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Thin-Walled Steel Hyperbolic Paraboloid Structures

Structures en tôle paraboloïdes hyperboliques

Dünnwandige hyperbolische Paraboloid-Stahltragwerke

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Attractive and economical hyperbolic paraboloid structures can be built of light gage steel elements (such as those shown in Fig. 1). An extensive investigation of the behavior of such structures is underway at Cornell University under the general direction of George Winter.

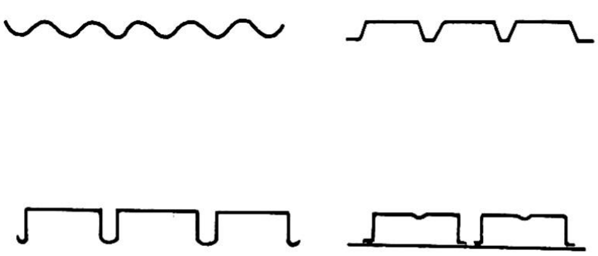

Fig. 1

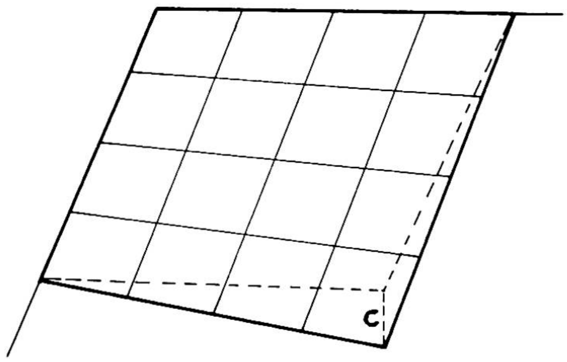

Fig. 2

Since a hypar is a warped surface generated by straight lines (Fig. 2), it can therefore be formed using light gage steel panels. These panels are laid side by side and warped individually. Welding or sheet metal screws are used to connect the panels to each other and the edges of the resulting shell decking to the edge members to form a hypar unit. Such units can be connected in a great variety of ways to produce interesting structures (Fig. 3). Often double layers are used with the deformations or corrugations of the two layers running perpendicular to each other. An example of a light gage hypar structure is shown in Fig. 4[1].

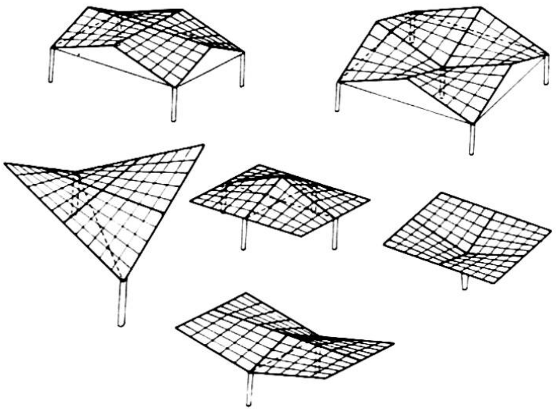

Fig. 3

Fig. 4

It is well known that, according to the membrane theory, hypars develop uniform shear stresses $S = pab/2c$ due to a uniform load p, where a and b are the side dimensions of the plan of a hypar unit and c is the rise of a corner with respect to an opposite edge as shown in Fig. 2. The equation describing the surface is $z = xyc/ab$. It was demonstrated in an extensive research program at Cornell University[2,3] (see also Report on Theme II/a) that light gage diaphragms can transmit uniform in-plane shear stresses. The ease of forming the surface and the satisfactory diaphragm action of the decking make the structure feasible and structurally efficient. With heavy decking and large edge member sizes, simple units of 50 ft by 50 ft (15 m by 15 m) or larger can be built. Thus, a 100 ft by 100 ft (30 m by 30 m) roof may be supported by a single central column. The dead load to live load ratio is exceptionally small for these structures.

Since the decking is connected at varying angles along the edge members, tubular edge members are convenient. However, a flange plate can be welded to other shapes and then warped (Fig. 5). For small structures edge members with an angle shape may be sufficient. The decking transmits uniform unit shear forces along the con-

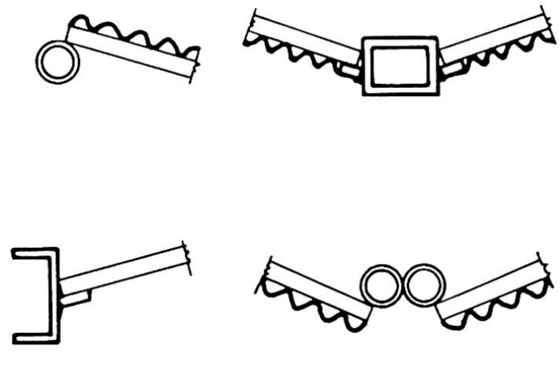

Fig. 5

nection, thus the edge member is loaded by an axial force varying linearly from zero at one end to a maximum tension or compression force at the other end.

Therefore, the preliminary design of light gage hypars consists of (1) the selection of the deck sizes to obtain the required rigidity, (2) the design of all the connections (either welds or screws) between the panels of the deck and between the deck and the edge members to transmit the membrane shears, (3) the design of the edge members. The forces in the edge members are easily calculated for the membrane stress condition. Care must be taken in applying the connections to insure that the welds or screws can take the shear forces.

The membrane stresses are usually small and the structure can carry remarkably heavy uniform loads. A small scale model of a hypar structure under loading is shown in Fig. 6.

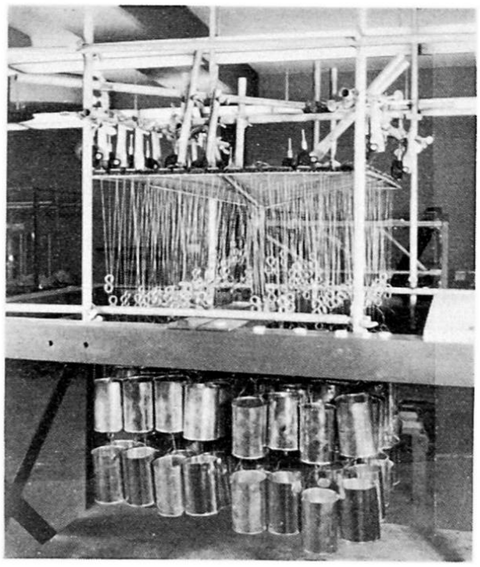

Fig. 6

The major problems in the design of light gage hypars involve deflections and buckling. The present investigation at Cornell University is concerned with these questions.

Light gage hypars may be rather flexible due to the small bending rigidity of the edge members and the possible low shear rigidity of the deck. Since both of these elements resist the deflections of the structure, it is not easy to determine the sizes required to limit deformations. For example, for an inverted umbrella shell (Fig. 7) the external corner deflection (A) is caused primarily by loads near this corner. This point shows practically no deflection for loads away from this corner. Usually the deflections of a free corner (such as point A) and the deflections of the center of the deck are of interest.

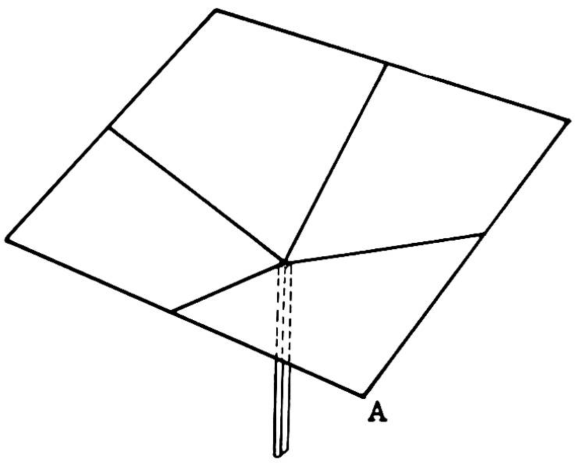

Fig. 7

It was found that for practical member sizes these deformations depend strongly on the effective shear rigidity of the deck (in addition to the proportions of the structure). The effective shear rigidity of a light gage steel deck is defined as the uniform edge force per unit length required to cause a unit angular distortion of the deck. Unless the rise (c) is relatively large (say $a/c < 2$), the shear rigidity obtained from flat diaphragm tests[3] may be used in the calculations.

The effective shear rigidity of light gage steel decking depends on several factors. If a single layer of sheet is used for the decking, the effective shear rigidity depends somewhat on the panel to panel connections and primarily on the spacing of the connectors (screws, welds) to the edge members (especially in the weak direction, i.e. transverse to the ribs or corrugations). It also varies with the plan dimensions. If two layers are used with the ribs of one layer running transverse to the ribs of the other layer, then the effective shear rigidity of the two layers also depends on the method of connection of the two layers to each other and to the edge members. If the edge members utilize warped flange plates for connection (Fig. 5) then each layer may be connected directly to the plate (one on each side of the plate) and the effective shear rigidity of the two layers in this case is approximately twice that for a single layer. However, if a single tubular edge member is used without flange plate, then only one layer of the decking can be attached directly to the tube and the second layer has to be attached to the first layer. In this case, the effective shear rigidity of the two layers is only about 50% larger than that for one layer because of the larger "eccentricity" of the shear being transferred to the edge members. Built-up box type edge members may be used so that each layer is connected directly to the edge members[4]. Box type edge members have the advantage of larger torsional rigidity.

The amount of interconnection of the two layers also strongly influences the shear rigidity of the deck. Tests were performed on 5 ft by 5 ft (1.5 m by 1.5 m) shells with continuously supported edge members. For each of the three rise to span ratios investigated (1/8, 1/5, 1/3) there was a 25% reduction in maximum deflections of the deck for the fully connected decks (sheet metal screws spacing equal to the pitch of the corrugated decking) as compared with the unconnected decks. It must be remembered, that it is usually necessary to interconnect the layers to some degree to prevent chatter due to wind.

If a single layer deck is used, large deformations may occur under localized loads since the deck cannot transmit the local loads by in-plane shear stresses and thus it has to distribute the load by plate bending along the ribs. Double layers are recommended if large local (concentrated) loads are expected. To illustrate: For each of the three rise to span ratios tested, the maximum deflection under a load covering 8 in by 12 in (20 cm by 30 cm) area at the center of the deck for two unconnected layers was only one third that for a single layer.

In the analysis of light gage steel hypars it is more important to separate the action of the edge members and the deck than it is in the analysis of reinforced concrete hypars. The eccentric forces on the edge members have to be considered. Two approaches have been followed in the investigation at Cornell University: the finite element method, and the numerical solution of the shallow shell equations together with the differential equations of flexure

$$R_1 \partial^4 F/\partial y^4 + R_2 \partial^4 F/\partial x^2 \partial y^2 + R_3 \partial^4 F/\partial x^4$$
$$= Eh[(2c/ab)\partial^2 w/\partial x \partial y + (\partial^2 w/\partial x \partial y)^2 - (\partial^2 w/\partial x^2)(\partial^2 w/\partial y^2)]$$

$$D[R_4 \partial^4 w/\partial x^4 + R_5 \partial^4 w/\partial x^2 \partial y^2 + R_6 \partial^4 w/\partial y^4] = -p$$
$$+ (\partial^2 w/\partial x^2)(\partial^2 F/\partial y^2) + (\partial^2 w/\partial y^2)(\partial^2 F/\partial x^2)$$
$$- 2(c/ab + \partial^2 w/\partial x \partial y)(\partial^2 F/\partial x \partial y)$$

and torsion for the edge members. Both of these approaches include the orthotropic properties of the shell. For the finite element procedure a twelve term polynomial and rectangular plate elements are used. The flexural rigidity of the edge members and the eccentricity of the edge shears are included.

For the calculation of the approximate deflection of the deck relative to simply supported edges the shallow shell equations or energy methods can be applied. However, the deflection of the edge members under the eccentric tangential shear forces is complicated and thus the above numerical-computer procedures are used. As a result of these analyses, the required deck sizes and connection spacings can be determined to limit deflections and to achieve the necessary strength.

Buckling of hypar structures may occur in one of two ways: either the deck buckles and the edge members remain essentially straight; or the compression edge members may buckle which actually means the collapse of the structure. Strictly speaking the buckling of the edge members is an inelastic beam-column deformation or stress phenomenon rather than an eigenvalue problem. However, the failure is abrupt. For example, an inverted umbrella roof (Fig. 7) may suddenly turn inside out.

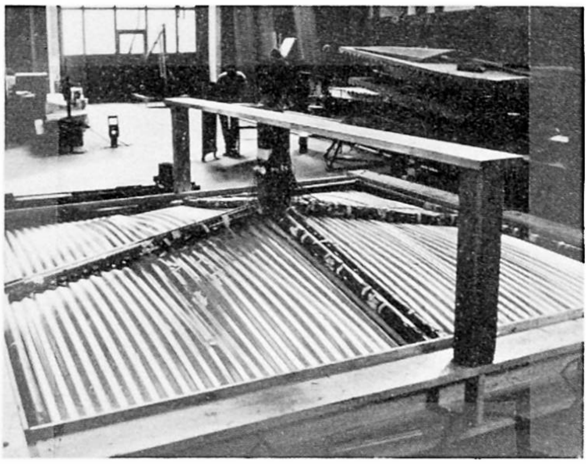

Fig. 8

The hypar surface is made up of two sets of orthogonal parabolas, one set behave as arches, the other as cables. If the load reaches the critical value, the shell buckles in a number of waves along the compression arches (Fig. 8). The buckling load for uniform loading and simply supported isotropic shells (rigid edge members) was calculated by Reissner[5]. His theory was generalized for orthotropic shells to be applicable for light gage steel hypars. The resulting equation has the following form

$$p_{cr} = 4(c/ab)^2 \sqrt{EhD} \sqrt{(R_4+R_5+R_6)/(R_1+R_2+R_3)}$$

where h is the thickness of the shell, E is the modulus of elasticity, D is the plate flexural rigidity, and the constants R depend on the shear and flexural rigidities of the orthotropic deck. For isotropic shells $R_1+R_2+R_3 = R_4+R_5+R_6 = 4$. It was found that the boundary conditions and the edge member flexural rigidity about either axis are not very significant. According to the work by Leet[6] on plastic models, the axial stiffness of the edge member does have an appreciable influence on the initial membrane buckling load. The effective shear rigidity of the deck is a very important factor and must be known with reasonable accuracy.

Considerable post-buckling strength exists provided that the deck is well fastened to the edge members which must have sufficient bending rigidity in the plane tangent to the shell at the edge; in such a case an appreciable tension field can develop in the shell along the ribs.

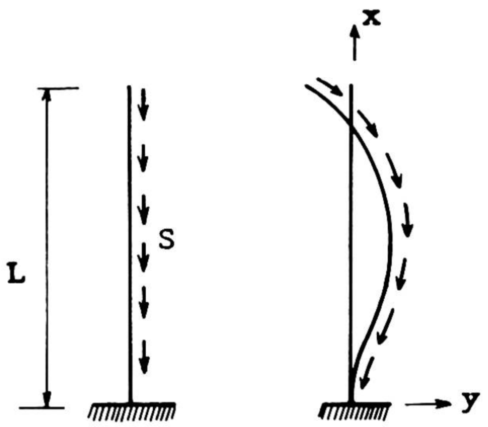

Fig. 9

The "buckling" of the compression edge members is caused by the eccentric tangential shear forces. The stability of a column under the so-called "follower forces" (Fig. 9) is a non-conservative problem and dynamic analysis is necessary. The differential equation is $EI.\partial^4y/\partial x^4 + S(L-x)\partial^2y/\partial x^2 + m\partial^2y/\partial t^2 = 0$. The results of such calculations for various limiting boundary conditions were found to bracket the experimental collapse loads for the deck and edge member sizes tested.

The finite element method or the numerical solution of the orthotropic equations give the load-deflection curves and also the smaller of the deck buckling or collapse loads. Therefore, design curves can be prepared based on deflection and stress limitations and on adequate safety factors against deck buckling or collapse. Extensive use of light gage steel hyperbolic paraboloid structures is expected in the future for such applications as service stations, small office buildings, vacation homes, factories, schools and canopies.

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