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Inelastic Behaviour of Reinforced and Prestressed Concrete Beams under Combined Bending and Torsion

Comportement non élastique des poutres en béton armé et en béton précontraint sollicitées à la fois à la flexion et à la torsion

Unelastisches Verhalten armierter und vorgespannter Betonbalken bei gleichzeitiger Wirkung von Biegung und Torsion

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Introduction

Pure torsion is generally avoided in concrete construction. Torsional stresses of varying magnitude nevertheless, occur in many structures, invariably as a secondary effect of bending — due to asymmetrical loading and the monolithic nature of concrete construction, whether it be reinforced, prestressed or composite. Although torsional stresses are seldom primary, they influence considerably the strength of a beam since the effects of bending and twisting occur simultaneously and are closely linked together, and because of the sudden and destructive character of torsion failures.

With the increased use of precast and in-situ structural concrete, hollow beams are also being extensively used for building, piling and bridging work. The trends of modern design for ultimate strength with reduced load factors make the problem of combined bending and torsion of great practical significance in many fields of structural analysis, and particularly in long span bridges with box-girders.

The existing test data [2] show that the failure of concrete under combined stresses is very complex indeed and not well-defined. Although the primary cause of such failures is either the tensile or compressive strength of concrete, depending upon the ratio of bending to twisting moment, the essential conditions producing such failures are not yet fully understood.

Test Results

The behaviour and strength of reinforced and prestressed concrete beams in pure bending and in pure torsion are well-established. Failure in bending is initiated either through yielding of the reinforcing bars or through primary compression failure due to the limited elongation of steel. In either case failure occurs by the crushing of concrete on the compression face. The ultimate strength of such beams can be very much greater than the initial cracking load.

Torsion failures, on the other hand, are violent, destructive and occur without warning if not adequately reinforced for diagonal tension. Such cleavage fractures result when the principal tensile stress due to torsional shear exceeds the tensile strength of concrete. When prestressed, the precompression must be eliminated before the concrete can be stressed in tension; prestressing consequently secures large increases in the torsional resistance of a beam.

Under combined bending and torsion, therefore, two different modes of failure are possible [1, 2].

Bending Failures

In construction practice bending effects usually predominate. In beams therefore subjected to large ratios of bending to twisting moments, failure results by the crushing of concrete on the compression face with or without steel yielding. The diagonal compression due to torsional shear increases the direct compression due to bending so that the effect of the addition of a small amount of torsion is to reduce the bending strength, although slightly. The crushing failure is always accompanied by the formation of debris and considerable disintegration due to energy release and is generally caused by the diagonal shear due to compression.

The presence of small amounts of torsion does not, however, deprive such beams of their reserve load carrying capacity after the beam has cracked in the tensile zone. The failure load can thus be much higher than the initial

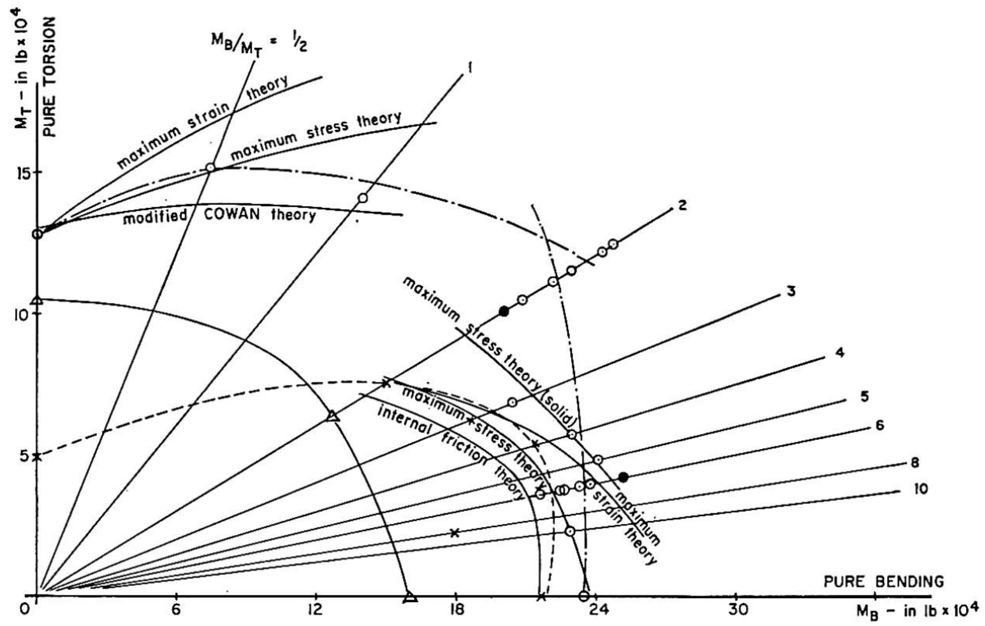

Fig. 1. Variation in the strengsh of prestressed solid and hollow beams at the formation of the first crack.

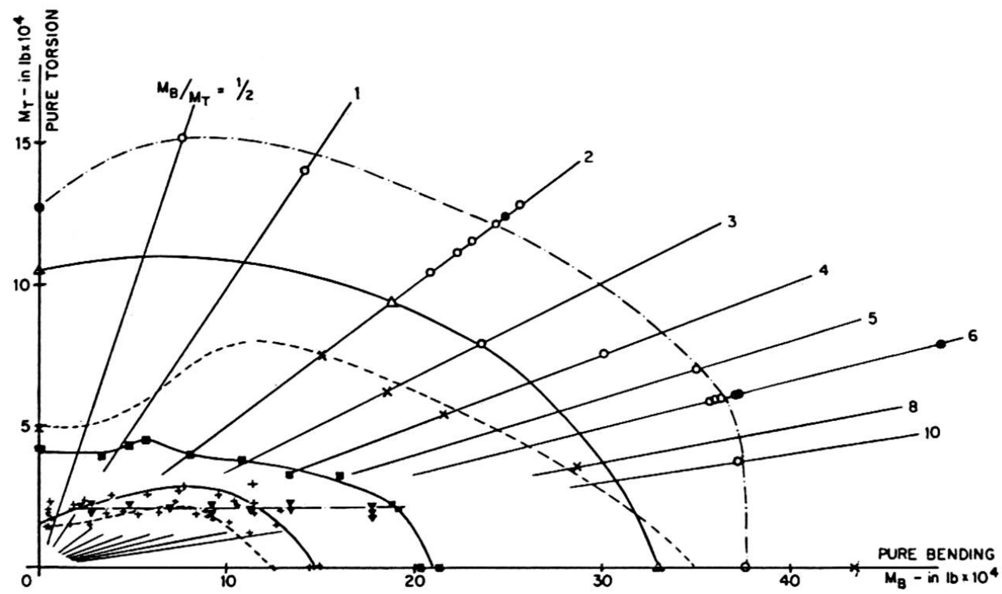

Fig. 2. Failure envelopes of prestressed solid and hollow beams.

M_0 = fully plastic yield moment for pure bending.
T_0 = fully plastic yield torque for pure torsion.
M = actual applied moment producing yield.
T = actual applied torque producing yield.
○ = 9 × 9 in hollow uniformly prestressed.
⊗ = 9 × 9 in hollow uniformly prestressed: web reinforcement.
⊠ = 9 × 9 in hollow uniformly prestressed: circular hollow.
□ = 9 × 9 in hollow uniformly prestressed — transition failure.
● = 9 × 9 in solid uniformly prestressed.
△ = 6 × 9 in solid uniformly prestressed (Cowan).
× = 6 × 8 in solid eccentrically prestressed (Jones).
■ = 5 × 8 in solid eccentrically prestressed (Rowe).
▼ = 5 × 8 in I eccentrically prestressed (Gardner).
+ = 6–10 × 8 in T eccentrically prestressed (Reeves).
▲ = 6 × 9 in solid reinforced concrete (Cowan).

cracking moment. This is, however, in marked contrast to the strength of beams subjected to ratios of bending moment to twisting moment (M_B/M_T) less than six. Below this limit even a small increase in torsion reduces the ultimate strength substantially with consequent loss of load-capacity after cracking (Figs. 1 and 2).

In hollow beams even small amounts of torsion are undesirable as they lead to serious rupture producing large and wide cracks at failure which often penetrate into the thickness of the webs and flanges (Fig. 3). Diagonal strains may in such cases be critical at more than one face. The provision of torsional reinforcement then becomes important and essential for such beams and even nominal steel helps to minimise the disruptive action of torsional stresses and hold the beam together.

Tests in reinforced concrete [3] show that there is actually an increase in

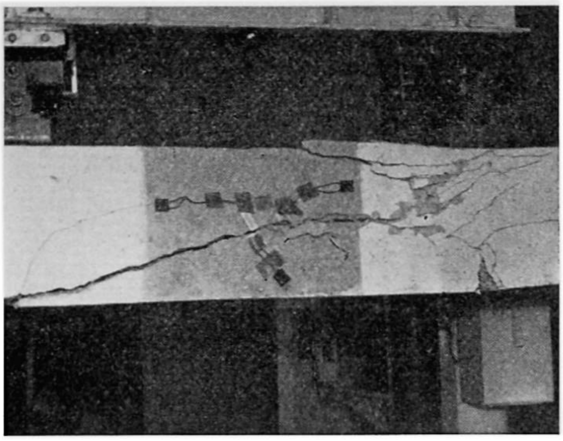

Fig. 3. Bending failure of a prestressed concrete hollow beam subjected to a twisting moment = $^1/_6$ of bending moment.

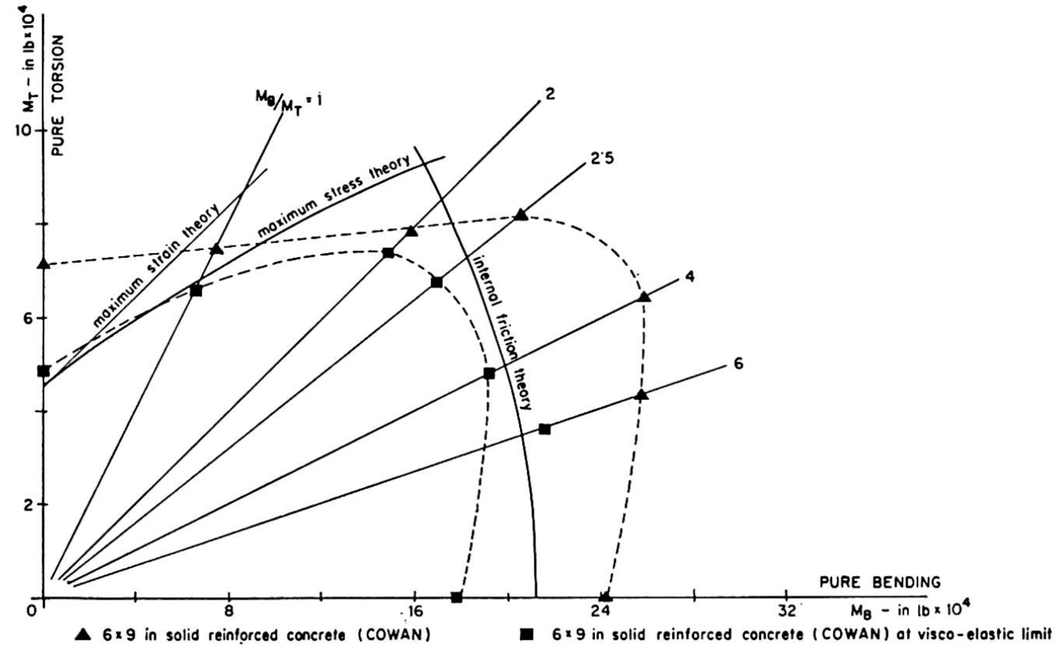

Fig. 4.

bending strength upto a ratio of $M_B/M_T = 4$, below which an increase in torsion reduces the bending strength (Fig. 4). Small torsional stresses thus enhance the factor of safety against bending failure. Where cracks traverse the torsional steel, yield strains can be developed. Failure may also occur in bond between concrete and steel.

Torsional Failures

Very small ratios of M_B/M_T, with the consequent predominance of the twisting effect are not generally common in concrete construction. They contribute, however, to a better understanding of the failure of concrete under combined stresses. Failure under such loadings is generally due to diagonal

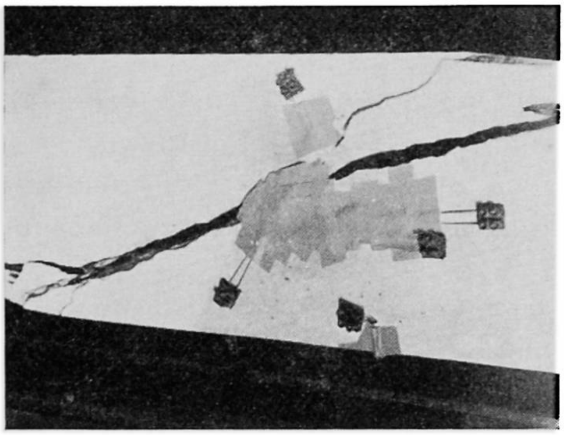

Fig. 5. Cleavage fracture of a prestressed concrete hollow beam subjected to equal bending and twisting moments.

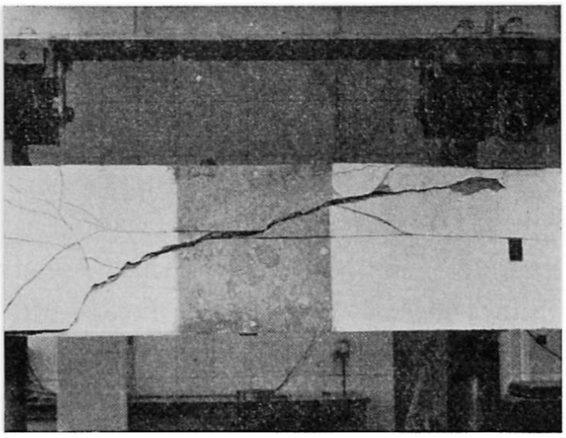

Fig. 6. Torsion fracture of hollow beam subjected to a twisting moment = ½ of bending moment.

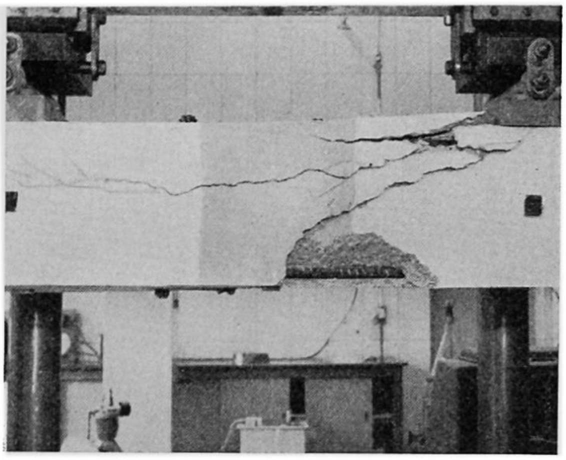

Fig. 7. Torsion failure of solid beam subjected to a twisting moment = ½ of bending moment.

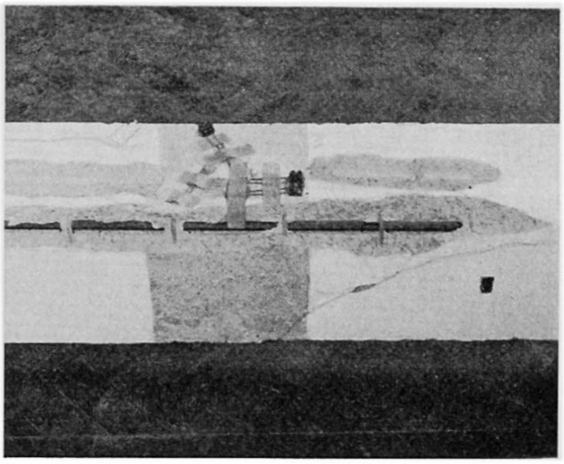

Fig. 8. Torsion failure with vertical stirrups. Twisting moment = ½ of bending moment.

tension due to shear and sudden and destructive, with debris "exploding" out of the section. Cracks formed are then clean and well-defined and free from any signs of crushing (Fig. 5); they, however, form a continuous chain right round the surface with a diagonal compression crack on the opposite face. These beams failed when the principal tensile stress on the compression face of the beam exceeded the tensile strength of concrete. Failure does not take place at the bottom face due to a redistribution of internal stresses.

The effect of the compressive stress due to bending is to reduce the principal tension due to torsional shear, and these beams were thus generally able to sustain a higher twisting moment than beams in pure torsion. Tests show that hollow [4] and T-beams [9] may be loaded upto 65% and 80% respectively of the ultimate bending moment before their strengths fall below the pure torsional strength (Figs. 6 and 7).

In normal reinforced concrete beams bending augments resistance to torsional deformation over a wide range of M_B/M_T ratios upto $2^1/_2$ to 3, beyond which any increase in bending actually reduces the torsional strength (Fig. 4). The gain in torsional load of prestressed solid beams is within narrower limits. In hollow beams the maximum increase in torsional resistance is obtained with a ratio of $M_B/M_T = {}^1/_2$ (Fig. 2). Any increase in applied bending moment beyond this limit reduces the torsional strength at a fast rate. Test data [4, 5] show that the addition of bending does not produce phenomenal increases in torsional strength. The presence of large compressive stresses has a dual action: while retarding the development of critical tensile stresses in some regions, it also affects adversely the tensile strain capacity in others.

In pure torsion the greatest increase in strength is obtained by uniform prestressing. It is, however, more economical in construction practice to provide an optimum stress gradient under dead load and the prestressing force. Under combined loading, greater increases in torsional strength can then be obtained. In such beams, initial failure occurs on the top face, and small amounts of bending cause negligible increase in the torsional strength. When the bending is sufficiently large so as to produce a uniform stress distrir bution, the ultimate strength is increased considerably. Tests on rectangula- beams [6, 7] show that depending on the stress gradient, this increase in torsional strength occurs for loads from one half to the full working load and as the applied bending moment increases the critical section changes from top to the bottom face via the middle of the sides.

In hollow beams subjected to low ratios of M_B/M_T and without torsional reinforcement, the cracks formed and extended rapidly and caused a sudden and immediate fracture. The initial cracking moments were also then the ultimate failure moments. Solid beams were able to sustain loads higher than the initial cracking loads. The presence of torsional steel lessens the violence of fracture and changes its character into a gradual and mild one preceded by cracking and deformation (Fig. 8).

Transition Failures

Between the crushing failure of concrete due to compression and the cleavage fracture due to torsion there is a transition stage. This occurs when the torsional shear stress becomes large enough to produce the critical principal tensile strain for the cleavage fracture of concrete. Failure in this intermediate stage is complex and the result is a dual failure — an initial cracking due to bending but an ultimate torsion failure. The initial vertical cracks inclined with increase in load, and failure occurred along these diagonal tensile cracks, often masking the bending cracks (Fig. 9).

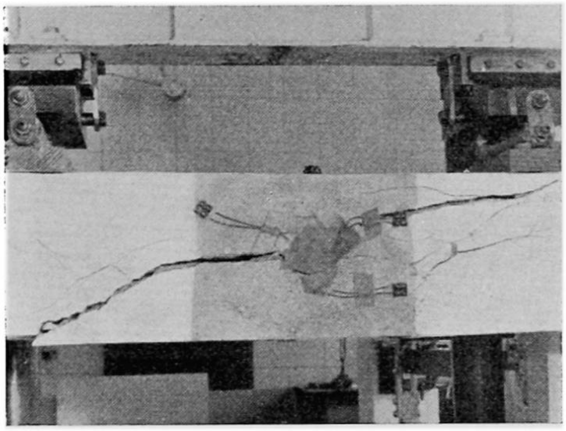

Fig. 9. Transition (initial bending, final torsion) failure: twisting moment = $^1/_5$ of bending moment.

In prestressed beams M_B/M_T ratios of 3 to 5 represent this change from torsion to bending failure. As the applied bending moment increased, the process of failure became gradual, with greater warning of impending fracture and increased load carrying capacity beyond initial cracking. The failure was quite sudden when it occurred. The ratio 5 represents the dividing line between ultimate torsion and ultimate bending failures: above this ratio a crushing failure occurs: at and below this, beams fail in ultimate torsion.

Tests on hollow beams show that they are more efficient and economical to resist torsional loads. There is a particular size of the hollow for which the maximum increase in strength may be obtained.

Tests on I and T-beams

These beams were initially loaded with upto 80% of the ultimate bending moment producing extensive cracks and then twisted to failure. Tests on eccentrically prestressed I beams [8] show that there is virtually no interaction between bending and torsion. This result is not altogether surprising. The I-section is basically weak in torsion, and the addition of bending moment is unable to overcome this inherent weakness of the section. The torsional strength of T-beams, on the other hand, is greatly influenced by the magnitude of the applied bending moment and the shape of the compression flange [9]. Compared with other cross-sections the greatest increase in torsional strength — as much as 1.6 times the pure torsional strength — is obtained with T-beams. The large compression area enables a more efficient redistribution of the very high shear stresses present thus increasing the load-capacity of the beams. Grouting also assists in this redistribution since it produces a better configuration of bending cracks.

Failure Criteria

The tests of the author and those of others are compared in Figs. 1, 2 and 4. The experimental data show that the customary elastic theories of failure cannot satisfactorily and adequately explain the essential differences between a bending failure and a torsion fracture. It is possible to use certain criteria such as a constant maximum stress or a constant maximum strain to compute the initial cracking load of a beam under combined stresses but no single criterion gives consistent results for all stress combinations. Nor does there seem to be an acceptable simple mathematical basis to predict the failure load for all stress ratios. A similar situation also exists with respect to the strength under combined stresses of other brittle materials like cast iron and of even mild steel.

These experiments also show that the mechanism of failure of concrete is complex and not clearly defined for many load combinations. They reveal that a cleavage type fracture can sometimes be preceded by substantial amounts of inelastic deformation. Thus torsion failures may be associated with relatively ductile behaviour. This phenomenon of considerable plastic behaviour before fracture under certain conditions of loading is also exhibited by other brittle substances. These tests confirm that under a superimposed bending stress a beam may carry large shear stresses without critical tensile stresses developing — thus concrete in torsion may exhibit significant plasticity under favourable loading conditions.

But torsional failures in any form are extremely undesirable in concrete. A tension crack is like a lever of very high mechanical advantage: once formed, propagation is extremely rapid and very little energy is expended to keep such a fracture going. Even with adequate torsional reinforcement, the reserve strength in torsional failure after initial cracking is considerably less than that in a similar bending failure. Only one major crack is needed for fracture in contrast to many needed for inelastic behaviour.

In any case, plastic deformation of concrete in torsion (and hence in tension) is insufficient to be able to develop full plasticity at failure [10, 11]. The substantial amounts of inelastic deformation found in torsional failures of certain stress combinations must therefore be associated with the compressive strength of concrete.

Although the diagonal tensile stress is undoubtedly the initiator of torsional fractures, the magnitude of these stresses and the character of the fracture is influenced by a number of factors including the distribution of shear stresses and the inelastic deformation preceding failure. An initially clean crack normal to the principal tension may thus develop into an oblique rough crack into the region of high compression and may show signs of crushing. It is not precisely known from these experiments where and how such tensile stresses are developed and more basic information on the deformation characteristics and stress and rotation capacities of concrete under combined stresses is needed to make the exact nature of these failures clear.

Strains and Rotations

As the applied bending moment increased these beams displayed a great capacity for torsional rotation from 2 to 4 times the ultimate value in pure torsion. Fig. 10 shows the measured rotation characteristics at or near the ultimate load. While there is no conclusive pattern, certain general trends may be seen to exist. As the mode of failure changed from torsion to bending, the deflection, strains and rotations all showed a gradual transition from the elastic into the inelastic stage.

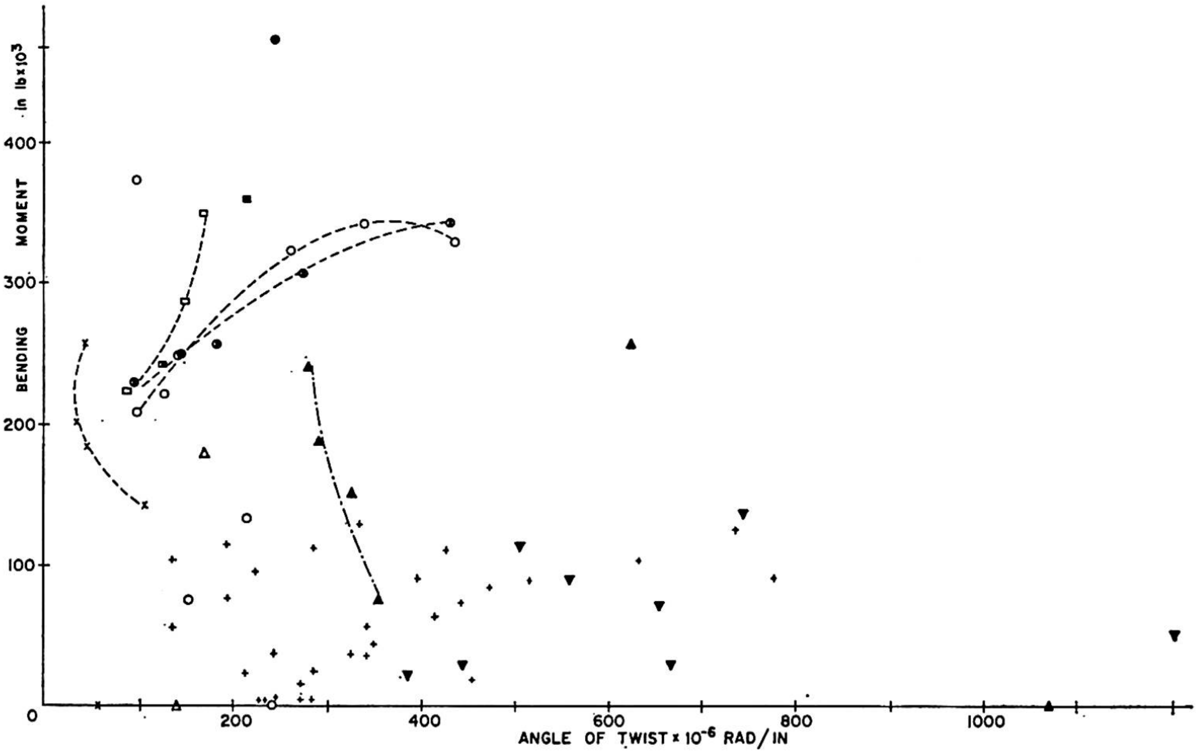

Fig. 10.

In bending failures the principal concrete compressive strains were considerably high. In torsion failures the principal tensile strains were all critical, of the order of 1.8 to 3.4×10^{-4} and were much larger in beams with nominal reinforcement. These and other results [9, 12] reveal the enhanced tensile strain capacity of concrete under favourable combined loadings.

With increase in the applied bending moment the beams showed increased ability to sustain very high shear stresses. These stresses were far in excess of the normal failure strength of concrete under shear or diagonal tensile stress.

A considerable redistribution of shear stresses must have therefore taken place for the concrete to be able to withstand such excessive shear stresses. The presence of very large compressive stresses is thought to be responsible for this redistribution, and this accounts for the increased torsional load carrying capacity with increasing bending stresses.

Interaction Curve

By treating concrete as an ideally plastic-rigid material, it is possible to obtain upper and lower approximations to the interaction curve of the bending and twisting couples at yield for the combined bending and twisting of various sections [13].

The various test results are plotted in Fig. 11 and show good agreement

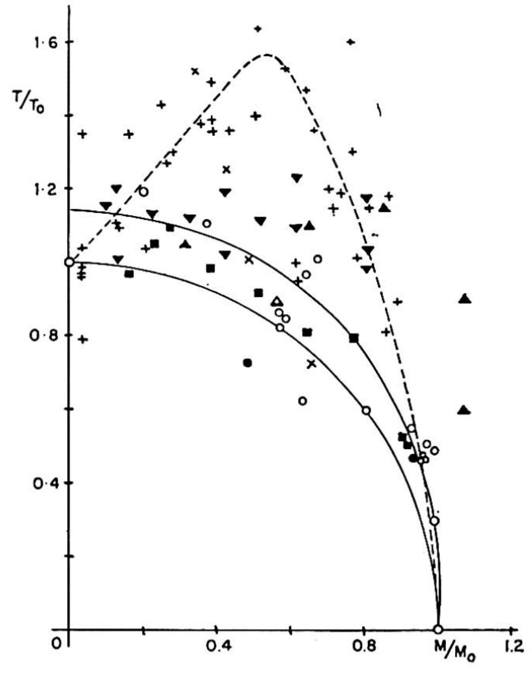

Fig. 11.

for hollow and solid prestressed beams but considerable scatter for reinforced concrete and I-beams. The results for T-beams follow a well-defined pattern

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Summary

Tests on reinforced and prestressed concrete beams subjected to combined bending and twisting are reported. Depending on the relative magnitude of the two stresses, beams may fail in diagonal tension or in compression. Between these two types of failure there is a transition from cleavage to crushing failure. The direct stresses due to the applied bending moment greatly augment the tensile strain, rotation and shear stress capacity of the beams which results in an increased torsional resistance.

The tests show no consistent agreement with the existing failure criteria. Failure loads may, however, be related to interaction curves.

Résumé

L'auteur présente des essais exécutés sur des poutres en béton armé et en béton précontraint sollicitées à la fois à la flexion et à la torsion. Selon la grandeur relative des deux efforts, c'est la compression ou l'effort de traction oblique qui provoque la rupture. Entre ces deux types de rupture existe une transition qui est celle de la rupture par fissuration à celle de la rupture par écrasement dû à la compression. Les contraintes directes dues au moment fléchissant qui est appliqué augmentent considérablement la capacité de déformation sous tension des poutres, leur capacité de rotation et leur résistance au cisaillement, ce qui accroît la résistance à la torsion.

On ne constate aucune concordance satisfaisante entre les résultats des essais et les critères de rupture actuellement admis. On peut néanmoins rapporter les charges de rupture aux courbes d'interaction flexion-torsion.

Zusammenfassung

Der Verfasser berichtet über Belastungsversuche an armierten und vorgespannten Betonbalken bei gleichzeitiger Biege- und Torsionsbeanspruchung. Je nach der relativen Größe der Beanspruchungsart brechen die Balken entweder infolge der schiefen Zug- oder Druckspannungen. Zwischen diesen beiden Brucharten kann ein Übergang vom Aufreißen zum Stauchen beobachtet werden. Die direkten Spannungen zufolge des Biegemomentes vergrößern die Dehn- und Verdrehungsfähigkeit der Balken und führen damit zu einer erhöhten Torsionstragfähigkeit.

Die Versuche zeigen keine konsequente Übereinstimmung mit den bekannten Bruchkriterien. Die Bruchlast läßt sich jedoch mit Kurven, die die Wechselwirkung von Biegung und Torsion berücksichtigen, angeben.