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## Vibrations of Framed Structures on Scale Models

*Vibrations de charpentes sur des modèles réduits*

*Schwingungen an Modellen von Rahmenkonstruktionen*

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The vibrations of complicated framed structures, such as framed foundations of turbine-generator sets cannot be reliably ascertained with the present means and calculating methods. Model tests offer the possibility of ascertaining the dynamic properties of the structure also with regard to the influences which in the calculation are mostly not taken into account (influence of shearing and longitudinal deformations of the beam, influence of the fixing, etc.).

In the Mechanical Engineering Research Institute of the Czechoslovak Academy of Sciences, a method for the measurements of the dynamic properties of framed structures was elaborated, and detailed investigations were carried out on several types of these structures. It was the purpose of this work to determine the influence of the individual parameters upon the position of the natural frequencies and the shape of the resonance curves.

In the first part of our investigations we studied the vibrations of various types of plane frames, at which the motion of the individual points takes place, on the one hand, in the plane of the frame and, on the other hand, perpendicularly to this plane. By measuring of the first eight natural frequencies we obtained for each frame diagrams which indicate the dependence of these frequencies on the ratio of the characteristic lengths of the frame, on the ratio of the torsional and the bending rigidity of the individual beams, etc. For instance, it was observed that at vibrations perpendicular to the plane of the frame the natural frequencies corresponding to antimetric and higher symmetric modes depended to a great extent on the torsional rigidity of the beam (changes up to 50% at  $0 < \frac{GJ_k}{EJ} < \infty$ ); other natural frequencies, such as the first symmetrical vibration, are practically not influenced by the torsional rigidity. The dependence of the frequencies on the parameters is plotted in the diagrams in dimensionless magnitudes.

Fig. 1 shows the course of the dimensionless magnitude

$$\lambda_2 = l_2 \sqrt[4]{\frac{\bar{\mu} \omega^2}{EJ}},$$

where denote:  $l_2$  characteristic length [cm],  
 $\bar{\mu}$  mass of the beam length of 1 cm  $\left[\frac{\text{kp s}^2}{\text{cm}^2}\right]$ ,  
 $EJ$  bending rigidity of the beam  $[\text{kp cm}^2]$ ,  
 $\omega$  natural frequency  $\left[\frac{1}{\text{s}}\right]$ ,

as a function of the length ratio  $\frac{l_2}{l_1}$  for the four lowest natural frequencies of the vibrations perpendicular to the plane of the frame and for  $\frac{GJ_k}{EJ} = 0,393$ . The course of the curve is in general continuous, only the dependence for the second symmetric form of the vibration exhibits in the surroundings of  $\frac{l_2}{l_1} = 1$  a break and the rise of the frequency is then smaller.

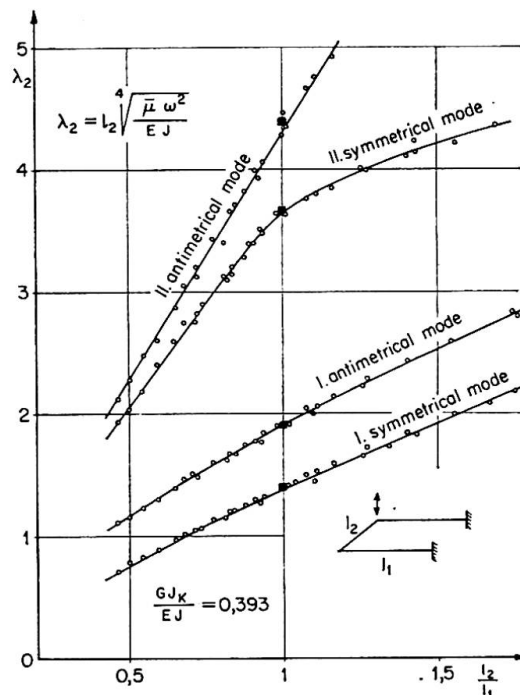


Fig. 1.

By comparison of the measuring results and the control calculations for some simple cases a sufficient accuracy of the model tests was proved (maximum error about 1%). In Fig. 1, the calculated points are marked by black squares.

The investigation of the plane frames was followed by measurements on spatial frames of the type of simplified frame foundations for turbosets. The accuracy of these model tests was again very good and the applied method of examination proved satisfactory. For this reason, it was also used for the investigation of the behaviour of actual foundations on models.

The scale of these models was 1 : 20. The material of construction was Umaplex of Czechoslovak manufacture (polymethyl methacrylate), dynamic modulus of elasticity  $E_{dyn} = 43\,600 \text{ kp cm}^{-2}$  in the frequency range of 20—400

c/s, accuracy in the construction of the models (wall thickness)  $\pm 0,01$  mm. The models of the foundation and of the machine were connected by means of an adhesive. The scale of the frequencies is

$$\frac{f_{model}}{f_{actual}} = \frac{n_{model}}{n_{actual}} = 7,22,$$

i. e. to an actual speed of 3000 rpm correspond 21 600 rpm on the model. By an appropriate selection of the force scale it was possible to attain that the amplitudes of the vibrations on the model and on the full-sized machine were equal and, consequently, easily measurable. The stress was remaining in the linear part of the characteristic. Vibrations in the measurement of the foundation were produced by an electromagnetic vibrator, in the measurement of the foundation with the mounted machine the vibrations were generated in the same way as on the full-sized machine, i. e. by rotating unbalanced rotors. These were constructed so that their critical speed corresponded to the critical speed of the actual machine, and thus it was possible to ascertain the mutual influence of rotors and foundations. The drive of the rotor was effected by a high-speed electric motor through an elastic coupling. The stator of the machine was so modelled, so as to fulfil the requirements corresponding to the mass, to the centre of gravity and the principal main moments of inertia, and roughly also to the rigidity of the casing and the bearing supports. The photograph of the model is shown in Fig. 2. For the measurements of the vibrations, inductive pickups Phillips, and tensometric and piezoelectric pickups of own manufacture were employed.

The measurements on the model were verified by measurements on the actual steel foundation. The positions of the natural frequencies on the model and the actual structure are roughly in agreement (max. deviation about 5%).

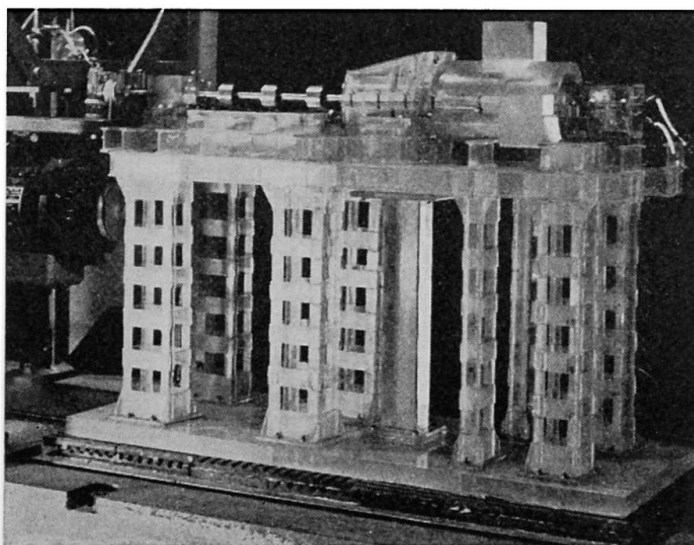


Fig. 2.

The shapes of the resonance curves were in agreement, too. The investigation on the model was mainly directed to the attainment of resonance amplitude curves in different points of the foundation and the machine, at different locations of the unbalanced mass on the rotors. In this way we obtained diagrams indicating the dynamic influence factors between the exciting rotating force and the amplitude in the main points of the structure. One of these diagrams is shown in Fig. 3. Here is plotted the dependence of the amplitude of horizontal vibrations of the front bearing of the turbine produced by the unbalance on the turbine rotor.

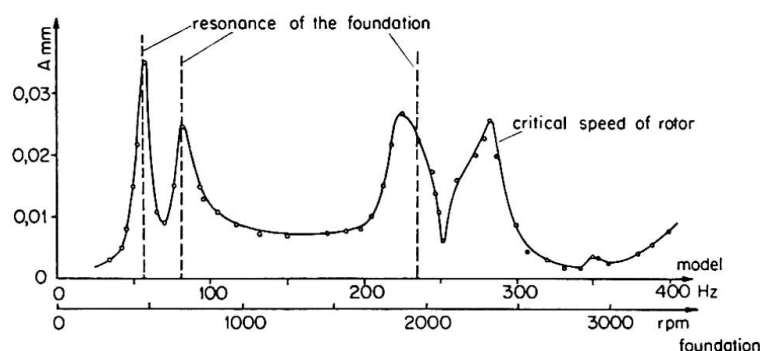


Fig. 3.

The influence of the elastic soil foundation on the behaviour of the machine foundation was ascertained on the foundation with the under plate rigidly attached to a concrete block, then with the inserted rubber-layer and with two of these layers. (In Fig. 2 there is a photograph of the foundation supported by two layers of rubber.) The rigidity of mentioned layers roughly corresponded to the rigidity of the soft soil. One layer had the rigidity  $C_{z\text{ mod.}} = 20 \text{ kp cm}^{-3}$  ( $C_{z\text{ real}} = 3 \text{ kp cm}^{-3}$ ), two layers  $C_{z\text{ model}} = 10 \text{ kp cm}^{-3}$  ( $C_{z\text{ real}} = 1.5 \text{ kp cm}^{-3}$ ). These relatively low values were chosen, so that the shifting of frequencies could be sufficiently expressive. The change of frequencies was mainly shown in lower modes by horizontal vibrations, which remarkably decreased. This fact is apparent from the diagram in Fig. 4, where positions of resonance peaks are plotted in dependence on frequency of exciting force  $f$  (Hz). The positions of peaks are signed with vertical abscissas. The magnitude of the abscissas is proportional to the greatest resonance amplitude. The diagram is ordered so as the frequencies of vibrations has been plotted in the horizontal direction. In the vertical direction every line corresponds to one response curve, signed by the number of the measurement. Individual measurements were differed by location of unbalances and by place, where the amplitudes of vibrations has been picked up. From the diagram it could be seen, that the first "horizontal" eigen-frequency was lowered from the value  $F_{H1}^{(0)} = 56 \text{ Hz}$  corresponding to rigidly attached under plate (Zero elastic layers) to  $F_{H1}^{(1)} = 47 \text{ Hz}$  with one rubber layer and to  $F_{H1}^{(2)} = 44 \text{ Hz}$  when situated on two rubber layers. The second eigen-frequency  $F_{H2}^{(0)}$  moved from 80 Hz to

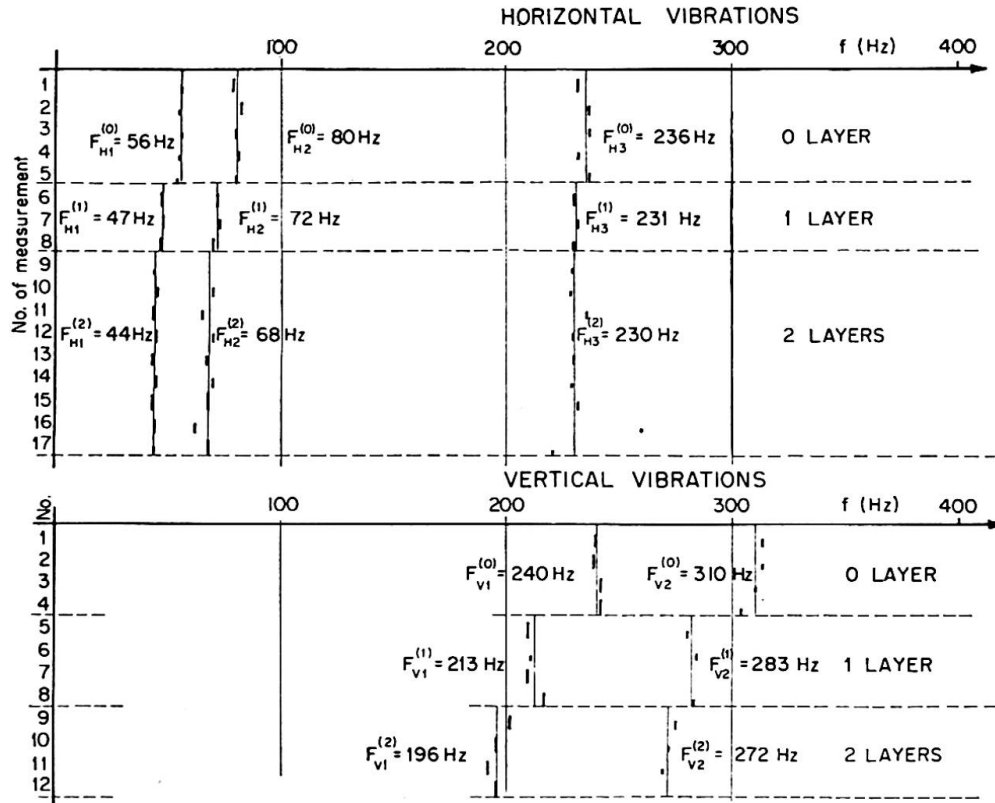


Fig. 4. Influence of rigidity of soil foundation on the resonance peaks' frequency of structures.

72 Hz and 68 Hz in the case of one and two rubber layers. The shift of eigen-frequencies, by  $F_{H1}$  and  $F_{H2}$  is therefore 16 and 21% (9 and 15% respectively). The third eigen-frequency of horizontal vibration  $F_{H3}$  on the contrary remained unchanged, because the change of  $F_{H3}^{(0)}$  from 236 Hz of rigidly attached under plate, to  $F_{H3}^{(1)} = 231$  Hz and  $F_{H3}^{(2)} = 230$  Hz by situating of under plate on one or two rubber layers is very small and is only about 2—2,5%.

This result can be explained by the mode of vibration, which corresponds to the defined frequency. During this vibration the elastic forces from pillars influencing the under plate, are roughly balanced out. This adequately rigid plate is practically in rest and none of displacements are transmitted to the soil foundation. The first "vertical" mode of vibration is mostly influenced by the under-layer-rigidity. The corresponding eigen-frequency by rigid under plate is  $F_{V1}^{(0)} = 240$  Hz. When posing the under plate on one layer, this frequency has been lowered to  $F_{V1}^{(1)} = 213$  Hz, and by posing on two layers the frequency was  $F_{V1}^{(2)} = 196$ , this is a decrease of 11% and 18%. This is roughly the same decrease as with the lower eigen-frequencies of "horizontal" vibrations. The second eigen-frequency  $F_{V2}$  shows a comparatively shift from 310 Hz to 283 and 272 Hz, this is a decrease of 8,5 and 12%.

From the mentioned results it follows, that by computing of the lower eigen-frequencies the rigidity of soil foundation must be respected, on the

contrary in the case of higher modes the rigidity of the soil foundation has almost no influence. The reason of this phenomena is the balancing out of the elastic and inertial forces in the structure itself and therefore none force is transmitted to the under plate. It should be emphasized, that the influence of the soil rigidity has been ascertained on a scale-model for a considerable change of the soil rigidity to the highest value of  $10 \text{ kp cm}^{-3}$  (the corresponding value by an actual foundation is  $1,5 \text{ kp cm}^{-3}$ ). The foundations are often situated on sufficiently rigid soils, the bulk modulus of which is several times higher and therefore the influence of the foundation rigidity will be lower, too.

### Summary

A method for measurements of dynamic problems of machine-foundations is described. The vibrations of various types of plane frames were studied. The accuracy of the model-tests was proved to be about 1%. The investigation of the model of actual steel turbine-foundation shows the influence of the rigidity of soil foundation on the frequency of resonance peaks. The change of frequency depends on the mode of vibration.

### Résumé

L'auteur décrit une méthode de mesure du comportement dynamique des fondations de machines. Il a étudié les vibrations de divers types de portiques plans. La précision des essais sur maquette s'est avérée être de l'ordre de 1%. L'étude d'une maquette d'un socle de turbine en acier montre l'influence de la rigidité du sol sur les fréquences des oscillations naturelles. La modification de la fréquence dépend du mode de vibration.

### Zusammenfassung

Der Verfasser beschreibt ein Verfahren zur Untersuchung des dynamischen Verhaltens von Maschinenfundamenten. Er hat die Schwingungen von verschiedenen Arten ebener Rahmen studiert, wobei die Voraussage auf Grund des Modellversuchs in der Regel um nicht mehr als 1% von der Wirklichkeit abwich. Die Untersuchung des Modells einer Turbinenfundament in Stahl zeigt den Einfluß der Steifigkeit des Untergrundes auf die Eigenfrequenzen. Dabei stellte sich heraus, daß die Veränderung der Eigenfrequenzen von der Schwingungsform abhängig ist.