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Computer Analysis of Folded Plate Structures

Etude à la calculatrice électronique des toits plissés

Berechnung von Faltwerken mit Hilfe elektronischer Rechengeräte

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Introduction

Continued and, indeed, apparently increasing interest in folded plate construction suggests that the application of electronic computers to the analysis and design of such structures be investigated. Two methods of analysis, both developed for electronic digital computers, are described in the present paper. The methods consider prismatic folded plate structures which are simply supported at their ends. Both methods are founded upon the basic equations of classical plate theory and classical two-dimensional elasticity theory [1, 2] [1]) and therefore imply only the assumptions and limitations inherent in those theories.

The first method is essentially a transliteration to matrix form of a theory described in a previous paper [3]. The matrix form is especially convenient for use on an electronic digital computer and a program is easily prepared which requires for input only the basic geometry (length, widths, thicknesses, and inclinations of the individual slabs and beams), material properties and loading. The program then constructs appropriate matrices and finally produces, as output, displacements and tractions at desired points within the folded plate structure.

In the second method, using a technique described previously [4], the basic equations of plate and elasticity theory are transformed into sets of first-order ordinary differential equations in the intrinsic variables; namely, Fourier coefficients of four components of displacement and of four tractions. These equations are in a form which is convenient for numerical integration on an electronic digital computer.

Since each of the slabs which form the folded plate is assumed to be simply-supported at the ends ($x=0, a$), it is convenient in both methods to represent loadings, displacements and tractions as compatible generalized half-range Fourier series. Thus, for example,

[1]) Numbers in brackets refer to items in the Bibliography.

$$w(x,y) = \sum_{m=1}^{\infty} w_m(y) \sin\frac{m\pi x}{a}, \qquad u(x,y) = \sum_{m=1}^{\infty} u_m(y) \cos\frac{m\pi x}{a}. \tag{1}$$

Loadings p and q, displacement v, and tractions M_x, M_y, N_x, N_y and Q_y are also expanded into sine series. Loading in the x-direction, and tractions M_{xy}, N_{xy} and Q_x are expanded into cosine series.

While Eqs. (1) postulate infinite series, only a few terms or harmonics of the several series are needed to provide the accuracy required in design. The equations and discussion in the following sections pertain to a single arbitrary harmonic of the general solution except as stated, and the dependent variables are the amplitudes or coefficients of the m-th terms in the several pertinent series.

First Method

Let i, j, k be three successive joints of a folded plate system (Fig. 1). For the m-th harmonic, let d_j^* be the column vector of the four components of displacement at joint j, the directions being associated with a reference plane

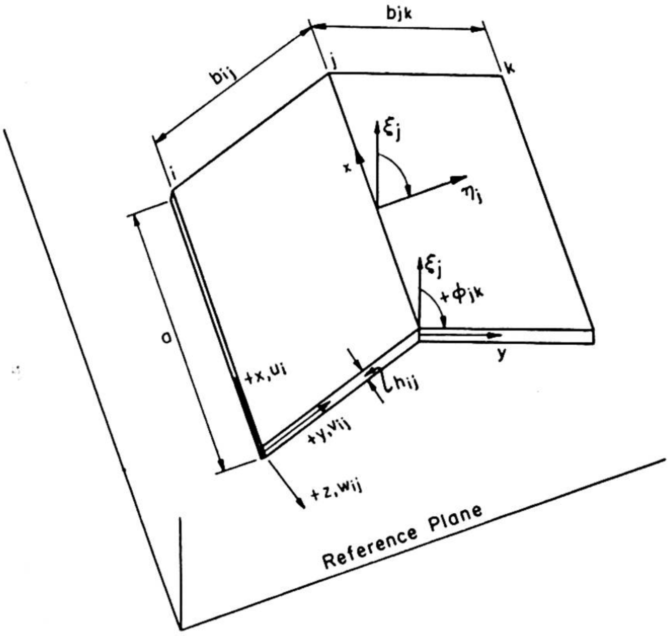

Fig. 1. Portion of folded plate structure.

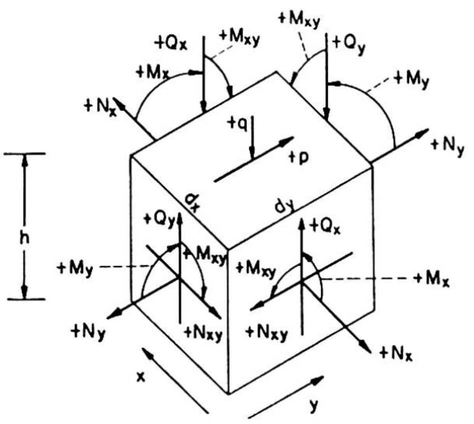

Fig. 2. Differential element showing positive tractions.

(conveniently taken as a horizontal plane) as shown in the figure. The amplitudes of the four tractions at edges j of slabs jk and ij are represented by the column vectors F_{jk}^* and F_{ji}^* and are given by the matrix equations

$$\begin{aligned} F_{jk}^* &= T_{jk} K_{jk} T_{jk} d_j^* + T_{jk} K_{kj} T_{kj} d_k^* + T_{jk} \overline{F}_{jk}, \\ F_{ji}^* &= T_{ji} K_{ji} T_{ji} d_j^* + T_{ji} K_{ij} T_{ij} d_i^* + T_{ji} \overline{F}_{ji}. \end{aligned} \tag{2}$$

Here T_{jk}, K_{jk}, $\overline{F}_{jk}$ are, respectively, a directional transformation matrix, a stiffness matrix, and a fixed-edge traction matrix. For a beam at and parallel to joint j, the four tractions may be written, similarly, in matrix form as

$$F_{jj}^* = T_{jj} K_{jj} T_{jj} d_j^* . \tag{3}$$

The several matrices are defined in Appendix I.

Since joint j is in equilibrium, the sum of Eqs. (2) and (3) must vanish. This yields the matrix equation,

$$\begin{aligned}[T_{ji} K_{ij} T_{ij}]\, d_i^* + [T_{ji} K_{ji} T_{ji} + T_{jj} K_{jj} T_{jj} + T_{jk} K_{jk} T_{jk}]\, d_j^* \\ + [T_{jk} K_{kj} T_{kj}]\, d_k^* = - T_{ji} \overline{F}_{ji} - T_{jk} \overline{F}_{jk} .\end{aligned} \tag{4}$$

Similar equations may be written at each joint. If there are n joints with unknown displacements, there will be essentially $4n$ algebraic equations in the same number of unknown displacements. Simultaneous solution may be effected by any suitable technique such as, for example, the Gauss-Jordan algorithm or matrix iteration.

Having determined the displacements at each joint, these may be substituted into the following formulas to obtain the tractions at desired points within each slab. For a typical slab, jk,

$$d_{jk} = T_{jk}^{-1} d_j^* , \qquad d_{jk} = T_{kj}^{-1} d_k^* , \tag{5}$$

where d_{jk} and d_{kj} are the slab displacement column vectors at j and k respectively, e.g., the elements of d_{jk} are θ_j, w_{jk}, v_{jk} and u_j,

$$\begin{aligned} M_y = & - R_1 (1-\nu) \{\beta_3 [S_1(y') + (\delta_3 - c^*) C(y')] - \delta_2/2\} \\ & - D_1 \overline{m}_1 [\theta_j \{-\beta_1 [S_1(y') + (\delta_1 - t^*) C(y')] + \beta_2 [C_1(y') + (\delta_1 - c^*) S(y')]\} \\ & \quad + \theta_k \{\beta_1 [S_1(y') + (\delta_1 - t^*) C(y')] + \beta_2 [C_1(y') + (\delta_1 - c^*) S(y')]\}] \\ & + D_1 \overline{m}_2 [w_{jk} \{\beta_3 [S_1(y') + (\delta_3 - c^*) C(y')] - \beta_4 [C_1(y') + (\delta_3 - t^*) S(y')]\} \\ & \quad + w_{kj} \{\beta_3 [S_1(y') + (\delta_3 - c^*) C(y')] + \beta_4 [C_1(y') + (\delta_3 - t^*) S(y')]\}] , \end{aligned} \tag{6}$$

$$\begin{aligned} M_x = & \; R_1 (1-\nu) \{\beta_3 [S_1(y') - (\delta_3 + c^*) C(y')] + \delta_1/2\} \\ & - D_1 \overline{m}_1 [\theta_j \{\beta_1 [S_1(y') - (\delta_2 + t^*) C(y')] - \beta_2 [C_1(y') - (\delta_2 + c^*) S(y')]\} \\ & \quad - \theta_k \{\beta_1 [S_1(y') - (\delta_2 + t^*) C(y')] + \beta_2 [C_1(y') - (\delta_2 + c^*) S(y')]\}] \\ & + D_1 \overline{m}_2 [w_{jk} \{-\beta_3 [S_1(y') - (\delta_3 - c^*) C(y')] + \beta_4 [C_1(y') - (\delta_3 + t^*) S(y')]\} \\ & \quad - w_{kj} \{\beta_3 [S_1(y') - (\delta_3 + c^*) C(y')] + \beta_4 [C_1(y') - (\delta_3 + t^*) S(y')]\}] , \end{aligned} \tag{7}$$

$$\begin{aligned} M_{xy} = & - R_1 (1-\nu) \beta_3 [C_1(y') - c^* S(y')] \\ & - D_1 \overline{m}_1 [\theta_j \{-\beta_1 [C_1(y') + (1 - t^*) S(y')] + \beta_2 [S_1(y') + (1 - c^*) C(y')]\} \\ & \quad + \theta_k [\beta_1 C_1(y') + (1 - t^*) S(y')] + \beta_2 [S_1(y') + (1 - c^*) C(y')]\}] \\ & + D_1 \overline{m}_2 [w_{jk} \{\beta_3 [C_1(y') - c^* S(y')] - \beta_4 [S_1(y') - t^* C(y')]\} \\ & \quad + w_{kj} \{\beta_3 [C_1(y') - c^* S(y')] + \beta_4 [S_1(y') - t^* C(y')]\}] , \end{aligned} \tag{8}$$

$$\begin{aligned} Q_y = & \; R_2 \beta_3 S(y') + D \overline{m}_2 \{\theta_j [-\beta_1 S(y') + \beta_2 C(y')] + \theta_k [\beta_1 S(y') + \beta_2 C(y')]\} \\ & - D \overline{m}_3 \{w_{jk} [\beta_3 S(y') - \beta_4 C(y')] + w_{kj} [\beta_3 S(y') + \beta_4 C(y')]\} , \end{aligned} \tag{9}$$

$$Q_x = -\frac{R_2}{2}[2C(y')\beta_3-1]-D\overline{m}_2\{\theta_j[-\beta_1 C(y')+\beta_2 S(y)]+\theta_k[\beta_1 C(y')+\beta_2 S(y')]\}$$
$$+D\overline{m}_3\{w_{jk}[\beta_3 C(y')-\beta_4 S(y')]+w_{kj}[\beta_3 C(y')+\beta_4 S(y')]\}, \qquad (10)$$

$$N_y = -R_4\beta_6[C_1(y')-(c^*+\delta_1)S(y')]$$
$$-D_2\overline{m}_1\,[v_{jk}\{\beta_5[S_1(y')-(\delta_5+t^*)C(y')]-\beta_6[C_1(y')-(\delta_5+c^*)S(y')]\} \qquad (11)$$
$$-v_{kj}\{\beta_5[S_1(y')-(\delta_5+t^*)C(y')]+\beta_6[C_1(y')-(\delta_5+c^*)S(y')]\}]$$
$$+D_2\overline{m}_1\,[u_j\{-\beta_7[S_1(y')+(\delta_6-c^*)C(y')]+\beta_8[C_1(y')+(\delta_6-t^*)S(y')]\}$$
$$-u_k\{\beta_7[S_1(y')+(\delta_6-c^*)C(y')]+\beta_8[C_1(y')+(\delta_6-t^*)S(y')]\}],$$

$$N_x = R_4\beta_6[C_1(y')-(c^*-\delta_2)S(y')]$$
$$-D_2\overline{m}_1\,[v_{jk}\{-\beta_5[S_1(y')+(\delta_4-t^*)C(y')]+\beta_6[C_1(y')+(\delta_4-c^*)S(y')]\}$$
$$+v_{kj}\{\beta_5[S_1(y')+(\delta_4-t^*)C(y')]+\beta_6[C_1(y')+(\delta_4-c^*)S(y')]\}]$$
$$+D_2\overline{m}_1\,[u_j\{\beta_7[S_1(y')+(\delta_7-c^*)C(y')]-\beta_8[C_1(y')+(\delta_7-t^*)S(y')]\} \qquad (12)$$
$$+u_k\{\beta_7[S_1(y')+(\delta_7-c^*)C(y')]+\beta_8[C_1(y')+(\delta_7-t^*)S(y')]\}],$$

$$N_{xy} = \frac{R_4}{2}\{2\beta_6[S_1(y')-(c^*+\delta_6)C(y')]+1\}$$
$$+D_2\overline{m}_1\,[v_{jk}\{\beta_5[C_1(y')-(\delta_6+t^*)S(y')]-\beta_6[S_1(y')-(\delta_6+c^*)C(y')]\} \qquad (13)$$
$$-v_{kj}\{\beta_5[C_1(y')-(\delta_6+t^*)S(y')]+\beta_6[S_1(y')-(\delta_6+c^*)C(y')]\}]$$
$$-D_2\overline{m}_1\,[u_j\{-\beta_7[C_1(y')+(\delta_5-c^*)S(y')]+\beta_8[S_1(y')+(\delta_5-t^*)C(y')]\}$$
$$-u_k\{\beta_7[C_1(y')+(\delta_5-c^*)S(y')]+\beta_8[S_1(y')+(\delta_5-t^*)C(y')]\}],$$

in which

$$y' = \frac{b}{2}-y; \qquad \alpha_m = \frac{m\pi b}{2a}; \qquad t^* = \alpha_m \tanh\alpha_m; \qquad c^* = \alpha_m \coth\alpha_m;$$

$$\overline{m}_1 = \frac{m\pi}{a}; \qquad \overline{m}_2 = \left(\frac{m\pi}{a}\right)^2; \qquad \overline{m}_3 = \left(\frac{m\pi}{a}\right)^3;$$

$$S(y') = \sinh\frac{m\pi y'}{a}; \qquad S_1(y') = \frac{m\pi y'}{a}\sinh\frac{m\pi y'}{a};$$

$$C(y') = \cosh\frac{m\pi y'}{a}; \qquad C_1(y') = \frac{m\pi y'}{a}\cosh\frac{m\pi y'}{a};$$

$$D_1 = \frac{E h^3}{24(1+\nu)}; \qquad D_2 = \frac{E h}{2(1+\nu)};$$

$$R_1 = \frac{4q}{\overline{m}_3 a}; \qquad R_2 = \frac{8q}{\overline{m}_2 a}; \qquad R_3 = \frac{4p}{(1+\nu)\overline{m}_2 a}; \qquad R_4 = \frac{8p}{\overline{m}_2 a};$$

$$\delta = \frac{3+\nu}{1+\nu}; \quad \delta_{1,5} = \frac{2}{1\mp\nu}; \quad \delta_{2,4} = \frac{2\nu}{1\mp\nu}; \quad \delta_3 = \frac{1+\nu}{1-\nu}; \quad \delta_6 = \frac{1}{\delta_3}; \quad \delta_7 = \frac{3+\nu}{1+\nu}.$$

Similar formulas may be deduced from Reference [3] for computation of displacements within the various slabs, if desired.

The equations should be used for a sufficient number of harmonics. For uniform loading, only the odd harmonics ($m = 1, 3, 5$, etc.) are involved and, usually, two to four of these will provide sufficient accuracy for design purposes.

Second Method

It has been shown in a previous paper [4] that, for slabs simply-supported at their ends, the equations which govern the deformations and tractions may be written as a system of eight first-order differential equations. In the present case these become

$$\frac{dw}{dy} = \theta; \qquad \frac{d\theta}{dy} = -\frac{M_y}{D} + \nu\left(\frac{m\pi}{a}\right)^2 w; \qquad \frac{dv}{dy} = \frac{N_y}{H} + \nu\frac{m\pi}{a}u;$$

$$\frac{du}{dy} = \frac{2N_{xy}}{H(1-\nu)} - \frac{m\pi}{a}v; \qquad \frac{dN_y}{dy} = \frac{m\pi}{a}N_{xy} - \frac{4p}{m\pi};$$

$$\frac{dN_{xy}}{dy} = -\frac{m\pi}{a}N_x; \quad \frac{dM_y}{dy} = V_y - \frac{2m\pi}{a}M_{xy}; \quad \frac{dV_y}{dy} = \left(\frac{m\pi}{a}\right)^2 M_x - \frac{4q}{m\pi};$$

$$M_{xy} = D(1-\nu)\frac{m\pi}{a}\theta; \qquad M_x = \nu M_y + D\left(\frac{m\pi}{a}\right)^2(1-\nu^2)w; \tag{14}$$

$$N_x = \nu N_y - H\frac{m\pi}{a}(1-\nu^2)u,$$

in which $$H = \frac{Eh}{1-\nu^2}, \qquad D = \frac{Eh^3}{12(1-\nu^2)}.$$

Eqs. (14) together with the transformations, Eqs. (16), form a system of eighth order subject to four boundary conditions at the initial edge and to four additional conditions at the terminal edge of the complete structure. If, for example, an edge is unsupported, the conditions are

$$N_y = N_{xy} = V_y = M_y = 0. \tag{15}$$

If the edge is elastically restrained by, say, an edge beam, the tractions at the edge are related to the displacements through the stiffness matrix of the beam (i.e., K_{jj}) with an appropriate directional transformation matrix. Alternatively, one may consider the edge beam itself as an additional slab which augments the folded plate structure and which, at its outer edge, is subjected to the conditions represented by Eqs. (15).

Given specified initial values of the eight intrinsic variables, the equations are readily integrated with the aid of an electronic computer and using a suitable numerical procedure such as the Runge-Kutta fourth-order process [5]. At each joint, the transformations

$$v^+ = v^- \cos x_j + w^- \sin\alpha_j; \qquad w^+ = w^- \cos\alpha_j - v^- \sin\alpha_j;$$
$$N_y^+ = N_y^- \cos\alpha_j + V^- \sin\alpha_j; \qquad V^+ = V^- \cos\alpha_j - N_y^- \sin\alpha_j; \qquad \alpha_j = \phi_{ji} - \phi_{ij} \tag{16}$$

are made within the computer, and the integration then proceeds with the new variables appropriate to the slab under consideration.

The complete and correct solution consists of the nonhomogeneous solution plus a linear combination of four homogeneous solutions, each obtained by integrating Eqs. (14) with appropriately chosen initial values and using

Eqs. (16) at the joints. The case of a structure with free edges will serve to illustrate the method.

For this case, one constructs four homogeneous solutions (Solutions 1 to 4 in Table 1) by deleting the p and q terms in Eqs. (14) and integrating. The initial conditions are Eqs. (15) and the values of the initial displacements are shown in the table. With the p and q terms restored to Eqs. (14), one integrates these equations to obtain the nonhomogeneous case (Solution 5 in the Table).

Table 1

Solution	Initial Values				Terminal Values			
	u	v	w	θ	N_y	N_{xy}	M_y	V_y
1	1	0	0	0	$N_y^{(1)}$	$N_{xy}^{(1)}$	$M_y^{(1)}$	$V_y^{(1)}$
2	0	1	0	0	$N_y^{(2)}$	$N_{xy}^{(2)}$	$M_y^{(2)}$	$V_y^{(2)}$
3	0	0	1	0	$N_y^{(3)}$	$N_{xy}^{(3)}$	$M_y^{(3)}$	$V_y^{(3)}$
4	0	0	0	1	$N_y^{(4)}$	$N_{xy}^{(4)}$	$M_y^{(4)}$	$V_y^{(4)}$
5	0	0	0	0	$N_y^{(5)}$	$N_{xy}^{(5)}$	$M_y^{(5)}$	$V_y^{(5)}$

Each of these solutions yields numerical values for all dependent variables at the terminal boundary. If this boundary is free, the calculated tractions are of interest and have values as indicated in the table.

One now writes the terminal boundary conditions in the form

$$\begin{aligned} C_1 N_y^{(1)} + C_2 N_y^{(2)} + C_3 N_y^{(3)} + C_4 N_y^{(4)} &= -N_y^{(5)}, \\ C_1 N_{xy}^{(1)} + C_2 N_{xy}^{(2)} + C_3 N_{xy}^{(3)} + C_4 N_{xy}^{(4)} &= -N_{xy}^{(5)}, \\ C_1 M_y^{(1)} + C_2 M_y^{(2)} + C_3 M_y^{(3)} + C_4 M_y^{(4)} &= -M_y^{(5)}, \\ C_1 V_y^{(1)} + C_2 V_y^{(2)} + C_3 V_y^{(3)} + C_4 V_y^{(4)} &= -V_y^{(5)} \end{aligned} \tag{17}$$

and solves these equations for the C's. These are, in fact, the correct values of the initial displacements. Now, with the starting values

$$u = C_1, \qquad v = C_2, \qquad w = C_3, \qquad \theta = C_4 \tag{18}$$

a final integration of Eqs. (14) is made with the loading terms included and using Eqs. (15) and (16). This calculation yields the amplitudes of displacements and tractions over the entire structure for the harmonic under consideration. The entire process is repeated for as many harmonics as are necessary to attain any desired accuracy.

Example

Fig. 3 shows the cross section and loading of one unit of a north light roof. The two narrow edge members were treated as beams having bending and torsional resistance. Curves for a typical quantity, the transverse bending

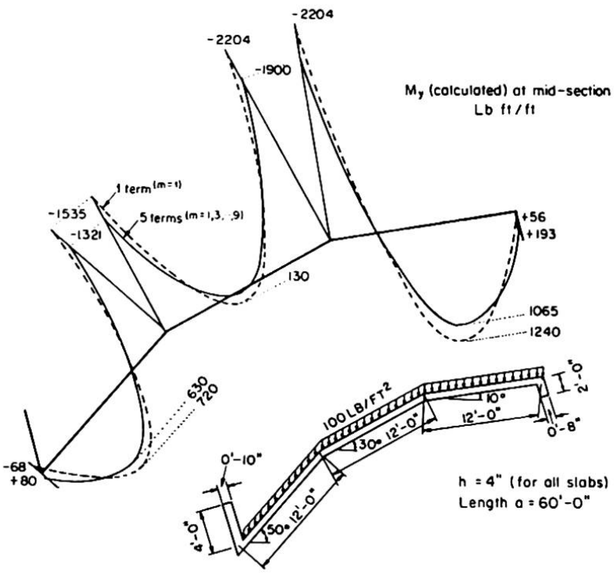

Fig. 3. North light folded plate structure.

moment at the section midway between the ends, are shown. One curve shows these moments computed by the first method using only the first harmonic ($m = 1$). The other curve presents the practically correct solution obtained as the sum of several harmonics ($m = 1, 3, \ldots, 9$). Comparison of the two curves indicates that it may be inadvisable to use, as is sometimes suggested, only the first harmonic as a basis for design and analysis.

The computer programs produce complete information on each of the tractions and displacements for as many transverse sections as may be desired. The programs, when used on a sufficiently large machine, are entirely self-contained. They accept as input the basic information such as geometry, material properties, desired number of harmonics and points at which final quantities are desired. The first method program then calculates the fixed-edge tractions and the edge coefficients for each slab and beam, forms the joint equations and solves for the joint displacements. It then computes tractions and displacements at as many sections and interior points as desired. The present example required approximately 0.75 minutes on an IBM 7090 computer to produce results for five sections. The second method program is similarly self-contained.

Appendix I

As used in Eq. (4), the displacement column vector, d_j^*, and the transformation matrix, T_{jk}, for slab jk at joint j are

$$d_j^* = \begin{bmatrix} \theta_j \\ \eta_j \\ \xi_j \\ u_j \end{bmatrix} \quad \text{and} \quad T_{jk} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_{jk} & \sin\phi_{jk} & 0 \\ 0 & -\sin\phi_{jk} & \cos\phi_{jk} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{19}$$

The stiffness matrices, K_{jk} and K_{kj}, for joint j of slab jk are written as

$$K_{kj} = \begin{bmatrix} S_1^{jk} C_1 & S_1^{jk} C_3 & 0 & 0 \\ -S_2^{jk} C_5 & -S_2^{jk} C_7 & 0 & 0 \\ 0 & 0 & -S_3^{jk} C_9 & S_3^{jk} C_{11} \\ 0 & 0 & S_3^{jk} C_{13} & -S_3^{jk} C_{15} \end{bmatrix};$$

$$K_{kj} = \begin{bmatrix} S_1^{kj} C_2 & -S_1^{kj} C_4 & 0 & 0 \\ -S_2^{kj} C_6 & S_2^{kj} C_8 & 0 & 0 \\ 0 & 0 & S_3^{kj} C_{10} & -S_3^{kj} C_{12} \\ 0 & 0 & S_3^{kj} C_{14} & S_3^{kj} C_{16} \end{bmatrix},$$

in which, according to Reference [3],

$$\begin{aligned}
S_1^{jk} &= \frac{E\, h_{jk}^3\, m}{12\,(1-\nu^2)\, a}, \qquad S_2^{jk} = \frac{m}{a} S_1^{jk}, \qquad S_3^{jk} = \frac{E\, h_{jk}\, m}{(1+\nu)^2 a}, \\
C_{1,2} &= \pi\,(\pm\beta_1 \cosh\alpha_m - \beta_2 \sinh\alpha_m), \qquad C_{5,6} = \pi^2\,(\pm\beta_3 \cosh\alpha_m - \beta_4 \sinh\alpha_m), \\
C_{7,8} &= \frac{m\,\pi^3}{a}\,(\pm\beta_3 \sinh\alpha_m - \beta_4 \cosh\alpha_m), \qquad C_{9,10} = \pi\,(-\beta_5 \cosh\alpha_m \pm \beta_6 \sinh\alpha_m), \\
C_{13,14} &= \pi\,(\pm\beta_7 \cosh\alpha_m - \beta_8 \sinh\alpha_m), \qquad C_{15,16} = \pi\,(\mp\beta_7 \sinh\alpha_m + \beta_8 \cosh\alpha_m), \\
C_3 &= \frac{m}{a} C_5, \qquad C_4 = \frac{m}{a} C_6, \qquad C_{11} = C_{13}, \qquad C_{12} = C_{14}, \qquad\qquad (21) \\
\beta_{1,4} &= (\alpha_m \operatorname{sech}\alpha_m \pm \sinh\alpha_m)^{-1}, \qquad \beta_{2,3} = (\alpha_m \operatorname{csch}\alpha_m \mp \cosh\alpha_m)^{-1}, \\
\beta_{5,8} &= \left(\alpha_m \operatorname{sech}\alpha_m \mp \frac{3-\nu}{1+\nu} \sinh\alpha_m\right)^{-1}, \qquad \beta_{6,7} = \left(\alpha_m \operatorname{csch}\alpha_m \pm \frac{3-\nu}{1+\nu} \cosh\alpha_m\right)^{-1}.
\end{aligned}$$

When $\pm$ or $\mp$ are indicated, the first subscript is associated with the upper sign and the second subscript with the lower sign.

The matrices for the slab ji are

$$K_{ji} = \begin{bmatrix} S_1^{ji} C_1 & -S_1^{ji} C_3 & 0 & 0 \\ S_2^{ji} C_5 & -S_2^{ji} C_7 & 0 & 0 \\ 0 & 0 & -S_3^{ji} C_9 & -S_3^{ji} C_{11} \\ 0 & 0 & -S_3^{ji} C_{13} & -S_3^{ji} C_{15} \end{bmatrix};$$

$$K_{ij} = \begin{bmatrix} S_1^{ij} C_2 & S_1^{ij} C_4 & 0 & 0 \\ S_2^{ij} C_6 & S_2^{ij} C_8 & 0 & 0 \\ 0 & 0 & S_3^{ij} C_{10} & S_3^{ij} C_{12} \\ 0 & 0 & -S_3^{ij} C_{14} & S_3^{ij} C_{16} \end{bmatrix}.$$

For the beam at the joint j the stiffness matrix is

$$K_{jj} = \begin{bmatrix} B_1^{jj} D_1^{jj} & +2\, B_1^{jj}\, b_{jj} \dfrac{m^2\,\pi^2}{a^2} & 0 & 0 \\ -B_2^{jj}\, b_{jj} & -2\, B_2^{jj} & 0 & 0 \\ 0 & 0 & \dfrac{-2\,\pi\, m\, b_{jj}}{a} B_3^{jj} & 3\, B_3^{jj} \\ 0 & 0 & \dfrac{\pi\, m\, b_{jj}}{a} B_4^{jj} & -2\, B_4^{jj}\, m_3 \end{bmatrix}, \qquad (23)$$

where

$$B_1^{jj} = \frac{E\,h_{jj}^3\,b_{jj}}{48\,m_1}\,\frac{m^2\,\pi^2}{a^2}, \qquad B_2^{jj} = \frac{E\,h_{jj}^3\,b_{jj}}{24\,m_1}\,\frac{m^4\,\pi^4}{a^4}, \qquad B_3^{jj} = \frac{E\,h_{jj}\,b_{jj}^2\,m^3\,\pi^3}{6\,m_2\,a^3},$$

$$B_4^{jj} = \frac{E\,h_{jj}\,b_{jj}\,m^2\,\pi^2}{2\,m_2\,a^2}, \qquad D_1^{jj} = \frac{16\,G}{E}\left(1-0.63\,\frac{h_{jj}}{b_{jj}}\right)m_1 + \frac{m^2\,\pi^2}{a^2}\,b_{jj}^2,$$

$$m_1 = \left(1+\frac{E\,h_{jj}^2\,m^2\,\pi^2}{10\,G\,a^2}\right), \quad m_2 = \left(1+\frac{2\,E\,b_{jj}^2\,m^2\,\pi^2}{5\,G\,a^2}\right), \text{ and } m_3 = \left(1+\frac{E\,b_{jj}^2\,m^2\,\pi^2}{10\,G\,a^2}\right).$$

For uniform load, the fixed edge traction column vectors $\bar{F}_{jk}$ and $\bar{F}_{ji}$, at the joint j for the slab jk and ji are

$$\bar{F}_{jk} = \begin{bmatrix} -R_1^{jk}(2\,\beta_3 \operatorname{Cosh}\alpha_m - 1) \\ R_2^{jk}\,\beta_3 \operatorname{Sinh}\alpha_m \\ 4\,R_3^{jk}\,\beta_6 \operatorname{Sinh}\alpha_m \\ R_3^{jk}\,[4\,\beta_6 \operatorname{Cosh}\alpha_m - (1+\nu)] \end{bmatrix}; \quad \bar{F}_{ji} = \begin{bmatrix} R_1^{ji}(2\,\beta_3 \operatorname{Cosh}\alpha_m - 1) \\ R_2^{ji}\,\beta_3 \operatorname{Sinh}\alpha_m \\ 4\,R_3^{ji}\,\beta_6 \operatorname{Sinh}\alpha_m \\ -R_3^{ji}\,[4\,\beta_6 \operatorname{Cosh}\alpha_m - (1+\nu)] \end{bmatrix}.$$

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Summary

Two methods for the analysis of folded plate roof structures are presented. Both methods have been developed for solution with the aid of an electronic computer. The methods are "exact" in the sense that they are based upon classical elasticity theory and classical plate theory.

One procedure is formulated along the lines of the displacement method of structural theory and takes the form of a set of algebraic equations in which the unknown quantities are the four generalized displacements at each joint or edge. The coefficients and constants of the set of equations may be generated within the computer.

The second procedure is based upon numerical integration of an appropriate eighth-order set of differential equations. This procedure differs from the usual methods for handling shell problems in that the dependent variables are taken

to be the intrinsic quantities. The output of the procedure is thus directly in terms of the quantities of interest, including displacements, rotations and stresses at all points on the antinodal lines.

Résumé

Il est présenté deux méthodes de calcul des couvertures en voiles polygonaux; leur conception présuppose l'emploi d'une calculatrice électronique dans la pratique. Ces deux méthodes sont «exactes» en ce sens qu'elles se fondent sur la théorie classique de l'élasticité et celle des plaques.

La première procède de la méthode des déformations et prend la forme d'un système d'équations algébriques dans lesquelles les inconnues sont les quatre déplacements généralisés à chaque joint ou arête. Les coefficients et constantes du système peuvent être déterminés par la calculatrice.

La seconde méthode est basée sur l'intégration numérique d'un système approprié d'équations différentielles du 8e ordre. Elle diffère des méthodes habituellement utilisées dans les problèmes relatifs aux voiles en ce que les variables dépendantes sont prises comme grandeurs intrinsèques. Ce sont ainsi des grandeurs d'intérêt immédiat que les résultats fournis représentent: déplacements, rotations et contraintes en tous les points des lignes anti-nodales.

Zusammenfassung

In der Arbeit werden zwei Berechnungsmethoden für als Faltwerke ausgebildete Dachkonstruktionen erläutert. Beide Methoden sind für die Lösung mit Hilfe elektronischer Rechengeräte entwickelt worden. Die Methoden sind «exakt», in dem Sinne, daß beide auf der klassischen Elastizitätstheorie und der klassischen Plattentheorie aufbauen.

Die eine Art des Vorgehens basiert auf der Deformationsmethode der Tragwerktheorie und erscheint in der Form eines Systems algebraischer Gleichungen, in welchem die Unbekannten die 4 verallgemeinerten Verschiebungen an jedem Rand oder an jeder Kante darstellen. Die Koeffizienten und Konstanten des Gleichungssystems können mit dem Elektronenrechner gewonnen werden.

Die zweite Methode basiert auf der numerischen Integration eines entsprechenden Systems von Differentialgleichungen der 8. Ordnung. Dieses Vorgehen unterscheidet sich von den üblichen Methoden zur Behandlung von Schalenproblemen, indem die abhängigen Variablen als Eigenwerte betrachtet werden.

Diese Methode liefert so direkt die gesuchten Werte für Verschiebungen, Drehungen und Spannungen an allen Punkten der Gelenklinien.