IVa1

On the Reduction of Shear Reinforcement as Derived From the Stuttgart Shear Tests 1961—1963

Sur une réduction de l'armature de cisaillement basée sur les essais de Stuttgart (1961—1963)

Über die Verringerung der Schubbewehrung auf Grund der Stuttgarter Schubversuche von 1961—1963

FRITZ LEONHARDT

o. Professor, Technical University of Stuttgart

Notation

t	total depth of section.
d	distance from extreme compression fiber to centroid of tension reinforcement.
z	distance from T to C, leverarm of internal forces.
b'	width of web.
b	width of compression flange.
s	spacing of stirrups or bent up bars in a direction parallel to the longitudinal reinforcement.
h	amount of horizontal movement of the M/z-line to get the T-line.
l	span length.
a	distance of single loads from support.
M	bending moment.
C	internal compression force in compression chord of "truss" due to bending.
C_s	compression force of the inclined struts of the "truss".
T_x	tension force of the longitudinal reinforcement (tension chord of "truss") due to bending.
T_y	tension force of stirrups
P	load on test beam.
P_p	prestressing force.
V	total shear force.
V_i	shear force carried by the compression members of the truss.
V'	shear force carried by the web reinforcement.
$v = \dfrac{V}{b'z}$	shear stress.
$v_i = \dfrac{V_i}{b'z}$.	

f_v tensile stress in web reinforcement.

f'_c compressive cylinder strength.

f_p compressive stress in p. c. due to prestressing force after all losses at centroid of cross section.

f_y yield strength of reinforcement.

t_1, t_2 principal stresses according to bending theory of beams.

A_v area of web reinforcement within the length s of the beam.

A_c gross area of section.

α angle between web reinforcing bars and longitudinal axis of beam.

β angle between compression strut and longitudinal axis of beam in truss analogy.

$r = \dfrac{A_v}{b's}.$

η factor of shear coverage, related to shear reinforcement according to MÖRSCH's analysis.

κ second reduction factor for shear coverage for short beams or loads close to support.

1. Introduction

The Stuttgart shear tests, which have been published in seven reports [1], [2], [3], [4], [5], [6] and [7] have brought considerable new knowledge due to the fact, that the many parameters, which have influence on the ultimate shear load, have been studied each separately. It has been found e. g. that the most unfavourable shear condition is given by one or two concentrated loads per span with a moment shear ratio of $\dfrac{M}{Vd} = 2.4$ to 3.5. All other load patterns can be considerably more favourable. The following paper deals only with this unfavourable loading, short beams or loads near the support with $\dfrac{M}{Vd} < 2.4$ are treated separately.

It must also be mentioned that all the Stuttgart tests have been made with high tensile steel as reinforcement with a yield point of $f_y = 60\,000$ psi with deformed bars.

2. The Variability of the Mechanism in the Cracked State

The classical shear analysis of W. RITTER and E. MÖRSCH explain the shear strength in the cracked state with the truss analogy, using a truss with parallel chords, compression struts at an angle of 45° and vertical or with $-45°$ inclined tension bars (fig. 1). It is assumed that the shear force V will be fully carried by the web members, which leads to the following percentage of shear reinforcement in the web for the length s

$$r = \frac{v}{f_v} = \frac{V}{b'z\sin\gamma f_v} \quad \text{with} \quad r = \frac{A_v}{b's}.$$

This amount of shear reinforcement may be called full shear coverage, a reduced amount will be defined by the reduction factor $\eta < 1$, referring to this full coverage.

with vertical stirrups, n - fold truss, $n = \frac{z}{s}$

stirrup forces $T_y = V$
diagonal compression forces $C_S = V\sqrt{2}$ } related to the beam length $\Delta \ell = z$

with inclined stirrups

diagonal forces $T_S = C_S = \frac{V}{\sqrt{2}}$ related to the beam length $\Delta \ell = z$
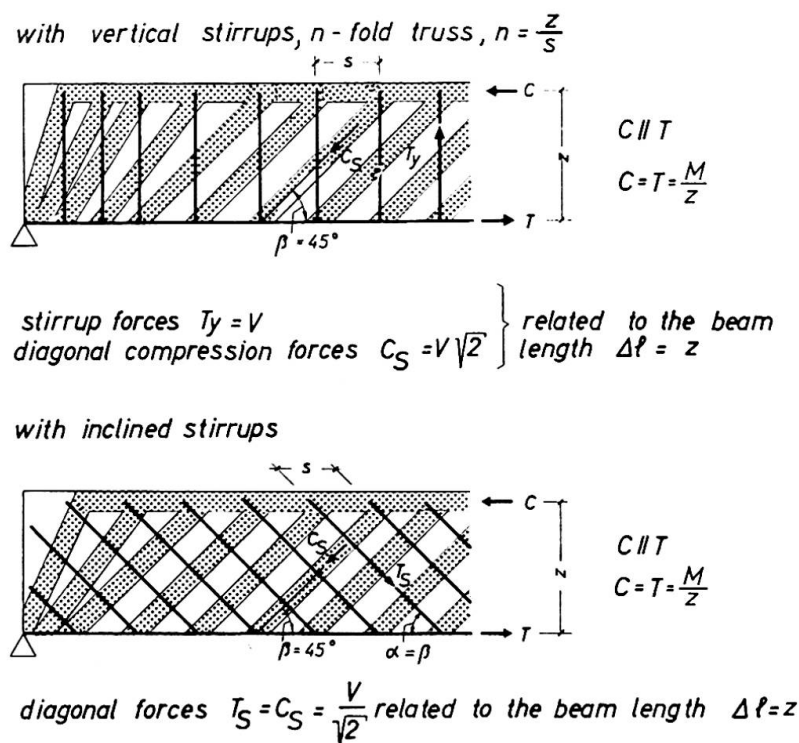
Fig. 1. Assumptions of classical truss analogy according to E. MÖRSCH.

With this classical truss analogy, the equilibrium conditions for the inner forces are fulfilled, but not the compatibility conditions, which are badly hurt, because the compression members of concrete are 3 to 20 times stiffer than the tension members of thin steel bars. Nature does not know this truss analogy and acts according to the law of the minimum of deformation work and, therefore, the inner forces in reinforced concrete beams change considerably as soon as cracks appear, strongly depending upon the stiffness ratio between the concrete and the steel members. A beam with a thick web will show considerably different internal forces compared with a beam of equal chord members, but with a thinner web. Even the crack pattern changes with this stiffness ratio and has influence on the mechanism. We shall try to explain these mechanisms by our test results, using again trusses as an analogy.

2.1. Mechanism for Slabs and Beams with Thick Webs (fig. 2)

In beams or slabs with rectangular cross-section we have the mechanism of a tied arch and the shear forces are carried totally by the inclined upper chord (arch or strut). The tie force decreases towards the support by the unreliable bending resistance of the concrete teeth between the cracks, as shown by G. KANI in [17], unreliable, because this resistance decreases to

almost zero, when the ratio between crack spacing and crack depth becomes small in beams with $t > 15''$. For ultimate load, this decrease of the tie force can become small, therefore, the tie should continue to the supports with full area and be well anchored.

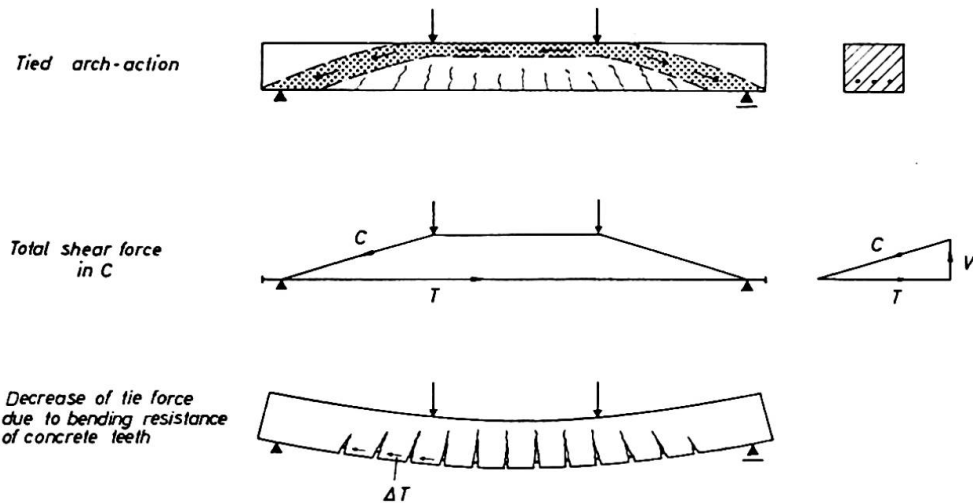As soon as cut-off bars are used in rectangular cross-sections with their

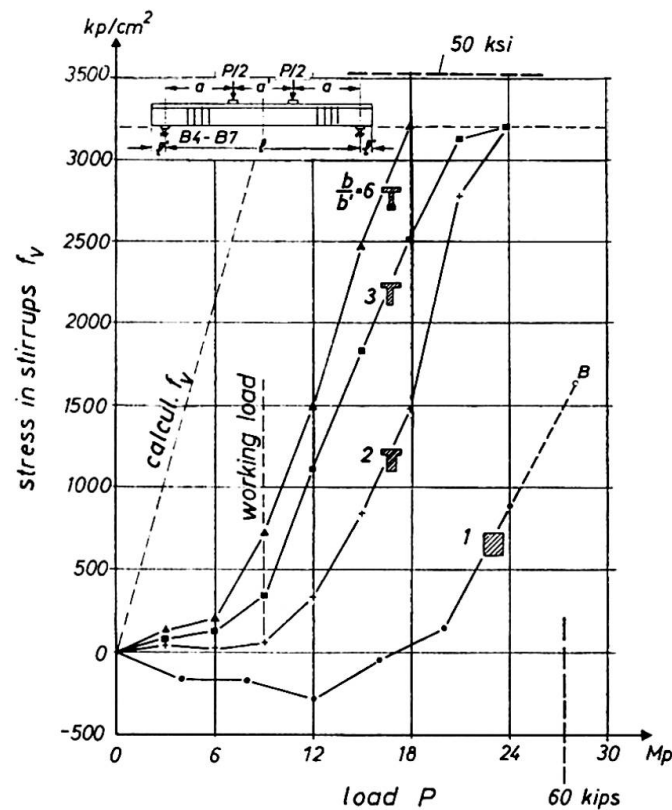Fig. 2. Tied arch action in beams with thick webs.

Fig. 3. Average stresses in stirrups, measured in beams with the one variable $b/b' =$ width of compression zone to width of web, with equal and constant shear reinforcement ($\eta = 0,38$) compared to stresses calculated according to MÖRSCH.

length determined by the moment diagram, then the shear strength will be reduced considerably [6].

A shear reinforcement in such cross-sections does not change this mechanism under working load, stirrups remain without tension up to very high load intensities and they begin to carry only after shear cracks enter the inclined compression chord C. The test results as shown in fig. 3 prove, that the stresses in stirrups are strongly influenced by the thickness of the web and in beams with rectangular cross-section they remain very small or are even negative (compression) up to $0.7\,P_u$ (ultimate load).

2.2. The Change of the Mechanism by the Reduction of the Web-thickness

If in T-beams, the compression and tension chords are kept equal and only the thickness of the web is reduced, then the tensile stresses of the equal stirrups grow almost proportional to the ratio b/b' (fig. 3). The measured tension forces in stirrups, however, remain far below those calculated with MÖRSCH's truss analogy even for webs as thin as $2''$ with $b'/b = 1/6$. This means that even in beams with very thin webs, a part of the shear force must be carried by the chords and not by the web members. In fact, we have measured tensile stresses on top of the compression flange near the supports. Therefore, even in T-beams the resultant compression force C, which can be considered as the top chord of an imaginated truss, must be inclined. Fig. 4 shows that a slope of 1/12 to 1/20 between the load and the upper end of the last shear crack is easily possible and has been observed. This slope means that 25% to

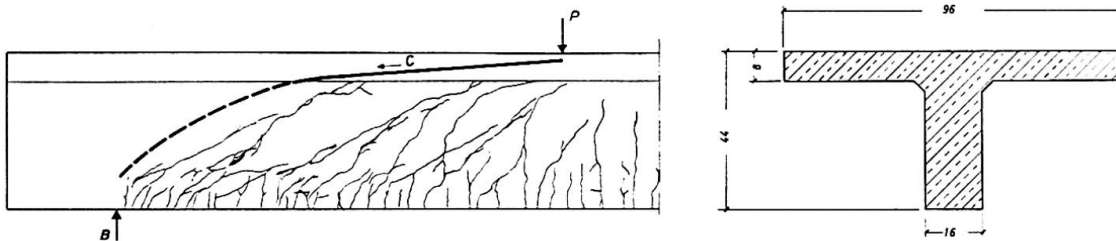Fig. 4. In T-beams, the resultant force C in the compression chord can be inclined by 1 : 12 to 1 : 20 in spite of shear cracks covering the whole depth of the web.
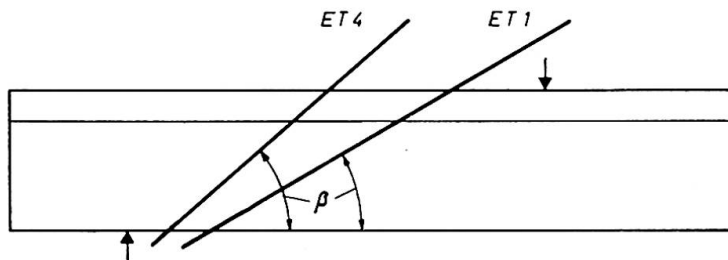

Fig. 5. Crack pattern of rectangular beam ET 1 with $b/b' = 1$ compared with pattern of T-beam ET 4 with $b/b' = 3$. Both beams have equal longitudinal and shear reinforcement. Average angles β of shear cracks differ considrably.

Nr.	truss-system	tension forces	
		in the web T_y related to z	in the chord T_x near the support
1		0	3,0 V
2		0,60 V	1,5 V
3		1,0 V	1,0 V
4		$T_{45°}$ $\frac{1}{2}\sqrt{2}\, V$ = 0,71 V	0,5 V
5		0,67 V	1,5 V
6		0,57 V	1,73 V

Fig. 6. The influence of an inclination of the top chord or of the angle $\beta \leqq 45°$ of the compression struts in simple trusses on the vertical tension force T_y and on the tie force T_y near the support.

15% of V is carried by the compression chord in a beam with a shear span
of 3. In the bottom chord (longitudinal reinforcement), a small part of V
might be carried by the dowel effect of the reinforcing bars; however, this
effect is surely over-estimated in [15], as soon as stirrups are used.

Further, we have observed that the inclination of the shear cracks is not
simply 45°, but varies considerably with the stiffness ratio expressed by b'/b
and can be as low as 30° (beam ET 1 in [2]) (fig. 5). This means that the dia-
gonal struts of our imaginated truss are less inclined and, therefore, a smaller
quantity of vertical tension bars, the stirrups, will be sufficient for the equi-
librium. Simultaneously, the tension force in the bottom chord near the
support will be larger than in a truss with 45° struts. Fig. 6 explains by simple
truss analysis, how the vertical tension forces T_y and the chord tension T_x
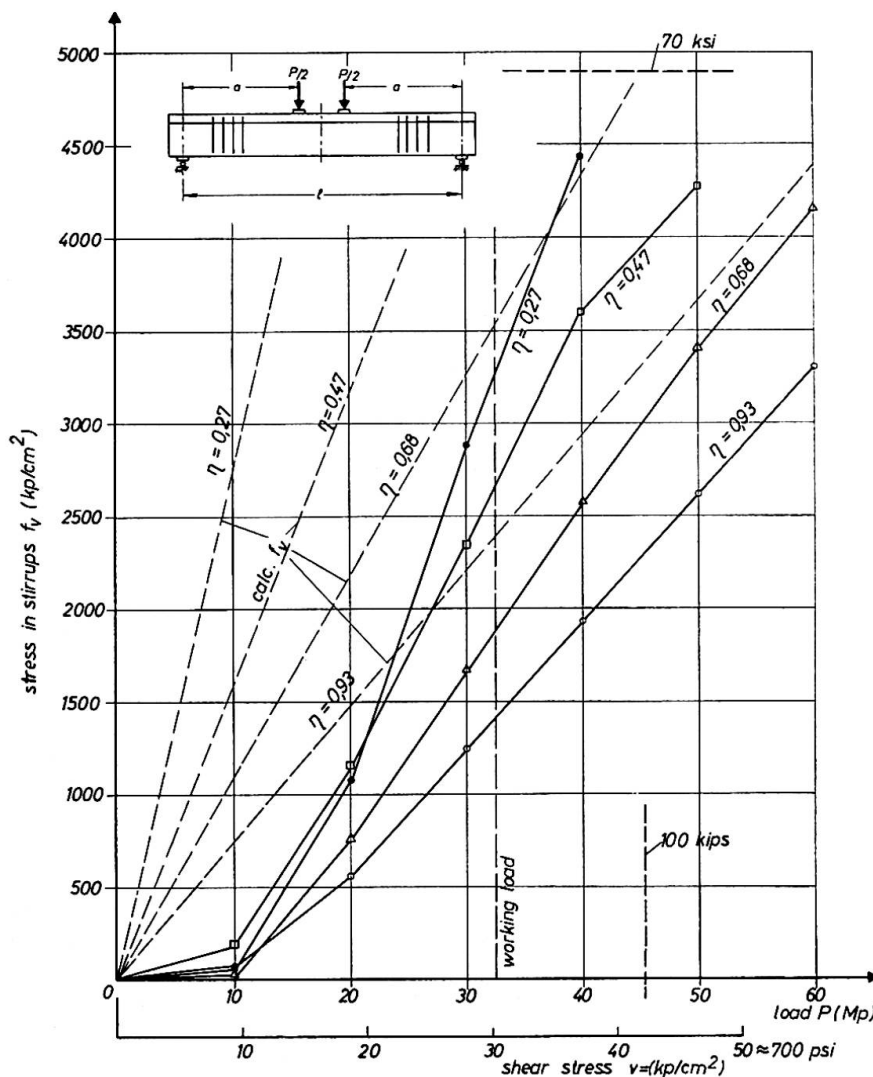are influenced, when the inclination of the top chord varies from 1 : 3 to zero

Fig. 7. Average stresses in stirrups, measured in beams with constant $b/b' = 6$ and A_s,
variable only the percentage of shear coverage η, compared with calculated stresses
according to MÖRSCH.

or the angle β of the diagonal struts varies from 45° to 30°. T_y can be between zero and V, related to the beam length $\varDelta l = z$.

The inclination of the shear cracks can also be influenced to a smaller extent by the percentage of shear coverage, expressed by η. We have found that for equal $b'/b = 1/6$, the angle β of shear cracks can vary between 37° for $\eta = 0.25$ and 44° for $\eta = 1.0$, the average variation however is small.

On the other side, η has no influence on the cracking load or on the distance between lines of calculated and actual steel stresses in stirrups as shown in fig. 7 for T beams with constant b'/b, only η variable.
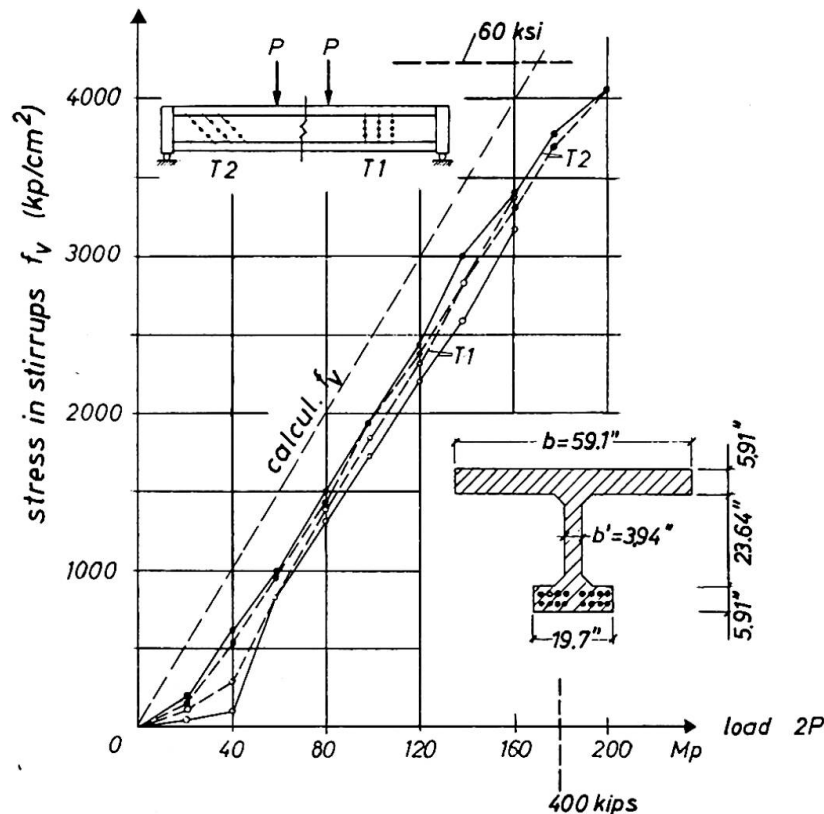

Fig. 8. Measured and calculated stresses in stirrups of beams with extremely thin webs ($b/b' = 15$), shear stresses v reaching 2500 psi. Beam T 1 had vertical stirrups, T 2 inclined stirrups.

Even in large beams with extremely thin webs and a ratio of $b'/b = 1/15$, as we have tested in our beams T 1 and T 2 [1] [5], where the angle of the cracks was about 45°, the forces in the stirrups do not reach the values of MÖRSCH's truss analogy, fig. 8. This means that even for highest shear stresses and in I-beams, a part of the shear force is carried by a slight inclination of the resultant compression force in the top chord and by "frame action" between the web-members and the chords.

Fig. 9 shows, how the portion of the shear force, which is carried by stirrups, is varying with increasing load for different ratios b'/b and two grades of shear coverage. These lines show, that the load intensity has considerable influence

on the mechanism, the web-members increase their portion of the shear with increasing load intensity and only ultimate load conditions tell the safety and give the design criteria.
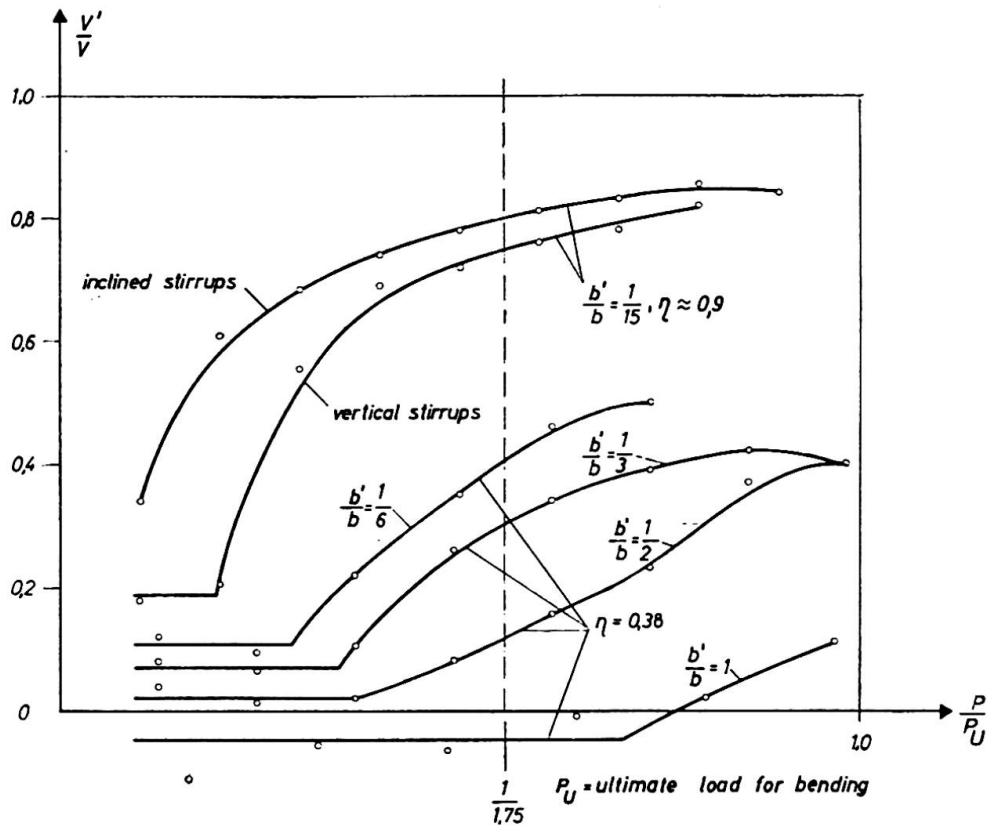
Fig. 9. Portion of shear force V' carried by web-reinforcement for different widths of webs expressed by b'/b and for different shear coverage η plotted against load intensity P/P_u.
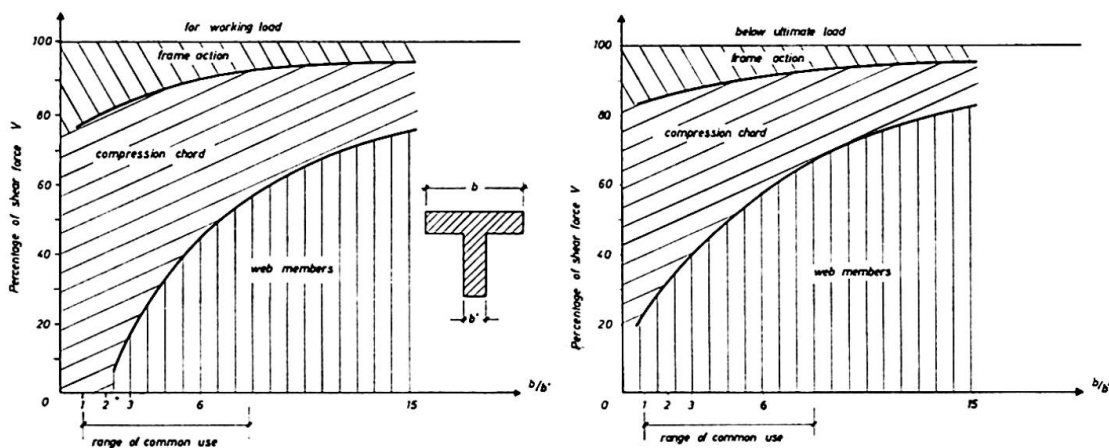
Fig. 10. Distribution of the shear force V

1. on web members = stirrups and struts,
2. on compression chord carrying V_c by inclination,
3. on frame action due to stiffness of junction between web- and chord members
    depending on b/b' for working load and shortly below ultimate load.

In fig. 10, the influence of the ratio between web stiffness and chord stiffness expressed by b'/b on the mechanism is shown in a different way; the lines give the distribution of the shear force on the web-members, the compression chord and on the "frame action" due to fixed connection between struts and chords. In the normal T beams with b/b' between 3 and 6, the web members carry less than half of the total shear force even under ultimate load conditions.

The result is, that the mechanism changes with decreasing web thickness and increasing load intensity from the tied arch to a truss with a curved or inclined top chord and with diagonal struts, of which the inclination varies between 30° and 45°.
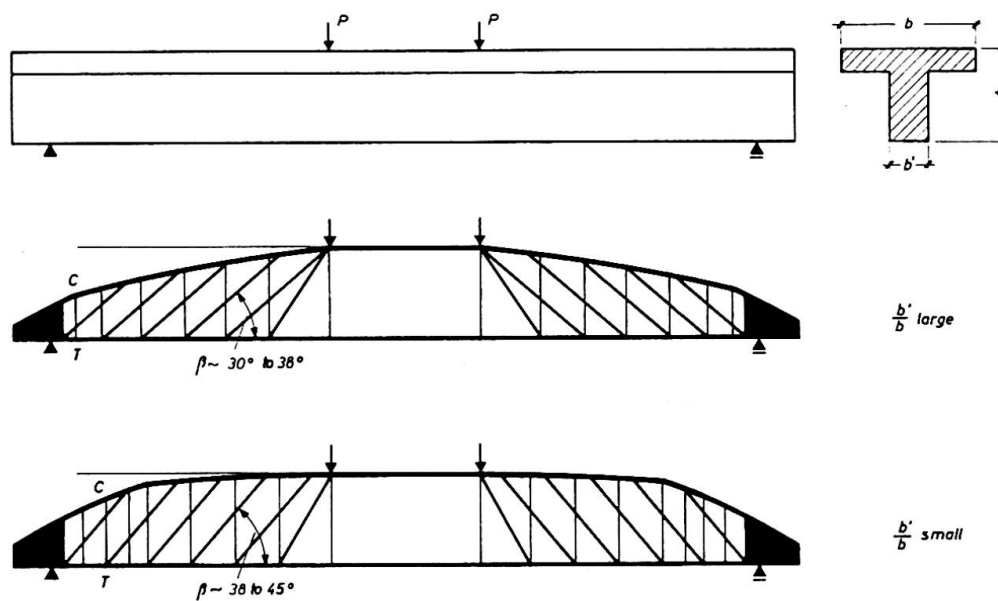

Fig. 11. The actual mechanism of internal forces can be compared with such trusses with inclined compression chords and diagonal web struts with angles $\beta \leq 45°$, the angles of chord and struts mainly depending on b/b' (extended truss analogy).
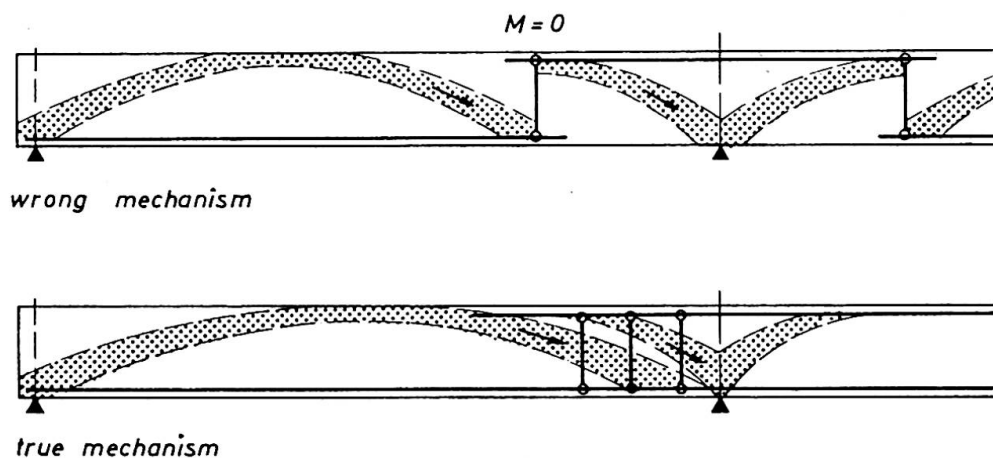

Fig. 12. Common idea of the mechanism of continuous beams compared with the findings by tests for rectangular cross sections.

This means, that the trusses which we must have in mind in order to understand the shear problem, must have variables as shown in fig. 11.

2.3. *Mechanism of Continuous Beams*

The system of internal forces in continuous beams is generally assumed to have a simple beam between the points of inflection ($M = 0$) suspended from a cantilever beam (fig. 12). The Stuttgart tests and US-tests [8] [9] showed, that for rectangular cross-sections or T-beams with thick webs, the inclined compression chord extends directly to the support so that the bottom tie reinforcement must also here be extended partially till to this support. Correspondingly, there are very flat shear cracks near the inner support with
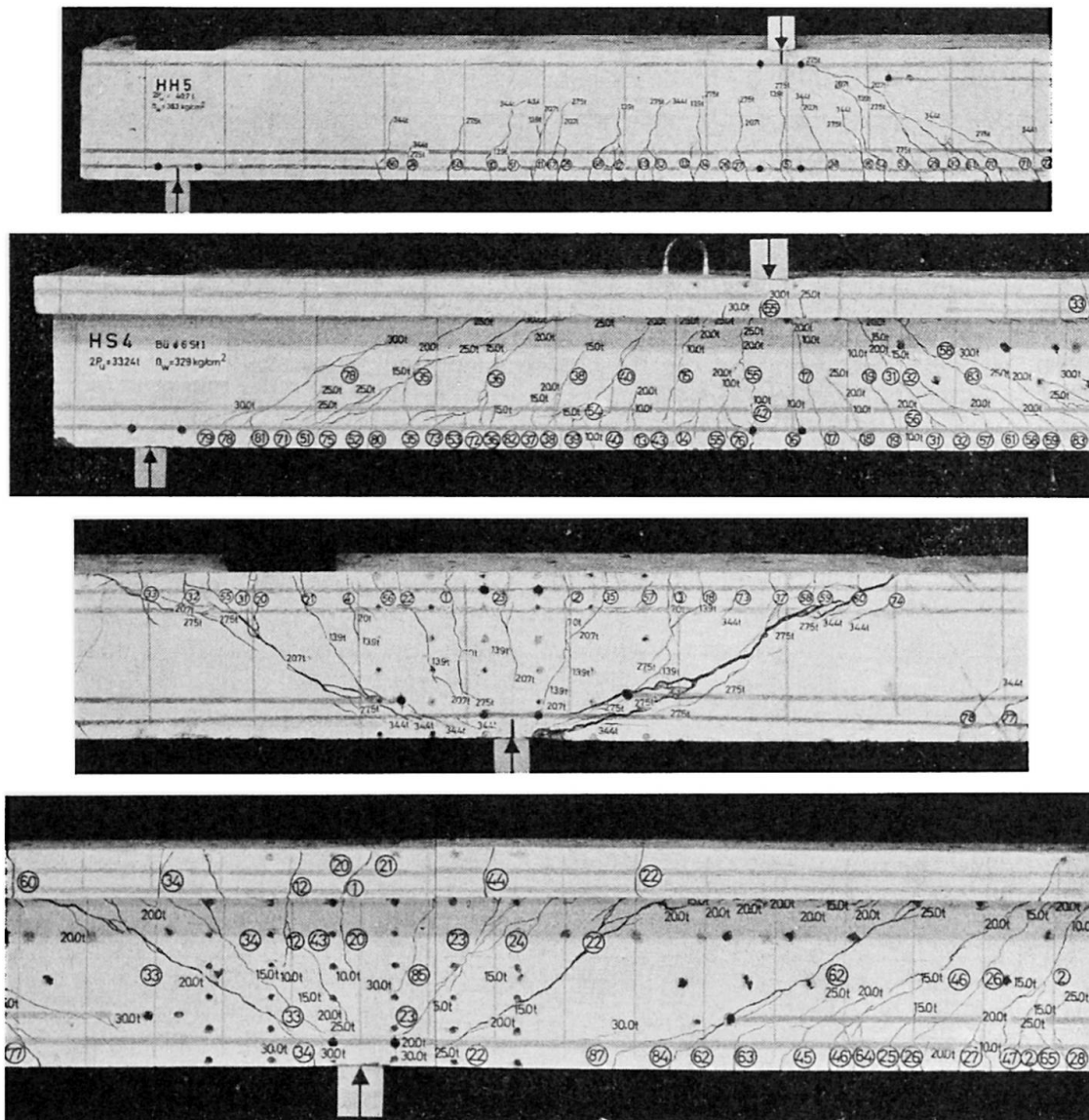

Fig. 13. Small inclination of shear cracks of continuous beams near inner support, if $b/b' \leq 3$.

angles as low as 30° (fig. 13). As a consequence, stirrups are more necessary near the support than in the region of the small moments, where we have no cracks up to high load intensities, so that stirrups in this region did show only small stresses.

Also in continuous beams, the mechanism changes with the stiffness ratio b'/b in a similar way as we have shown for single span beams. In beams with thin webs, there is truss action under high loads with a smaller influence of the inclination of the chords but a larger influence of the angle β of the struts near inner supports. The ratio of V'/V is larger there than near outer supports.

3. Calculation of the Reduction of Shear Reinforcement

3.1. Slender Beams and Slabs

The described mechanisms make it clear that a full shear coverage according to MÖRSCH's theory is not necessary, because the tensile forces in the web are considerably smaller and their magnitude depends primarily upon the stiffness ratio b'/b. The sound design principle for reinforced concrete, to carry all tensile forces by steel, can, therefore, be fulfilled by a reduced shear coverage $\eta < 1$ corresponding to the requirements of the ACI Code 318-63.

The measured stresses in stirrups showed in all tests a characteristic pattern as drawn in fig. 14. The line of the stresses in the stirrups as a function of the load runs almost parallel to the one calculated with MÖRSCH's truss analogy, however, in a horizontal distance equal to the load P_{crack}. This load P_{crack} is the one under which a shear crack has reached the stirrup. We have defined P_{crack} as the load which is found by continuing the measured line straightly towards the abscissis. To this load corresponds to a shear stress v_i, which we
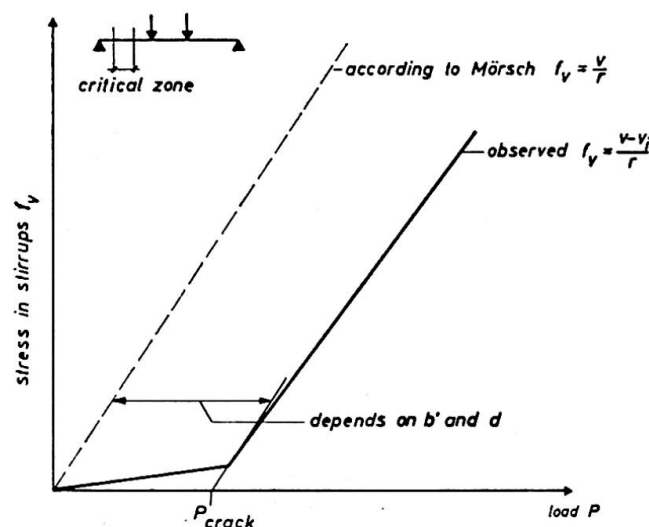

Fig. 14. Characteristic line of stresses in web reinforcement, compared with classical analysis of MÖRSCH.

found to be different from v_c as found for beams without shear reinforcement. The US results give $v_c = f(\sqrt{f_c'})$ influenced by the tensile strength of the concrete. The value v_i, however, is influenced by the portion of $V_i = P_{crack}$ which represents the portion of V which is carried by the compression members of the truss and, therefore, shows a straight line function with the compressive strength f_c' (fig. 15). We have found this value to be different for single and continuous beams:

$$\text{for single span beams} \quad v_i = \frac{1}{16} f_c',$$

$$\text{for continuous beams} \quad v_i = \frac{1}{22} f_c'.$$

(1)

P_{crack} increases with increasing b' and z, because the corresponding shear force is $V_i = v_i b' z$. The thicker the web, the larger the distance between the actual stress line of the stirrups and the calculated one according to MÖRSCH. This is caused by the inclination of the compression chord and the angles $\beta < 45°$ for the diagonal struts. From the characteristic line of fig. 14 we derive the formula for the stresses of the stirrups

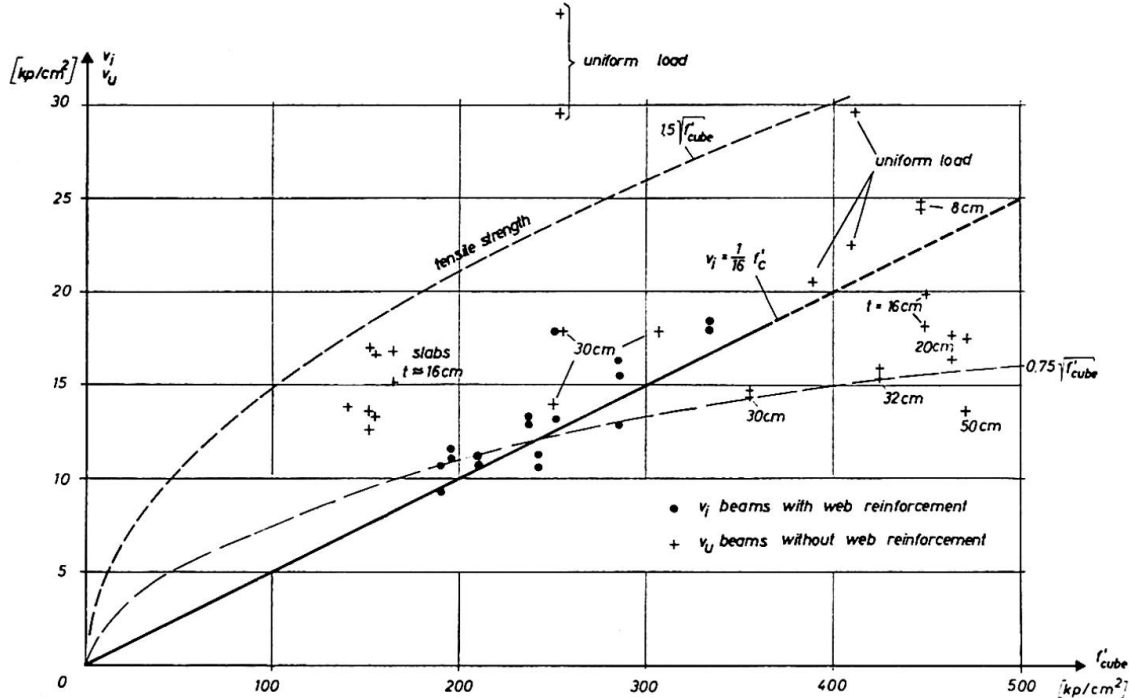$$f_v = \frac{v - v_i}{r}.$$

(2)

Fig. 15. Shear stress V_i due to portion of shear force carried by compression members of single span beams with shear reinforcement plotted against the cube strength of concrete, compared with the ultimate shear stress V_u of beams without shear reinforcement for different depth d. Only beams with $3,2 < \dfrac{M}{v\,d} > 2,5$.

Consequently, the factor for the necessary percentage of shear coverage can be written

$$\eta = \frac{\nu\,v_{D+L} - v_i}{\nu\,v_{D+L}} \qquad (3)$$

with $D+L$ = working load, ν = factor of safety.

This formula corresponds in principle to the ACI Code 318-63 and the Stuttgart tests confirm fully the soundness of this new US-Code concerning shear.

If we plot the v_u/f_c' of all our test results for the mentioned range of a/d and the results of [9] against the percentage of shear coverage η, we get fig. 16. The two lines for $v_i = 1/16 f_c'$ and $1/22 f_c'$ are below all corresponding points with two exceptions; three values of continuous beams, which did not fail by
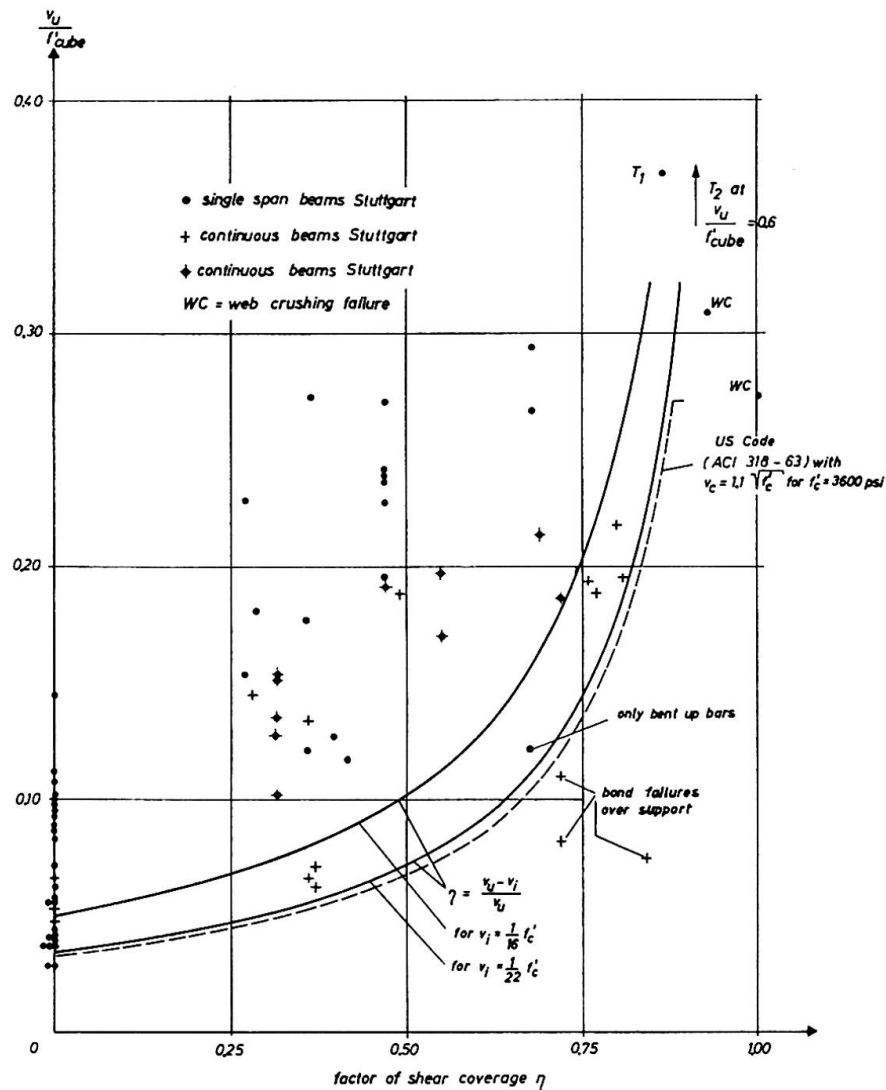
Fig. 16. The ultimate shear stresses of test beams with shear failures plotted against percentage of shear coverage compared with curves $\eta = \dfrac{v_u - v_i}{v_u}$.

shear in the web but by bond failure of the top chord bars, which have only about half the bond strength of bottom bars due to the sedimentation of the concrete after compaction. One value concerns a simple span beam which was web-reinforced with bent up bars only, which is not allowed.

The graph proves, that the percentage of shear coverage η calculated with eq. (3) using v_i according to (1) gives sufficient safety; v_{D+L} refers to the maximum value of V of the beam and η should be constant for the shear zone belonging to this max V.

An upper limit of v must be chosen in order to avoid failure by diagonal compression in the web. In our tests we found that the diagonal compression is not only influenced by the direction of the web reinforcement (as can be derived from the truss analogy), but it is further influenced by the angle of the shear cracks. We found that for vertical stirrups, the principle diagonal compression stress for high shear intensity can reach

$$t_2 = 2.4\,v \text{ (for low shear stresses, it can be } t_2 \to 3.5\,v).$$

For inclined stirrups, $(\alpha = 45°)$, closely spaced, it reaches

$$t_2 = 1.7\,v.$$

Therefore, using a factor of safety of 2.1 and a reduction for sustained load, the upper limit of the shear stresses due to the working load can be chosen
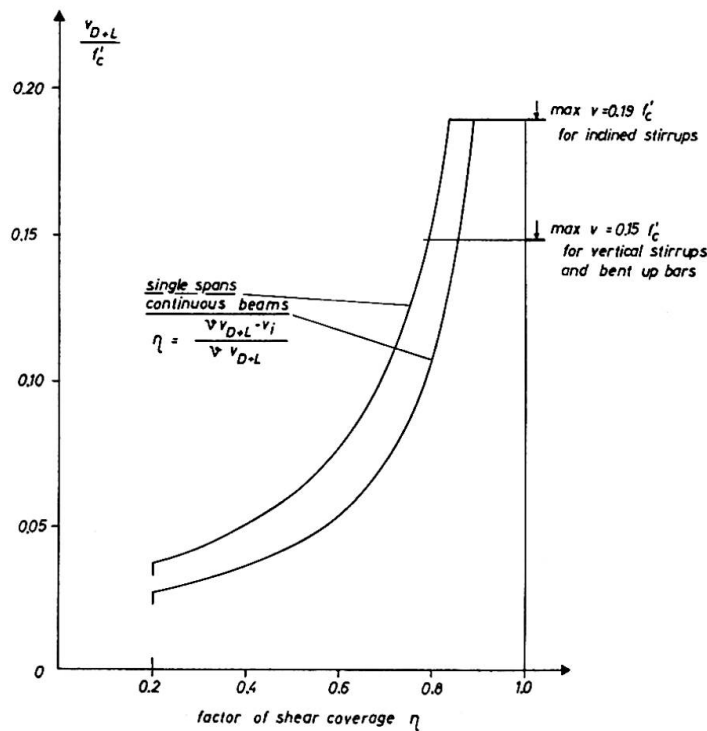

Fig. 17. The required shear coverage for working load design and upper limits of allowable shear stresses.

for vertical stirrups and stirrups combined
   with bent up bars $\max v_{D+L} = 0.15 f'_c$

for inclined stirrups $\alpha = 45°$ to $55°$ $\max v_{D+L} = 0.19 f'_c$

(since compression is critical and not tension, we refer v to f'_c and not to $\sqrt{f'_c}$).

The maximum shear stresses for working load give the necessary percentage of shear coverage η for single span and continuous beams according to fig. 17. The upper limits are governed by web compression, the lower limits show the low values which can be allowed without web reinforcement and with constant ties. For T beams with $b'/b < 1/3$, the minimum requirements for shear reinforcement must be observed.

3.2. Further Reduction of the Shear Coverage for Short Beams, Short Slabs or Brackets

The Stuttgart tests have confirmed the results of other investigators [12], concerning the influence of $\dfrac{M}{v\,d}$ or of a/d for single loads and of the slenderness ratio l/d for distributed loads as shown in fig. 18. According to these test
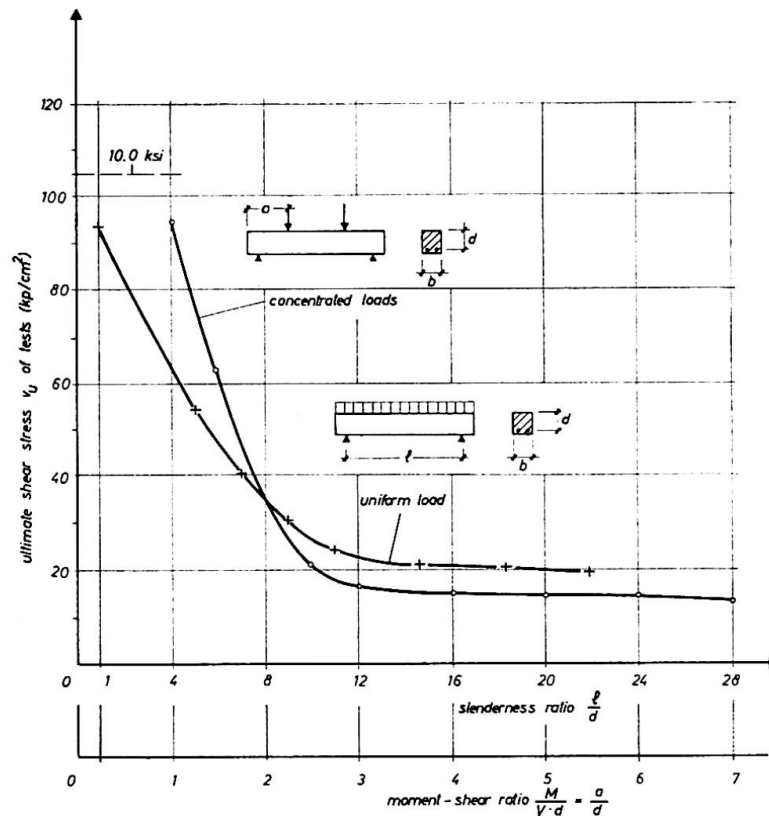

Fig. 18. The increase of shear carrying capacity without shear reinforcement in beams
with the decrease of the slenderness
ratio l/d or of the shear span,
ratio a/d for single loads.

results, the shear strength of beams without shear reinforcement increases rapidly for concentrated loads as soon as $a/d < 3$ or for distributed loads as soon as $l/d < 12$. For deep beams with $l/d = 1$ such high shear stresses had been obtained, that a shear reinforcement would make no sense, as several tests have actually proved [18] [7].

This fact can be fully explained by the arch action of the concrete, it allows us to further reduce the shear coverage by the factor κ, which can be taken from the straight line given in fig. 19. The necessary shear reinforcement will finally be calculated by

$$A_v = \kappa\,\eta\,\frac{V s \sin \alpha}{z f_v}$$

or with the percentage of shear reinforcement:

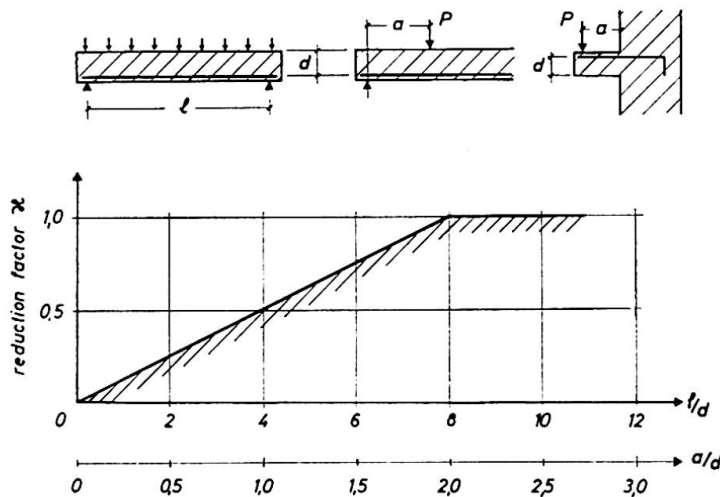$$r = \kappa\,\eta\,\frac{A_v}{b' s \sin \alpha} = \kappa\,\eta\,\frac{v}{f_v}.$$

Fig. 19. Reduction factor κ for the portion of the shear force to be carried by web reinforcement or for the shear coverage depending
on l/d for uniform load or
on a/d for contribution of concentrated loads.

4. Structural Conditions for Reduced Shear Reinforcement

4.1. Preference of Stirrups

The reduced shear coverage should preferably be made with stirrups in close spacing, the spacing decreasing with increasing values of the shear stress from $a < t/2$ to $t/6$. Bent up bars of high strength steel are less suitable and lead to larger width of shear cracks. Inclined stirrups show the best efficiency (fig. 20). Bent up bars are acceptable in continuous beams in the region of

small moments for changing some bars from bottom to top, avoiding herewith the large anchorage length especially in the top region.

The stirrups must be well anchored, preferably with hooks (fig. 21). Stirrups made of a welded wire fabric with a wire spacing of 2 to 6'' and with a top anchorage by welded longitudinal wires, are especially suited for ribs, joists, and small beams in buildings.
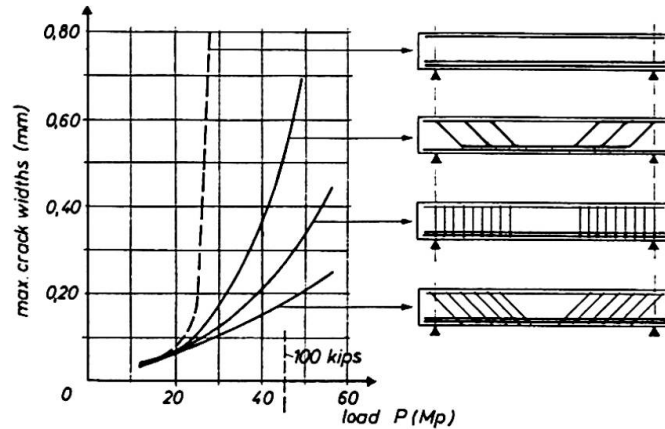

Fig. 20. Maximum crack width of shear cracks for average types of shear reinforcements. Comparison for equal cross-sections A_c and A_s.
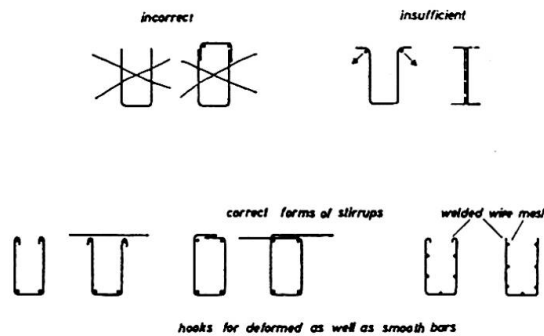

Fig. 21. Value of different forms of stirrups, the upper anchorage with a short length is very important.

4.2. The Determination of the Points for Cut Off and Bent up Bars

It is known that the tie force T is not proportional to the bending moment, as soon as the shear stress is low. For higher shear stresses the distribution of the tie force depends also upon the direction of the shear reinforcement. For low values of v and rectangular cross sections or slabs, the tensile force in the longitudinal reinforcement can be as high as $T = \dfrac{M}{z} + 1.5\,V$ near the support. MÖRSCH's truss analogy gives for vertical stirrups a tie force $T = \dfrac{M}{z} + \dfrac{V}{2}$. This value increases according to the described mechanisms.

The length of longitudinal bars can therefore not be found with the M/z-line, but with a T-line, which is found by moving the M/z-line horizontally by the amount of h (fig. 22). This h must be made depending upon v. Since the

shear stress v is connected with the factor of shear coverage η according to fig. 17, it will be sufficient to determine h from η and the direction of the web reinforcement.

The two following values cover practical needs (fig. 23):

$$h = (1.2 - 0.9\,\eta)\,d \geqq 0.5\,d \text{ for vertical stirrups or vertical stirrups combined with}$$
$$\text{bent up bars,}$$
$$h = (1.2 - 1.2\,\eta)\,d \geqq 0.2\,d \text{ for inclined stirrups } \alpha = 45° + 55°.$$
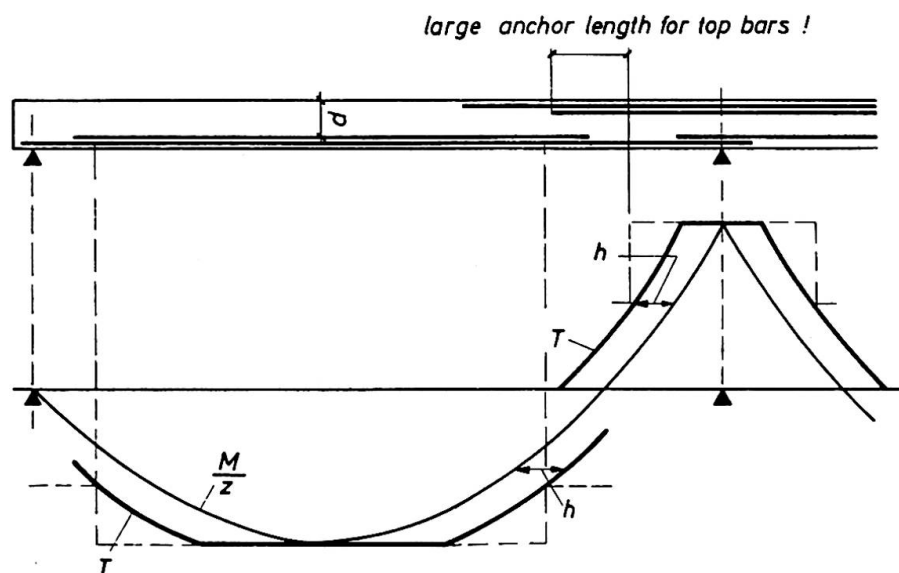

Fig. 22. The determination of the length of cut off bars must be made for a tie force diagram with the T-line found by moving the M/z line horizontally with the length h, found by the diagram of fig. 23.
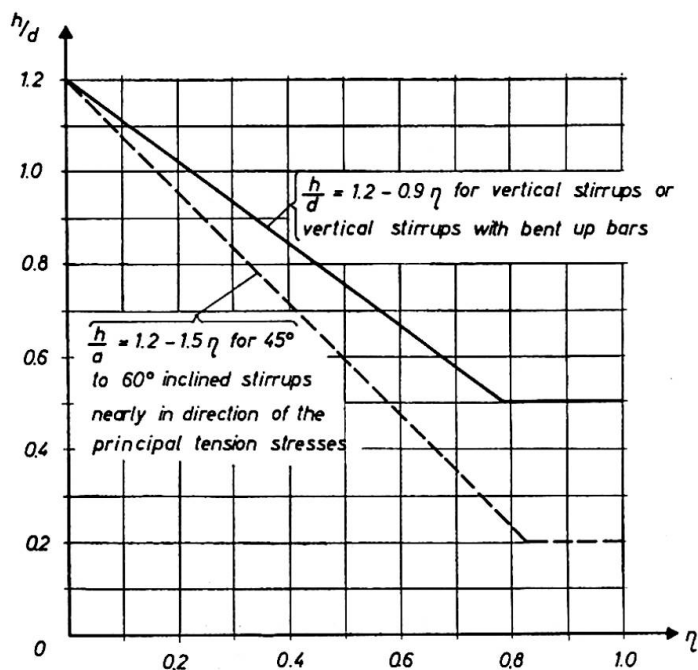

Fig. 23. Values of h for T-line, depending on the factor of shear coverage η and the type of web reinforcement.

Top bars for negative moments over intermediate supports of continuous beams must get a larger length of anchorage than bottom bars, if the beams are concreted in normal position, because the bond strength of top bars is low due to the sedimentation of the compacted concrete. The bond stress of top bars should always be calculated and it will be found, that often small diameters must be used and well distributed horizontally into the slab (fig. 24).
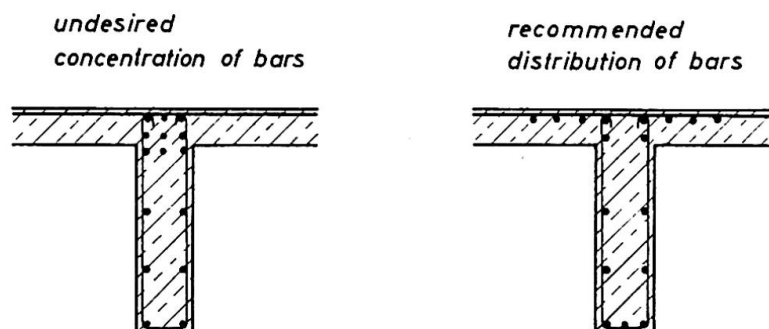

Fig. 24. Top bars for negative moments of continuous beams should be distributed sidewards into the slab, using medium diameters of bars.

4.3. Anchorage

In beams and slabs without shear reinforcement, especially in beams with a slenderness of $l/d < 8$ or with concentrated loads close to the supports with $a/d < 2$, a safe anchorage of the longitudinal bars must be provided. In many cases the bond length of straight deformed bars is here inadequate and hooks may be necessary, preferably hooks horizontally placed or loops or special anchor pieces.

5. Torsion and Prestressing

Two tests on large prestressed hollow box girders, of which F. LEONHARDT and R. WALTHER have reported in [16], have shown, that similar relations for reduced web reinforcement as shown here can be established for the shear forces due to torsion and for the reduction by prestressing forces. It was found, however, that the principal compression stress can become as high as 3.5-times the value calculated by usual analysis and, therefore, can become critical earlier than generally suspected. One must, therefore, be careful about the diagonal compression in webs. On the other side for torsion also principal tension has not to be limited for the design, because it is easily possible to take care of the tensile forces by reinforcement in the webs.

Fig. 25 shows a first tentative proposal for the determination of the necessary coverage of shear in prestressed concrete girders, depending upon the shear stress v due to loads and prestressing force and depending upon the average normal stress due to the final prestressing force $f_p = P_p/A_c$. These lines need further confirmation by tests.
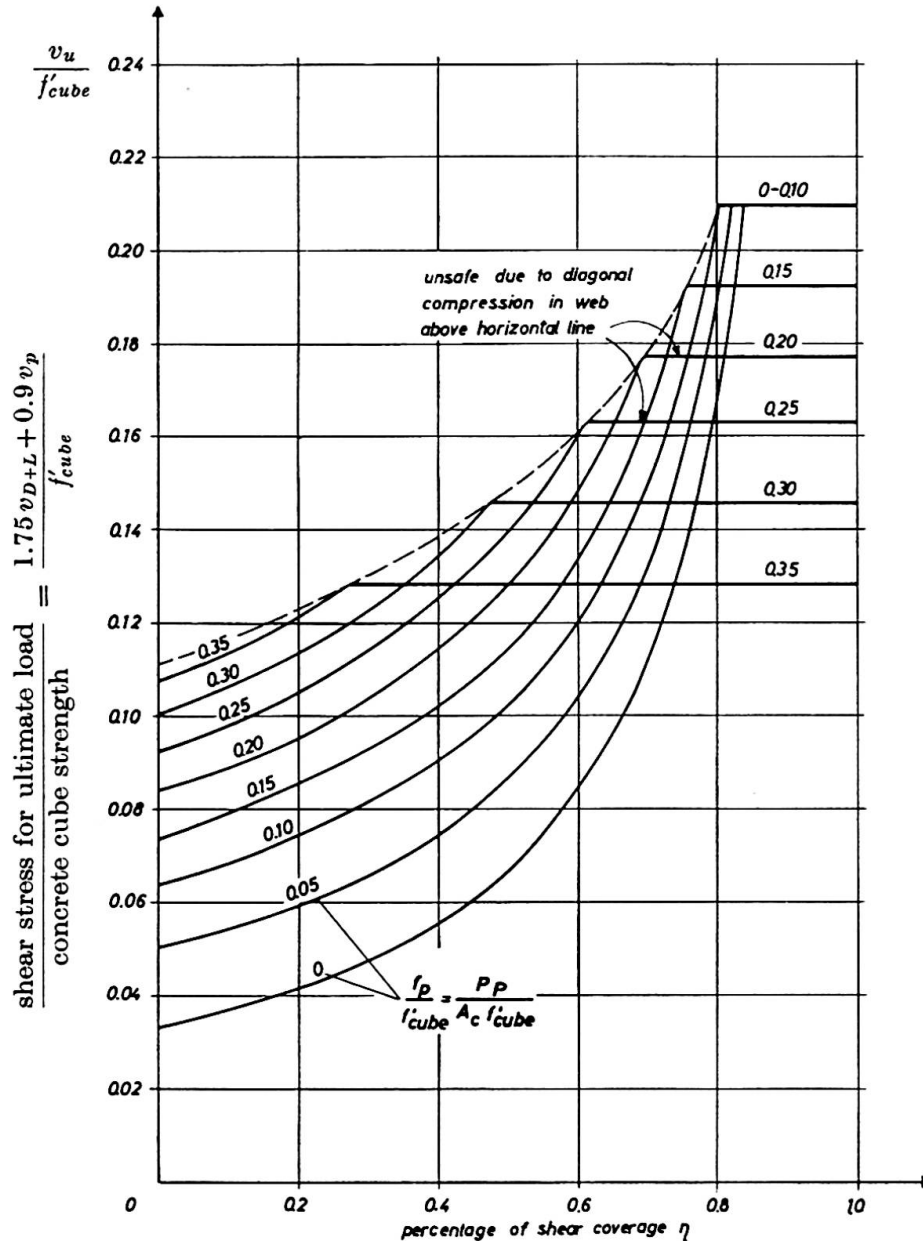
Fig. 25. The necessary shear coverage for prestressed concrete girders, considering the degree of prestressing by the average compression stress due to P_p depending on the maximum shear stress due to loads and P. Limits to prevent web crushing failures.

Literature

1. LEONHARDT, F. and WALTHER, R.: Versuche an Plattenbalken mit hoher Schubbeanspruchung. Heft 152 des DAfSt., Berlin 1962.

2. LEONHARDT, F. and WALTHER, R.: Schubversuche an einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung. Heft 151 des DAfSt. Berlin 1962.

3. LEONHARDT, F. and WALTHER, R.: Schubversuche an Plattenbalken mit unterschiedlicher Schubbewehrung. Heft 156 des DAfSt.

4. LEONHARDT, F. and WALTHER, R.: Schubversuche an Durchlaufträgern. Heft 163 des DAfSt.

5. LEONHARDT, F. and WALTHER, R.: Beiträge zur Behandlung der Schubprobleme im

Stahlbetonbau. Beton u. Stahlbetonbau 1961, Heft 12, 1962, Hefte 2, 3, 6, 7 u. 8, 1963, Hefte 8 u. 9 (7 Fortsetzungen).

6. LEONHARDT, F. and WALTHER, R.: Schubversuche an Platten mit geschweißten Bewehrungsmatten. Beton- u. Stahlbetonbau 1964, Heft 4 u. 5 (8. Fortsetzung zu «Beiträge zur Behandlung der Schubprobleme im Stahlbetonbau»).

7. LEONHARDT, F. and WALTHER, R.: Untersuchungen an wandartigen Trägern mit unterschiedlicher Bewehrung und Belastung. Heft 172 des DAfSt.

8. BRYANT, ROBERT H., BIANCHINI, ALBERT C., RODRIGUEZ, JOSÉ J. and KESLER, CLYDE E.: Shear strength of two-span continuous reinforced concrete beams with multiple point loading. ACI-Journal, Sept. 1962, Proc. V. 59.

9. RODRIGUEZ, JOSÉ J., BIANCHINI, ALBERT C., VIEST, IVAN M., KESLER, CLYDE E.: Shear strength of two-span continuous reinforced concrete beams. ACI-Journal, April 1959, Vol. 30.

10. FERGUSON, PHIL M.: Some implications of recent diagonal tension tests. ACI-Journal, Aug. 1956, Vol. 28.

11. ACI Building Code, 318–63, June 1963, ACI, Detroit, USA.

12. LAUPA, A., SIESS, CH. P., NEWMARK, N. M.: Strength in shear of reinforced concrete beams. University of Illinois Bulletin, No. 428.

13. GURALNICK, S. A.: Shear strength of reinforced concrete beams. Proceedings of the American Society of Civil Engineering, St. 1, Vol. 85, 1959.

14. FRANZ, G., and NIEDENHOFF, H.: Die Bewehrung von Konsolen und gedrungenen Balken. Beton- u. Stahlbetonbau 1963, Heft 5.

15. KREFELD, W. J. and THURSTON, C. W.: Studies of the shear and diagonal tension strength of simply supported reinforced concrete beams. Report Columbia University in the City of New York. Department of Civil Engineering and Engineering Mechanics, June 1962.

16. LEONHARDT, F. and WALTHER, R.: Torsions- und Schubversuche an vorgespannten Hohlkastenträgern. Festschrift der Beton- u. Monierbau AG, Düsseldorf, Okt 1964.

17. KANI, G. N. J.: The riddle of shear failure and its solution. ACI-Journal, April 1964.

18. CHOW, LI, CONWAY, H. D. and WINTER, G.: Stresses in deep beams. Proc. ASCE, Sep. No. 127, May 1952, Paper 2557.

Summary

The classical shear analogy of W. RITTER and E. MOERSCH, using a truss with parallel chords and compression struts at an angle of 45°, could be extended on the basis of the Stuttgart shear tests to trusses with an inclined compression chord and struts with angles between 30° and 45°, according to the observed inclined compression forces. The ratio between width of compression flange and width of web is of a particular influence on these inclinations of the truss members and therefore on the amount of forces in the tension bars. This allows to reduce the shear reinforcement by subtracting a stress τ_{0D} from the classical shear stress τ_0.

The consequent increase of the tensile force in the bottom chord between load and support can be found by moving the M/z-line horizontally by a portion of the depth, depending on the percentage of shear coverage, for which formulas are given. On this way, the tensile force is also found, for which the longitudinal bars must be anchored at end supports.

Practical recommendations are given for the design of the shear reinforcement in single span and continuous beams. The future development of the extended truss analogy on prestressed beams and beams with torsion load is indicated.

Résumé

L'analogie classique pour le calcul au cisaillement selon W. RITTER et E. MÖRSCH, qui admet des treillis à membrures parallèles et bielles de compression à 45°, peut être étendue à des treillis à membrure supérieure inclinée et bielles de compression inclinées de 30° à 45°. Le rapport de la largeur de la dalle de compression à celle de l'âme de la poutre influe fortement sur les efforts dans les armatures tendues, L'analogie élargie du treillis conduit à une diminution de l'armature de cisaillement parce qu'elle permet de réduire la contrainte de cisaillement classique τ_0 d'une constante τ_{0D}.

On tient compte de l'augmentation des efforts de traction longitudinaux qui en résulte dans la zone d'effort tranchant en déplaçant horizontalement la ligne M/z d'une fraction de la hauteur utile d, fraction dépendant du pourcentage d'armatures de cisaillement. En même temps, on obtient aussi l'effort de traction qui doit être ancré à l'appui.

L'auteur donne des règles pratiques pour le dimensionnement de l'armature de cisaillement dans des poutres simples et des poutres continues; il montre le développement futur de cette théorie pour des poutres sollicitées à la torsion et des poutres précontraintes.

Zusammenfassung

Die klassische Schubanalogie mit parallelgurtigen Fachwerken mit 45° Druckstreben von W. RITTER und E. MÖRSCH konnte auf der Grundlage der Stuttgarter Schubversuche zu Fachwerken mit geneigtem Obergurt und Druckstreben in Neigungen von 30° bis 45° erweitert werden. Das Verhältnis der Druckplattenbreite zur Stegbreite ist von besonderem Einfluß auf die Kräfte in den Zugstäben. Die erweiterte Fachwerkanalogie führt zur Verringerung der Schubbewehrung, indem ein Festwert τ_{0D} von der klassischen Schubspannung τ_0 in Abzug gebracht wird.

Die daraus folgende Zunahme der Zuggurtkraft im Querkraftbereich wird durch eine horizontale Verschiebung der M/z-Linie um einen vom Schubdeckungsgrad abhängigen Teil der Nutzhöhe d berücksichtigt. Man erhält damit auch die am Auflager zu verankernde Zuggurtkraft.

Für die Ausbildung der Schubbewehrung in Einfeld- und Durchlaufträgern werden praktische Hinweise gegeben. Die künftige Entwicklung der erweiterten Theorie auf Träger mit Torsionsbelastung und unter Vorspannung wird angedeutet.

Leere Seite

Blank page

Page vide