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Life Estimate of Fatigue Sensitive Structures

Estimation de la durée de service d'ouvrages sensibles à la fatigue

Lebensdauer von ermüdungsempfindlichen Tragwerken

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1. Introduction

Metal structures subject to a large number of repeated loads of statistically variable intensity S may fail in either of two modes:

a) By excessive deformation, instability or sudden fracture resulting from the single occurrence of an unexpectedly high rare load intensity.
b) By progressive damage produced by repeated loads of operational intensity, in the form of distributed micro-cracks coalescing into localized macro-cracks, terminated by the occurrence of a load of high, but not unexpected intensity by which the damaged structure is destroyed.

While the first mode is usually referred to as "ultimate load failure" and the second as "fatigue failure", the latter is, in essence, also an ultimate load failure but one involving the fatigue-damaged structure, and therefore occurring under a terminal load of considerably lower intensity and of much higher frequency of occurrence than the "ultimate load" producing failure in mode a).

In this differentiation it is implied that the spectrum of operational loads which produce fatigue damage differs from the spectrum of "ultimate loads" which produces both the ultimate load and the fatigue failure in such a way that the latter cannot be obtained from the former by simple extrapolation towards very low probabilities of occurrence. In essence the spectrum of ultimate loads could however be considered as a spectrum of extreme values of large samples of operational loads. By this assumption a quantitative relation between the two load spectra could be established.

It should be noted that this concept of "fatigue failure" only applies to repeated *variable* load intensities. If a constant load intensity is repeated, as in a conventional fatigue test, fatigue failure occurs when the progressive damage in $(N-1)$ load repetitions has reduced the resisting section to such an extent that it can no longer carry the load at its N^{th} repetition. In this case the statistical variation of the load intensity, which produces the formal similarity between ultimate-load- and fatigue-failures, vanishes as a design parameter and the statistical variation of the fatigue life is due entirely to the variation in the rate of progressive damage $\left(\frac{dD}{dn}\right)$ resulting from the "inhomogeneity" of the polycrystalline structure of the metal. It is assumed that this rate of damage per load cycle is proportional to the difference between the applied stress level and a "threshold stress" (endurance limit); under constant load intensity S the effective stress increases as the initial cross section A is reduced by progressive damage to $(A - A_r)$. Introducing $D = \left(1 - \frac{A_r}{A}\right)$, the stress increases therefore as $(1 - D)^{-k}$, where $0 < D < 1$ is a measure of fatigue damage and $1 < k < 2$ characterizes the effect of the reduced cross section on the resultant stress intensity. The damage rate can therefore be expressed by

$$\frac{dD}{dn} = f\left[\frac{S}{(1-D)^k}\right]. \tag{1}$$

2. Return Period of Ultimate Load Failure

The life estimate of structures failing in mode a) can be based on the evaluation of the "return period" of ultimate failure. Since the probability of such failure P_U is related to the safety factor ν considered as the quotient of two statistical variables [1]

$$\nu_U = \frac{R}{S}, \tag{2}$$

where R is the structural resistance under ultimate load conditions with distribution $P_1(R)$, and S the "ultimate load" with distribution $P_2(S)$, by the assumption that

$$P_U = \int_0^{\nu_U} p(\nu_U)\, d\nu_U = P(\nu_U) \quad \text{for} \quad \nu_U = 1 \tag{3}$$

the mean "return period" of ultimate failure

$$\overline{T}_U = P_U^{-1} \tag{4}$$

expresses the mean number of repetitions of the statistical load S required, on the average, to produce one failure in nominally identical structures of statistical resistance R. The probability distribution of the return period, which expresses the probability that failure will occur before T_U load repetitions

$$P(T_U) = 1 - (1 - P_U)^{T_U} \tag{5}$$

since $(1 - P_U)^{T_U}$ is the probability that failure will not occur in T_U load repetitions. If P_U is small and T_U is large Eq. (5) may be written

$$P(T_U) = 1 - \exp\left(-\frac{T_U}{\overline{T}_U}\right). \tag{6}$$

Hence the probability that failure with mean return period $\bar{T}_U$ will not occur before T_U load repetitions, which is the probability of survival

$$L(T_U) = [1 - P(T_U)] = \exp\left(-\frac{T_U}{\bar{T}_U}\right) \tag{7}$$

or, for small $P(T_U)$

$$P(T_U) = \frac{T_U}{\bar{T}_U}. \tag{7a}$$

If the structure is to survive the "mean return period" of ultimate failure with a probability $[1 - P(T_U)]$ the return period of failure to be used for design is

$$\bar{T}_{UD} \sim \frac{T_U}{P(T_U)} = [P_U\, P(T_U)]^{-1}. \tag{8}$$

In other words if the structure shall survive a specified return period of failure with a specified probability of $[1 - P(T_U)]$, its design return period $\bar{T}_{UD}$ should be associated with a safety factor that ensures a probability of failure of $[P_U\, P(T_U)] \ll P_U$. Hence the "risk of ultimate failure" r_U, which is the probability of ultimate failure of a structure that has survived T_U load repetitions

$$r_U = \frac{p(T_U)}{L(T_U)} = -\frac{d\,L(T_U)}{L(T_U)} = -\frac{d}{d\,T_U} ln\, L(T_U) = \bar{T}_U^{-1}$$

is constant. If the structure is designed for a return period of ultimate failure $\bar{T}_{UD} \gg \bar{T}_U$, the risk of failure is reduced by the factor of $[P(T_U)]^{-1}$.

3. Return Period of Fatigue Failure

While practically all metal structures subject to repeated loads will show fatigue damage if the number of repetitions is large enough, fatigue is a significant design criterion only if the "return period" of fatigue failure under repeated variable load intensity is considerably shorter than the return period of ultimate failure. The safety factor of a structure subject to fatigue damage is no longer a stationary statistical variable but decreases with increasing number n of load repetitions which gradually reduce the resistance R to ultimate load failure. Hence, instead of Eq. (2) where ν_U is independent of n, ν is now a function of n

$$\nu_F = R(n)/S = \nu(n) \tag{9}$$

through the fatigue damage $D(n)$ which expresses the reduction of the resistance R by changing the distribution $P_1(R)$ to a family of distributions

$$P_1[R(n)] = P_1[R[1 - D(n)]^k], \tag{10}$$

where $D(0) < D \leqq D(n)$, with $D(0) = 0$ and $D(n) = 1$.

The distribution of

$$\frac{R}{S}[1 - D(n)[^k = \nu_U [1 - D(n)]^k \quad (11)$$

necessarily differs from the distribution of the quotient (R/S) because $[1 - D(n)]^k$ is not a constant but a statistical variable due to the statistical character of the damage function $D(n)$. Only if the distributions of both ν_U and $[1 - D(n)]^k$ are logarithmic-normal, the distribution of ν_F is also logarithmic-normal.

Under the simplifying assumption of non-statistical linear damage accumulation $D(n)$ the resulting relation between the distribution functions

$$P(\nu_U) = P\left[\frac{\nu_F}{[1 - D(n)]^k}\right] \quad (12)$$

implies that the probability of fatigue failure P_F at which $\nu_F \leqq 1$, is at the abscissa of the function $P(\nu_U)$ at which $\nu_U \leqq [1 - D(n)]^{-k}$. Thus the distribution functions $P(\nu_U)$ computed under various assumptions [2] for the distribution functions $P_1(R)$ and $P_2(S)$ and for the "central safety factor" of the design $\nu_{U0} = R_0/S_0$, can be used to determine the probability of fatigue failure under the ultimate load spectrum S as a function $P_F[D(n)]$ of prior fatigue damage produced by the operational load spectrum. Since $[1 - D(n)]^{-k} > 1$, the probabilities $P_F > P_U$.

Thus, for instance, for logarithmic-normal distributions $P_1(R)$ and $P_2(S)$ with $\nu_{U_0} = (\breve{R}/\breve{S})$ being the ratio between the median values of R and S, the probability function

$$P(\nu_U) = \Phi\left[\frac{1}{\delta}\log\left(\frac{\nu_U}{\nu_{U_0}}\right)\right], \quad (13)$$

where the error integral

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}t^2\right) dt \quad (14)$$

and the resulting standard deviation

$$\delta = [\sigma(\log R) + \sigma(\log) S)]^{1/2}. \quad (15)$$

The probability of fatigue failure therefore according to Eqs. (12) and (11)

$$P_F = P\{[1 - D(n)]^{-k}\} = \Phi\left\{\frac{1}{\delta}\log[(1 - D)^{-k}(\nu_{U_0})^{-1}]\right\} = \Phi\left[\frac{1}{\delta}\log\left(\frac{1}{\nu_{F_0}}\right)\right]. \quad (16)$$

Since $\nu_{F_0} = [1 - D(n)]^k \nu_{U_0}$ is a function of $D(n)$, the probability of fatigue failure P_F become functions of the damage D, and thus functions of the number n of load cycles applied.

Using the diagram $P(\nu)$ computed [2] for logarithmic-normal distributions of R and S with $\sigma_s/\breve{S} = 0.30$ and $\sigma_R/\breve{R} = 0.10$ for various ultimate load design values ν_{U_0} between 1.0 and 4.5 (Fig. 1) the following approximate values are

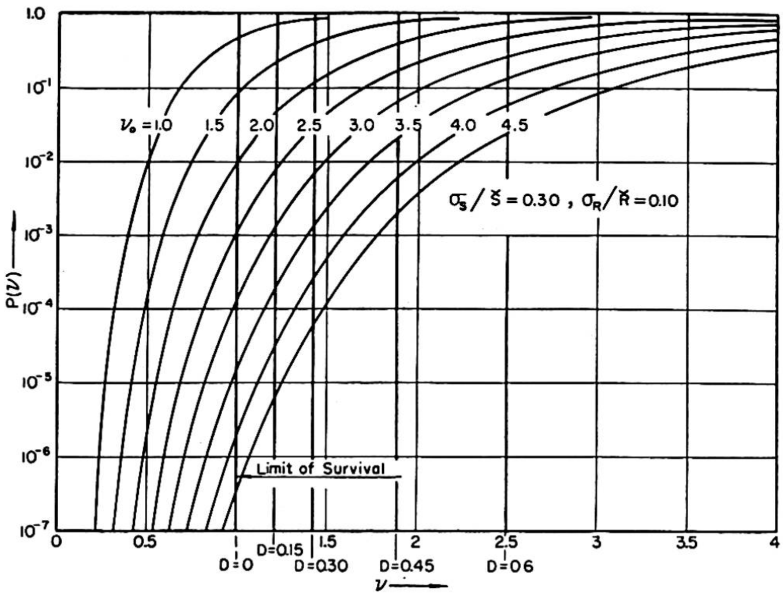

Fig. 1.

obtained for P_F as a function of D for the ultimate load design safety factors $\nu_{U_0} = 2$, 3 and 4:

Table I

D		0.0	0.15	0.30	0.45	0.60
P_F	$\nu_0 = 2$	10^{-2}	5×10^{-2}	10^{-1}	5×10^{-1}	9×10^{-1}
	$= 3$	2×10^{-4}	10^{-3}	8×10^{-3}	8×10^{-2}	3×10^{-1}
	$= 4$	3×10^{-6}	3×10^{-5}	3×10^{-4}	8×10^{-3}	8×10^{-2}

For the relation of D and the total number $n = \sum_i n_i$ of load cycles at the different stress levels S_i different assumptions can be made; the simplest is that of quasi-linear damage accumulation with stress interaction factors ω_i to compensate for the damaging ($\omega_i > 1$) or strengthening ($\omega_i < 1$) effect of interaction between high and low stress intensities [3] and with minimum lives N_{0i} delimiting the ranges of crack initiation and crack propagation [4]

$$D = \sum_i \left(\omega_i \frac{n_i - N_{0i}}{N_i - N_{0i}} \right) \quad \text{for} \quad N_{0i} < n_i < N_i. \tag{16}$$

For $n_i < N_{0i}$, $D = 0$.

With the aid of Eqs. (16) and (12) the relation between P_F and n can be established: with increasing value of damage D the probability of failure increases rapidly, as illustrated by Table I, and the mean return period of failure $\overline{T}_F$

$$\overline{T}_F(n) = [P_F(n)]^{-1} \tag{17}$$

decrease accordingly. Thus there is a mean return period of fatigue failure $\overline{T}_F[D(n)] = \overline{T}_F(n)$ associated with each damage level $D(n)$, and the ratio

$$\frac{P_F(n)}{P_U} = \frac{\overline{T}_U}{\overline{T}_F(n)} = f(n) > 1 \tag{18}$$

can be considered as a "fatigue sensitivity" factor of the structure. Obviously, the fatigue sensitivity increases with increasing damage, but for the same amount of damage also with increasing design safety factor for ultimate load design. This can be illustrated by converting Table I into a table of "fatigue sensitivity" factors f, dividing all rows by the value P_U which is identical with P_F for $D=0$

Table II

D		0.0	0.15	0.30	0.45	0.60
$f(D)$	$\nu_0 = 2$	1	5	10	50	90
	$= 3$	1	5	40	400	1500
	$= 4$	1	10	100	2670	26700

The return period of fatigue failure at constant damage has a distribution similar to that of ultimate load failure according to Eq. (6):

$$P(T_F) = 1 - \exp(-T_F/\overline{T}_F) \tag{19}$$

or

$$L(T_F) = \exp(-T_F/\overline{T}_F) \tag{20}$$

and therefore the "design return period" of fatigue failure will depend on the selected probability of surviving the mean return period, which might be considered as the specified design life of the structure.

The value of $\overline{T}_F$ depends strongly on the damage function $D(n)$ which, in turn, is strongly affected by the "minimum fatigue life" $N_{0i} = N_0(S_i)$, which delimits the fatigue initiation period. Since the length of the fatigue initiation period in relation to the total fatigue life at constant stress or variable stress is a characteristic of the structural material as well as of residual stress fields in the fatigue-critical parts of the structure produced by previous load history or arbitrary prestraining [5], both effects can be introduced into the damage factor and thus into the estimate of the probability of fatigue failure under the relevant spectra of operational and "ultimate loads".

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Summary

By defining fatigue failure as an "ultimate load failure" of a structure damaged in fatigue by operational loads, the estimate of fatigue life can be reduced to that of a "mean return period" of an ultimate load type of failure for which statistical methods of safety analysis have already been developed. By applying such methods in conjunction with a simple fatigue damage function the fatigue sensitivity of a structure can be evaluated in terms of the ratio of the return periods of fatigue failure and ultimate load failure.

Résumé

En considérant la rupture par fatigue comme la «ruine sous une charge limite» d'un ouvrage déjà fatigué par l'action des charges de service, on peut ramener l'estimation de la durée de service à celle du «nombre moyen de répétitions de charges données» relatif à un type de ruine sous une charge limite pour lequel on connaît des méthodes statistiques qui permettent le calcul de la sécurité. Si on applique ces méthodes en utilisant une fonction d'endommagement par fatigue mathématiquement simple, on pourra évaluer la sensibilité à la fatigue d'un ouvrage en fonction des nombres de répétitions des charges relatifs à la rupture par fatigue et à la ruine sous une charge limite.

Zusammenfassung

Durch die Definition des Ermüdungsbruches als einen «statischen» Bruch des durch Ermüdungsbeanspruchungen unter Betriebslasten geschädigten Tragwerkes kann die Abschätzung der Lebensdauer unter Ermüdungsbeanspruchungen zurückgeführt werden auf die Bestimmung einer mittleren «Rückkehrzeit» eines «statischen» Bruches, für welchen statistische Methoden der Sicherheitsberechnung bereits entwickelt wurden. Durch Anwendung dieser Methoden im Zusammenhang mit einer einfachen Ermüdungsschädigungsfunktion wird es möglich, die Ermüdungsempfindlichkeit eines Tragwerkes auszuwerten und als das Verhältnis der «Rückkehrzeiten» von Ermüdungsbruch und statischem Bruch darzustellen.