

**Zeitschrift:** IABSE congress report = Rapport du congrès AIPC = IVBH  
Kongressbericht

**Band:** 8 (1968)

**Artikel:** Approximate inelastic analysis of hear wall-frame structures

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**DOI:** <https://doi.org/10.5169/seals-8796>

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### IIIa

#### **Approximate Inelastic Analysis of Shear Wall-Frame Structures**

Analyse inélastique approximée pour des structures composées de portiques et de murs

Angenäherte unelastische Berechnung von Scheiben-Rahmentragwerken

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#### INTRODUCTION

Plastic design methods are available for structures in which sway displacements are completely prevented and for those which consist entirely of moment resisting frames <sup>(1)</sup>. Both methods are based on assumptions which make the design of tall structures feasible even using manual computation procedures.

Commonly, however, multi-story structures are neither completely braced nor unbraced but consist of frames coupled to flexural shear walls <sup>(2)</sup>. The shear walls have greater stiffnesses than do the frames and thus tend to dominate the behavior of the structure.

Under lateral loads the deflected shapes of the free frame and shear wall are shown in FIGS. 1 (a) and (b). Since the deformations of the two elements must be compatible, the final deflected shape of the structure will be that shown in FIG. 1 (c). In the top stories the shear wall exerts large shears on the frame. These shears are accounted for in present elastic design procedures which consider the interaction between the frame and shear wall <sup>(3)</sup>.

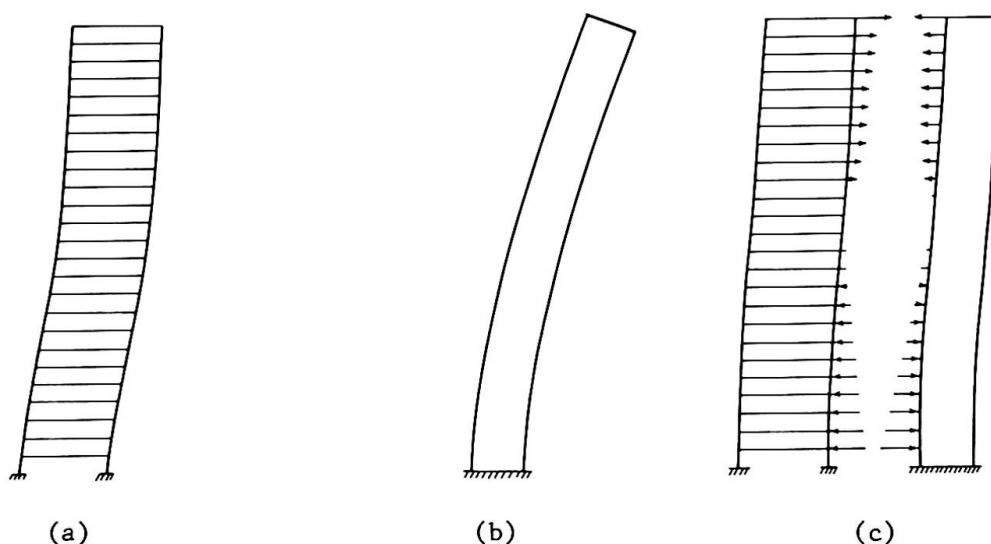


FIG. 1. DEFLECTED SHAPES

In the inelastic range, the frame-shear wall interaction may force plastic hinges to form in the frame early in the loading history, thus reducing the stiffness for additional load increments. The vertical loads on the structure acting through the lateral displacements produce "secondary moments" known as  $P-\Delta$  moments. The  $P-\Delta$  effect combined with the inelastic action of the structure may cause significant reductions in load-carrying capacity.

#### METHOD OF ANALYSIS

To reduce the analysis of the structure to manageable terms the actual structure is replaced by the model shown in FIG. 2. The shear walls and frames of the actual structure have been replaced by the systems shown. These are designed to have equivalent lateral stiffnesses and strengths <sup>(4)</sup>.

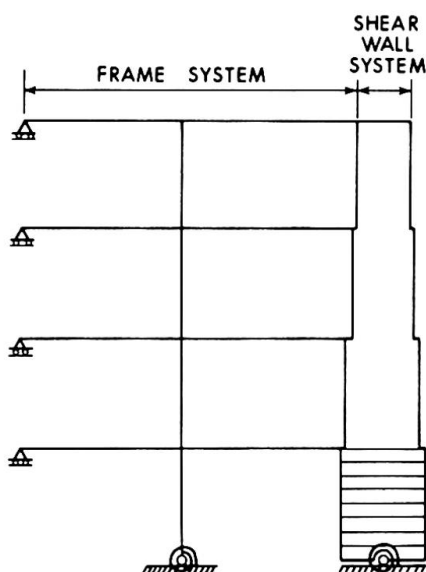


FIG. 2. ANALYTICAL MODEL

This procedure implies that the structures considered are reasonably symmetrical and do not exhibit significant torsional deformations. The lumping procedures used to form the analytical model are similar to those used for the elastic analysis of frame-shear wall structures <sup>(3)</sup>

To analyze the model for a given set of lateral loads, the loads are first applied to the shear wall and its free deflection is computed. The frame is then forced into a compatible set of deformations and the shears developed by the frame are computed. These are applied to the wall as corrective forces and a new deflected shape computed. The process is continued until the total shears developed are in equilibrium with the applied lateral loads <sup>(3)</sup>. To obtain the complete load-deformation relationship for the structure the lateral loads are increased and the above process is repeated.

At each step in the process the inelastic action of the frame and the shear wall is accounted for by using the inelastic moment-curvature ( $M-\theta$ ) relationships in the computation of deflections and the resulting forces <sup>(4)</sup>. An elastic perfectly plastic  $M-\theta$  relationship is used for the frame members with the plastic moment capacity of the columns reduced to account for axial loads. For the shear wall a bilinear  $M-\theta$  relationship is assumed.

The  $P-\Delta$  effect is also included at each stage of the process. The secondary moments in each story are computed from a knowledge of the deflected shape and vertical loads. The corresponding shears are then added at each floor level and the additional deflections computed. The process is continued until the deflected shapes converge.

#### FOURTEEN-STORY BUILDING

The first structure considered is a fourteen-story building, rectangular in plan, with nineteen bays of 11 feet 6 inches in the long direction and three bays of 20 feet 0 inches in the short direction. The building had been analyzed previously for a load of 20 psf. applied perpendicular to the long side of the building <sup>(3)</sup>. The results are presented only to check the validity of the analytical model shown in FIG. 2.

The properties of the original structure are given in REF. 3. The analytical model is shown in FIG. 3, the members have been lumped to form the

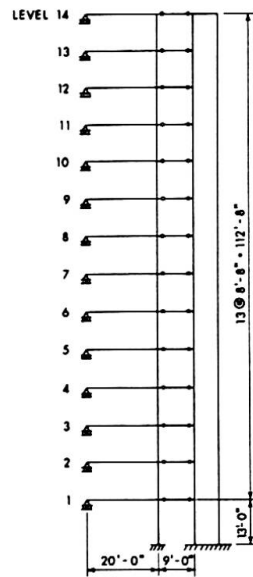


FIG. 3. FOURTEEN-STORY BUILDING

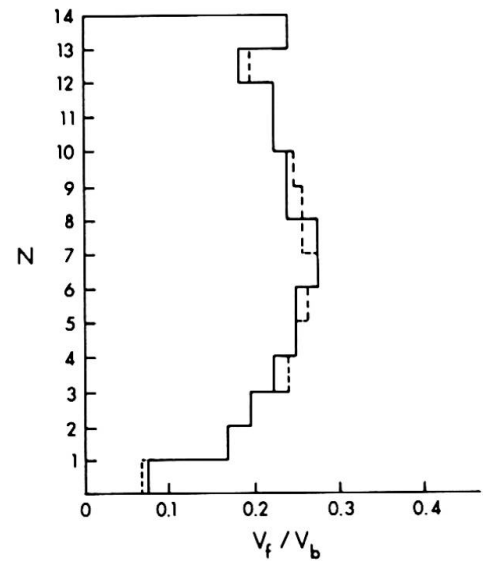


FIG. 4. SHEAR DISTRIBUTION

equivalent systems <sup>(4)</sup>. In FIG. 4 the story number,  $N$ , is plotted versus the proportion of the total base shear carried by the frame,  $V_f/V_b$ . The solid lines indicate the shears obtained using the model shown in FIG. 2, the dashed lines represent the results obtained previously <sup>(3)</sup>. The agreement is satisfactory. At this stage of loading the structure is elastic and the  $P-\Delta$  effect has little influence <sup>(4)</sup>.

In FIG. 4, the frame shear,  $V_f$ , is relatively constant in the top portion of the structure. The applied shear, however, increases linearly (approximately) from the top of the structure. Thus the top stories of the frame must carry shears in excess of those applied on the story due to the pull exerted by the shear wall. To study the influence of the wall stiffness on the shear distribution several additional analyses were performed. The structure was changed for each analysis by reducing the stiffness of the shear wall

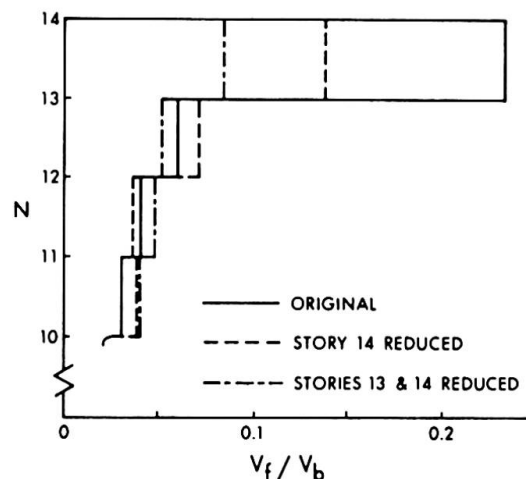


FIG. 5. VARIATION IN SHEAR DISTRIBUTION

to one-hundredth of the original value in the top stories. The results are summarized in FIG. 5 which plots  $N$  versus  $V_f/V_b$ . The results of the analysis of the original structure are shown as the solid lines. The dashed lines represent the values obtained when the top story stiffness is reduced to one-hundredth of the original value and the broken lines represent the results when the stiffnesses of the top two stories are reduced. As the stiffness of the wall is reduced the shears carried by the more flexible stories are also reduced, however, the shears carried by the other stories may be increased. The action of the lower portion of the structure is unchanged.

#### TWENTY-FOUR STORY BUILDING

The second example considered is a twenty-four story, three bay steel frame. The frame had been designed using both the allowable stress and plastic strength

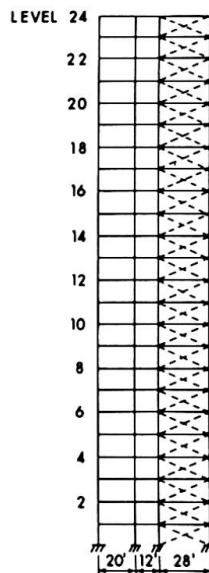


FIG. 6. TWENTY-FOUR STORY STRUCTURE

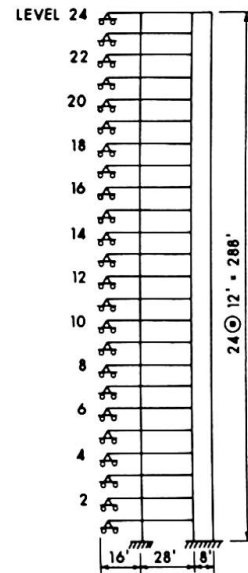


FIG. 7. LUMPED STRUCTURE

techniques, under the assumption that it was completely braced <sup>(1)</sup>. The original frame is shown in FIG. 6 and the analytical model in FIG. 7. The member properties and the vertical loads acting at each floor level are given in REF. 1; the structure was lumped according to methods used in REFS. 3 and 4 and the properties of the analytical model are given in REF. 4. No attempt was made to obtain a flexural shear wall corresponding to the truss shown in FIG. 6, instead several analyses were performed with varying shear wall stiffnesses. The ratio of the wall stiffness to the column stiffness,  $K_w/K_c$ , was held constant in each story. The plastic strength of the wall was chosen to bear a reasonable relationship to the stiffness; this strength/stiffness ratio was

maintained for each story. The shear wall was assumed to have a constant width of 8 feet 0 inches.

The model was subjected to vertical loads at each floor level and to concentrated lateral loads. The vertical loads were held constant for the analysis while the lateral loads increased monotonically. The lateral load at the roof level was one-half those at the other levels.

FIG. 8 shows graphs of the lateral force at the top of the frame,  $H$ , versus the top level column rotation,  $\rho$ . The frame has a ratio of wall stiffness to column stiffness of 50. The upper curve has been obtained from an analysis which neglects the  $P-\Delta$  effect. The first hinge in the frame is detected at point 'a'. The shear wall yields first at the base as shown by point 'b' on the graph. The structure is essentially, a

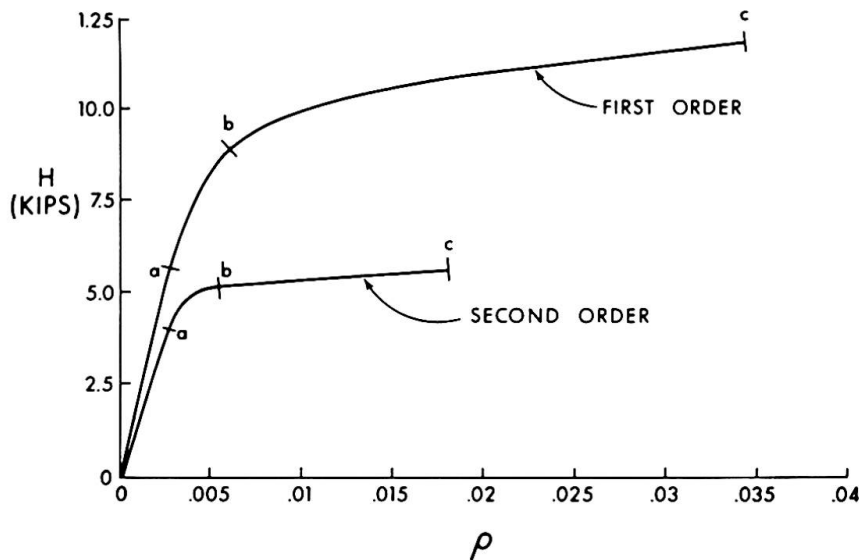


FIG. 8. LOAD-DEFLECTION CURVE

'weak beam-strong column' type and at point 'c' hinges have formed at the ends of all the beams. The only column hinge detected between points 'b' and 'c' occurs at the top of the column in the 24th story. Since a bilinear moment-curvature relationship has been assumed for the wall it will continue to accept increasing load. To demonstrate the  $P-\Delta$  effect, an analysis represented by the lower curve in FIG. 8 was performed. In this case, the structure was analyzed with reduced plastic moment capacities for the columns. The reduced capacities did not influence the results as the structure failed due to instability without hinges forming in the columns. At point 'a' on the lower curve, the first hinge in the structure was

detected. Up to point 'b' the rate of decrease of stiffness is moderate. Beyond point 'b' the deflection of the structure increases rapidly up to point 'c'. At this load level, the wall becomes inelastic. For the next increment of lateral load, the deformations become so large that the system does not converge. The load corresponding to 'c' has been taken as the ultimate load carrying capacity of the structure. In FIG. 8 the difference in load carrying capacity predicted by the two analyses is primarily due to the  $P-\Delta$  effect, since only one hinge forms in the columns.

FIG. 9 consists of several plots showing the deflected shape of the structure as the lateral load is incremented. The curves are obtained from the second order analysis corresponding to the lower curve. In FIG. 9 the

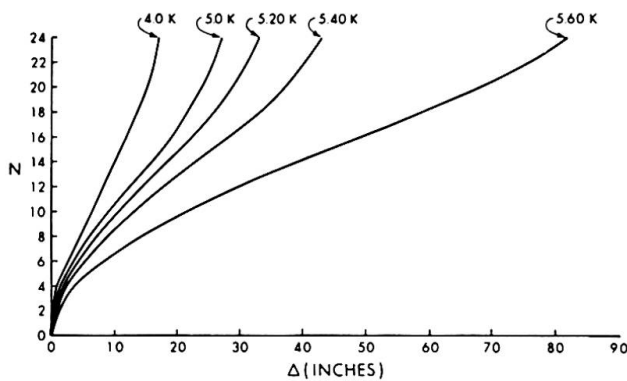


FIG. 9. DEFLECTED SHAPES

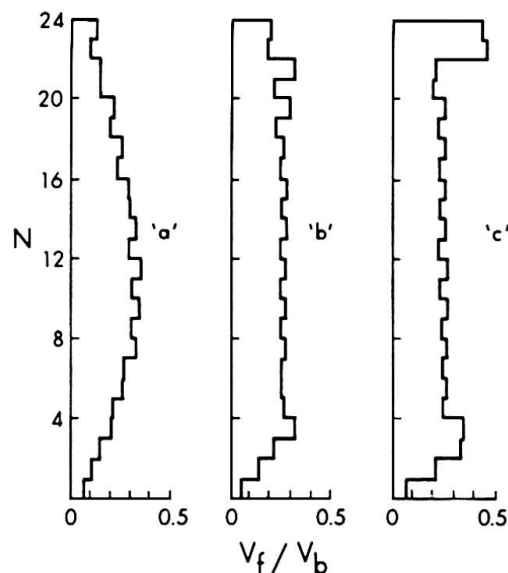


FIG. 10. SHEAR DISTRIBUTION

deflected shapes for lateral loads of 5.20 and 5.60 kips correspond to points 'b' and 'c' on FIG. 8. These two curves emphasize the rapid increase in deflection which occurs as the wall enters the inelastic range.

The shear distribution between the wall and the frame is of interest in this study.. FIG. 10 plots the story,  $N$ , versus the ratio  $V_f/V_b$  for three stages in the loading history. The stages correspond to points 'a', 'b' and 'c' of FIG. 8. The base shear at each stage includes the appropriate component of the  $P-\Delta$  effect. At the elastic limit (stage 'a') large shears act near mid-height of the frame. As the frame yields (between 'a' and 'b') the shears are redistributed and near the ultimate load (stage 'c') are largest near the top and bottom. Relative to the applied story shear,



large shears occur in the top stories at all stages and are accentuated by yielding in the frame.

The effect of varying the wall stiffness was also studied. FIG. 11 is a plot of the top level lateral force,  $H$ , versus the top level lateral deflection,  $\Delta$ . In all cases, the  $P-\Delta$  effect was considered and the reduced plastic moment capacities used for the columns. The curves are

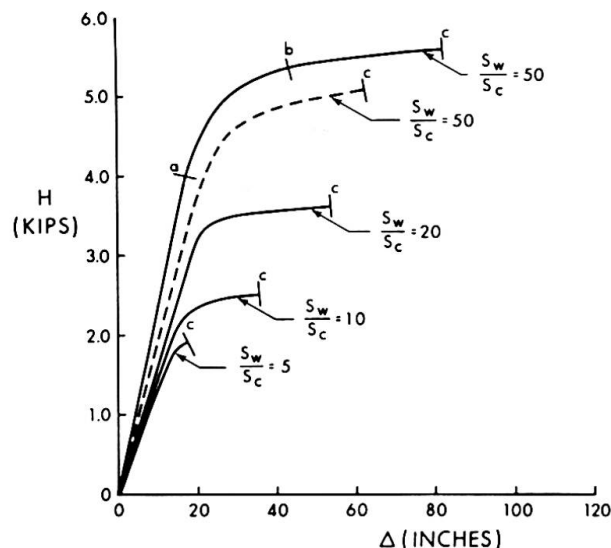


FIG. 11. LOAD-DEFLECTION CURVES

plotted for ratios of  $K_w/K_c$  varying from 5 to 50. The load carrying capacity of the structure decreases with a reduction in the shear wall stiffness. This is primarily due to the increased severity of the  $P-\Delta$  effect.

The structure having  $K_w/K_c = 50$  was reanalyzed assuming zero wall width. The result is shown as the dashed curve. In this case, the analysis (assuming zero wall width) yields conservative results, apparently since it neglects the extra restraining moment on the wall. This moment is the result of the shear at the wall end of the beam acting through half the wall width.

It can be observed from FIG. 11 that the difference in behavior due to the variation in wall to column stiffness is considerable. In all cases, the analysis did not converge beyond point 'c'. The loads corresponding to points 'c' have been taken as the ultimate load carrying capacities of the structures. Due to the procedure used in the analysis, the unloading branch of the load-deflection curve can not be obtained.

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## SUMMARY

A method has been presented for the approximate inelastic analysis of frame-shear wall structures. The method accounts for the wall-frame interaction and the  $P-\Delta$  effect. The results presented illustrate the shear distributions obtained and the reduction in load-carrying capacity due to the secondary effects.

## RÉSUMÉ

Une méthode a été présentée pour l'analyse inélastique approximée d'une forme de structures composées de murs et de câdres. La méthode tient compte de l'action réciproque du mur et du câdre, et de l'effet du  $P-\Delta$ . Les résultats présentés servent à démontrer les distributions de forces et les rapetissements de la charge limite qui résulte des effets secondaires.

## ZUSAMMENFASSUNG

Für die angenäherte unelastische Berechnung von Scheiben-Rahmentragwerken wird ein Verfahren vorgestellt. Diese Methode zieht das Zusammenwirken der Scheibenrahmen und des  $P-\Delta$ -Effekts in Betracht. Die Ergebnisse zeigen die Schubverteilung sowie die Traglastverminderung aus sekundärem Einfluss.

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