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### IIIa

#### Research on Plastic Design of Multi-Story Frames at Lehigh University

Recherche sur le calcul plastique des portiques multiétagés à l'Université de Lehigh

Forschung über das Traglastverfahren von Stahlhochbaurahmen an der Lehigh Universität

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#### 1) Introduction

The research team at the Fritz Engineering Laboratory of Lehigh University has extended a major effort since 1958 to the development of Plastic Design Methods for Planar Multi-Story Steel Frames. The principal motivation of this effort was the conviction that Plastic Design is both more rational and more economical than Allowable-Stress Design.

This research work can be subdivided into the following categories:

- 1) Research on Component behavior
- 2) Development of design and analysis methods
- 3) Experimental verification on frames

The summary of this research, as well as design methods and design examples for braced and unbraced multi-story frames, is given in Ref. 33 of the general report by Messrs. Steinhardt and Beer. The following brief comments are a review of the present status of this research and they are intended to provide additional information to that given in the General Report, especially emphasizing the Lehigh work.

#### 2) Studies on Component and Subassemblage Behavior

A knowledge of the load-deformation behavior of the components of a structure is basic information required for a structural analysis. All the various components of a structure, such as wide-flange beams and beam-columns and many types of connections, were studied in several major research programs. In addition, the load-deformation behavior of subassemblages consisting of several members was thoroughly investigated. The purpose of this research has been the need to define the geometric and material limits which must be fulfilled for a successful application of Plastic Design procedures.

In the inelastic range the load-deformation relationship is highly non-linear, and it depends on yielding, strain-hardening, geometry changes, initial imperfections and on lateral-torsional and local buckling. A typical moment-rotation curve for a beam (Fig. 1) illustrates the initial elastic behavior (OA), the reduction in stiffness due to yielding in the presence of strain-hardening (AB), the point of instability due to combined local and lateral-torsional buckling (C) and the reduction of moment capacity in the unloading range (CD). The relationship is usually idealized in plastic design as a rigid-plastic (EF) or an elastic-plastic (OGF) relationship. However, some analysis methods utilize a bilinear curve (OGI) (Ref. 1) and in some applications the actual complete M- $\theta$  curve of beam-columns is utilized (Ref. 2, for example).

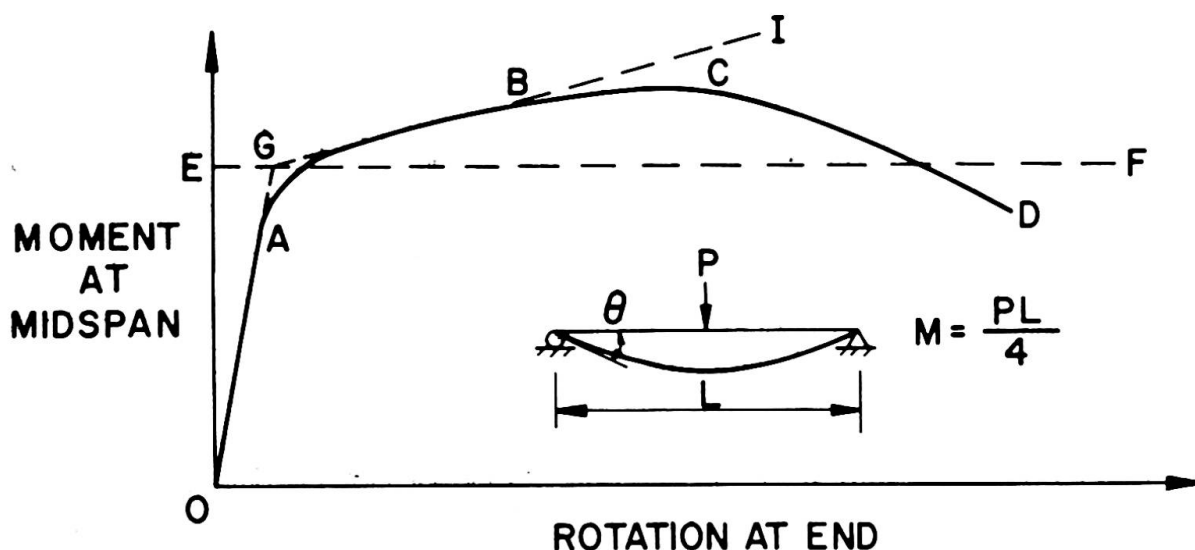


Fig. 1 Moment-Rotation Relationship of Beam

The research on components has concentrated on developing theoretical models whereby the whole M- $\theta$  curve, or significant portions of it, can be predicted. Many experiments were also performed to substantiate or complement these theoretical predictions.

#### Research on Beams

Numerous experiments were performed on wide-flange beams under uniform moment (Ref. 3) and moment gradient (Ref. 4), with various types of lateral bracing (Ref. 5), and on beams of high strength steel (Ref. 6), to study the post-yield behavior. Theoretical models, based on the concept that failure results when local and lateral-torsional buckling occur simultaneously, permitted a prediction of the limits of inelastic rotation capacity, and a definition of the required maximum flange and web width-thickness ratios and maximum bracing spacing (Refs. 7 through 12). For example, the maximum flange width-thickness ratios for steels with yield points of 36 and 50 ksi, respectively, were found to be 18 and 14. The corresponding maximum unbraced lengths for beams under uniform moment were determined to be, respectively,  $38r_y$  and  $28r_y$ , where  $r_y$  is the weak axis radius of gyration.

### Research on Beam-Columns

This work concentrated on the theoretical determination of the in-plane end moment-versus-end rotation curves of beam-columns, extending the work of Chwalla (Ref. 12.28 of the general report) to wide-flange members containing residual stress (Refs. 13 and 14; a summary of this work is given in Chap. 9 of Ref. 33 of the general report). The solution of the problem was achieved by numerical integration procedures, and non-dimensional curves for use in design are presented in Ref. 15. Experiments have given excellent verification of the theoretically obtained curves over a wide-range of the relevant parameters (Refs. 16 and 17). Theoretical studies on inelastic lateral-torsional buckling of unbraced beam-columns bent about their major axis have also shown good agreement with experiments (Refs. 18 and 19). Design procedures, based on this research, have been developed. These are summarized in Chap. 4 of Ref. 33 cited in the general report.

### Research on Connections

Extensive experimental programs were performed on various types of rigid corner connections and rigid beam-to-column connections (for a review of this work see Chap. 8 of Ref. 12.27 of the general report). Design procedures, based on this work, were developed to assure that connections have adequate rotation capacity and a greater moment capacity than the members to be joined. These procedures are summarized in Chap. 5 of Ref. 33 given in the general report.

### Research on Subassemblages

The basic design element for multi-story frames was found to be a "subassembly" consisting of one beam-column with a restraining beam framing into it at its upper and lower end. The load-deformation behavior of such a subassembly can be determined, using equilibrium, compatibility, and the moment-rotation relationships of its three elements (Chaps. 9, 10 and 17 in Ref. 33 of the general report and Refs. 2 and 20). Excellent correlation was noted between theoretically predicted and experimentally measured behavior (Ref. 17). These tests also provided experimental confirmation of individual beam, beam-column and connection behavior.

### 3) Frame Design Procedures

Following a phase of planning and layout, the design and analysis procedure can be divided roughly into three phases. These are (1) preliminary analysis, (2) selection of members, and (3) evaluation and revision of the preliminary design. The challenge in the preliminary analysis is to determine sets of forces on each member resulting from the expected loading which will permit selection of members in a straightforward fashion while using the knowledge which has been gained about component behavior. Frequently, the preliminary forces must be determined based on very limited information about the actual member sizes of the structure. It is also highly desirable that the preliminary force information be obtained in a form which permits selection of member sizes with a minimum of additional computation. Member selection should be followed by procedures for evaluation of the design or at least give a conservative measure of the relative performance of the structure. Evaluation procedures which will do an accurate job for a localized portion of the structure are especially

valuable. They permit revision of an unsuitable design before proceeding to additional parts of the structure.

#### 4) Design of Braced Frames

The development of preliminary analysis and design procedures for braced frames resulted in methods which were exactly what would have been expected on the basis of earlier methods for low buildings. Beams were designed to develop three-hinge mechanisms in the clear span between column flange faces under full factored gravity load. The end moments of the beams were distributed arbitrarily to the columns above and below each joint, but can be justified by some other distribution. Though this practice might seem questionable to some, the lower bound theorem may be interpreted as supporting it. If only one distribution of internal forces could support the applied loads without violating the plasticity conditions, it would have to be the correct distribution of forces.

Design procedures for x-type diagonal bracing were formulated on the assumption that slender members would be used which could only carry tension and would buckle in the elastic range under compression. The bracing members were assumed to carry all lateral shears and to resist all shears due to the  $P\Delta$  effect in simple truss action without assistance from the frame. Evaluation of the probable actual behavior of such a structural system would reveal that the frame must accept part of the lateral shear in order to deform sufficiently to allow the bracing members to deform enough to accept the lateral force. The usual design practice imposing a smaller load factor for gravity loads in combination with wind or earthquake loads allows the frame designed for gravity loads to retain some capacity for resisting lateral loads in combination with the diagonal bracing. Of course some revisions may be necessary in beams and columns adjacent to the diagonal bracing in order to resist axial force components imposed by the diagonal members.

Summaries of the design methods for braced frames are given in Refs. 21, 22 and 23.

#### 5) Tests of Braced Frames

Tests of four frames which approximate the concepts of Steinhardt and Beer's Fig. 4a and 5a have been made. (Refs. 24, 25 and 26) All four frames had three 10 ft. stories and two 15 ft. bays but they were subjected to different combinations of loading. Twelve inch deep beams and 6W columns were used in all frames. Fig. 2 shows a load-deflection curve of one of the tests and a comparison typical of all the tests with a theoretical prediction ignoring the  $P\Delta$  effect. The photograph in Fig. 2 shows the loading frame used to support the specimen laterally so a single plane frame could be tested alone. Also shown is the system of gravity load simulator devices which allow the application of truly vertical loads even though the frame sways laterally in its plane. The details of the experimental techniques are given in Ref. 27.

Conclusions of the test series were that lateral loading had no significant effect on the ability of the frames to reach the vertical load predicted on the basis of first order theory. Diagonal bracing carried most of the lateral load and the rigid frame was required to resist only 14 to 26 percent of the total lateral load. Very slender brac-

ing performed best when it was tightened during erection so that sag and slack were removed and diagonals subject to compression would remain in tension until the maximum expected lateral load was applied to the frame.

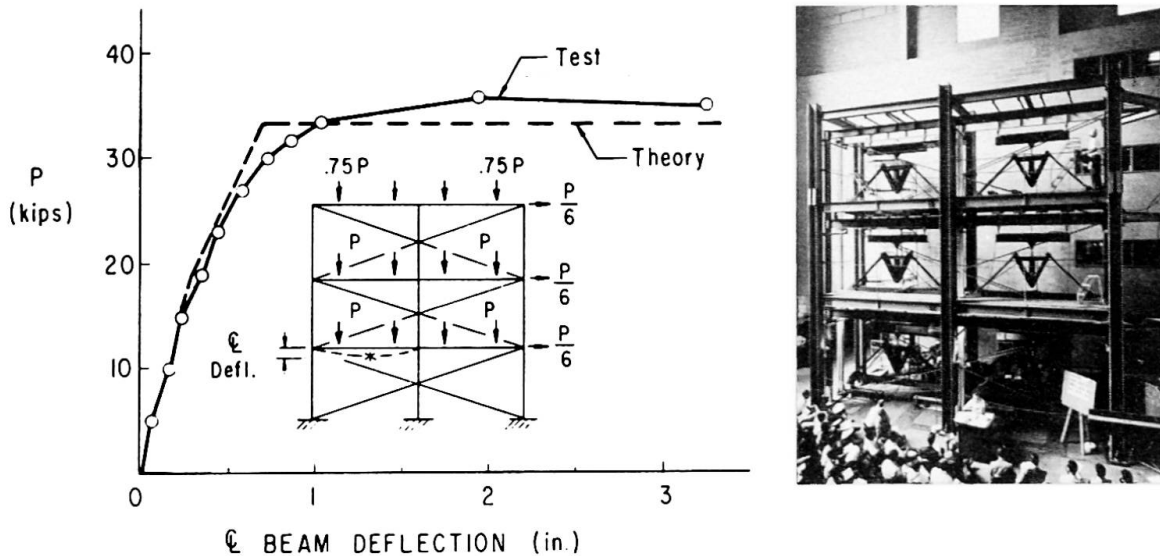


Fig. 2 Results of Braced Frame Test

#### 6) Design and Analysis Procedures for Unbraced Frames

Three key quantities based on elementary statics form the basis of preliminary design procedures for unbraced frames subjected to combined gravity and wind or earthquake loading. A sum of column end moments in a story can be based on the total shear in the story caused by lateral loads and the  $P\Delta$  effect initially estimated from an assumed story sway. The sum of girder end moments within one floor level can be based on an arbitrary reasonable assignment (such as one-half) of the sums of column end moments in the stories immediately above and below the floor. The third key quantity is the limiting end moment of a framed girder subjected to transverse loads in combination with the anti-symmetric moment pattern resulting from lateral loads. Many current design proposals avoid this issue by placing all loads at the joints, but this does not reflect the actual capacity of the members. For most loadings there will be a substantial beam moment at the lee column due to gravity load alone. Addition of the lateral loading causes this moment to increase to the plastic hinge value rapidly, thus revising the strength and stiffness characteristics of the beam for the duration of loading. Design charts giving important ordinates of the moment diagrams for usual loadings are available (Ref. 33 of the general report and Ref. 28).

The final operation of the analysis made for preliminary design purposes is a process called moment balancing. This process is primarily a "bookkeeping" method for assigning the general sums of column and girder end moments determined from prior steps to discrete locations within the story so that beams and columns may be selected each for their own separate force system.

Progress is being made in developing computer programs to parallel the manual computation procedures for preliminary design of unbraced frames. A program has been developed to handle the routine effort of tabulating forces on each member from tributary areas of floors, calcu-



lating story moments and shears, and performing the moment balance to determine all beam and column moments (Ref. 29). The designer then needs to select beams from standard economy tables for plastic design and columns from design charts of reduced plastic moments.

The trial design resulting from the preliminary procedure reflects a complete disregard of compatibility and an assumed story sway which is at best a guess. The subassembly method of analysis has been developed to give an insight into the probable behavior of the structure selected (Refs. 30 and 31). This method uses the properties of the actual members to determine both the load vs. lateral deflection behavior of individual subassemblies and the behavior of a larger assemblage consisting of a whole story. It is necessary to construct load-deflection curves for each column in a story from design charts and then combine the curves for a whole story in order to use the subassembly method manually. The complexity of this process is currently the largest barrier to practical application of plastic design of unbraced multi-story frames. Fortunately a computer program in the FORTRAN Language has been developed to compute the complete load-deflection curve for a story having known member sizes (Ref. 28).

Another computer program gives the elastic-plastic second order load-deflection curve of a complete unbraced multi-story frame up to maximum load (Ref. 32). It uses an iterative process for solution and requires surprisingly little computer capacity to handle frames up to thirty stories high and five bays wide. The disadvantage of such a program is that it ceases to function when it reaches the maximum load of the frame. Information as to the true relative suitability of less heavily loaded portions of the structure is lacking. This points up the main advantage of the other computer program based on the subassembly method which discloses maximum strength of each story as well as deflections in the elastic range.

#### 7) Tests of Unbraced Frames

Two series of unbraced frame tests were conducted. One series resulted from a frame stability investigation (Ref. 33). Two three-story frames 10 ft. wide were tested. Beams were 6 in. deep members and columns were 4 inch wide-flange members with strong axis slenderness ratios of 40 or 45. A typical test result is shown in Fig. 3.

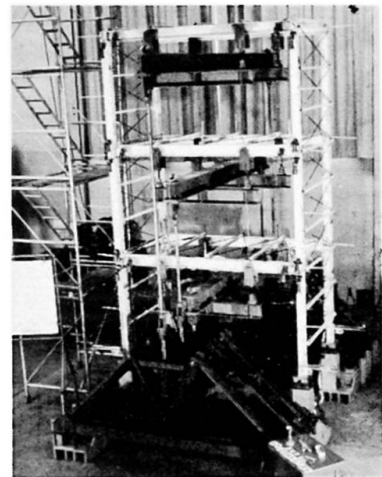
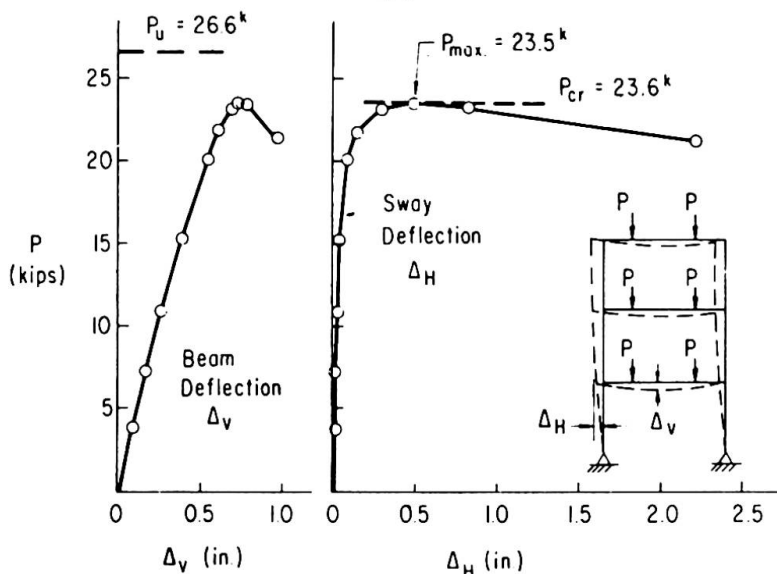


Fig. 3 Results of Frame Buckling Test

Results show that the reduction of the frame buckling load below the load to form a mechanism was predictable. The best method of prediction was a small lateral load method which used a load-deflection solution for the vertically loaded frame subjected to a simultaneous lateral load of one percent of vertical load or less. Both buckling test results exceeded the prediction based on an accidental lateral load equal to one-half percent of the vertical load.

The second series of unbraced frame tests was conducted on two three-story one-bay frames and one three-story two-bay frame subjected to combined vertical and horizontal loads in the plane of the frames (Ref. 34). All stories were 10 ft. high and all bays were 15 ft. wide. Various combinations of 8, 10 and 12 inch deep beams were used with 5 and 6 inch deep columns. A typical test result is shown in Fig. 4. The inadequacy of the first-order theory for prediction of behavior was de-

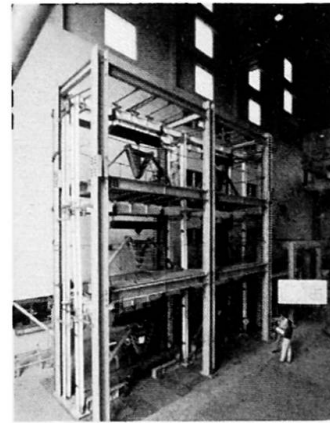
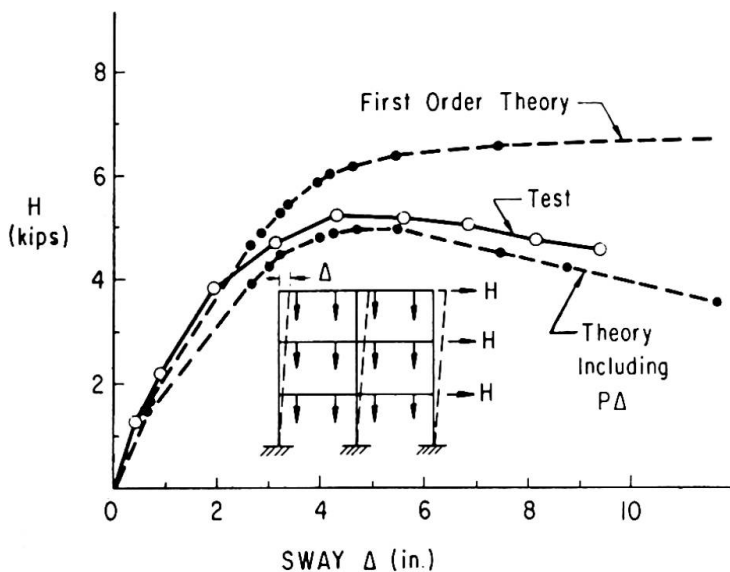


Fig. 4 Results of Unbraced Frame Tests

monstrated. Adequate predictions were obtained by using a linearized elastic-plastic second-order analysis which considered the  $P\Delta$  a small number of cycles of reversed static loading providing data to assist in future earthquake resistant design studies.

## 8. Summary

The brief review of the research on component behavior shows the importance which was attached to the attainment of an understanding of the limiting capacity of beams, beam-columns and connections. Without such an understanding, Plastic Design would be impossible. Fortunately, the limits imposed by the component capacities can be achieved in practical design without offsetting the advantages of economy gained by Plastic Design of the total structure.

The understanding of component behavior is by no means complete. Many problems remain to be solved. Among these the following are currently under study: biaxial bending of beam columns, subassemblies with unbraced beam-columns, the post-yield material properties of new types



of high-strength steel, web local buckling, and behavior of components under repetitive reversed loading into the plastic range. Furthermore, the behavior of connections and of beams under moment gradient is under renewed study.

The outlook for adoption of a plastic design method for braced multi-story frames is promising. The 1968 revision of the American Institute of Steel Construction Specification will extend plastic design coverage to braced multi-story steel frames will soon be published and distributed (Ref. 23). It is significant to note that the plastic design method has already been applied successfully to the design of a braced eleven-story apartment building (Ref. 35).

The continuing efforts to produce practical methods for unbraced frames should bear fruit in the not-so-distant future. Experience with and refinement of these new methods can be expected to eventually lead to the economic limit of ordinary beam and columns skeleton type framing. There is no need to apologize when these limits are reached. The knowledge gained about component behavior and new procedures will help when new and better framing systems are developed.

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## SUMMARY

The research team at the Fritz Engineering Laboratory of Lehigh University has extended a major effort since 1958 to the development of Plastic Design Methods for Planar Multi-Story Steel Frames. The highlights of this work are presented in this discussion. The following topics are described: 1) component behavior, 2) development of design and analysis methods, and 3) experimental verification on frames.

## RÉSUMÉ

L' équipe de recherche du Fritz Engineering Laboratory de l' université de Lehigh a porté, depuis 1958, son principal effort dans le domaine des méthodes de calcul plastique pour des structures multiétagées planes en acier. Les résultats de ces recherches avancées sont présentés dans cet exposé. Les sujets suivants sont décrits: 1) facteurs impliqués dans le comportement des structures, 2) exposé du calcul et des méthodes d'analyses, et 3) vérification expérimentale sur portiques.

## ZUSAMMENFASSUNG

Die Forschungsgruppe des Fritz Engineering Laboratory an der Lehigh Universität hat seit 1958 einen erheblichen Aufwand gemacht an der Entwicklung des Traglastverfahrens für ebene Stahlochbaurahmen. Die wichtigsten Erfolge dieser Forschung sind hier zusammengefasst. Die folgende Themen werden beschrieben: 1) das Verhalten von Fachwerkskomponenten, 2) die Entwicklung von Methoden für Dimensionierung und für Berechnung der Traglast, und 3) experimentelle Prüfung an Rahmen.

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