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**Inelastic Behaviour of Reinforced Concrete Shear Wall-Frame**

Comportement inélastique de structures en béton armé composées de murs et de portiques

Unelastisches Verhalten der Stahlbeton-Scheibenrahmen

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INTRODUCTION

This paper presents some results of a computer study of the behavioural characteristics of a plane reinforced concrete frame braced laterally by a shear wall. The method of analysis employed traces the load-deflection behaviour of the structure until failure occurs either by instability or by a collapse mechanism. The members are assumed to be elastic-perfectly plastic.

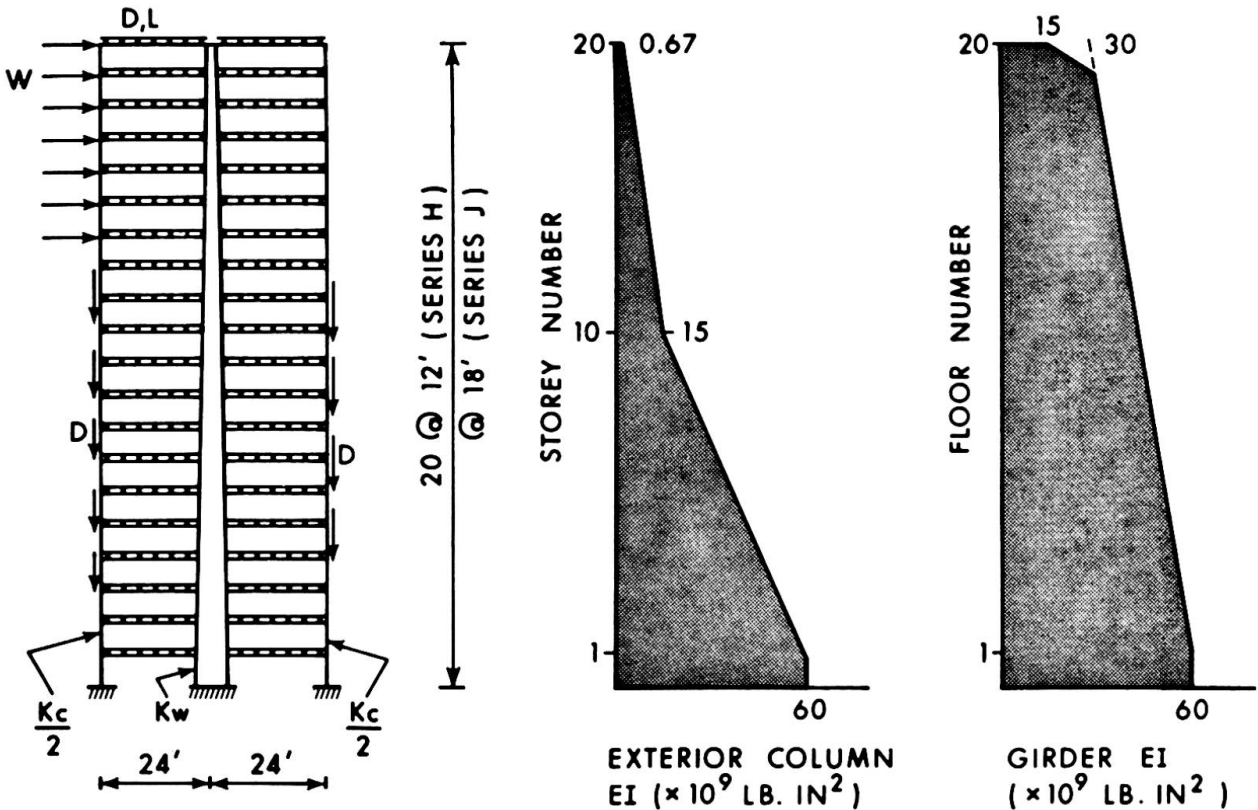
The behaviour of a twenty storey two bay reinforced concrete structure is taken as the basis for the discussion. The properties of the structure were adjusted to study the effects of variations in the wall stiffness, axial shortening of the wall and columns, and the secondary or  $P-\Delta$  moments due to lateral deflections.

METHOD OF ANALYSIS

Many first order elastic solutions exist for the case of a frame coupled with a shear wall subjected only to lateral loads. In addition, a number of elastic-plastic solutions for unbraced metal frames have been described in the literature. To date, however, the inelastic action of coupled shear wall-frame structures has not been studied extensively.

The analysis on which these results are based provides a second order elastic-plastic solution to the coupled shear wall-frame system <sup>(1)</sup>. An iterative procedure is used to solve slope-deflection equations modified to consider axial load effects and the finite shear wall width. As the loads are incremented and plastic hinges are detected in the structure, adjustments are made to the appropriate slope-deflection equations. Axial shortening of the columns and walls is considered although creep deflections have been ignored. Failure may be due to instability or the formation of a collapse mechanism. The structure is assumed to be braced against local and out-of-plane buckling. It is possible to consider any rectangular configuration of beams, columns and walls in a single plane.

For reinforced concrete members an elastic-perfectly plastic moment-curvature relationship has been derived for the girder, column and wall sections. This relationship considers the section geometry, material properties and the effects of axial loads. Deformations due to shear or inclined cracking have not been considered.



(a) Elevation of Frame

(b) Distribution of Member Stiffnesses

Figure 1. DETAILS OF FRAMES

STRUCTURE AND LOADING

The behaviour of two series of reinforced concrete frames will be discussed in this paper. The frames will be referred to by means of a letter denoting the frame series and a number denoting the ratio,  $K_w/K_c$ , of the EI of the shear wall to the sum of the EI values of the columns in each storey. Details of the frame geometry and the distribution of member stiffnesses are shown in Fig. 1.

The series H and J frames had a 12 foot and an 18 foot storey to storey height, respectively. In each case they consisted of square, symmetrically reinforced tied columns with a total longitudinal reinforcement ratio of 0.04 and rectangular beams reinforced in tension only with  $\rho_f/f'_c = 0.18$ . In all cases the yield strength of the reinforcement was taken as 60,000 psi and the concrete strength was assumed to be 4,000 psi.

To facilitate studies of the effects of variables, all the frames discussed in this paper had the same columns and beams. The structure H1 was designed by the ultimate strength procedures in the ACI Building Code assuming material understrength factors,  $\phi$ , equal to 1.0 for all members. The resulting member stiffnesses are plotted in Figures 1(b) and 1(c). The series J frames had the same member sizes as the series H frames.

The columns varied in size from 8.5 inches square in the top storey to 22 inches square in the bottom storey. The girders varied from 10 inches wide by 22 inches deep at the roof to 14 by 30 inches at the first floor.

In frames H1 and J1, designed as unbraced frames, the "shear walls" were solid tied column sections with 4 percent longitudinal reinforcement. These walls varied from 9.5 inches square in the top storey to 26 inches square in the bottom storey. To augment the shear wall stiffness in the other frames the moment of inertia and plastic moment capacity of the wall were increased by the appropriate value of  $K_w/K_c$ . The width of the wall in the plane of the frame was kept constant to eliminate the effects of wall-width from the basic study. Similarly, to minimize the effects on hinge formation of the relative axial shortening of the wall and columns, the wall area was held constant as  $K_w/K_c$  was varied.

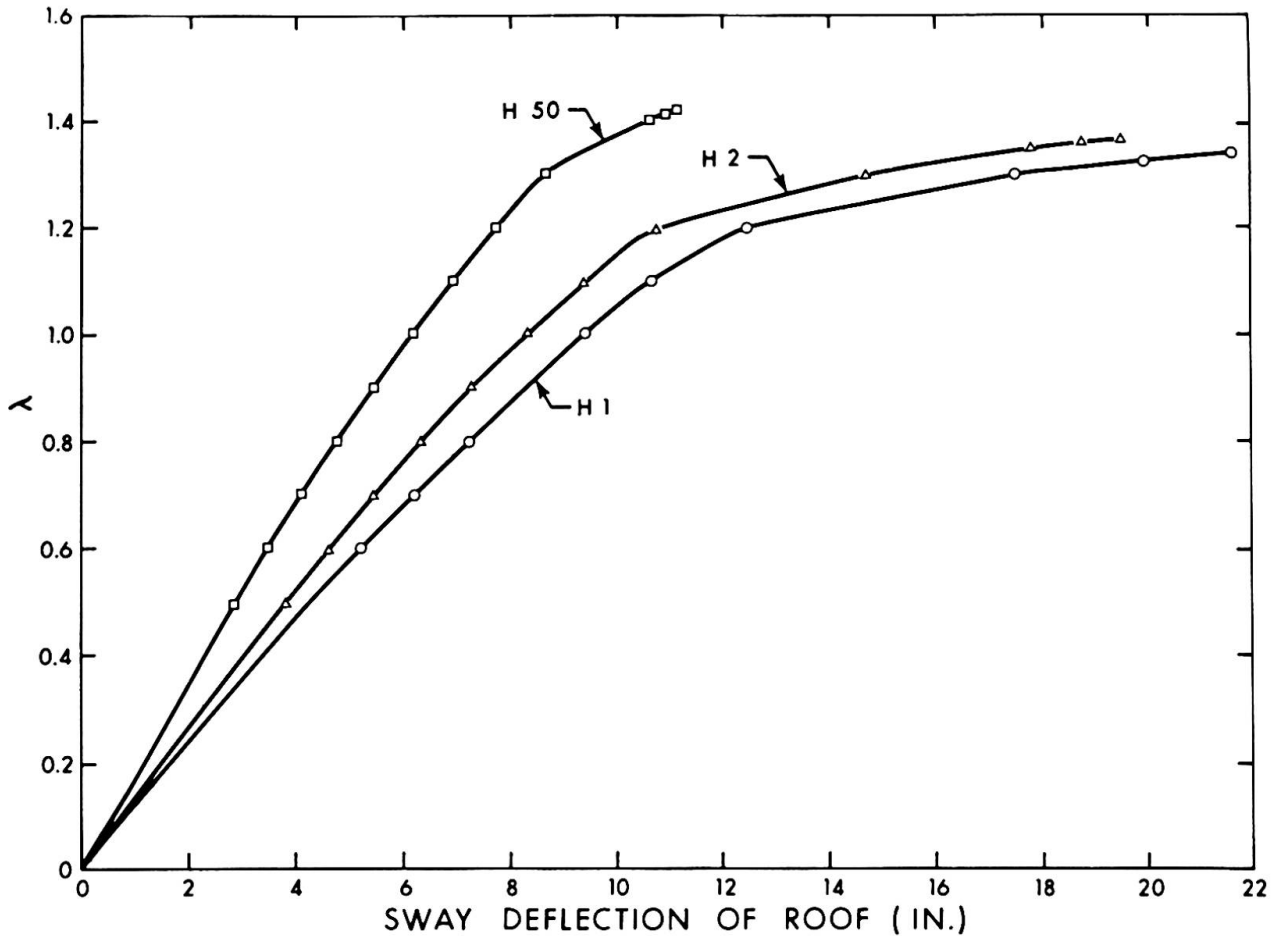


Figure 2. EFFECT OF WALL STIFFNESS ON SWAY DEFLECTION OF TOP OF FRAME

In all cases the loading was applied statically and was proportionally increased until failure occurred. In this paper results will be given in terms of the load factor  $\lambda$  in the expression  $\lambda(D + L + W)$  where  $D$ ,  $L$  and  $W$  represent reasonable working load values of the dead, live and wind loads for structures of the type considered.

#### EFFECTS OF VARIATION OF WALL STIFFNESS ON LOAD DEFLECTION BEHAVIOUR

Second order elastic-plastic analyses were carried out on Series H and J frames with relative wall stiffness values,  $K_w/K_c$ , of 1, 2, 6, 12, 20, and 50. Typical load-deformation plots from these analyses appear in Figure 2. Figure 3 shows bending moment diagrams for the walls in frames H1 and H50 at  $\lambda = 1.00$ , prior to the formation of any plastic hinges in the structure.

It is apparent that, although the wall and columns of Frame H56' possess a total lateral stiffness of 25.5 times that in H1, the overall frame stiffness does not increase in this proportion. This can be explained by the difference in behaviour between a portal frame and a cantilever wall. Inspection of the wall bending moment diagrams in Figure 3 indicates that the stiffening function of the wall is diminished as the structural action reverts to cantilever behaviour at the base of the wall, as indicated by a gradual shift of the initial point of contraflexure up the wall as the wall stiffness increases. Similar behaviour was noted in the Series J frames.

In considering the effect of wall stiffness on the portion of the total lateral load carried by the wall it was also evident that the loads carried by the wall did not increase in direct proportion to the increase in the relative stiffness of the wall.

The implication of this is that, except for structures with extremely stiff walls, the frame members may be underdesigned if the wall is assumed to carry the entire lateral load.

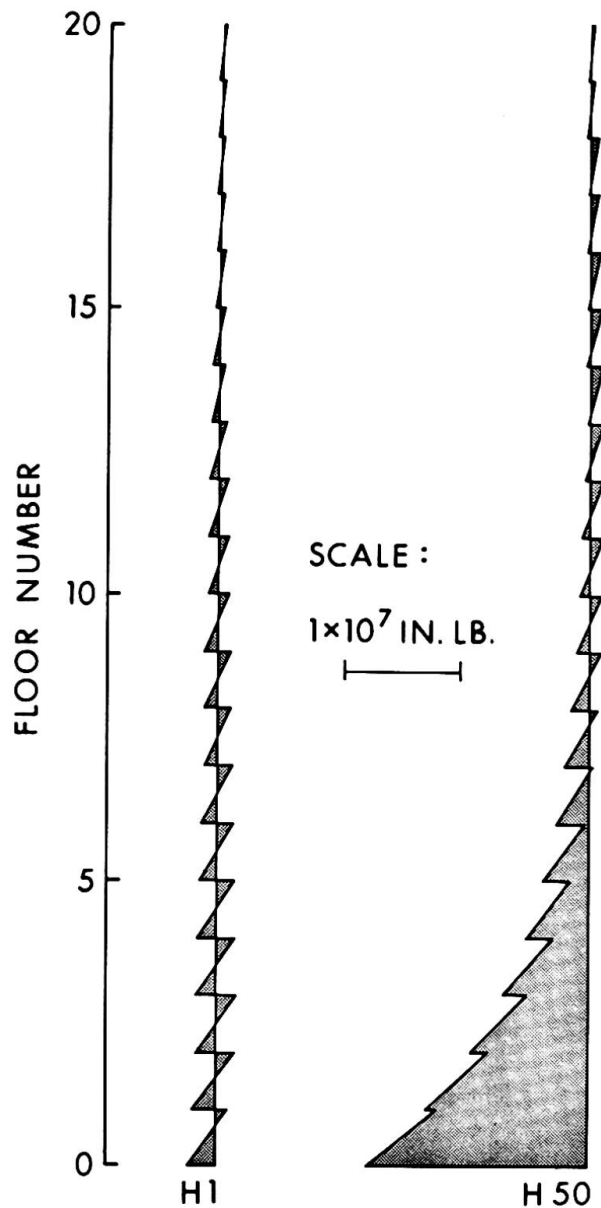


Figure 3. BENDING MOMENTS IN THE WALLS IN FRAMES H1 AND H50 AT WORKING LOADS

EFFECTS OF COLUMN AXIAL SHORTENING

Frame H1 was used as the basis for a study of the effects of relative axial shortening of the columns and wall in a frame. The load-deflection plots from three analyses of this frame are shown in Figure 4.

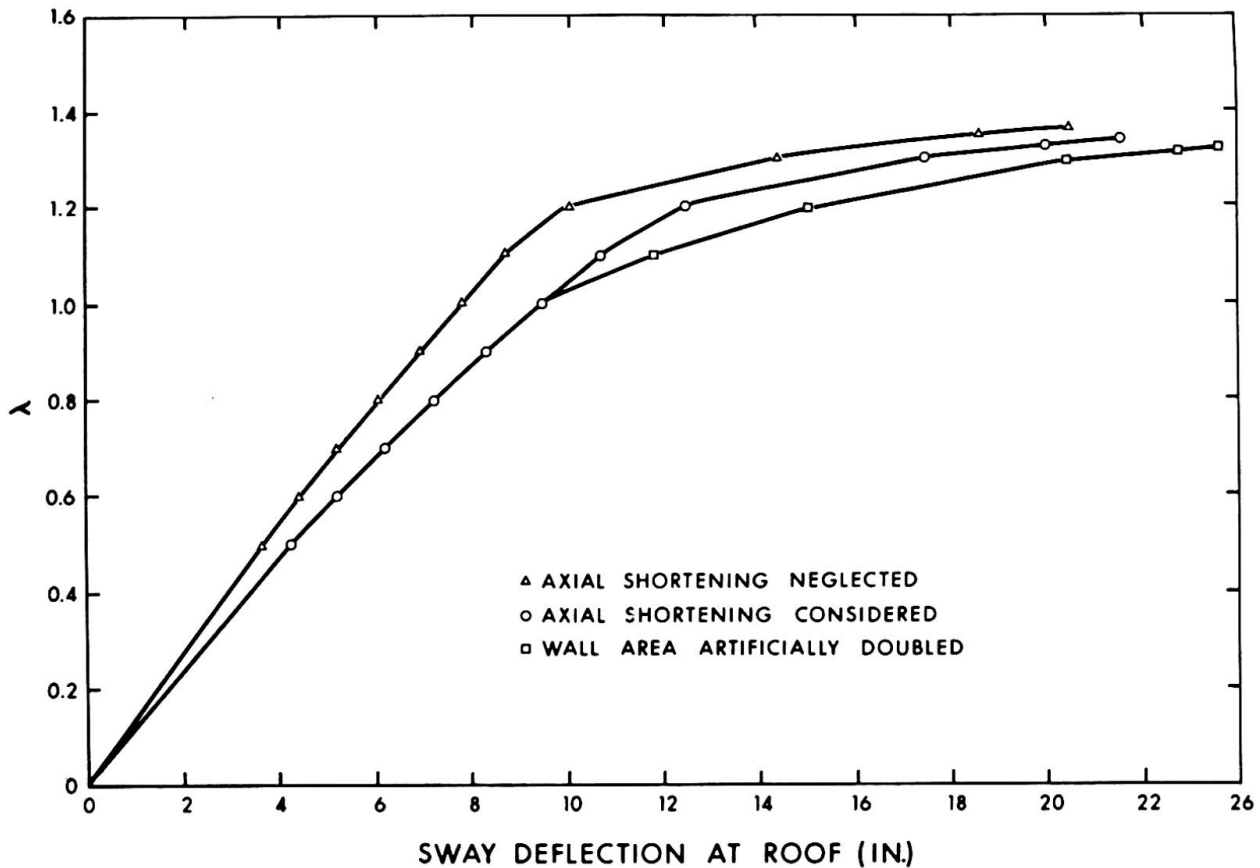


Figure 4. EFFECTS OF AXIAL SHORTENING IN FRAME H1

In this particular case the lateral deflection of the top storey at working loads was underestimated by 18 percent and the failure load was overestimated by 2 percent when the axial shortening of the members was neglected.

When the wall area was doubled without changing its EI or plastic moment capacity there was no change in the stiffness of the structure at low loads. However, the reduction in the axial shortening of the wall resulted in greater relative column to wall deflections and caused premature hinges in the girders near the top of the structure. This reduced the stiffness of the structure and led to earlier instability.

This effect was more pronounced when the ratio of shear wall stiffness to column stiffness was greater, especially if the shear wall section remained constant over the height of the building. These results have not been included here since in most practical designs the thickness of the core walls would be varied in accordance with the axial loads in the core. However, in the case of buildings with prismatic core walls as might be the case if the core was slipformed, for example, the relative axial shortening of the walls and columns should be considered.

#### TRANSITION FROM SWAY INSTABILITY TO BRACED FRAME INSTABILITY

In conjunction with the second order elastic-plastic analyses, first order elastic analyses were performed on the Series H and J frames. The resulting bending moment values were compared to observe the significance of the secondary ( $P-\Delta$ ) moments in the various frames. This effect was expressed in terms of a moment magnifier,  $F$ , equal to the second order analysis moment at any point divided by the corresponding moment from the first order analysis.

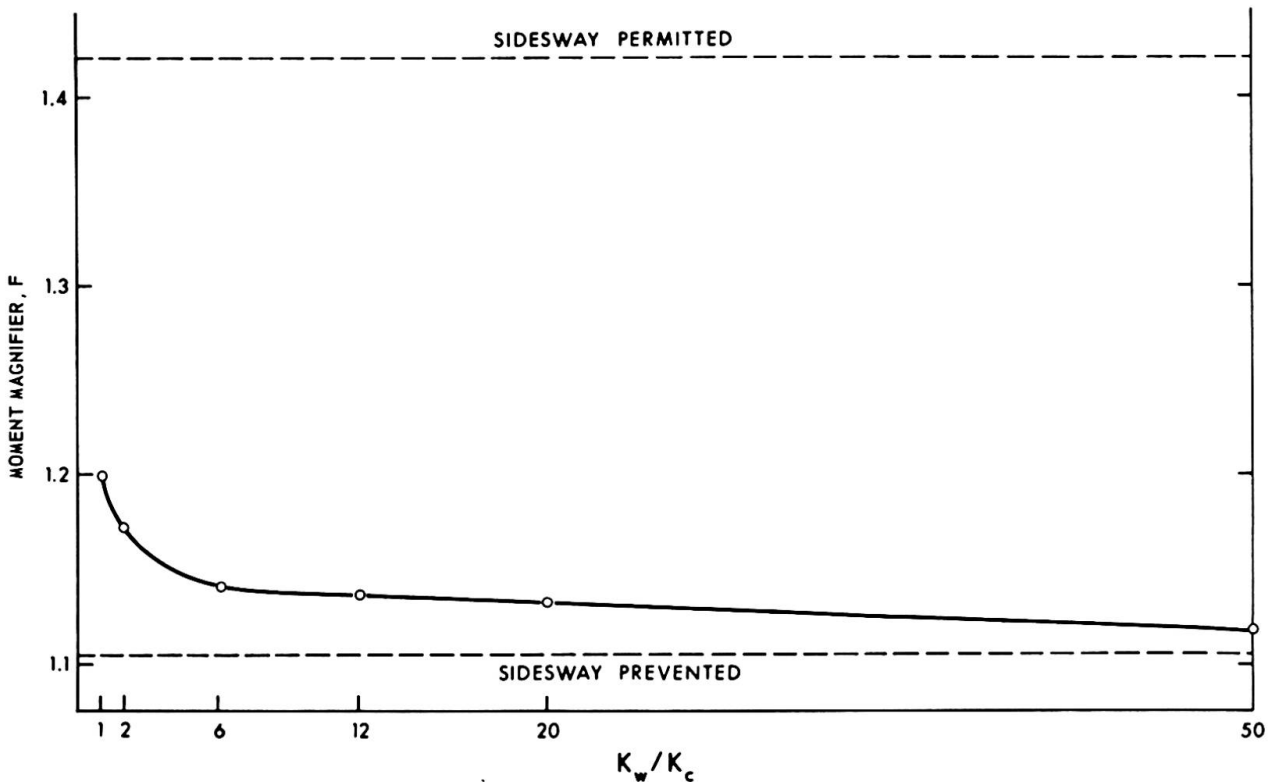


Figure 5. EFFECT OF RELATIVE WALL STIFFNESS ON MOMENT MAGNIFIER

The values of the moment magnifier,  $F$ , at working loads ( $\lambda = 1.00$ ) in the leeward column of the ninth storey of Series J frames are plotted as a function of  $K_w/K_c$  in Figure 5. It should be noted that at  $\lambda = 1.00$ , no hinges had formed in any of the structures considered. The two horizontal dashed lines appearing on this graph represent the values of  $F$  derived using the traditional moment magnifier relationship<sup>(2)</sup> given by Equation 1. The critical load,  $P_e$ , is based on a nomographic evaluation of effective length<sup>(2, 3)</sup>.

$$F = \frac{1}{1 - \frac{P}{P_e}} \dots \dots \dots (1)$$

The  $F$  values derived from this analysis never approached the values computed using Equation 1 assuming the frames are free to sway. Part of this discrepancy is caused by the fact that the values of the effective lengths used were derived assuming a typical interior column in an infinitely large, rectangular structure. In addition, the effective lengths were derived assuming only axial loads in the columns, while the frames considered here had both wind and uniformly distributed gravity loads.

On the other hand, however, for values of  $K_w/K_c$  greater than about 6, the moment magnifier values computed in this study did approach the  $F$  values computed using Equation 1 assuming a frame braced against sidesway. Similar effects were noted for the columns in other stories of the H and J frames.

The values of the moment magnifiers presented in Figure 5 suggest that a safe approximate design procedure for multi-storey frames would be:

- (1) Analyze forces and moments using a conventional first order elastic analysis.
- (2) Amplify the column moments, and where necessary the girder moments, using a moment magnifier given by Equation 1. The effective length for braced columns could be used in computing  $P_e$  if  $K_w/K_c$  is greater than 6, and that for unbraced columns if  $K_w/K_c$  is less than 6.
- (3) Design all sections to have a plastic moment capacity equal to or greater than the moments computed in Step 2.

NOTATION

- F = Moment Magnifier
- $f'_c$  = Compressive strength of the concrete
- $f_y$  = Yield strength of the reinforcement
- $K_c$  = Sum of  $EI/h$  values for all columns in any storey
- $K_w$  =  $EI/h$  of shear wall in any storey
- P = Axial load on column
- $P_e$  = Euler column buckling load

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## SUMMARY

An elastic-plastic analysis for reinforced concrete structures consisting of a coupled frame and shear wall has been derived and applied to several examples. The results of this analysis suggest that relative axial shortening of walls and columns may lead to a premature instability failure of the structure. In addition, the studies suggest that a relatively low shear wall stiffness is required to change the behaviour from that of an unbraced frame to that of a braced frame with respect to instability.

## RÉSUMÉ

L'analyse plastique-élastique pour des structures de béton armé composées d'un cadre et d'un mur accouplés a été dérivée et a été appliquée à plusieurs exemples. Les résultats de cette analyse suggèrent que le raccourcissement axial relatif des murs et des colonnes peut conduire à un affaiblissement de la structure dû à une instabilité prématurée. En addition, les études montrent qu'un mur d'assez petite rigidité suffit pour changer les propriétés de stabilité d'un portique seul en celles d'un portique renforcé d'un mur bien rigide.

## ZUSAMMENFASSUNG

Eine elasto-plastische Analyse für Stahlbetonhochhäuser bestehend aus einer Zusammenfassung von Rahmen und Schubwand ist präsentiert durch verschiedene Beispiele. Die Resultate dieser Analyse regen an, dass eine verhältnismässige Verkürzung der Wände und Stützen zu einem vorzeitigen Zusammenbruch des Bauwerks führen mögen. Des weiteren schlagen die Studien vor, dass eine verhältnismässig niedrige Schubwand-Steifigkeit erforderlich ist, um eine Veränderung des Verhaltens, mit Hinsicht auf die Unstabilität, zwischen einem unverankerten und verankerten Rahmen zu erzielen.