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Autor: Neville, A.M.

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Shear Strength of Continuous Reinforced and Prestressed Concrete Beams

Résistance au cisaillement des poutres continues en béton armé et précontraint

Die Schubfestigkeit kontinuierlicher Stahlbeton- und vorgespannter Balken

A. M. NEVILLE
Professor, M. C., T. D., M. Sc., Ph. D., A. M. I. C. E., A. M. I. Struct. E., M. ASCE, M. E. I. C.
Calgary, Canada

Notation

∅	=	diameter.
ton	=	English ton (= 2240 lb.).
r	=	percentage area of web reinforcement.
v	=	nominal shearing stress $= \frac{V}{b\,j\,d}$.
a	=	shear span.

Introduction

Extending previous studies on the shear strength of concrete beams, a part of which was reported at the 6th Congress [1], continuous beams have been investigated with a special reference to the moment-shear interaction at the ultimate. In continuous beams there are regions in which the bending moment changes from a high positive to a high negative value, with a concomitant high shear. In the vicinity of the point of contraflexure there exists the unique combination of a high shear and no applied direct stress. There is thus no flexural cracking there and shear design formulae derived for a cracked section cannot be used.

Beams with Orthogonal Web Reinforcement

As a continuation of a study of beams with orthogonal web reinforcement [2], continuous beams covering two spans of 4′—4 $\frac{1}{2}$″ each were tested. They had a cross-section of 4″ by 10″, with (plain) main tension and compression reinforcement area ratio, p, of 2.4%. The shear span-effective depth ratio, a/d, was 2.2. When $\frac{1}{4}$″ ∅ stirrups were spaced at 4″ centres no significant increase in ultimate load was caused by the addition of two $\frac{1}{4}$″ ∅ horizontal bars at the level of the neutral axis, all beams failing in diagonal tension. Nevertheless, the widening of the diagonal tension crack was delayed somewhat and a better distribution of cracks was obtained. The initial cracking load was not affected since, as is well-known, web reinforcement plays no role until diagonal cracking has started. On the other hand, shrinkage markedly affects the cracking load and has to be carefully controlled.

When the stirrups were spaced at $2\frac{1}{2}''$ centres (which corresponds to $r = 0.98\%$) in the zone between the load point and the centre support (where the shear was greatest), and at 4″ centres elsewhere, the addition of one layer of horizontal web reinforcement resulted in practically no improvement when the concrete strength was 4400 lb./in.^{2}. However, with a 5500 lb./in.^{2} concrete an increase in ultimate load of approximately 15% was observed on addition of $\frac{1}{4}'' \varnothing$ horizontal bars, and a further increase when $\frac{1}{2}'' \varnothing$ horizontal web reinforcement was used. This beam failed at a load equal to the ultimate flexural capacity (which is slightly increased by the additional steel) and a full redistribution of moments had taken place. Fig. 1 shows the failure patterns.

From a series of tests it seems that when stirrups are inadequate because of too large a spacing the addition of horizontal web reinforcement will not remedy the situation. With a smaller stirrup spacing [approximately $1/2d$ $(1-k)$] the use of orthogonal reinforcement is advantageous if very high strength concrete is used: the widening of the diagonal tension crack is delayed so that the horizontal steel plays a greater role in resisting inclined tension. The use of larger horizontal bars effects little improvement and in fact these bars did not yield while stirrups did, but arrangement of small bars in two layers, one at the level of the neutral axis, the other half-way towards the tension steel, is more efficient. It is worth noting that the weight of the $\frac{1}{4}'' \varnothing$ horizontal web reinforcement is less than 18% of the weight of the stirrups alone, and of course only a small fraction of the total weight of reinforcement. If the same weight of steel were used in the form of stirrups the influence on the shear capacity of the beam would be smaller. The fixing of horizontal bars is extremely simple so that no large labour cost is involved. It should be stressed though that horizontal bars are not a substitute for adequately spaced stirrups.

Referring to beams with stirrups only, it is interesting to observe that even though they failed in shear they withstood a higher nominal shearing stress (600 lb./in.^{2} when $r = 0.61\%$, and 670 lb./in.^{2} when $r = 0.98\%$) than simply supported beams of similar properties and with the same values of r (500 and 530 lb./in.^{2} respectively). We should note, however, that for the same applied shear the bending moment in a continuous beam is smaller than in a simply supported beam.

Beam B 19 of Fig. 1 failed in shear but reached its flexural load capacity. In this connection it is interesting to observe that under the action of moment and shear the concrete is subjected to biaxial compression and hence its

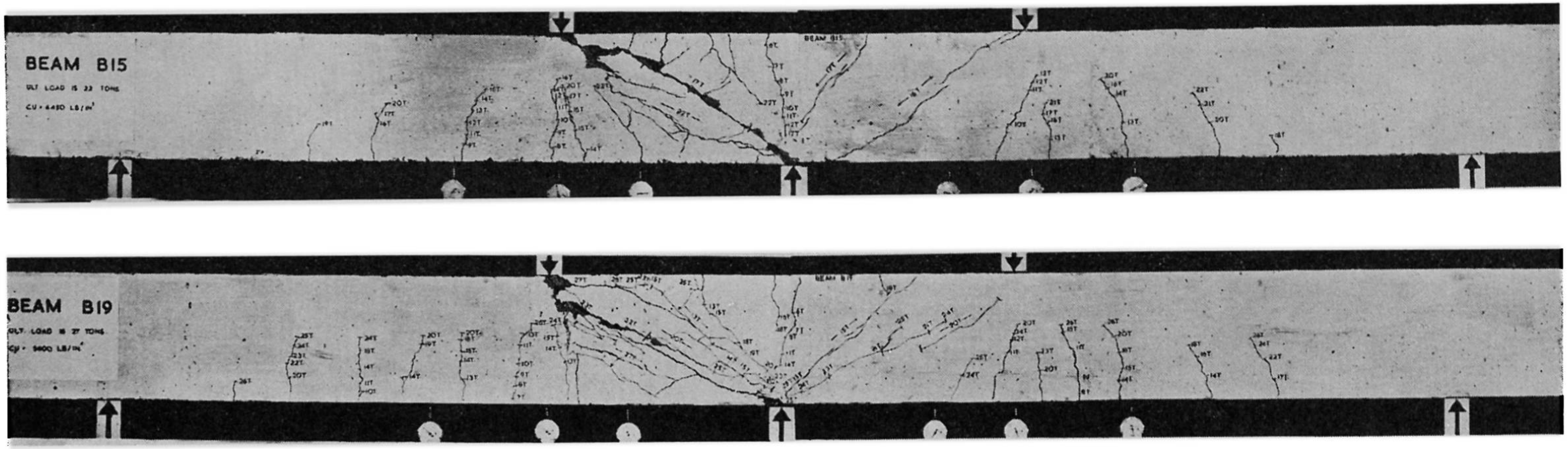

Fig. 1. Beams with stirrups at $2\frac{1}{2}''$ centres:
B 15 without horizontal web reinforcement; B 19 with two $\frac{1}{2}''$ ⌀ bars at the neutral axis.

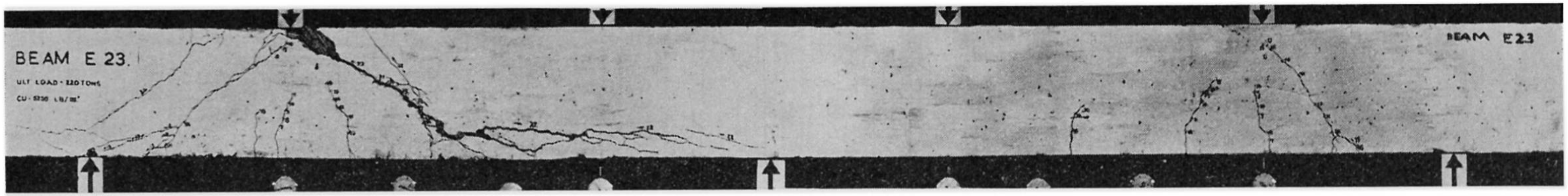

Fig. 3. A continuous beam with equal shears in all shear spans adjacent to supports, and with equal moment-shear ratios on the support-side of all loads.

resistance to compression is higher [3]. Thus failure in shear may occur at an increased compressive stress, possibly at a load in excess of the theoretical flexural capacity, and yet truly in shear. Loads acting on the surface of the beam also have the effect of inducing biaxial compression [1]. If closed stirrups are present they provide a form of binding of the compression zone, and thus permit it to develop a considerably greater strain before compression failure; this allows a greater rotation and may raise the shear capacity of the beam.

Moment-Shear Interaction in Reinforced Beams

Tests on continuous beams with constant longitudinal reinforcement (but without stirrups) and a/d varying between 1.5 and 3.5 have indicated an absence of a significant shear-moment interaction. Fig. 2 shows that the higher the bending moment over the centre support the higher the nominal shearing stress at failure, all beams failing in shear. The failing load was slightly lower the higher the a/d ratio (for the shear span nearest to the centre support); this is similar to the behaviour of single span beams and also of continuous prestressed beams (see infra).

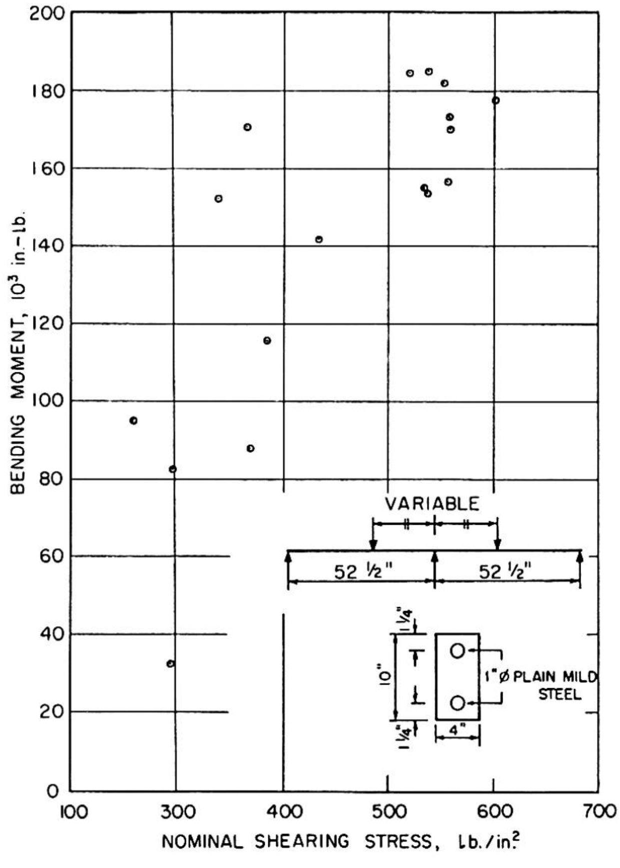

Fig. 2. Relation between the bending moment over the centre support and nominal shearing stress at failure.

To study the interaction further a continuous beam was tested under two point loading per span arranged so that the shear in each of the shear spans adjacent to a reaction was the same. The $\frac{M}{Vd}$ ratios were also sensibly equal: 1.72 on the outside of the outer load point, and 1.65 near the centre support. The elastic point of contraflexure was between the load points, less than 2″ from the inner load point. Fig. 3 shows the crack pattern at failure, which took place in the shear span with a shear equal to only 2/3 of the shear in the other shear spans so that $\frac{M}{Vd}=2.57$. Thus once again the factor equivalent to a/d appears to be of importance. It may also be noted that a lower moment gradient means that the zone of flexural cracking extends more rapidly with an increase in load, and this may lead to inclined cracking. There was no cracking near the point of contraflexure.

Prestressed Beams of Rectangular Cross-Section

Two-span beams with a varying linear transformation were tested. Failure occurred in all cases in diagonal tension near the centre support with a second hinge in the span. The diagonal tension crack formed suddenly and opened wide over a distance extending to within 1 to $1\frac{1}{2}$ in. of extreme fibres. With one exception this diagonal cracking load represented between 84 and 100% of the collapse load on the beam. The sudden character of shear failure in prestressed beams (which were post-tensioned and grouted), sometimes nearly explosive in character, is thus apparent, and it is clear that all important prestressed as well as reinforced beams must have web reinforcement. How early the warning of failure is, i.e. what is the increase in load between diagonal tension cracking and collapse (expressed as a percentage of the collapse load), depends on several factors. As in reinforced concrete beams, the a/d[1]) ratio has an important influence: the higher the value of a/d the less warning there is, and for high a/d collapse is simultaneous with the opening of the diagonal tension crack.

For a constant value of a/d, the difference between the cracking and collapse loads is greater the smaller the total prestressing force in the beam, F_s. Thus for a constant stress in wires at transfer, f_s, the difference is greater the smaller the steel area; conversely, for a constant steel area the difference increases with a decrease in f_s. These statements apply of course only to beams failing in shear but they seem to form a logical pattern as a smaller compression due to prestress means that cracking occurs under a lower load. This lowers the rigidity of the beam, and leads to a greater difference between the cracking load and the ultimate load irrespective of the actual value of the latter.

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[1]) d is the overall depth of section.

From the rather insufficient tests on beams with F_s constant, and f_s and the percentage steel area, p, variable, it seems that the difference between diagonal cracking and ultimate load increases with a decrease in f_s and therefore with the concomitant increase in p. This is because with a high value of p the loss in effective prestress due to creep of concrete is higher, and diagonal cracking occurs earlier.

Let us consider the effect on shear strength of f_s, p, and F_s; only two of these are independent. If f_s is sufficiently high a small increase in the load on the beam will bring the wires into the non-elastic zone and cause flexural cracking of concrete so that the deflection of the beam will increase at a high rate, and failure by crushing of concrete will follow. Conversely, if f_s is well below the yield stress of steel the increase in strain due to load is small, the final deflection is also small and failure takes place in shear before the full flexural capacity is reached. All beams with f_s greater than one-half the 0.1% proof stress failed in flexure, and in all of them the yield stress in the wires was reached. Beams with a slightly lower value of f_s failed in shear but at a load equal to the ultimate load in flexure. Nevertheless, the change in the pattern of failure from flexure to shear as a result of a reduction in the stress in the wires at transfer makes the use of beams with too low a stress in wires undesirable.

If f_s is constant and p increases, the flexural capacity of an under-reinforced

Table 1. Influence of Percentage Area of Reinforcement on Failure of Rectangular Prestressed Concrete Beams

p %	F_s lb.	Ultimate load ton	Mode of failure
0.196	8,300	20.6	Flexure
0.371	8,210	33.5	Shear
0.371	11,900	31.2	Flexure
0.557	11,870	31.0	Shear
0.196	5,070	20.5	Flexure
0.371	4,640	29.2	Shear
0.371	11,750	39.5	Flexure
0.742	12,030	29.5	Shear
0.371	11,750	39.5	Flexure
0.557	12,630	36.0	Shear
0.371	8,210	33.5	Shear
0.557	11,870	31.0	Shear
0.557	11,750	36.0	Shear
0.742	12,030	29.5	Shear

section increases. The shear load which can be carried increases at a lower rate so that the mode of failure changes from flexure to shear (Table 1). For beams failing in shear the one with a higher value of p fails earlier (see lower part of Table 1).

In the two cases considered in the preceding paragraphs an increase in the variable studied resulted in an increase in F_s, but F_s is not believed to be a primary factor influencing the type of failure.

A study of the relation between the maximum moment on the beam and the shear at cracking or at ultimate shows that moment-shear interaction is very small or absent. Fuller data were obtained from tests on I-beams.

Prestressed I-beams

Symmetrical, rather squat, I-beams of 9000 lb./in.^{2} concrete (measured on cubes) were tested under loading shown in Fig. 5: the a/d ratio was 3. The linear transformation of the cable, which varied between 0 and $2\frac{1}{2}''$ downwards at centre support, reduces the plastic moment of the section over the centre support but does not affect the ultimate strength of the beam as a whole, as shown in Fig. 4, where the slope of the regression line is not significant. There is, however, a secondary effect of transformation in that it decreases the vertical component of prestress, which reduces the net shear. However, even at the maximum transformation the decrease was less than 10% of the shear at failure. It is also worth noting that the vertical component of prestress affects the shear strength of a beam primarily when excessive principal tension in the web leads to cracking there [4]. When, however, diagonal cracks develop

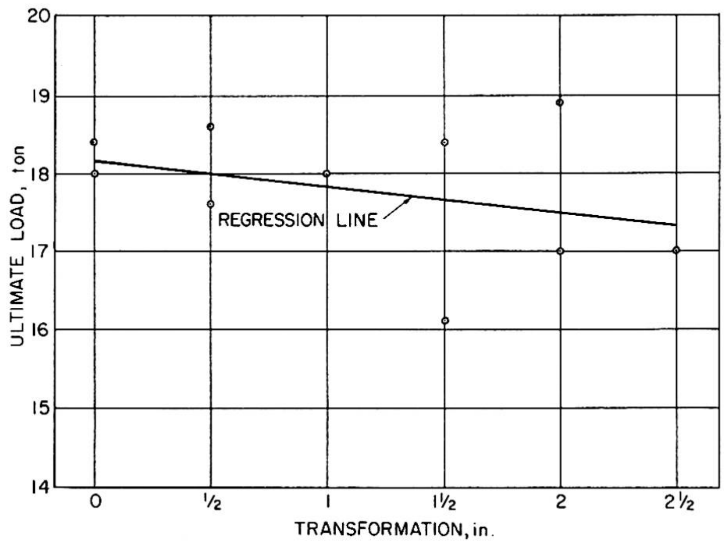

Fig. 4. Relation between ultimate load on the beam and linear downward transformation of tendon at the centre support.

from flexure cracks, the inclination of the tendon is of lesser importance than its position, as the latter governs the moment-capacity of the section.

Determination of the moment-load relation has shown departure from the "elastic" curve when a flexural crack forms over the centre support: the rate of increase in the moment falls off, i. e. a redistribution of moments takes place. However, as flexural or shear cracks open in the span the rate increases somewhat but the average rate remains below the rate prior to the beginning of the redistribution.

Fig. 5 shows the relation between the measured plastic moment over the centre support and the shearing force acting in the centre shear span at the

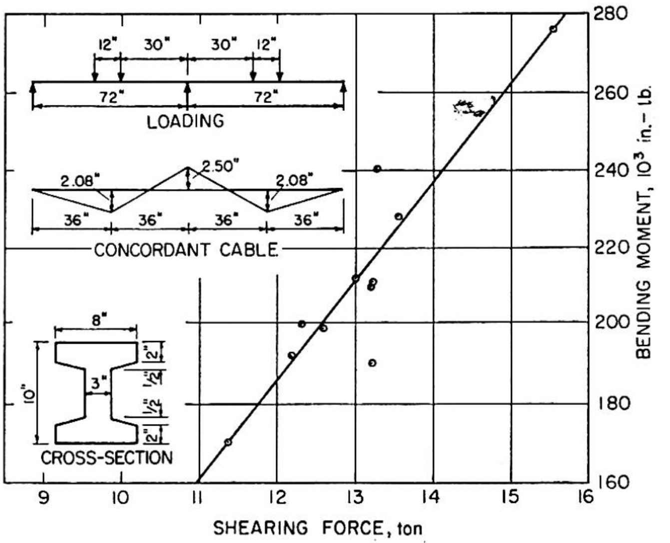

Fig. 5. Relation between bending moment over the centre support and shearing force at ultimate.

same stage. The change in the plastic moment is due to the varying transformation of the tendon, and the measured moments agree very closely with calculated values. An increase in the moment is accompanied by proportionately smaller increase in shear capacity (all beams failing in shear). This does not establish measurable shear-moment interaction. Furthermore, the failure in shear occurs perforce away from the section at which the plastic moment is developed and for the beams of the present series the diagonal tension crack is usually positioned so that at the level of the centroid of the beam the crack is half-way between the centre support and the near load point. The magnitude of the (collapse) bending moment at this intersection of the crack and the centroidal axis does not seem to affect the magnitude of the maximum shearing force that the beam can withstand. Moreover, the sign of the bending moment, positive or negative, at this intersection is of no consequence (see Fig. 6). This behaviour offers a strong argument against an influence of moment on shear capacity in continuous beams of the type considered. The influence of an applied shear on the moment that can be developed at a section is yet to be studied.

Determination of principal strains at the critical section has shown that their orientation changes to the direction of 45° to the beam axis more rapidly (i.e., under a lower load) in beams failing in shear. However, the magnitude of the principal tension in a beam is not a direct indication of its mode of failure. In I-beams with high or moderately high a/d ratios diagonal cracking occurs at a principal tension approximately equal to the tensile strength, but

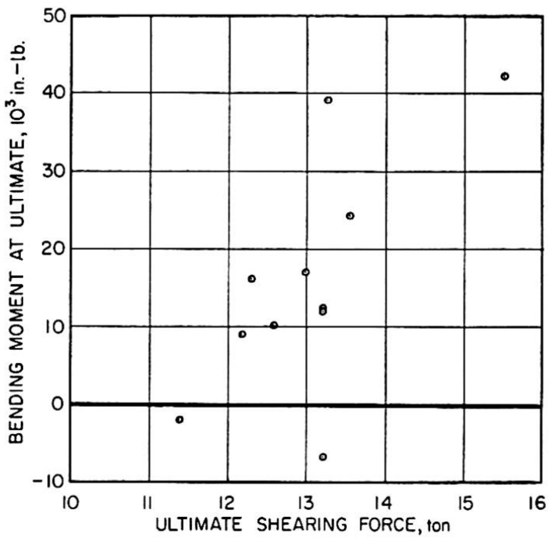

Fig. 6. Relation between the bending moment at the mid-point of the shear span adjacent to the centre support and the shearing force at ultimate.

in rectangular beams often at a considerably lower stress. The design of these beams should not, therefore, be based on a permissible principal tension. This behaviour is due to the influence of flexural stresses and development of flexure cracks from which the final shear failure develops: the origin of the shear failure is thus different. Maximum stress occurs nearer to the tension surface of the beam [4], and principal stress on the centroidal axis cannot serve as a design criterion. Thus the influence of the bending stresses on the capacity of a beam to resist shear is explained, but this is not tantamount to a true and substantial moment-shear interaction.

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Summary

Factors influencing the shear strength of continuous beams, reinforced or prestressed, are discussed, and it is shown that there is no substantial shear-moment interaction at failure. Tests on reinforced beams with orthogonal web reinforcement are described.

Résumé

On discute les facteurs qui influencent la résistance au cisaillement de poutres continues en béton armé ou précontraint, et on montre qu'il n'y a pas d'interaction importante entre l'effort tranchant et le moment fléchissant à la rupture. On décrit des essais sur des poutres en béton armé avec armatures de cisaillement du type orthogonal.

Zusammenfassung

Die Faktoren, die die Schubfestigkeit kontinuierlicher Balken aus Stahlbeton oder vorgespanntem Beton beeinflussen, werden zur Diskussion gestellt. Es wird gezeigt, daß es beim Bruch keine größere Wechselwirkung zwischen Querkraft und Biegemoment gibt. Versuche an Stahlbetonbalken mit orthogonaler Schubbewehrung werden beschrieben.