Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 8 (1968)

Artikel: Beams and columns braced by thin-walled steel diaphragms

Autor: Errera, S.J. / Apparao, T.V.S.R

DOI: https://doi.org/10.5169/seals-8758

IIa

DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Beams and Columns Braced by Thin-Walled Steel Diaphragms

Poutres et piliers renforcés par des tôles transversales minces

Durch dünnwandige Stahlquerscheiben versteifte Träger und Stützen

S.J. ERRERA and T.V.S.R. APPARAO
Department of Structural Engineering
Cornell University
Ithaca, New York

INTRODUCTION

Thin-walled corrugated sheets or ribbed steel panels are often used as side wall sheathing, roof decking or floor decking of steel framed buildings. An illustration of such a building is shown in Fig. 1, and some examples of the cross sectional shapes of available steel panels are shown in Fig. 2. These panels carry

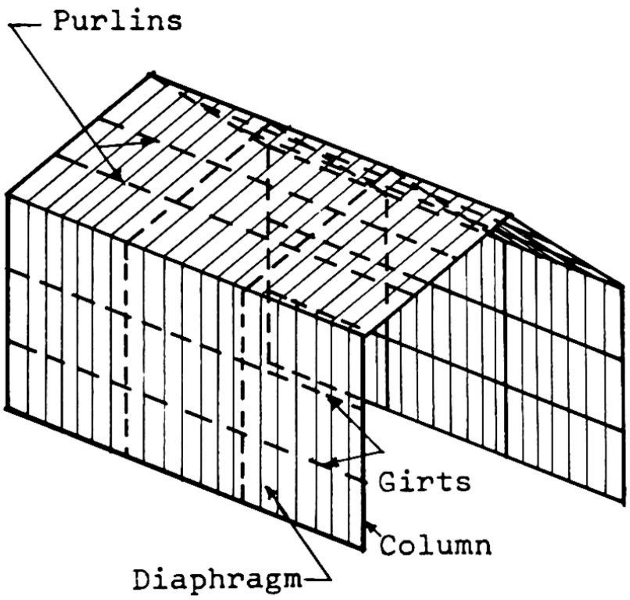


Fig. 1 A Steel Framed Building

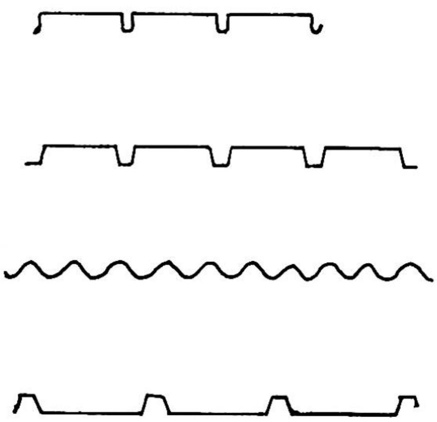


Fig. 2 Panel Configurations

loads normal to their planes (such as wind, snow or floor loads) by virtue of their bending strength in that plane, and transmit these loads to the building frame. In addition, as mentioned in

Professor Winter's report on Theme IIa, because of their in-plane shear rigidity, these interconnected panels act as shear-resistant diaphragms, and can provide restraint to the members of the steel frame against buckling in the plane of the diaphragm.

Fig. 3 shows a pair of beams or columns with attached dia-

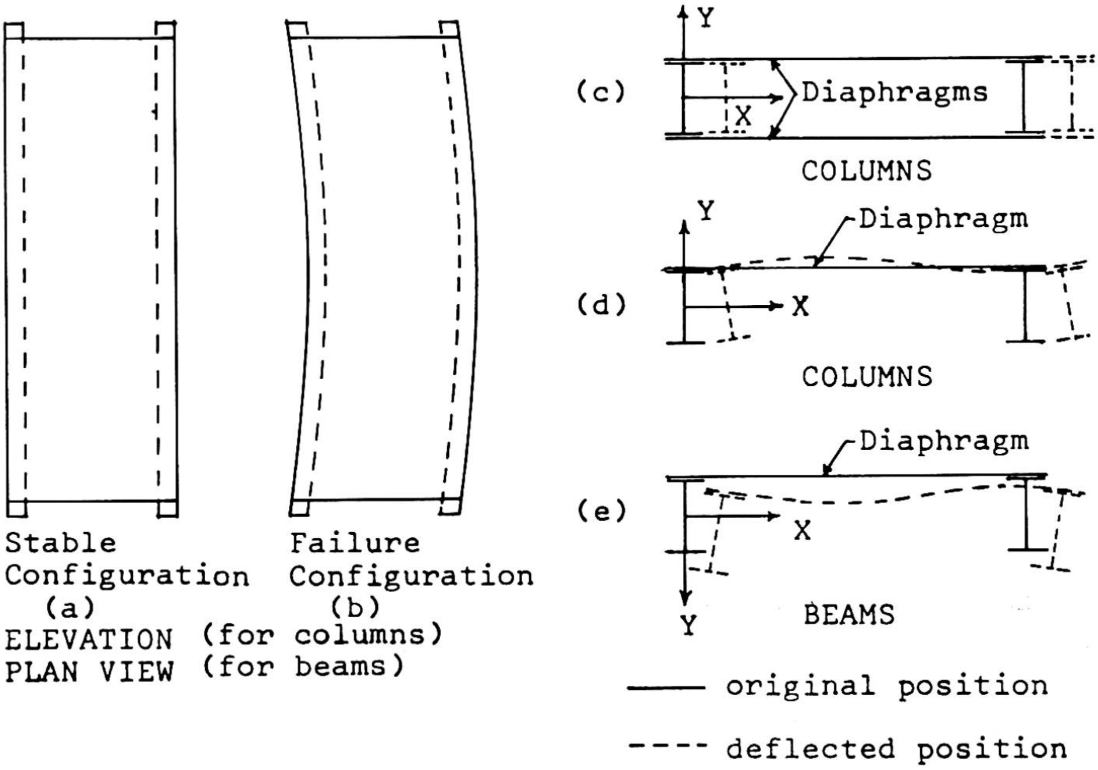


Fig. 3 Diaphragm-Braced Columns or Beams

phragm (a) with no load, and (b) in an assumed buckled shape. A diaphragm that is attached to two or more parallel members and constrained to deform with the members as they deflect laterally (Fig. 3b) is in a state of shear. Properly connected diaphragms are quite effective in resisting shear, and therefore can be very efficient in bracing beams, joists or purlins, against lateral torsional-flexural buckling. Similarly, buckling loads or failure loads of columns can be increased by either (1) bracing the columns along their length with diaphragms or (2) bracing the columns with girts which in turn are braced by a shear diaphragm, as shown in Figs. 1 and 4, and hereafter called diaphragm-girt bracing.

Fig. 4 Columns with Diaphragm-Girt Bracing

This report presents a summary of the results of an investigation at Cornell University on the effectiveness of thin-walled steel diaphragms as bracing for columns and beams. Comparisons of theoretically computed buckling loads and experimental failure loads are presented. Theoretically predicted deflections considering initial imperfections of columns, and based on small deflection theory, are compared with experimental deflections for a typical case.

CHARACTERISTICS OF THE DIAPHRAGM

In the present discussion, the two important characteristics of the diaphragm are its shear rigidity and its shear strength. No reliable theory has yet been developed to predict the rigidity and strength of a diaphragm; however, a standard procedure for testing a diaphragm in shear and for computing the rigidity and strength from its load-deflection relationship has been established.(1) (Superscripts indicate Reference Numbers).

For light gage steel diaphragms the effective shear modulus may be denoted as G_{eff},(2) expressed in force per unit of cross-sectional area. The shear rigidity Q is then expressed as

$$Q = A_d \, G_{eff} \tag{1}$$

where A_d is the cross-sectional area of the diaphragm contributing to the support of one member.

THEORY

<u>General</u> - Theory of diaphragm-braced members has been developed to cover the following:

1. the critical moment for diaphragm-braced I-,(2) channel(3) and Z-section(3) beams subjected to uniform moment
2. the buckling load for axially-loaded I-section columns(4),(5) directly braced by diaphragms on one or both flanges
3. the buckling load for axially loaded I-section columns with diaphragm-girt bracing(3)
4. load-deflection relationships(3) for diaphragm-braced members with initial imperfections
5. failure loads(3) of diaphragm-braced members with imperfections, based on stipulated failure criteria.

Two analysis methods will be illustrated by particular cases; one based on equilibrium, the other based on an energy method.

Diaphragm-Braced Beams (Equilibrium Method) - Diaphragm-braced I- and channel section beams under uniform moment in the YZ plane (Fig. 3e) bend in the Y direction before they buckle laterally. Due to initial imperfections, real, imperfect beams deflect laterally and twist even at moments below the buckling moments. To determine the precritical deflections considering these initial imperfections, equilibrium equations of a diaphragm-braced I or channel beam may be written as:

$$EI_x v^{iv} = 0 \tag{2}$$

$$EI_y u^{iv} - Q(u''+e\beta'') + M_x(\beta''+\beta_o'') = 0 \tag{3}$$

$$GK\beta'' - E\Gamma\beta^{iv} + Qe(u''+e\beta'') - F\beta - M_x(u''+u_o'') = 0 \tag{4}$$

where EI_x is the bending rigidity about the x-axis
EI_y is the bending rigidity about the y-axis
$E\Gamma$ is the warping rigidity
GK is the torsional rigidity
F is a parameter of the cross-bending rigidity of the diaphragm(2)
e is the distance from the center of gravity of the beam section to the plane of the diaphragm
M_x is the moment about the X-axis
u_o and β_o are the initial deflections in the X-direction and about the Z-axis respectively
v is the deflection in the Y direction
and u and β are the additional deflections corresponding to u_o and β_o respectively.

Knowing the initial deflections, the above equations 2, 3 and 4 can be solved to obtain load-deflection relationships of the diaphragm-braced beam. Further, if the initial imperfections are set equal to zero, the non-trivial solution of the resulting homogeneous equations gives the lateral torsional-flexural buckling moment of the diaphragm-braced beam. Following the above procedure, the buckling moment M_{cr}, the amplitude of the additional lateral deflection C of the centroidal axis in the X-direction, and the amplitude of the additional twist D of a diaphragm-braced I- or channel beam are given by:

$$M_{cr} = \sqrt{\{EI_y(\tfrac{n\pi}{L})^2 + Q\}\{E\Gamma(\tfrac{n\pi}{L})^2 + GK+Qe^2\}} + Qe \tag{5}$$

$$C = \frac{M\delta_\beta\{E\Gamma(\frac{n\pi}{L})^2 + GK + Qe^2\} + M\delta_u\{M-Qe\}}{\{EI_y(\frac{n\pi}{L})^2 + Q\}\{E\Gamma(\frac{n\pi}{L})^2 + GK + Qe^2\}-(M-Qe)^2} \tag{6}$$

$$D = \frac{M\delta_u\{EI_y(\frac{n\pi}{L})^2 + Q\} + M\delta_\beta(M-Qe)}{\{EI_y(\frac{n\pi}{L})^2 + Q\}\{E\Gamma(\frac{n\pi}{L})^2 + GK + Qe^2\}-(M-Qe)^2} \tag{7}$$

where L is the total length of the member

n = 1 or 2 depending on whether the ends of the beam are "simply supported" or "fixed" against lateral deflection

δ_u and δ_β are the amplitudes of the initial lateral deflection and twist of the beam respectively (initial deflections are assumed to be affine to the buckled shape of an ideal beam)

and M is the moment under consideration.

The cross-bending rigidity of the diaphragm is neglected in Eqs. 5, 6 and 7.

Diaphragm-Braced Columns (Energy Method) - The change in total energy U from the compressed stable position to the compressed and deflected unstable position of the portion of the system related to one column of a diaphragm-braced column assembly is formulated below. The total energy U consists of strain energy V of the column, potential energy U_w of the axial load, and energy B_s due to shear in the diaphragm. Thus the total energy is given by

$$U = \frac{1}{2}\int_0^L \{EI_y u''^2 + GK\beta'^2 + E\Gamma\beta''^2 - P(u_t'^2 - u_o'^2) - P\frac{I_P}{A}(\beta_t'^2 - \beta_o'^2) + Q\,(u'^2 + e^2\beta'^2)\}dz \tag{8}$$

where $u_t = u_o + u$, and $\beta_t = \beta_o + \beta$. The energy due to the cross-bending rigidity of the diaphragm is neglected. Deflections of a diaphragm-braced column at precritical loads are obtained by solving the equations resulting from minimization of U using the Rayleigh-Ritz technique.

If the diaphragm bracing is symmetric (Fig. 3c) the terms with derivatives of β in Eq. 8 will vanish, and the load-deflection relationship and buckling load P_{cr} of a column are given by:

$$\{EI_y(\frac{n\pi}{L})^2 - P + Q\}\ C = PE_{10} \tag{9}$$

$$P_{cr} = EI_y\left(\frac{n\pi}{L}\right)^2 + Q \tag{10}$$

where E_{10} is the amplitude of initial lateral deflection of the centroidal axis and P is the axial load on the column.

If columns are braced on one flange only (Fig. 3d), the load-deflection relationship for the column is given by:

$$\begin{bmatrix} EI_y\left(\frac{n\pi}{L}\right)^2 + Q-P & Qe \\ Qe & E\Gamma\left(\frac{n\pi}{L}\right)^2 + GK + Qe^2 \end{bmatrix} \begin{Bmatrix} C \\ D \end{Bmatrix} = P \begin{Bmatrix} E_{10} \\ \frac{I_p}{A} F_o \end{Bmatrix} \tag{11}$$

where F_o is the amplitude of initial twist of the column. The buckling load P_{cr} of the column is obtained by letting the initial deflection equal zero and solving the resulting homogeneous equations for the non-trivial solutions; the smaller value of P from the solutions gives P_{cr}. Eqs. 9 and 11 are obtained by assuming either sine or cosine functions for the deflection pattern in the first mode, depending on whether the ends of the column are "hinged" or "fixed" respectively.

Columns with Diaphragm-Girt Bracing - The method of solution to obtain buckling loads and deflections at precritical loads for this case is similar to that employed in the case of diaphragm-braced columns, and is described in detail in Reference 3. For a column with j intermediate girts there are (j+2) possible modes of buckling, including sidesway. The actual mode of buckling depends on the diaphragm rigidity Q, the bending stiffness of the girts, m, the eccentricity e of bracing, and the geometry of the column assembly. For example, for a column with one intermediate girt and with "hinged" ends (i.e. flexurally hinged, and warping unrestrained) possible modes of buckling are shown in Fig. 5 and possible types of elastic behavior of the column are shown in Fig. 6. In Fig. 6 each curve in each graph represents a particular failure mode. The governing critical load and corresponding critical mode are given by the solid curves.

Inelastic Theory - In all cases the equations obtained in the elastic range can be modified by replacing the modulus of elasticity E by a suitable reduced modulus E_r, and the shear modulus G by a corresponding reduced modulus G_r to describe the behavior in the inelastic range.[3]

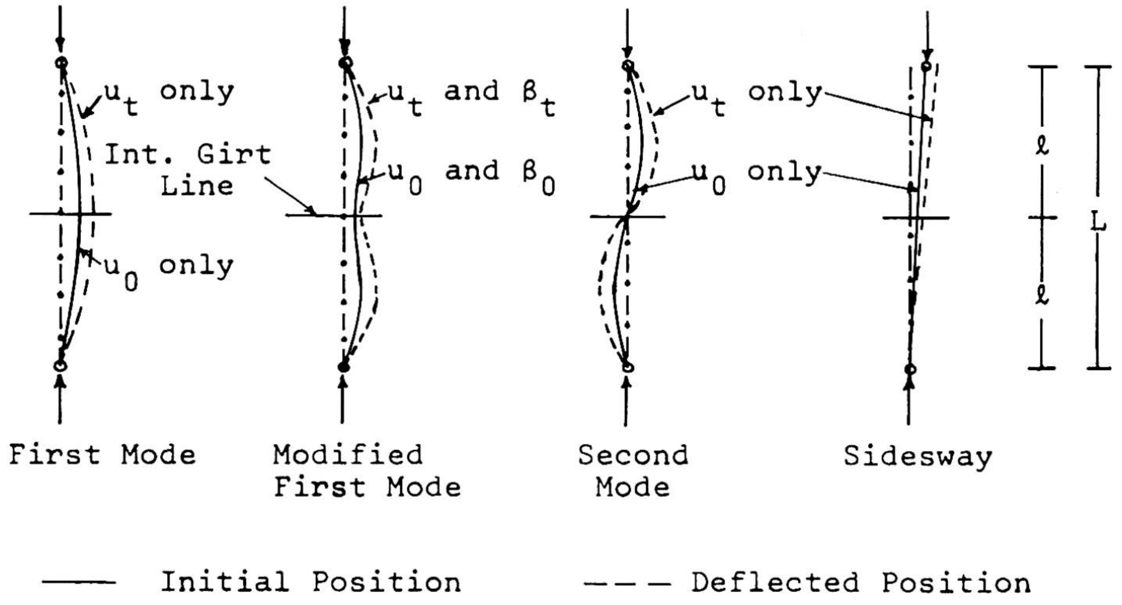
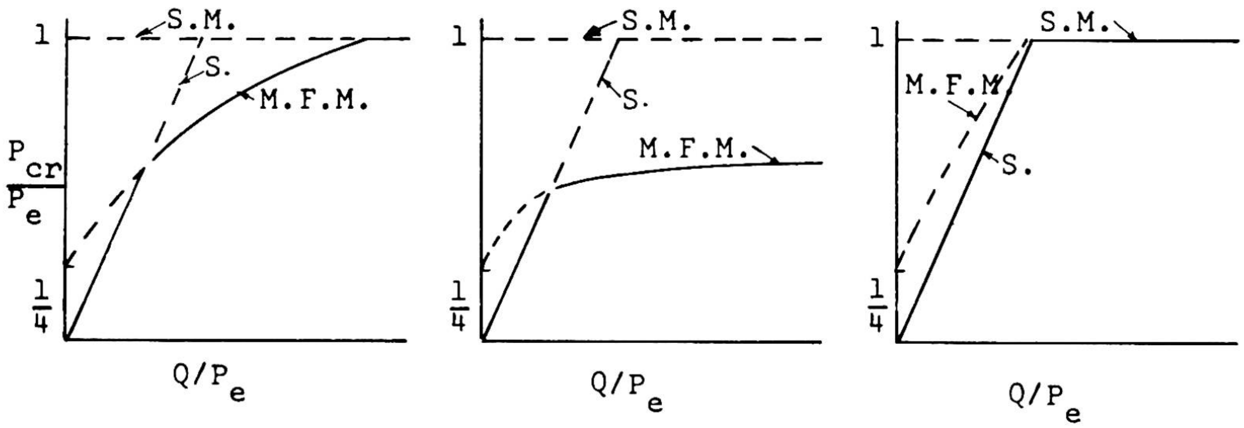


Fig. 5 Buckling Pattern of an Imperfect Column with One Intermediate Girt



Legend: M.F.M. - Modified First Mode, S.M. - Second Mode, S. - Sidesway, as shown in Fig. 5; $P_e = \frac{\pi^2 EI_y}{\ell^2}$

Fig. 6 Possible Types of Behavior of an Elastic Column with one Intermediate Girt

THEORETICAL FAILURE LOADS

Actual failure loads of beams and columns are usually smaller than the theoretical buckling loads because of initial imperfections and consequent deflections at precritical loads. The deflections of diaphragm-braced beam and column assemblies can be computed using the methods described above; therefore, theoretical failure loads can be computed if appropriate failure criteria are established for the beams, columns, diaphragm and girts. For example, it may be assumed that a column with continuous diaphragm bracing fails when

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \geq 1 \tag{12}$$

where f_a is the computed axial stress, P/A

f_b is the computed bending stress at the critical section, including the effect of additional deflection under axial load

F_a is the buckling stress

and F_y is the yield stress.

Then, knowing the deflections of a column at a given load, one can compute the axial and bending stresses in the column, and compare the results with Eq. 12 to determine if column failure will occur at that load. Similarly, the computed deflections can be used to determine the shear strain in the diaphragm, which can be compared with the shear strain that would cause failure of the diaphragm. Computation of theoretical failure loads of beams and columns is illustrated in Ref. 3.

EXPERIMENTAL RESULTS

General - To corroborate the theoretical results, tests were conducted on 35 diaphragm-braced beam and column assemblies all using 50 ksi nominal yield strength steel. Several types of light gage steel diaphragms were used; these were fastened to the members by power driven pins or sheet metal screws. In each case, G_{eff} for the diaphragm was determined experimentally, and the shear rigidity Q was calculated from Eq. 1. To obtain a conservative estimate of the theoretical deflections at pre-failure loads, the amplitudes of the initial deflections used in the calculations were the largest measured values irrespective of their location.

Diaphragm-Braced Beams - Beam tests included pairs of 8JR6.5 and 10B17 I-sections, 8[6.2 sections, and an assembly of four 8JR6.5 beams. Beam spacing ranged from 1'-3" to 3'-8", (38 cm to 112 cm) and lengths ranged from 20' to 30' (6.1 m to 9.15 m) with ends fixed against lateral bending. A comparison of the theoretical

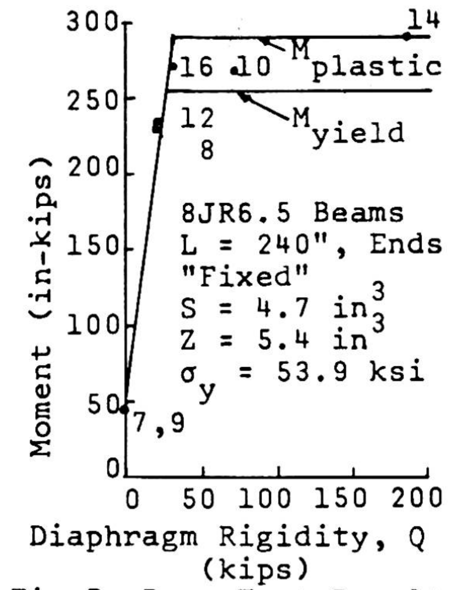


Fig.7 Beam Test Results and Predicted Values

buckling loads with test failure loads for 8JR6.5 assemblies is shown in Fig. 7. The moments sustained by the diaphragm-braced beams were always more than 80% of predicted buckling values for ideal members, and as high as six times the buckling moments of corresponding unbraced beams.

Diaphragm-Braced Columns - Tests were conducted on assemblies of pairs of 4I7.7 columns braced by diaphragms on one or both flanges. Spacing of columns ranged from about 14" to 17" (36 cm to 43 cm), and slenderness ratios ranged from 280 to 50. A comparison of theoretically predicted buckling loads with the test failure loads is shown in Fig. 8. Failure loads were 80% to 98% of predicted buckling loads for ideal members and as high as ten times the buckling loads of unbraced columns.

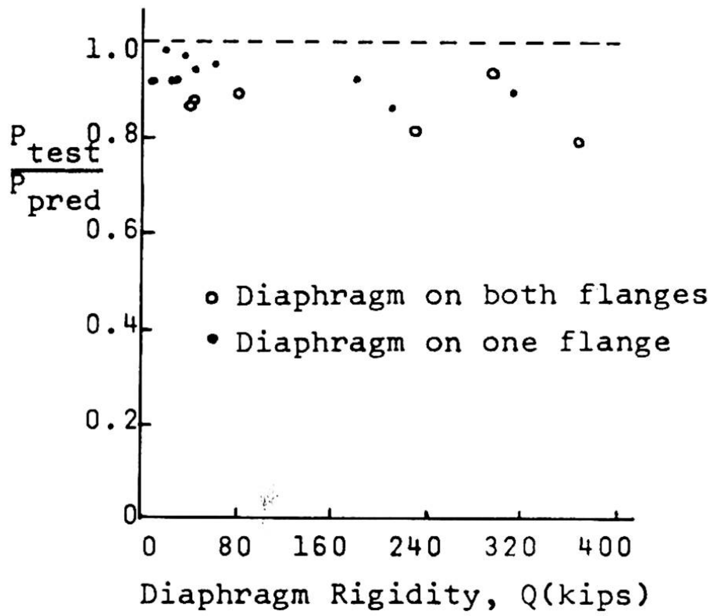


Fig. 8. Comparison of Theory and Column Test Results

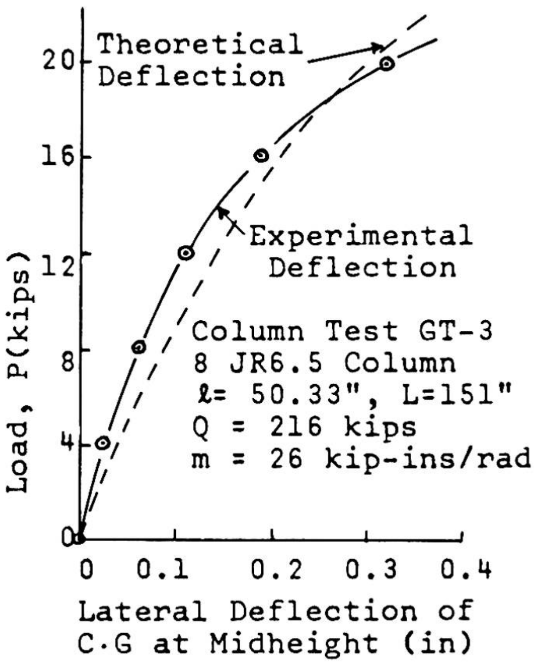


Fig. 9 Load-Deflection Curves

Columns with Diaphragm-Girt Bracing - Tests were made on three assemblies of axially-loaded pairs of 8JR6.5 I-section columns braced by two intermediate girts and a 26 gage standard corrugated steel diaphragm. Columns were spaced 6' (1.8 m) apart and were

12'-7" (3.8 m) high. Theoretical buckling loads and experimental failure loads for the three assemblies are given in Table 1, and

TABLE 1. Comparison of Test Failure Loads with Predicted Buckling Loads of Columns with Diaphragm-Girt Bracing

Test	Twist Restraint, m(kip-in/rad.)	Failure Mode	Distance, e (in)	Predicted Critical Load (kips)	Max Test Load (kips)	Test/Pred.
GT-1	0	Modified First Mode (Tor-Flex)	6	21.4	17.7	0.84
GT-2	7750	Third Mode (Flexural)	10	39.9	37.3	0.94
GT-3	13	Modified First Mode (Tor-Flex)	6	29.6	25.5	0.86

the load-deflection relationships for a particular assembly are shown in Fig. 9. Failure loads were 84% to 94% of predicted buckling loads, and as high as nine times the buckling load of an unbraced column. Deflections predicted by the theory give a conservative estimate of the experimental deflections (Fig. 9), except at very high loads where secondary effects of large deflections began to prevail. A photograph of assembly GT-2 after test appears in Fig. 10. It is evident that failure was by buckling of the column over one-third its total length.

FURTHER STUDIES

Recent efforts have been directed toward assessing the practical implications and applications of diaphragm bracing. The dimensions and member sizes of the column-girt-diaphragm assembly of a typical steel building with metal sheathing is shown in Fig. 11. As ordinarily fastened, the diaphragm bracing has a Q-value of approximately 1000 kips. Calculations show that this combination of girts and panels is adequate to fully brace the columns, where "full bracing" is equivalent to the bracing that would be provided by an infinitely rigid diaphragm. In other cases, minor

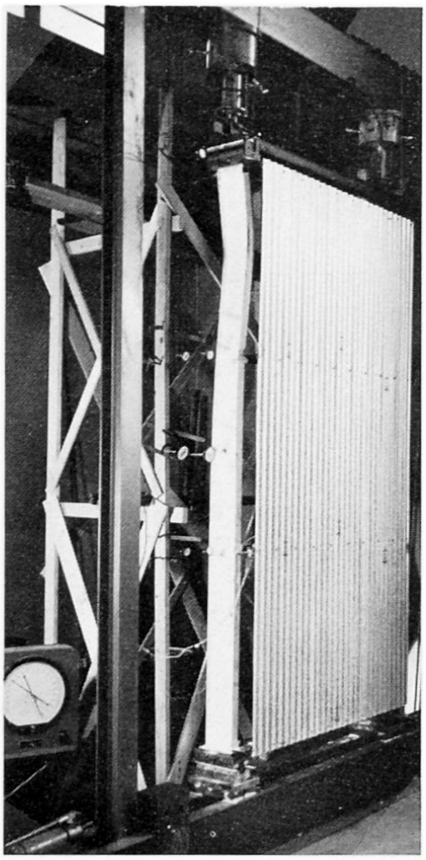


Fig. 10 Column-Girt-Diaphragm Assembly After Test GT-2

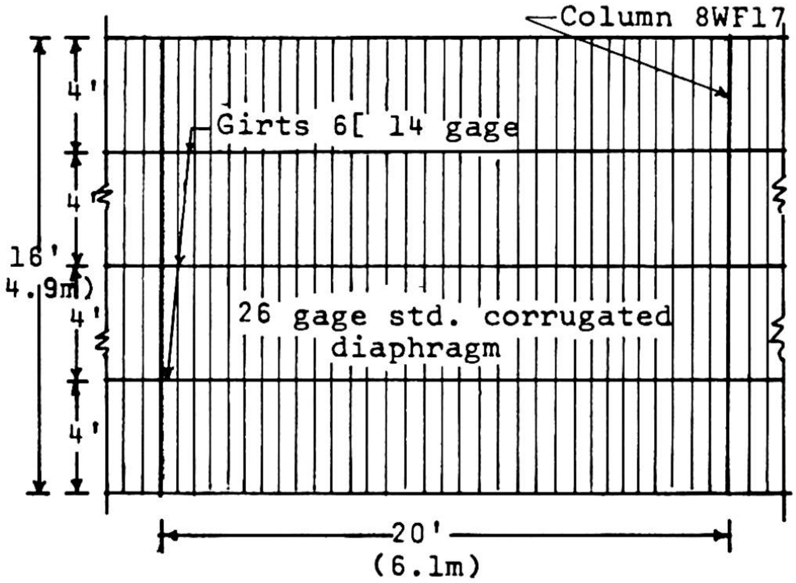


Fig. 11 Column-Girt-Diaphragm Assembly of a Typical Steel Building

modifications in construction practice may be required to achieve full bracing. Other studies have shown similar results for most practical sizes of beams braced by diaphragms in this type of construction. In such cases design loads may be increased, or other types of lateral bracing can be safely omitted.

CONCLUSIONS

Theory and test results both indicate that properly connected diaphragms can be very effective as bracing for beams and columns. In tests of actual assemblies, 80% to 100% of the theoretical critical load was obtained, with failure loads up to 10 times the critical load for the same member without diaphragm bracing.

This information permits the more economical design of: wall columns connected directly to steel sheathing; wall columns connected to girts which in turn are connected to steel sheathing; roof trusses connected directly or through purlins to roof decking; and floor beams and joists connected to light steel flooring. In all these cases buckling in the plane of the panels can be prevented by utilizing the shear rigidity of the ribbed thin diaphragm which, as wall, roof or floor, must be present in any event and, therefore, is available at no extra cost.

ACKNOWLEDGMENTS

This paper is an outcome of an investigation sponsored at Cornell University originally by the American Institute of Steel Construction and later by the American Iron and Steel Institute.

REFERENCES

1. "Design of Light Gage Steel Diaphragms", by the American Iron and Steel Institute, New York, N.Y.
2. Errera, S. J., "Buckling of Beams and Columns Braced by Shear-Rigid Diaphragms", Ph.D. Dissertation, Cornell University, 1965.
3. Apparao, T.V.S.R., "Problems in Structural Diaphragm Bracing", Ph.D. Dissertation, Cornell University, 1968.
4. Pincus, G., "The Performance of Columns and Beams Continuously Braced with Diaphragms", Ph.D. Dissertation, Cornell University, 1963.
5. Errera, S. J., Pincus, G. and Fisher, G. P., "Columns and Beams Braced by Diaphragms", Proceedings ASCE, Vol. 93, ST1, February 1967.

SUMMARY

Thin-walled corrugated sheets or ribbed steel panels often are used as side wall sheating, roof decking or floor decking of steel framed buildings. These interconnected panels act as shear-resistant diaphragms, and can restrain the members of the steel frame against buckling in the plane of the diaphragm. Theory and confirming test results are presented for columns and beams braced directly by diaphragms, and also for columns braced by girts which in turn are braced by diaphragms.

RÉSUMÉ

Souvent des tôles ondulées ou profilées servent au revêtement des fassades ou à la couverture des toits et planchers des portiques à étages multiples en acier. Ces tôles, solidement liées entre elles, forment une dalle stabilisante, retenant la construction et pouvant empêcher un flambement dans leur plan. La théorie et les résultats expérimentaux sont présentés ici pour les deux cas possibles, les tôles stabilisant directement poutres et piliers, ou stabilisant seulement les poutres, lesquelles retiennent les piliers à leur tour.

ZUSAMMENFASSUNG

Häufig werden Well- und Rippenblech als Fassadenbekleidung, Dach- und Fussbodeneindeckung von Stahlrahmenkonstruktionen verwendet. Diese Bleche wirken als stabilisierende Querscheiben und können das Knicken der Stahlrahmen in ihrer Ebene verhindern. Theorie und Testresultate werden dargestellt sowohl für direkte Verstärkung von Stützen und Balken als auch für verstärkte Träger, die ihrerseits dann auch die Stützen halten.