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Plastic Design

Calcul en plasticité

Plastische Bemessung

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It is appropriate to consider at the outset the basic principles underlying the theory of plastic design of steel structures and to compare this theory with its predecessor, the elastic theory. The elastic design is based on the working loads, a conservative but realistic set of loads that may be actually applied to the structure, and the allowable unit stresses, which must not be exceeded under the most unfavourable load combinations. The stress analysis is supposed to be conducted in conformity with the acceptable theory and the current engineering practice, and this of course implies a tacit acceptance of some degree of error. The allowable stress forms a certain fraction of the yield stress of structural steel of the particular grade used, and the reciprocal of this fraction is usually called the factor of safety. This factor is in effect the factor of ignorance covering a multitude of uncertainties and faults of all kinds associated with design, detailing, fabrication, construction, loads, materials, etc. It covers also, to some extent, mistakes which may be expected in design, as in all human activities.

Plastic design, on the other hand, restricted in its application to statically indeterminate flexural frames, is concerned not with the working but with the failure condition of the structure, which is defined as a state of a very large deformation. This condition is expected to be attained under a load whose intensity exceeds the working load by a quantity known as the load factor or, more correctly, the overload factor. This factor is the equivalent of the factor of ignorance of the elastic design. Nominally the overload factor provides for no uncertainties other than in loads; actually, of course it does provide for them in an indirect way, and in doing this it ceases to be a measure of overload in view of the variability of the other relevant factors involved. The load factor is thus another variety of the factor of safety-factor of ignorance, somewhat misnamed, and in no way better in principle than its conventional counterpart.

The implications involved in the existence of the two acceptable but different factors of safety were apparently not appreciated at the time of incorporation of plastic design into the American and Canadian specifications. As it stands now, a structure may be found as underdesigned by the elastic standards and overdesigned by the plastic. An elastic designer could justify the same structure on the basis of a higher allowable stress,

but the specifications would not permit this. Yet if he changes his approach to plastic the weak design becomes acceptable, —a situation hardly making any sense. A coexistence of two distinct and contradictory systems is no more rational in engineering practice than in any other realm of human endeavours.

The pioneers of plastic design claimed the advantages of their method over its elastic counterpart in the simplicity of calculation, requiring no recourse to indeterminate theory, and in the economy of the resultant structure. This, however, was before they fully realized that the mechanism theory, used in plastic design, while elegant in its simplicity, is insufficient for practical purposes, and that the examinations of instability of the structure and of the change in its geometry are all-important. (This is the field of the so-called non-rigid plastic theory). With instability occurring under partly elastic and partly inelastic conditions the plastic theory suddenly became a most complex assembly of numerous assumptions and hypotheses claimed to be justified by experimental evidence, which however strikes an independent observer as limited and questionable. The alleged economy of the theory also became doubtful in certain areas. Here are some other major uncertainties of plastic theory.

Flexural strength of a member is proportional to its yield strength, but this property of the material is highly indeterminate, varying by more than 50% from the average value for the same grade of structural steel.

Realistic treatment of live and other variable loads, comparable to the procedures used in the elastic design, is not available. There exist highly complicated plastic theories of incremental failure and alternating plasticity but the value of the load factor with which these theories must be associated is unknown apart from the fact that it should be smaller than the one used in the conventional plastic design, because failure under live load requires numerous applications of the load of limiting intensity, while a heavy steady load causes failure in a single application. To the writer's knowledge no attempts to correlate the two plastic load factors have ever been made.

There are no procedures or methods in existence of the non-rigid plastic analysis, as distinct from design; in other words there is no way to determine the load factor of a structure not conforming to the empirical formulae prescribed for prevention of different types of instability.

It is appropriate at this stage to make reference to the common criticism of the elastic theory advanced by the plasticians, that the allowable stress used in this theory is a fiction, because it excludes several participation stresses, (i.e. non-load-carrying stresses), like local stress concentrations, residual stresses etc. This criticism is invalid because the exclusion of the participation stresses is intentional. The non-load-carrying stresses must be excluded, because such

is the nature of the elastic theory and not because the theory is incomplete or inaccurate. A comprehensive review of the weaknesses of plastic theory may be found in the writer's papers (50), (51), (52).

The preceding discussion has been directed mostly at the plastic design of low frames. Tall or multi-storey frames, restrained from sidesway by rigid cores or diagonal bracing, are not too different in their action from the low frames.

In the design of tall frames with sidesway it is necessary to contemplate not only the instability of the individual members but also of the whole frame or its major parts. This compounds the difficulties and calls for more assumptions. The situation may be illustrated on the approach proposed by Professor Horne. He uses an empirical relation in which the true load factor is expressed through two others: the rigid plastic, which ignores instability, and the fully elastic. Since the latter is impossible to find, it is replaced by pseudo-elastic factor based on an imaginary rigid-plastic-rigid stress-strain relationship of the material. Apparently the proponent of the method expects designers to use it for all structures including the ones involving human occupancy. The writer can hardly share this view. His detailed appraisal of the method is found in his discussion of the Horne's paper (53).

The statement made earlier to the effect that the elastic and plastic factors of safety and overload are two different but equally legitimate in principle factors of ignorance, must be re-evaluated now. From all that has just been said, the writer feels that the difficulties encountered in the development of plastic theory have proved unsurmountable and the theory failed signally to live up to its claims.

There is however, something to say in favour of the plastic theory of low frames. Firstly, it assists in understanding structural behaviour of frames by giving an insight into their action at failure, and secondly, it points to desirability of using a variable allowable stress in the conventional elastic design and provides information for establishment of its numerical values. As an alternative to the elastic method of design the plastic theory is unnecessary. Its alleged rationality and economy are pure fictions, and its existence alongside the elastic design merely exposes a deficiency of logic in the specifications. Attempts to apply plastic design to multi-story buildings have no justification. Elastic design of a tall building, allowing for the deformation of the structure under load is complex enough even with the use of electronic computer and iteration procedure. The same problem under elasto-plastic conditions appears insoluble, and the attempts at its solution with the assistance of the proposed simplifying assumptions, seem unreliable.

The inclusion of plastic theory in the design specifications is mostly the work of the American plasticians, and their failure to meet and to counter, if possible, the closely defined objections of the opponents tends to cast further doubt on the validity and the applicability of their

theory. The continued research activity in the field of plastic design is no proof of its soundness, but is merely a testimonial to the tenacity of its proponents and to the availability of liberal funds.

The writer feels that the author's characterization of the method of plastic design as "by no means complete" is much too moderate.

- (50) A. Hrennikoff. Weaknesses of the Theory of Plastic Design. The Engineering Journal (Engineering Institute of Canada, Montreal), November 1961 and July 1962.
- (51) A. Hrennikoff. Plastic and Elastic Designs Compared. Preliminary Publication. Seventh Congress, Rio de Janeiro, 1964. International Association for Bridge and Structural Engineering.
- (52) A. Hrennikoff. The Present Status of Plastic Design. The Engineering Journal (E.I. of Canada, Montreal), November 1965 and April 1966.
- (53) A. Hrennikoff. Discussion. Generalized Approximate Method of Assessing the Effect of Deformation on Failure Loads by M.R. Horne. Seventh Congress, Rio de Janeiro, 1964. I.A.B.S.E.

SUMMARY

Although in principle the plastic design in steel is comparable to the conventional elastic design, in actuality it is inferior to it for several reasons, including the inability to analyze different types of buckling failure, the difficulty with the live load action and the wide variability of the plastic properties in the same grade of the material. The existence in the specifications of two distinct but equally acceptable methods of design the elastic and the plastic, leading to different solutions, is unsound.

RÉSUMÉ

En principe, l'analyse plastique et l'analyse élastique conventionnelle se valent dans la construction métallique. En fait, l'analyse plastique est inférieure à bien des égards: Par exemple par son incapacité d'analyser plusieurs types de ruine par voilement, la difficulté qu'on a avec l'action de la charge de service et les grandes divergences des propriétés plastiques dans une même qualité de matériau. Il n'est donc guère justifié de parler du calcul élastique et du calcul plastique comme de deux méthodes également valables, mais conduisant à des résultats différents.

ZUSAMMENFASSUNG

Obwohl die plastische Berechnungsmethode im Stahlbau im allgemeinen mit der konventionellen, elastischen Methode vergleichbar ist, so ist sie ihr doch aus verschiedenen Gründen unterlegen, inbegriffen die Unfähigkeit, verschiedene Bruchformen aus Beulen zu analysieren, sowie die Schwierigkeit der Verkehrslastbewegung und die weite Streuung der Plastizitätswerte desselben Materials. Es ist also nicht stichhaltig, von zwei gleich annehmbaren, ebenbürtigen Berechnungsmethoden zu reden, nämlich der elastischen und der plastischen, die zu verschiedenen Ergebnissen führen.

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