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## IV a 4

### A Theory for the Combined Action of Bending Moment and Shear in Reinforced and Prestressed Concrete Beams

Short review of a treatise by Dr. MOGENS LORENTSEN<sup>1)</sup>

*Théorie sur l'action combinée de la flexion et du cisaillement dans les poutres en béton armé et en béton précontraint*

*Theorie über das Zusammenwirken von Biegung und Schub in armierten und in vorgespannten Betonträgern*

GEORG WÄSTLUND

Professor, head of the institution

This is a short review of a doctor thesis by MOGENS LORENTSEN: "Shear and bond in prestressed concrete beams without shear reinforcement." LORENTSEN's full paper is rather comprehensive. It contains among other things test results from 30 beams, all without shear reinforcement but of various design as to the shape of cross section, some are reinforced and other prestressed, some with bond and others without bond.

The paper further gives a conception of the mechanism of shear failure and also the bases for computation of the critical shear force. The theory has a great advantage in that it is applicable to ordinary reinforced concrete as well as to prestressed concrete.

LORENTSEN has applied his theory to test results published by earlier authors as well as to test results of his own. The agreement is satisfactory.

His formulas are to some extent similar to those given in the new American Building Code Requirements. It is interesting that LORENTSEN's formulas are based on a conception of the mechanism of failure, whereas the American ones are more empirical. In the following some main points of the theory are given.

Fig. 1 shows in principle a reinforced beam with bond between reinforcement and concrete. The beam is supposed to go to fail in shear. The figure shows the approximative distribution of the tension force just before what LORENTSEN calls "the critical shear cracking". As seen, here the tension force in the reinforcement drops abruptly to a low value in the uncracked part of the beam. That will mean very high bond stresses on a short length, denoted  $c$  in the figure.

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LORENTSEN has simulated this case in an exaggerated manner in a test beam, see Fig. 2. That is a T-beam with only one reinforcement bar. There is no bond between bar and concrete, but anchorages at both ends and at two intermediate points. The intermediate anchorages correspond to very con-

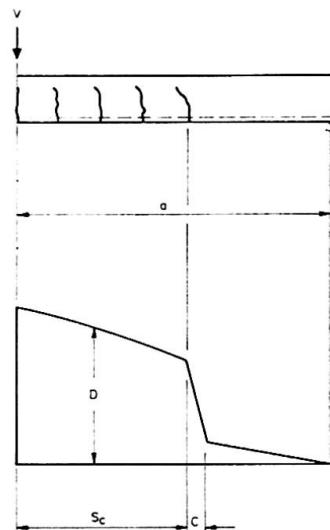


Fig. 1. Approximative distribution of tension force  $D$  in the reinforcement bar.

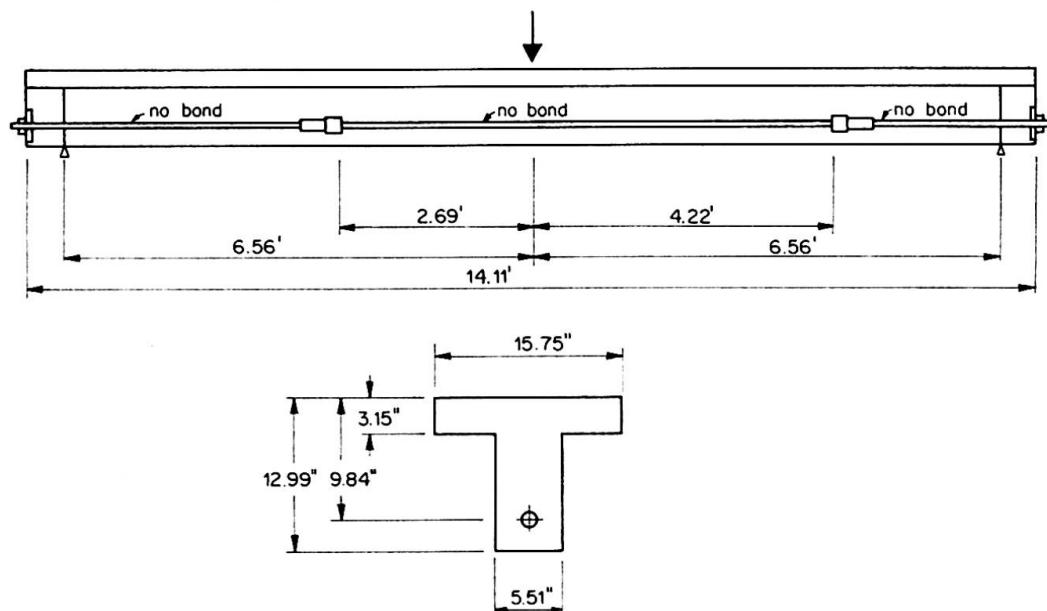


Fig. 2. Test beams with intermediate anchorages.

centrated bond forces in a cracked reinforced beam with the anchorages placed just at the outermost bending cracks.

Fig. 3 shows the behaviour at failure of that beam. First came bending cracks, one in the middle and two at the intermediate anchorages. At further loadings those outermost bending cracks were extended, in the compressed zone near horizontally in the direction towards the load, in the tensile zone

horizontally towards the supports causing splitting of the concrete. The lower part of the figure shows the measured strain distribution in the bar.

In an ordinary reinforced beam the mechanism of failure has been found to be quite similar. Fig. 4 shows such a T-beam, like that in Fig. 2, but now

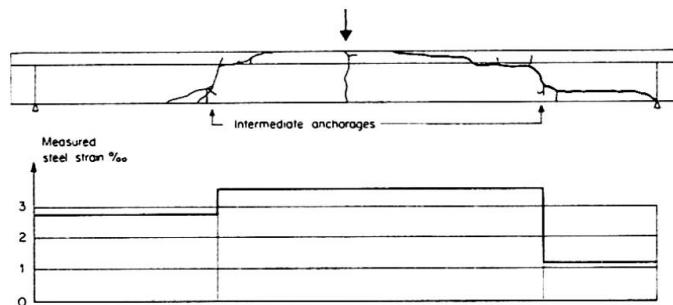


Fig. 3. Failure of a beam, shown in Fig. 2.

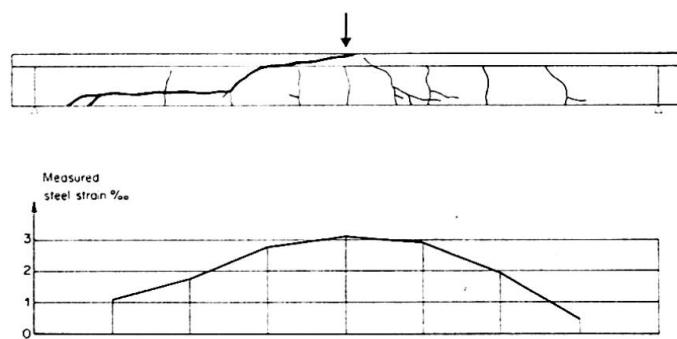


Fig. 4. Failure of a beam with normal bond.

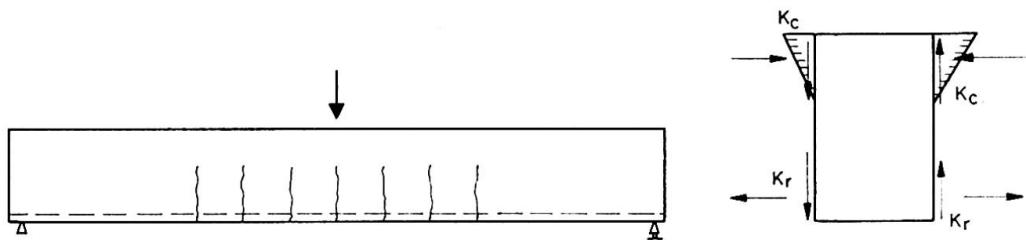


Fig. 5. Shear forces in a beam with pure beam action.

with normal bond before cracking all along the bar. The beam was tested in the same manner, first the bending cracks appeared. One of the outermost cracks then bent near horizontally proceeding further on both in the upper and lower parts of the beam.

LORENTSEN gives the following conception of the mechanism of failure. Fig. 5 shows schematically how a beam gets bending cracks and how the shear in this state is carried by a "crack lamella". This state represents a pure beam action. The shear force then is composed of two components  $K_c$  and  $K_r$ .  $K_c$  is a component, carried by the uncracked concrete zone, and  $K_r$  is a compo-

ment, carried by the reinforcement bar due to its dowel effect. Similar thoughts have recently been published by KANI<sup>2</sup>).

The beam action is supposed to be complete up to the stage, when the shear force attains the so called lamella strength. When the shear force exceeds the strength of the lamella, the load will be carried by a combined beam and arch action. Fig. 6 shows this case, with forces here acting on a very short

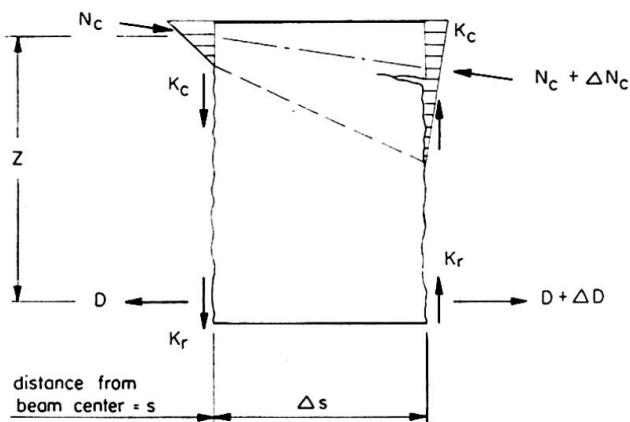


Fig. 6. Shear forces in a beam with combined beam and arch action.

lamella. The pluses are of pure nominal kind and can in reality be minus. The arch action is represented by the slope of the compression forces. The maximal shear then will be composed of three forces,  $K_c$  and  $K_r$ , as before, and in addition the vertical component of  $N_c$ .

The mechanism of the shear failure is now assumed to be as follows. The bending crack in the "section of critical shear" is considered. In the figure this crack is represented by the right hand side of the lamella. The force  $K_c$  will have its maximal value, when the *vertical* tensile stress in the lamella initiates a near horizontal crack in the lamella. This crack will then proceed further on in the beam in the direction towards the load.

As soon as this crack will start to open, the dowel effect of the reinforcement will come to action. When the dowel effect is fully utilized and reaches its maximal value  $K_r$ , the bar will split the concrete along the bar in the direction towards the support.

Simultaneously as the bond diminishes the arch action will start, which is marked on Fig. 6 through sloping compression forces  $N_c$ .

LORENTSEN has deduced a value of  $K_c$  corresponding to the creation of a horizontal tensile crack in the lamella, the deduction made in accordance with the theory of elasticity, see Fig. 7. In this case the lamella was divided

<sup>2</sup>) G. N. J. KANI: "The Riddle of Shear Failure and its Solution", Journal of ACI, April 1964.

in elements, and for the numerical calculations a computing machine was used. The right part of the figure shows the calculated vertical tensile stresses (relative values) in three horizontal sections. The lamella concrete strength  $K_c$  also was studied by testing rectangular concrete elements, simulating the lamellas.

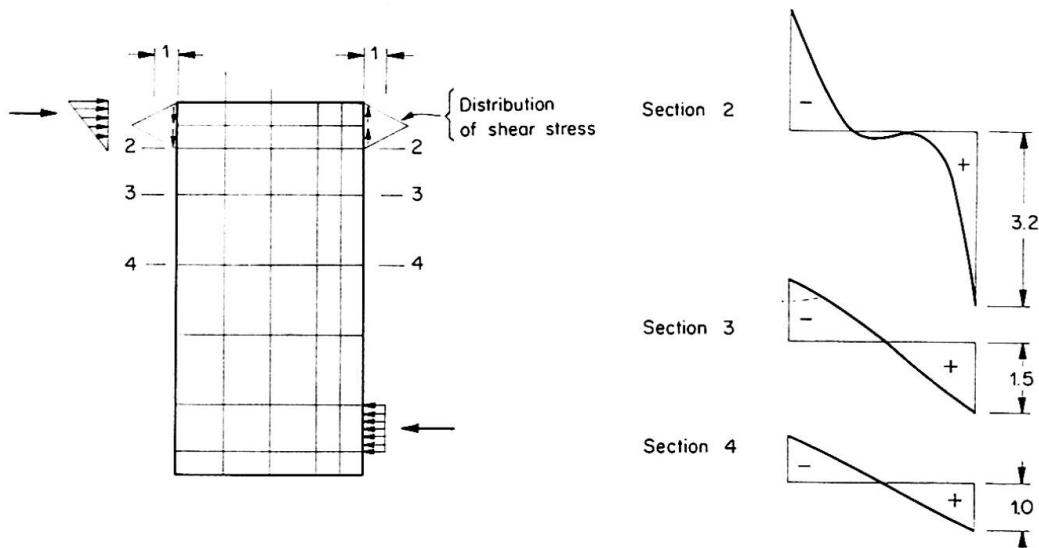


Fig. 7. Calculated vertical tensile stresses (relative values) in three horizontal sections of a lamella, the stresses caused by the shear force  $K_c$  in the compressed zone.

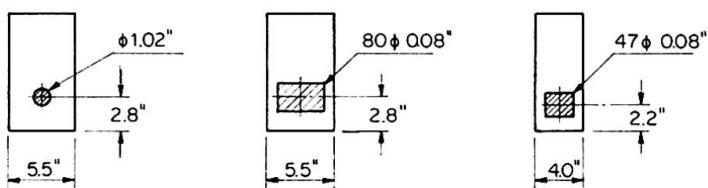
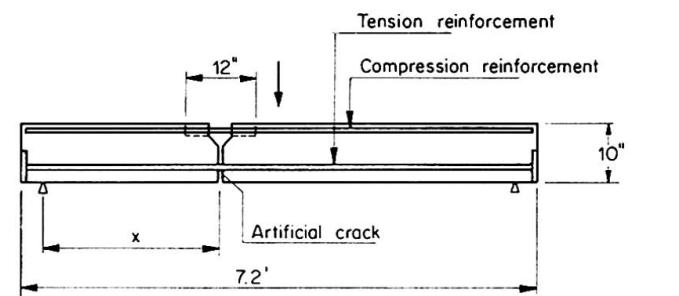


Fig. 8. Test beams for the study of dowel effect.

LORENTSEN also to some extent studied the value of  $K_r$ , the dowel effect, partly through own tests, partly through evaluating tests by other authors. Fig. 8 shows his test beams. The beams had an artificial crack, where the shear mainly was transferred through the dowel effect. Different tension reinforcement was used.

In order that the expressions might be comparable with the American formula the expressions are given in English units.

LORENTSEN found the following expressions

$$K_c = 2.3 b k d \sqrt{f'_c},$$

$$K_r = 0.95 b H \sqrt{f'_c},$$

$$K = K_c + K_r \text{ "lamella strength"},$$

where  $d$  and  $H$  are shown in Fig. 9,

$f'_c$  = prism strength,  
 $k \cong \sqrt{n p}$ ,  
 $n = E_r/E_c$  relation between elastic moduli,  
 $p$  = percentage of reinforcement.

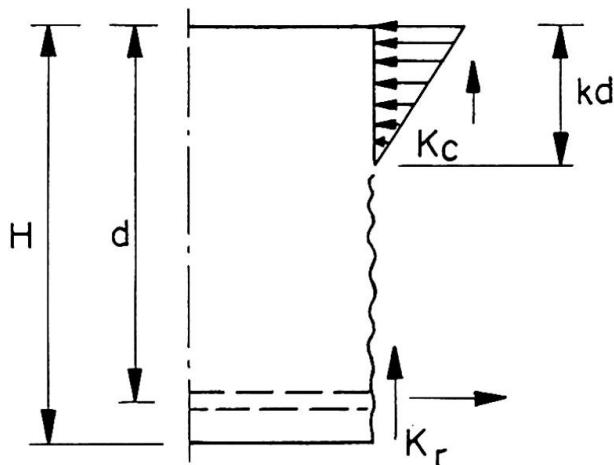
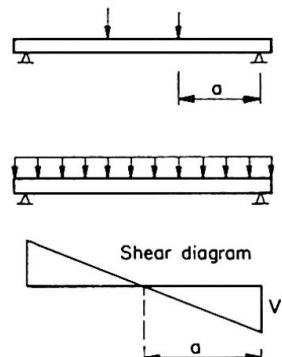
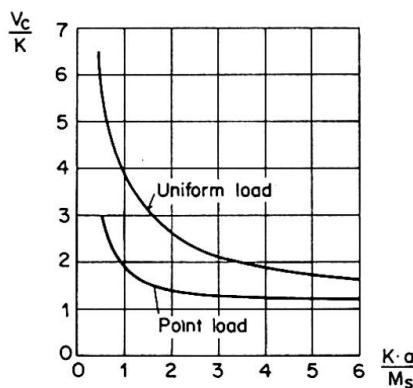


Fig. 9. Notations.

Critical shear strength  $V_c$

$$V_c = K \cdot \Phi \left( \frac{K \cdot a}{M_s} \right) = \gamma \cdot K > K$$



$M_s$  = Moment giving bending crack, prestressing taken into account.

Fig. 10. Magnification factor  $\gamma$  due to combined beam and arch action.

The value of  $K$  represents the main part of the critical shear force. Fig. 10 gives the value, if in addition to  $K$  the arch action is taken into account. A magnification factor  $\gamma$  is introduced, which is dependant on the parameter  $K a/M_s$ , where  $M_s$  is the moment giving bending cracks in the beam, in case there is prestressing also this taken into account. The  $\gamma$ -value is given in diagram, Fig. 10 for beams with uniform load and with point loads. It is seen that uniform load is more favorable. Favorable is also a short shear span  $a$  and a high value of  $M_s$ , crack moment, all in accordance with known facts. The same formulas thus are given for both ordinary reinforced concrete and prestressed concrete. For use in practice some further development might be necessary.

### Summary

The paper comprises a short review of a doctor thesis by MOGENS LORENTSEN, giving a conception of the mechanism of shear failure in reinforced and prestressed concrete beams without shear reinforcement. This conception also gives the bases for computation of the critical shear force, as shown.

### Résumé

L'auteur donne un court aperçu de la thèse de MOGENS LORENTSEN, thèse qui traite le mécanisme de la rupture par cisaillement dans les poutres en béton armé et en béton précontraint, sans armatures de cisaillement. Ce travail donne aussi les bases pour le calcul de l'effort de cisaillement critique.

### Zusammenfassung

Dieser Beitrag enthält eine kurze Übersicht über die Doktorarbeit von MOGENS LORENTSEN, welche den Schubbruchmechanismus in armierten und in vorgespannten Betonträgern ohne Schubarmierung behandelt. Diese Arbeit gibt auch die Grundlagen für die Berechnung der kritischen Schubkraft.

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