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Non Linear Plastic Analysis of High Strength Steel Plane and Space Frameworks

Analyse plastique non-linéaire de système de portiques dans le plan et dans l'espace en aciers de haute résistance

Nichtlineare, plastische Analyse ebener und räumlicher Stahl-Rahmentragwerke hoher Festigkeit

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I_Introduction.

The plastic analysis of framed structures requires the determination of the collapse mechanism under the action of proportional loads. Although the collapse mechanism is simple in concept, it depends on too many factors. In rigid plastic theory the collapse mechanism can be obtained by static and kinematic theorems[1] or following the successive formation of plastic hinges until the failure of the structure. Whenever a plastic moment is attained at any cross section, a plastic hinge forms at this section, and it can undergo rotation of any magnitude as long as the bending moment stays constant at the fully plastic value. However there are some discrepancies between the assumptions of rigid plastic theory and the actual behavior of the structure. The plastic hinges develop along a plastified zone, and the strain hardening assures that the plastic hinges will extend over increasing lengths of the member even before the extensive ductility is exceeded[2]. The structure being loaded beyond the elastic limit of its material, the moment curvature diagrams are not linear and the deformations and also the effect of the deformations upon the equilibrium equations are more accentuated than the linear elastic analysis. The fully plastic moment is subject to variation by the slenderness ratio of the member[2][3] by the rotational angle change[3] and also by the member axial and shear forces[1].

Computer programs have been developed for plastic analysis. The program proposed by Wang[4] will trace definitely the location and the sequence of formation of all plastic hinges until collapse, yields the cumulative load factor and the deflections and moments at all nodal points at the time of formation of each plastic hinge.

Wang's program has been modified by Harrison[5] in order to include the finite deformation effects. Rubinstein and Karagozyan[6] have given a solution for minimum weight design.

Intensive experiments have been evolved to show the agreement between the theory and the actual behavior, primarily in two centers: the Cambridge University[7], England, and Lehigh University[8], U.S.A.

In the present paper an attempt has been made to solve the space or plane framework structure accounting for the finite deformation effects, the reduction of plastic moments due to axial member forces, the change in flexural stiffness caused by member axial forces and the influence of shear forces to the deflections[9]. But the effect of strain hardening[10] and the reduction of plastic moment due to member shear forces are neglected. Also the spread of plastic zone and the residual stresses due to live loads[11] are ignored.

The computer program gives as results the collapse mode, that is, whether the collapse occurred by a plastic collapse mechanism or by the instability of whole structure or by a member instability. With an out-of-core Cholesky routine a big structure with more than 2000 unknowns may be handled without any increase in the capacity of the computer. The required computer time is much higher than the non-linear analysis of the same structure, since a non-linear analysis is performed at the formation of each new hinge.

Numerical examples have been given in order to compare the results obtained with those already worked out experimentally or theoretically and one more to illustrate the behavior of the space structures.

2_Non_Linear Analysis of Framed Space Structure.

An iteration procedure[12] is applied to framed space or plane structure to determine its deformed configuration. The basic idea in this procedure is to perform a standard linear analysis under the action of a given set of external loads and then calculate the member end forces using the deformed geometry. If the member end forces at a joint are not in equilibrium with the given external loads, the out-of-balance forces are applied on to the deformed geometry to yield another set of deformations and forces. If the new forces do not satisfy the joint equilibrium, the linear analysis continues with the latest geometry and with the latest out-of-balance forces. This procedure is repeated until equilibrium is reached at every joint.

The stiffness method has been used as a standard linear analysis procedure. The member center line is chosen as the y-axis while the two principal inertia axes of the section constitute the x and z axes of a cartesian co-ordinate system. These axes are called the "member axes" and are referred to a general stationary XYZ cartesian co-ordinate system (Figure 1). The joint deformations obtained from the linear analysis are relative to the generalized XYZ co-ordinate axes. In evaluating the member end forces, it is extremely convenient to work with the member end

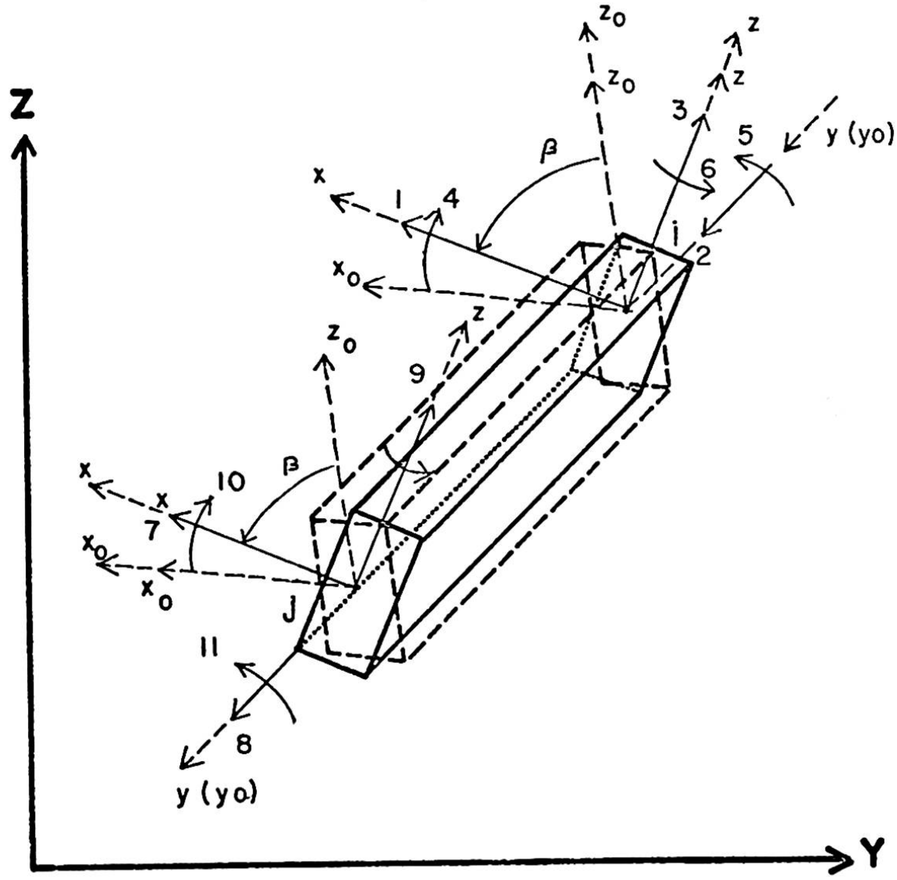


Figure 1. Co-ordinate Axes.

deformations relative to the deformed member axes which are obtained from

$$\{\delta\}_{xyz} = [T]\ \{\delta\}_{XYZ}$$

where,

$\{\delta\}_{xyz}$ = the column vector of member end deformations relative to the deformed member axes,

$\{\delta\}_{XYZ}$ = the column vector of generalized XYZ co-ordinates

$[T]$ = the orthogonal transformation matrix involving the direction cosines of the deformed member axes

The member forces relative to the deformed member axes, (Figure 2) may be written as follows

$$F_2 = -F_8 = (L_0 - L')AE/L_0$$

$$F_4 = (4EI_x/L'(1+\phi_x))\theta'_4 S_{1x} + (2EI_x/L'(1+\phi_x))\theta'_{10} S_{2x}$$

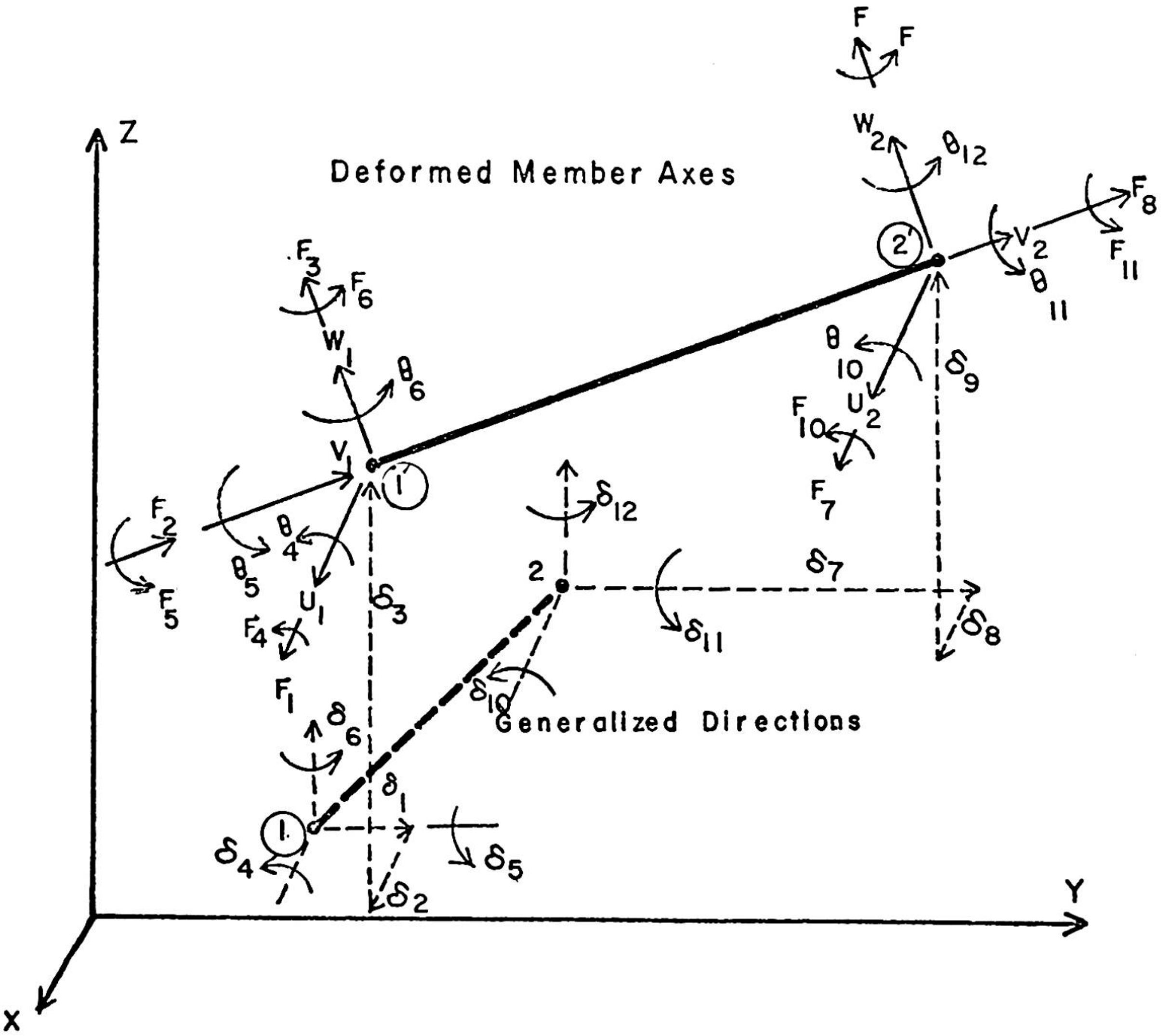


Figure 2. A Beam-Column Member in Space

$$F_{10} = (4EI_x/L'(1+\phi_x))\theta'_{10}S_{1x} + (2EI_x/L'(1+\phi'_x))\theta'_4 S_{2x}$$

$$F_6 = (4EI_z/L'(1+\phi_z))\theta'_6 S_{1z} + (2EI_z/L'(1+\phi_z))\theta'_{12}S_{2z}$$

$$F_{12} = (4EI_z/L'(1+\phi_z))\theta'_{12}S_{1z} + (2EI_z/L'(1+\phi_z))\theta'_6 S_{2z}$$

$$F_5 = -F_{11} = (GJ/L')(\theta_5 - \theta_{11})$$

$$F_7 = -F_1 = (F_6+F_{12})/L'$$

$$F_3 = -F_9 = (F_4+F_{10})/L'$$

where

$$L_0 = \left[(X_2-X_1)^2 + (Y_2-Y_1)^2 + (Z_2-Z_1)^2\right]^{\frac{1}{2}}$$

$$L' = \left[(X'_2-X'_1)^2 + (Y'_2-Y'_1) + (Z'_2-Z'_1)^2\right]^{\frac{1}{2}}$$

$$\theta'_4 = \theta_4+\theta_w \quad : \qquad \theta'_{10} = \theta_{10}+\theta_w$$

$$\theta'_6 = \theta_6+\theta_u \quad : \qquad \theta'_{12} = \theta_{12}+\theta'_u$$

$$\theta_u = \text{Arcsin}\left(\frac{u_2-u_1}{L_0}\right)$$

$$\theta_w = \text{Arcsin}\left(\frac{w_1-w_2}{L_0}\right)$$

In the above expressions, u_1, u_2, w_1 and w_2 are the translations and θ_4, θ_{10}, θ_6, θ_{12}, θ_5 and θ_{11} are the rotations of the member ends relative to the deformed member axes as obtained from the linear analysis. S_{1x}, S_{2x}, S_{1z}, S_{2z}, are the correction factors to include the influence of the axial force on the member flexural stiffness coefficient and ϕ_x, ϕ_z are the correction factors to include the influence of member shear forces on to the displacements.

3. Plastic Analysis of Framed Structures.

To determine the collapse mechanism, a non-linear analysis as mentioned above is performed after each succesive hinge formation. The rotation of all previously formed hinges is checked, and if the rotation of one of the previous hinges decreases, this hinge is locked again. The collapse may occur with the formation of a new hinge or within the non-linear cycles. In the former case, the collapse is caused by a plastic mechanism, and in the latter case it is caused either by a member instability or by the instability of the whole structure.

The member instability and the effect of the member shear forces are taken into account by introducing proper factors in the member flexural stiffness coefficients. At every step the value of the member plastic moment is modified, depending on the member axial force. For I beams both cases[1] are considered separately, that is, whether the neutral axis lies in the web or in the flange.

No allowance for strain-hardening is made. Also the spread of plastic zone, reduction of plastic moment due to member shear forces and the residual stresses due to live loads are ignored.

Structures having members with variable moments of inertia can also be solved.

4_ Computer Programming.[13]

The data values fed in computer are respectively the characteristic of the structure such as: the Young modulus E, the Poisson ratio μ, the co-ordinates of the joints referred to XYZ generalized co-ordinate axes, the two end joints of each member, moments of inertia in two principal inertia axes, polar moment of inertia, area of the section, the web area and depth if it is an I section, plastic moments in two principal inertia axes, redundant joints information, joint and member loads if any. All the other operations are performed automatically in computer.

The computer gives as output the displacements and rotations at every joint and the member end reactions of all the members for first linear analysis, then the same information for non-linear analysis. This pattern is repeated after formation of each new hinge until the collapse of the structure. The collapse occurs either by the singularity of structure stiffness matrix or by the large joint deformations. The load factors for each step and the cumulative load factor are also pointed out.

5_ Examples of Analysis

The dimensions and member characteristics of the first two examples are selected from previous studies in order to compare the results. An example of space frame is also given. The parameters NL and NC show respectively whether the reduction of plastic moment due to member axial forces and the stability correction factors are taken into account or not. These parameters may have the value equal either to zero or one which means respectively that the corresponding correction factors are included or not in the analysis. QP is the cumulative load factor.

5_1. Portal Frame.

The dimensions and member characteristics are given in Figure 3. The successive hinge formation and their location, cumulative load factors, the horizontal displacement at joint B, (Δ_1), and H, (Δ_3), and the vertical displacement at joint E, (Δ_2), are also given in the tables for linear and non-linear analysis.

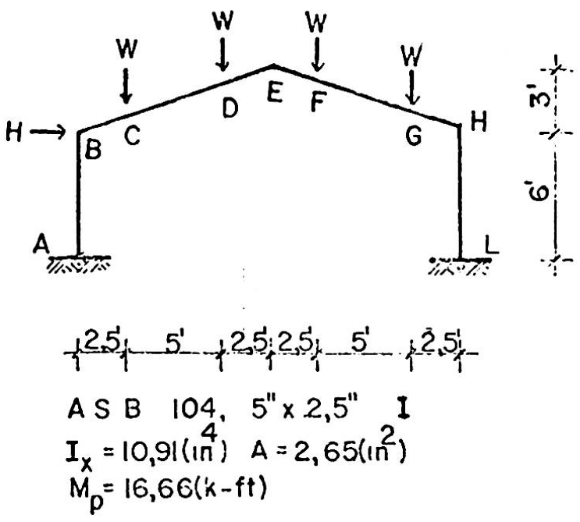


Figure 3

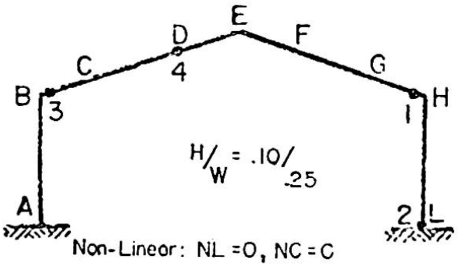
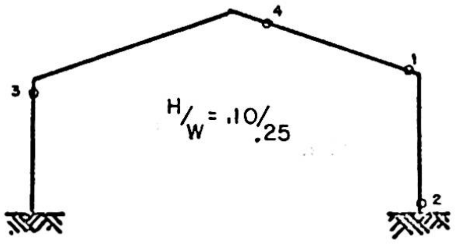


Figure 4

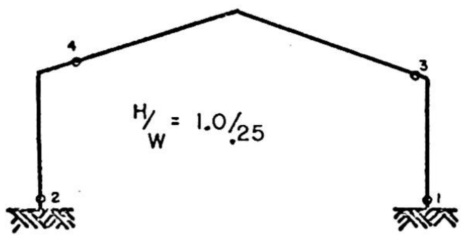


Linear:

Non-Linear NL=1, NC=0

NL=1, NC=1

Figure 5



Linear

No-Linear NL = 0, NC = 0

NL = 1, NC = 0

NL = 1, NC = 0

Figure 6

Table I H/W = .10/.25

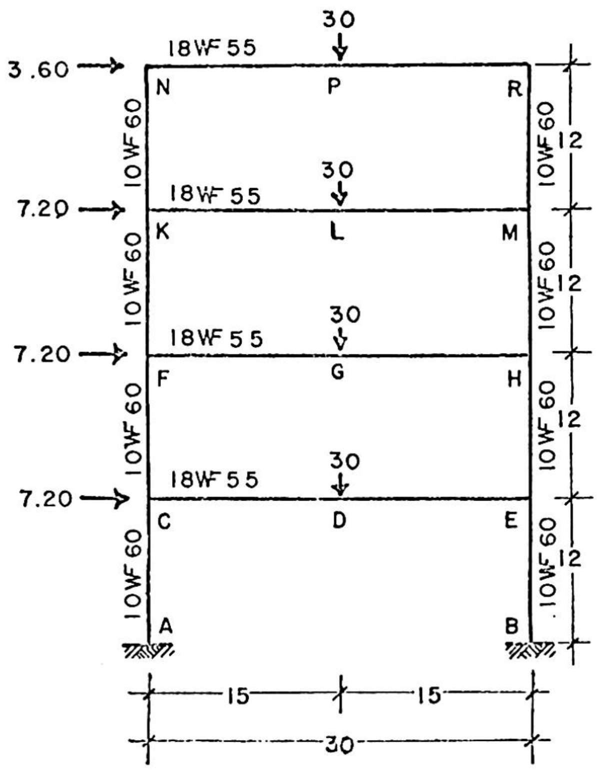
		LINEAR ANALYSIS	NON-LINEAR ANALYSIS NL=0 NC=0	NL = 1 NC = 0	NL = 1 NC = 1
PLASTIC HINGE ORDER	1	14.051	13.85	13.428	13.40
	2	14.555	14.38	13.937	13.89
	3	16.161	15.90	15.717	15.66
	4	18.330	18.02	17.703	17.69
DEFORMATION (in)	Δ_1	0.172	0.178	0.220	0.220
	Δ_2	3.253	3.272	3.264	3.324
	Δ_3	2.097	1.996	2.034	2.065

Table II H/W = 1./.25

		LINEAR ANALYSIS	NON–LINEAR ANALYSIS NL=0 NC=0	NL=1 NC=0	NL=1 NC=1
PLASTIC HINGE ORDER	1	6.627	6.559	6.386	6.367
	2	8.666	8.562	8.474	8.463
	3	9.008	8.911	8.724	8.729
	4	10.286	10.077	9.920	9.922
DEFORMATION (in)	Δ_1	4.127	4.141	4.162	4.224
	Δ_2	2.363	2.555	2.579	2.617
	Δ_3	5.521	5.461	5.507	5.586

5_2, Four Storey Frame.

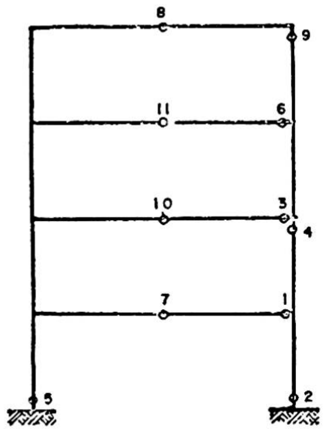
The dimensions and member characteristics are given in Figure 7. The successive hinge formations and their location, cumulative load factors, the horizontal displacement at joint C(Δ_1), N(Δ_3), R(Δ_5), the vertical displacement at joint D(Δ_2) and P(Δ_4) are also given in the tables for linear and non-linear analysis.



	$Ix(in^4)$	$Iz(in^4)$	$J(in^4)$
10WF60	343.0	116.5	2.15
18WF55	888.9	42.0	2.65
18WF60	984.0	47.1	2.94

	$A(in^2)$	Mpx(k.ft)	Mpz(k.ft)
10WF60	17.60	213.73	99.68
18WF55	16.19	317.69	54.52
18WF60	17.64	349.10	60.94

Figure 7



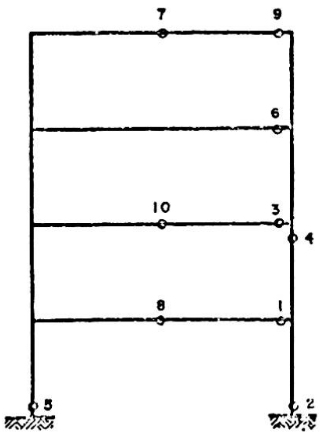
Linear.

Figure 8

PLASTIC HINGE ORDER	LINEAR ANALYSIS	NON-LINEAR ANALYSIS		
		NL = 1, NC = 0	NL = 0, NC = 0	NL = 1, NC = 1
1	1.739	1.739	1.739	1.739
2	1.903	1.759	1.906	1.769
3	1.916	1.851	1.914	1.812
4	1.991	1.941	1.996	1.891
5	2.147	2.059	2.139	1.975
6	2.148	2.123	2.146	2.038
7	2.158	2.135	2.151	2.127
8	2.162	2.141	2.154	2.133
9	2.189	2.158	2.177	2.138
10	2.210	2.160	2.190	2.148
11	2.230	2.161		2.143
12		2.186		

Table III

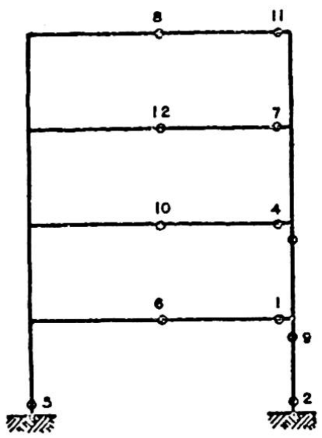
PLASTIC HINGE ORDER	NON-LINEAR ANALYSIS (NL = 0, NC = 0)				
	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
1	0.899	0.731	2.889	1.023	2.867
2	1.088	0.922	3.396	1.113	3.371
3	1.103	0.939	3.427	1.107	3.402
4	1.249	1.047	3.889	1.042	3.862
5	1.522	1.273	4.721	1.228	4.685
6	1.557	1.287	4.781	1.229	4.762
7	1.588	1.300	4.862	1.229	4.832
8	1.604	1.306	4.899	1.234	4.869
9	2.608	2.595	6.824	1.546	6.793
10	3.136	3.271	8.001	1.691	7.964



Non-Linear. NL=0, NC=0.

Figure 9

Table IV



Non-Linear NL=1, NC=C:

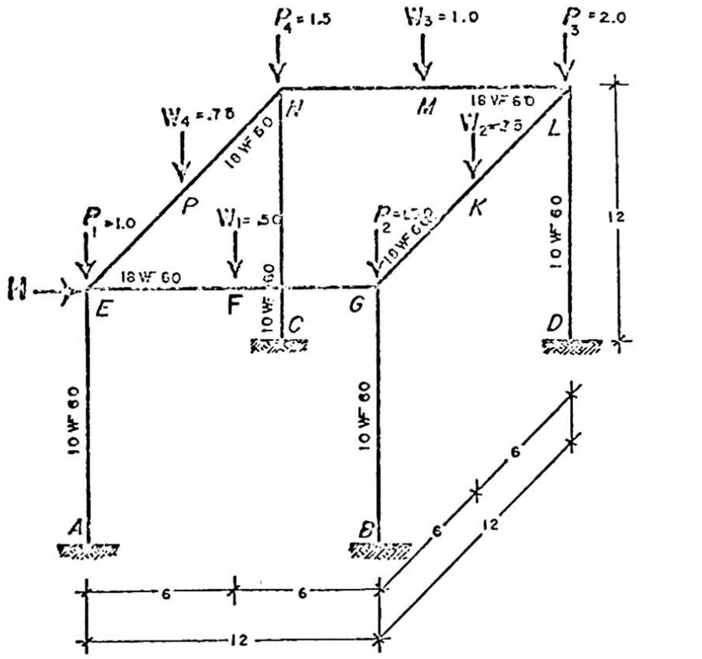
Figure 10

PLASTIC HINGE ORDER	LINEAR ANALYSIS				
	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
1	0.898	0.730	2.885	1.017	2.868
2	1.083	0.921	3.383	1.114	3.364
3	1.103	0.938	3.429	1.121	3.410
4	1.253	1.063	3.890	1.169	3.880
5	1.524	1.288	4.735	1.254	4.713
6	1.530	1.291	4.746	1.255	4.724
7	1.582	1.312	4.867	1.261	4.846
8	1.681	1.448	5.065	1.264	5.044
9	2.311	2.314	6.328	1.360	6.306
10	2.806	2.992	7.563	1.803	7.541
11	4.150	4.731	11.344	2.720	11.322

.Table V

5-3. Space Frame.

The dimensions and member characteristics are given in Figure 10. The successive hinge formation and their location, cumulative load factors, the horizontal displacement at joints E(Δ_1), G(Δ_3) and vertical displacement at joint F(Δ_2) are also given in the tables for non-linear analysis.



Same steel section characteristics as Four Storey Frame

Figure 11

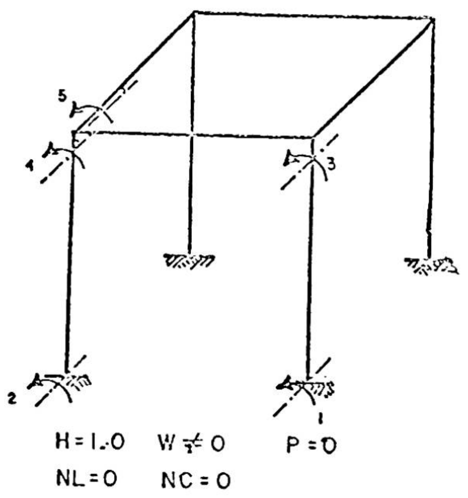
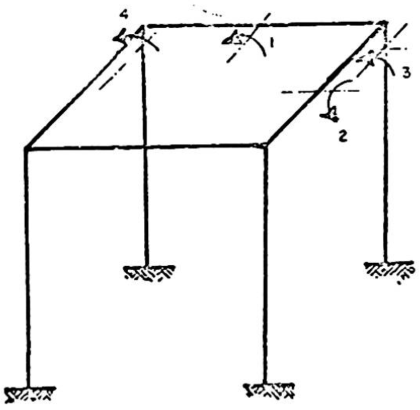


Figure 12

		NL = 0		NC = 0		NL = 1		NC = 0	
		QP	Δ_1	Δ_2	Δ_3	QP	Δ_1	Δ_2	Δ_3
PLASTIC HINGE ORDER	1	67.8013	-0.0051	0.9448	-0.0639	67.8184	-0.0051	0.9450	-0.0639
	2	71.9553	-0.0071	1.0379	-0.0682	70.1642	-0.0062	0.9976	-0.0663
	3	72.0123	-0.0089	1.0407	-0.0645	71.5020	-0.0081	1.0367	-0.0679
	4	76.4655	-0.0137	1.4552	-0.0931	76.0266	-0.0129	1.4596	-0.0970
	5	90.3714	-0.0170	5.4564	-0.2097				

Table VI



H= .10 W ≠ 0 P=0

NL=0 , NC=0

NL=1 , NC=0

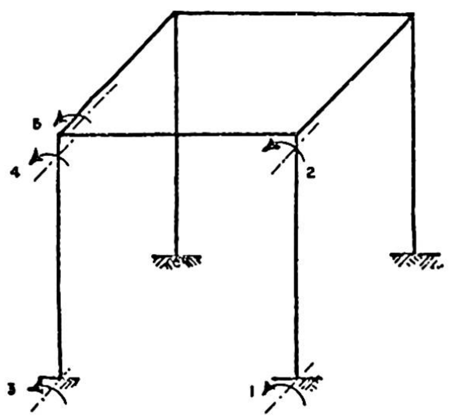
Figure 13

		PLASTIC HINGE ORDER			
		1	2	3	4
NL=0 , NC=0	QP	146.243	171.4713	186.4638	187.3258
	Δ_1	. 0.0051	-0.0058	-0.0066	-0.0066
	Δ_2	0.0111	0.0130	0.0140	0.0416
	Δ_3	- 0.2569	- 0.5581	- 0.7379	-0.7792
NL=1 , NC=0	QP	146.2433	171.4620	178.6744	179.2672
	Δ_1	-0.0051	- 0.0058	- 0.0062	-0.0063
	Δ_2	0.0111	0.0130	0.0135	0.0325
	Δ_3	-0.2569	- 0.5580	- 0.6444	- 0.6727
NL=1 , NC=0	QP	146.243	179.603	180.296	
	Δ_1	-0.0051	+0.0087	+0.0084	
	Δ_2	0.0111	-0.0040	- 0.0069	
	Δ_3	- 0.2569	-0.7373	-0.7719	

Table VII

PLASTIC HINGE ORDER	NL = 0 NC = 0: Qp	Δ_1	Δ_2	Δ_3
1	67.695	- 0.0098	0.9475	- 0.0876
2	71.840	- 0.0120	1.0370	- 0.0956
3	72.040	- 0.0139	1.0451	-0.0898
4	76.385	- 0.0189	1.3656	- 0.1199
5	88.249	- 0.0246	5.4655	- 0.2455

Table VIII



H=1.0 W ≠ 0 P=0

NL=1 NC=0

Figure 14

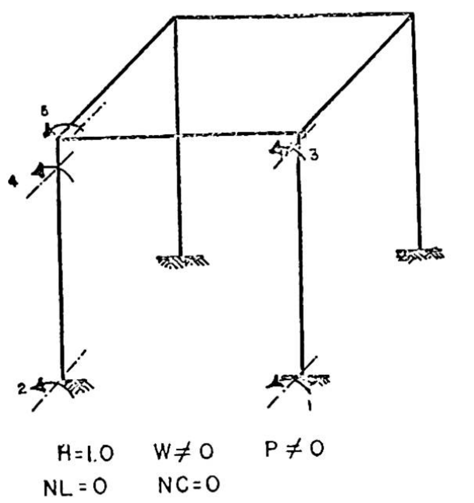


Figure 15

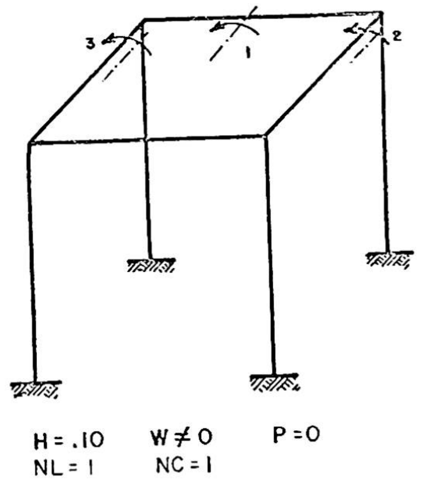


Figure 16

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SUMMARY

In the present paper an attempt has been made to solve the space or plane framework structures accounting for the finite deformation effects, the reduction of plastic moments due to axial member forces, the change in flexural stiffness caused by member axial forces and the influence of shear forces to the deflection. But the effect of strain hardening and the reduction of plastic moment due to member shear forces are neglected. Also the spread of the plastic zone and the residual stresses due to live loads are ignored.

RÉSUMÉ

Cette étude essaie de résoudre les systèmes de portiques plans ou dans l'espace en tenant compte des effets des déformations finies, de la réduction des moments plastiques et de la variation de rigidité à la flexion dues aux efforts axiaux et de l'influence des efforts de cisaillement sur la déformation. On a négligé cependant l'effet du durcissement ainsi que la réduction du moment plastique due aux efforts de cisaillement. De même on ne tient pas compte de l'extension de la zone plastique ni des tensions résiduelles dues à la charge de service.

ZUSAMMENFASSUNG

In diesem Beitrag ist der Versuch unternommen worden, ebene und räumliche Rahmentragwerke mit Berücksichtigung der Wirkung endlicher Verformungen, der Abminderung der plastischen Momente unter Achsiallasten, des Wechsels der Biegesteifigkeit aufgrund der Stabachsialkräfte sowie des Einflusses der Querkräfte auf die Durchbiegungen zu lösen. Hingegen sind die Wirkung der Verfestigung und die Abminderung des plastischen Moments infolge Stabquerkräfte vernachlässigt worden. Ebenso sind die Ausbreitung der plastischen Zone und die Eigenspannungen infolge Verkehrslast unberücksichtigt.