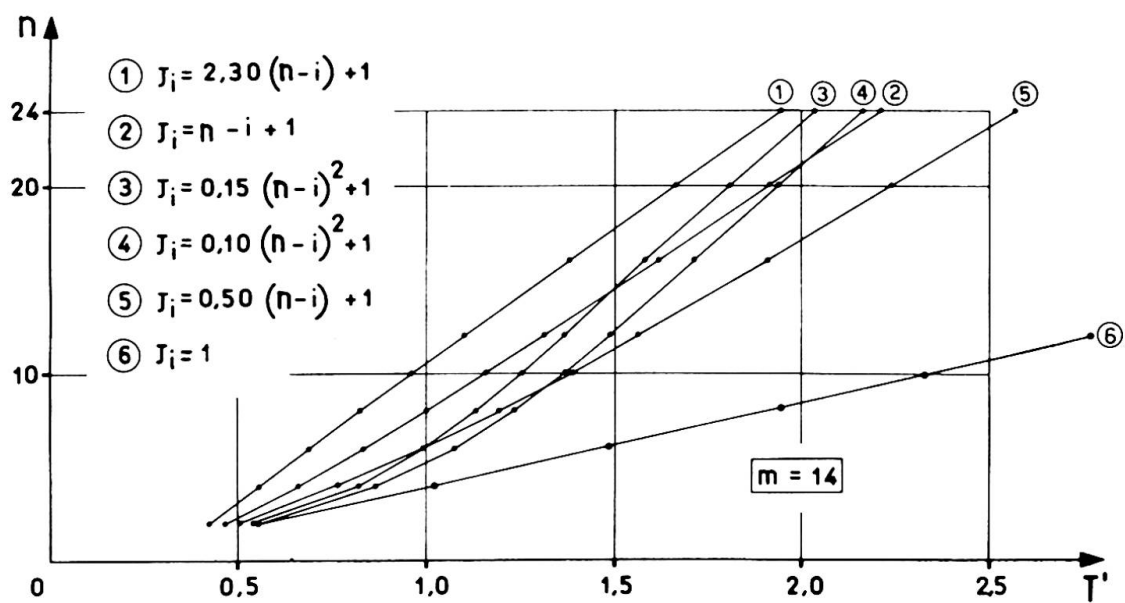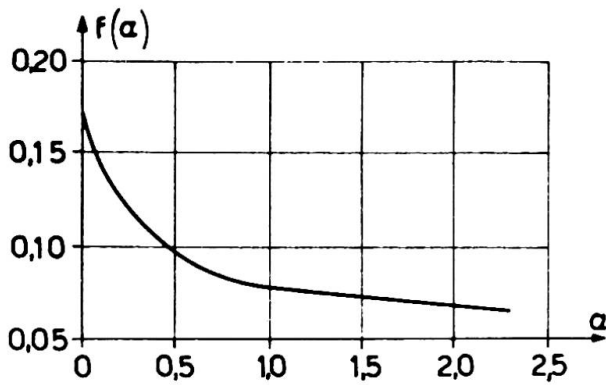
Fig. 1

Fig. 2

Fig. 3

gested by Housner and Brady:

$$T = 1,08 \sqrt{n} - 0,86$$

and with that one by Ifrim (1) previously mentioned (fig.4 and 5).

In the same graphs concerning 1 and 14 spanned frames, both tha values obtained through calculations and those derived by using (2) are shown. Finally a qualitative graph shows what one might obtain if beams were infinitely rigid.

Later the investigation has been extended to braced frames with beams arranged according to "St. Andrew's Cross". It is assumed that beams are weightless and so slender that cannot absorb any thrust.

The procedure which has been followed is practically the same of that for ordinary frames. The difference consists in the fact that the matrix of stiffnesses of which are sought the eigenvalues is obtained as summation of two distinct matrices: the first one concerning the rods of structures and derived as before; the second one regarding the rods of bracing. The latter is tridiagonal because is independent on the rotation of joints and is invariant during calculations iterations. Such matrix can be immediately calculated and depends on the cross section area of rods as well as on the geometric dimensions of the mesh.

Introducing the equivalent moment of inertia:

$$J_e = \frac{A\, l^2 \sin^3 \phi}{12}$$

where:

A is the area of cross section of bracing rod;
l the width of the mesh;
ϕ the slope of bracing rods with respect to the horizontal, one has that the second stiffnesses matrix looks formally the same than that relative to frames with infinitely rigid beams.

Also in this stage several numerical examples have been carried out. Investigations have been performed on 1 to 14 spanned frames and with number of floors ranging from 2 to 24. Besides different variation laws of the moment of inertia on the height of the building and different stiffness ratios have been considered.

From the analysis of the results it seems still possible to apply the (2) if J_n is replaced by the fictitious moment of inertia

$$\bar{J}_n = J_n + \frac{\Sigma J_{e,n}}{m+1}$$

$\Sigma J_{e,n}$ is relative to the equivalent moments of inertia of all bracing rods of the last floor.

Similarly the parameter a will be still defined through the (3) where J_i and J_n must be replaced by $\bar{J}_i$ and $\bar{J}_n$ In such a way the equivalent moment of inertia of bracing rods has been "distributed" to all the columns at each floor, making still valid the (2).
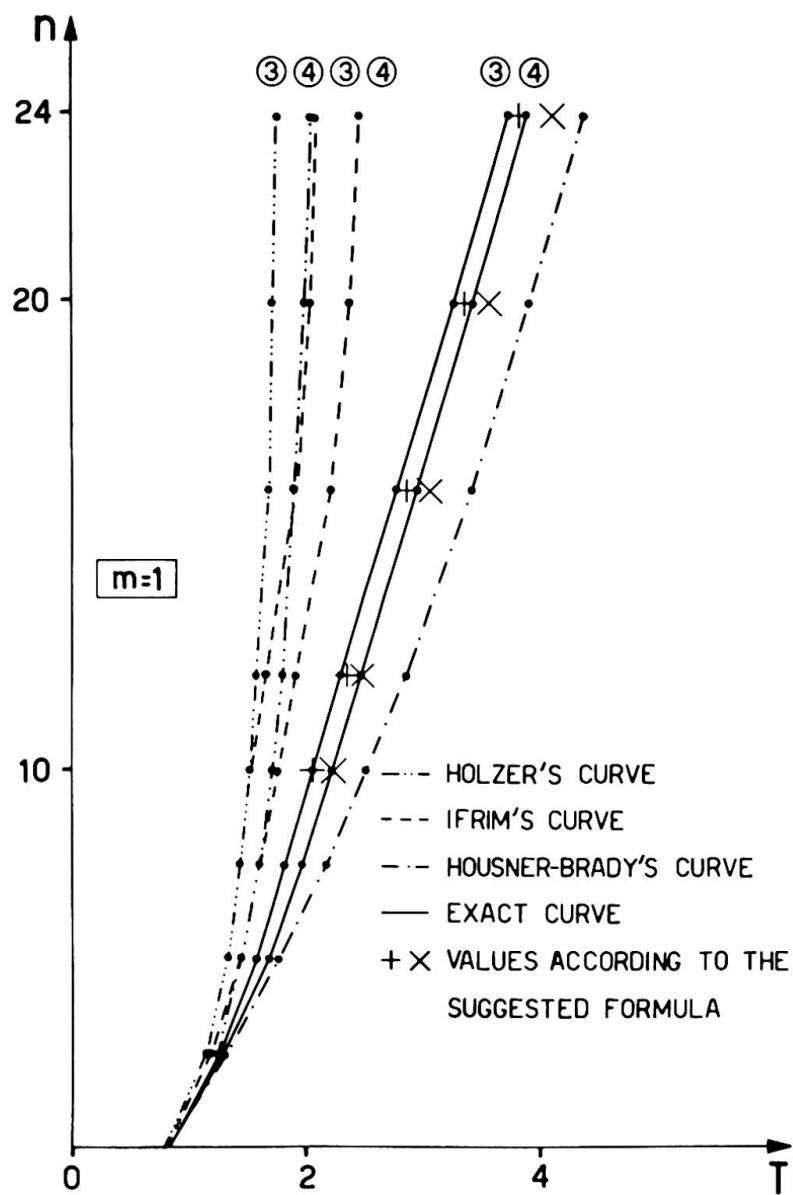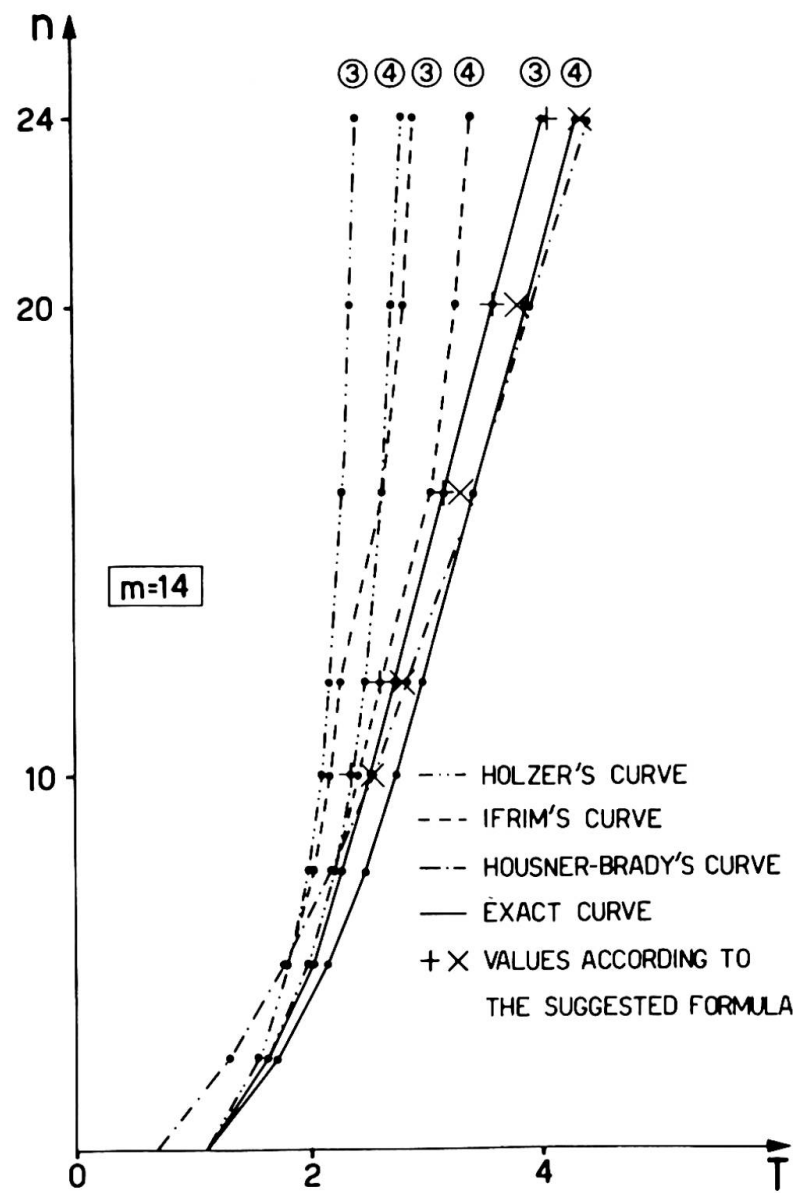
Fig. 4

Fig. 5

Doing in this way it has been implicitely assumed that stiffners stiffness may change with joints rotation. Since this does not correspond to the real phenomenon it seems reasonable to introduce a reduction factor k. On the other hand since the ratio beams stiffness over columns stiffness increases with the number of spans, one may conclude that vibrations modes for multi-spanned frames usually exhibit smaller joints rotations. Therefore it can be predicted that factor k increases with the number of spans. Numerical results confirm this assumption giving values of k ranging from 0,65 and 1,00 for a number of spans between 1 and 14. The error is smaller than 20% also in limit cases.

In conclusion the following expression for the period of braced frames can be proposed:

$$T = 2 \pi \, l_o^2 \sqrt{\frac{\mu_o}{E \, J_n}} \; \sqrt{\frac{m + 1}{m+1 + \dfrac{\Sigma J_{e,n}}{J_n}}} \; \kappa T'$$

2) - Space Structures

The problem of dynamics of space frames has been set up and analyzed in a second stage of studies.

The following fundamental hypotheses have been assumed:

a - linear elasticity of all members of structure;
b - infinite rigidity of each horizontal structure with regard to the deformations in the plane;
c - negligible torsional stiffness of resisting members of the structure.

Under these hypotheses for the generic space frame with in horizontal structures (such as the one shown in fig. 6) the dynamic deformed configuration is determined if the components:

$$u_i, \quad v_i, \quad \phi_i \quad (i = 1, 2 \ldots \ldots n)$$

for each horizontal structure are known. The u_i, v_i, ϕ_i correspond respectively to a rigid translation in the x direction, in the y direction and to a rigid rotation around the origin 0 of the i-th horizontal structure coinciding with its centroid.

The elastic response of the structure to a generic system of displacements and rotations given to each horizontal member can be represented therefore in the following form:

$$X_j = \sum_{i=1}^{n} \Lambda_{xx,ij} \, u_i + \sum_{i=1}^{n} \Lambda_{xy,ij} \, v_i + \sum_{i=1}^{n} \Lambda_{x\phi,ij} \, \phi_i$$

$$Y_j = \sum_{i=1}^{n} \Lambda_{yx,ij} \, u_i + \sum_{i=1}^{n} \Lambda_{yy,ij} \, v_i + \sum_{i=1}^{n} \Lambda_{y\phi,ij} \, \phi_i \qquad (4)$$

$$M_j = \sum_{i=1}^{n} \Lambda_{\phi x,ij} \, u_i + \sum_{i=1}^{n} \Lambda_{\phi y,ij} \, v_i + \sum_{i=1}^{n} \Lambda_{\phi\phi,ij} \, \phi_i$$

where X_j, Y_j, M_j are respectively the elastic reactions in the x
and y direction and the torsional reaction around 0 arising at the
level of the j-th horizontal structure. The coefficients A_{ij} re-
present the influence coefficients of the structure, i.e the elas-
tic reactions at the j-th level due to displacements u_i, v_i, ϕ_i
of the i-th level. They can be easily deduced if for each plane
frame belonging to the space structure the reactions r_{ij} are known.
Such reactions are due to the unit displacement of the generic i-th
beam for all other beams prevented from translation.

In dynamics, if σ is the generic frequency of the structure in
free vibrations, the displacements of the generic horizontal member
turn out to be:

$$u_i = \bar{u}_i \sin \sigma t$$

$$v_i = \bar{v}_i \sin \sigma t$$

$$\phi_i = \bar{\phi}_i \sin \sigma t$$

Assumed that at each horizontal structure the mass is uniformely
distributed, the inertial reactions in the x and y direction and
the torsional reaction around 0 will be:

$$\bar{X}_j = - m_j \frac{d^2 u_j}{dt^2} = m_j \sigma^2 \bar{u}_j \sin \sigma t$$

$$\bar{Y}_j = - m_j \frac{d^2 v_j}{dt^2} = m_j \sigma^2 \bar{v}_j \sin \sigma t \qquad (5)$$

$$\bar{M}_j = - m_j \rho_j^2 \frac{d^2 \phi_j}{dt^2} = m_j \rho_j^2 \sigma^2 \bar{\phi}_j \sin \sigma t$$

where m_j is the total mass of the horizontal structure and ρ_j is
the polar radius of giration of its area around 0.

If columns masses are neglected, the equations of dynamic equi-
librium are immediately obtained by equating (4) to (5). Setting
the determinant of the coefficients of the system equal to zero
one gets the algebraic equation of 3n degree in σ^2 whose roots
σ_1^2, ... σ_n^2 allow the evaluation of the 3n free frequencies of
the multi-story space structure.

It is obvious on the other hand that an iterative process is
necessary if one wants to improve the values obtained by taking
into account the mass of members. One should in fact recalculate
the coefficients A_{ij} starting from the σ_i computed in the first
cycle and taking into account the inertial forces of the beams and
columns. Then the solution of the system of equations of dynamic
equilibrium will lead to new values σ_i. The iteration process
should be carried out as long as two consecutive solutions give al-
most the same value for the frequencies.

It should be born in mind, however, that in almost all cases
the masses of horizontal structures are much larger that those ones
of the columns. Consequently the values of σ_i given by the first
iterative process are practically exact.

Fig. 6

Fig. 7

1° mode

$\overline{T} = 1,7830$

2° mode

$\overline{T} = 1,0335$

3l

3l

l

l

x

y

O

u

v

φ

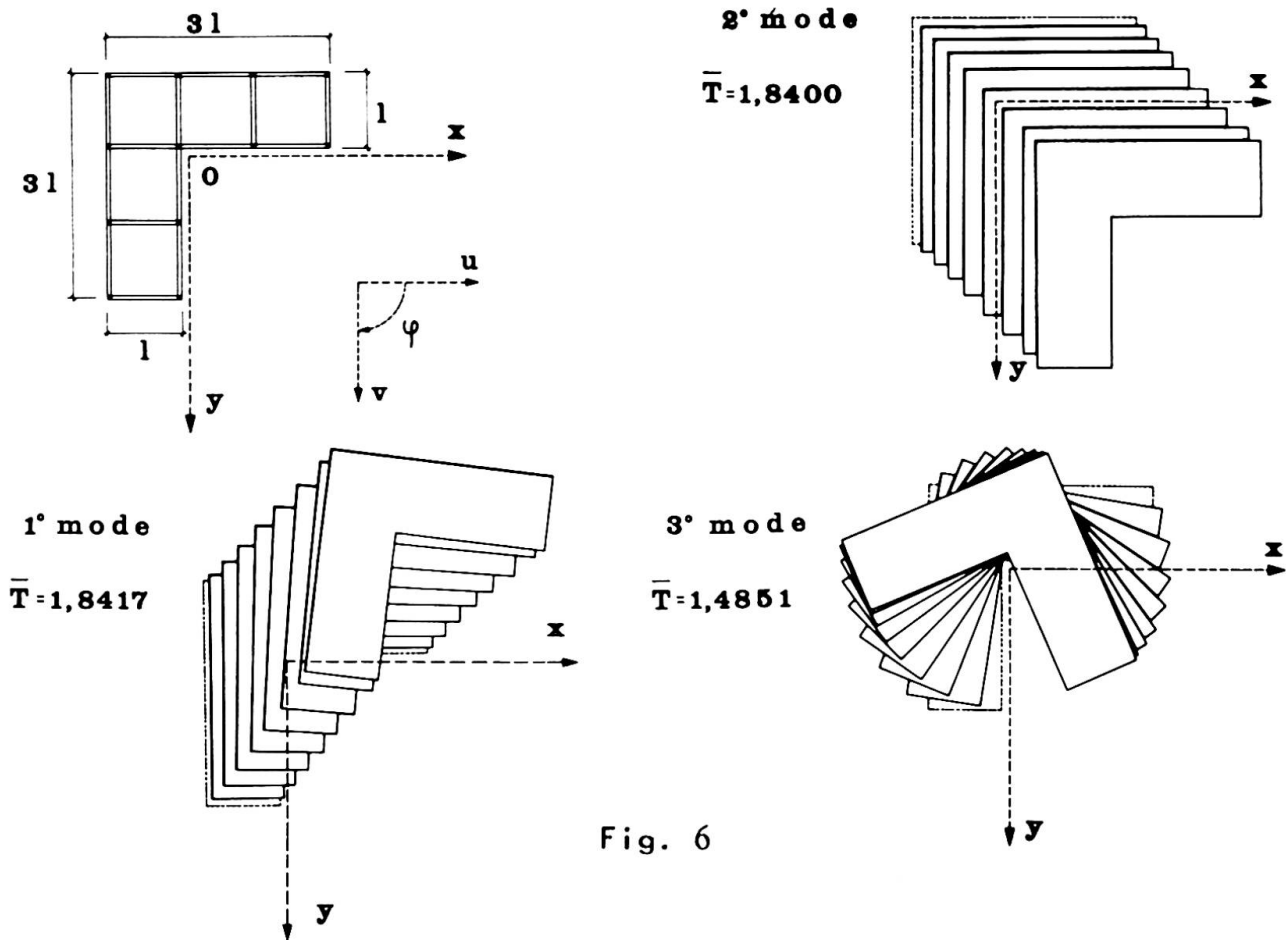3° mode

$\overline{T} = 0,7036$

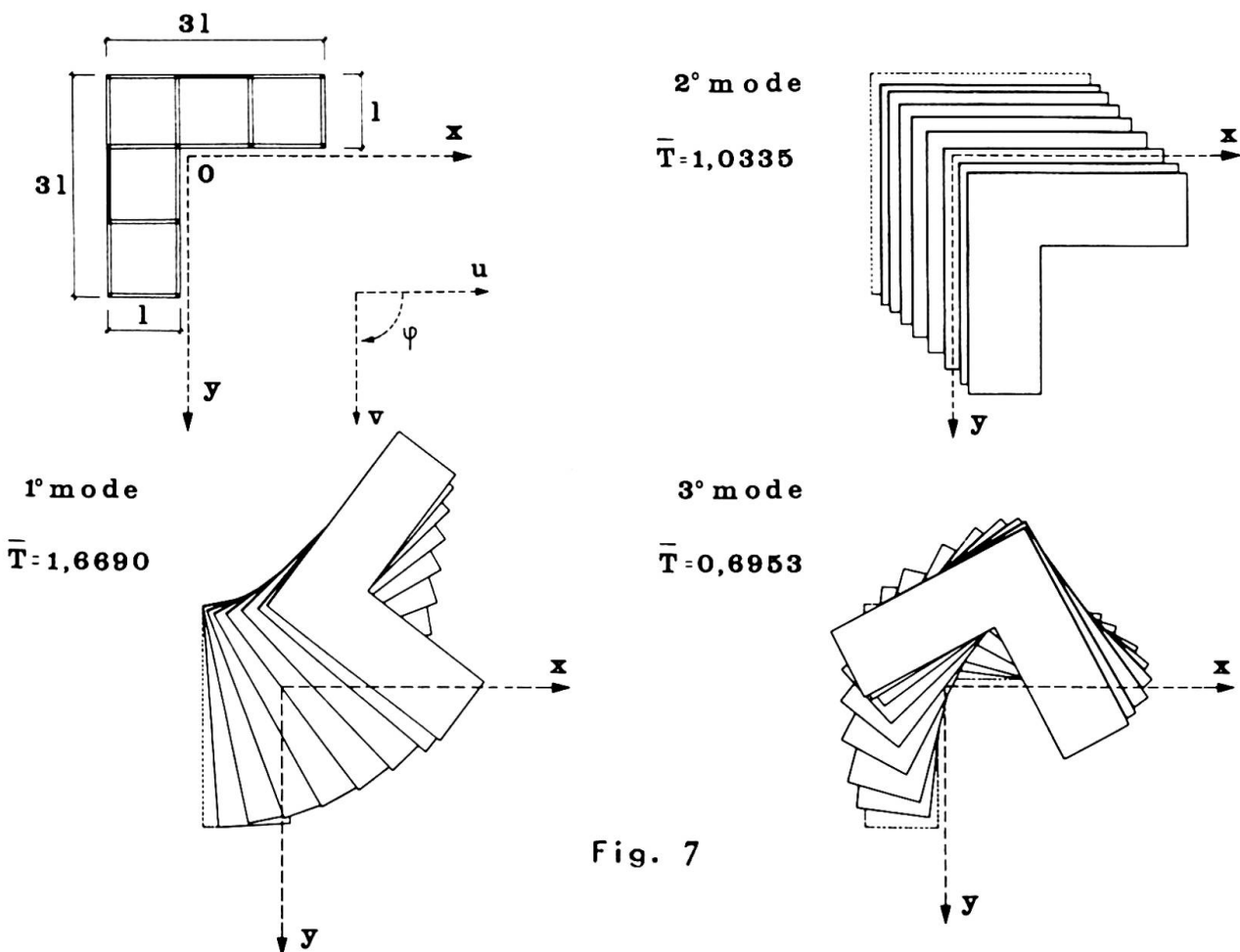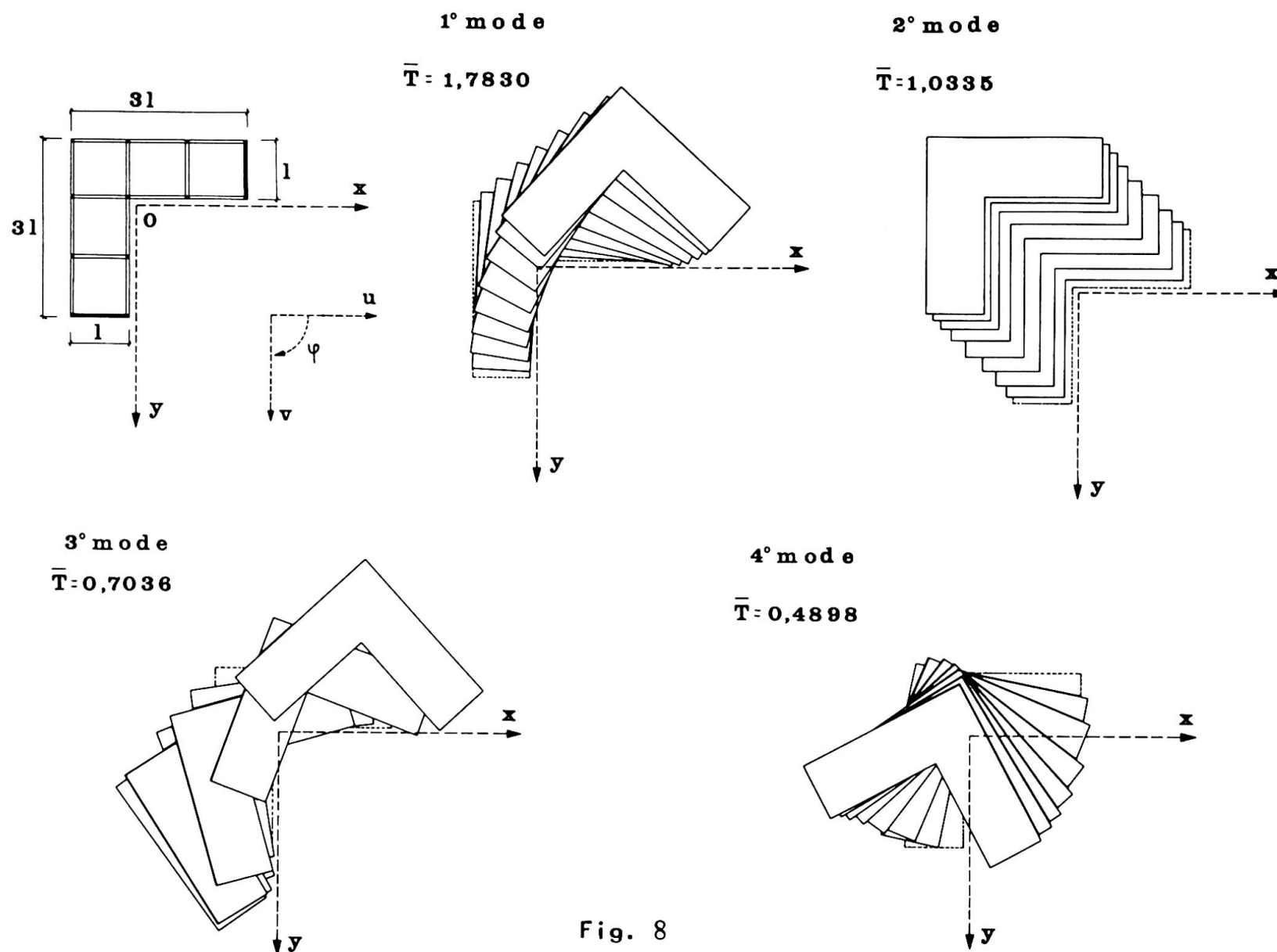4° mode

$\overline{T} = 0,4898$

Fig. 8

The foregoing procedure has been applied to a steel building
with L plan and eight horizontal structures and with two orders of
mutually orthogonal frames.

Three structural schemes of the building have been considered:
the first one without windbracing (fig.6) and the other two with
windbracing arranged as shown in figs. 7 and 8.

The dynamic analysis of the three space schemes has been carried
out with the computer.

In the graphs previously mentioned have been represented the mo-
des of vibration of $\overline{T}$ related to the period T by:

$$\overline{T} = \frac{T}{\sqrt{\dfrac{M_o \, L_o^3}{E \, J_n}}}$$

A comparison has been made between the results obtained in this
way and the other ones given by decomposing the space structure   in
plane frames.

It can be concluded that the period of vibration obtained in the
hypotheses of plane behaviour is usually smaller than the real value.

SUMMARY

It is briefly reported on the works of the Commission XIII of the
Convention of European Constructional Steelwork Associations concer-
ning the recommendations for designing steel structures in seismic
area. The results of a study on the dynamic behaviour of plane and
space framed structures are also shown. Finally some simplified for
mulae obtained through a large numerical investigation are suggested.

RÉSUMÉ

L'article résume très brièvement les travaux de la Commission de
la Convention Européenne des Associations de la Construction Métalli
que, concernant les recommandations pour le calcul des bâtiment dans
les zones seismiques. On expose aussi les résultats d'une étude sur
le comportement dynamique de structures en portique dans l'espace.
On donne l'expression de formules simplifiés pour le calcul de la pé
riode de vibration.

ZUSAMMENFASSUNG

Es wird ein kurzer Abriss von der Arbeit der Kommission XIII der
Europäischen Konvention der Stahlbauverbände gegeben bezüglich der
Empfehlungen zur Bemessung von Stahlbauten in Erdbebengebieten. Eben-
so wird eine Studie über das dynamische Verhalten ebener und räumli-
cher Rahmentragwerke aufgezeigt. Schliesslich werden einige verein-
fachte, durch erweiterte numerische Nachforschungen erzielte Formeln
für die Schwingungsdauer angegeben.

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