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Autor(en): **Olszak, W. / Stepie, A.**

Objekttyp: **Article**

Zeitschrift: **IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht**

Band (Jahr): **7 (1964)**

PDF erstellt am: **26.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-7898

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Time-dependent Effects in Compressed Bound Elements

Phénomènes retardés en pièces comprimées frettées

Zeitabhängige Erscheinungen in umschnürten Druckelementen

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1. If a structural element is subjected to the action of external loads maintained over long periods of time, time-dependent effects occur (creep and relaxation). These phenomena result in a redistribution of interior forces and, for this reason, are essential for the determination of the actual factor of safety of the structural element under consideration. Previous research work has been mainly concerned with the redistribution of interior forces (chiefly bending moments) produced by plastic and time-dependent deformations in structures such as beams, frames, arches and, partly, plates. However, pronounced "migration" of interior forces occurs also in reinforced concrete elements loaded axially which are of basic importance in the domain of structural engineering. Here, the existing research work is related nearly exclusively to compressed elements with longitudinal reinforcement.

The present paper is devoted to the problem of redistribution of interior forces, produced by the time-variable creep of the concrete core in spirally bound elements, as well as to the role of these phenomena in the determination of the limit load of such elements and their real factor of safety.

2. The bound element under consideration is assumed to be composed of a core (radius $r=a$) with rheological properties and a binding which, for simplicity, is treated as a substitute continuous coating (thickness g), covering the core. We also assume (in agreement with reality) that the spiral binding is not capable of transferring longitudinal loads. The axis of the core coincides with the axis z of a cylindrical coordinate system r, φ, z. The labels r, φ, z of σ and ϵ denote radial, circumferential and longitudinal stress and strain, respectively.

With the above assumptions, we have, for the elastic range, the following relations between the strains and stresses in the core:

$$\epsilon_p = \frac{1-\nu}{E}\,p - \frac{\nu}{E}\,\sigma_z, \qquad \epsilon_z = -\frac{2\nu}{E}\,p + \frac{1}{E}\,\sigma_z, \tag{2.1}$$

where E denotes Young's modulus of the core and ν its Poisson's ratio. We have taken into consideration the equalities $\sigma_r = \sigma_\varphi = p$ and $\epsilon_r = \epsilon_\varphi = \epsilon_p$.

To assess the elastic as well as the rheological phenomena of the core, we adopt N. KH. ARUTYUNYAN's model of a body, [1], which for concrete is in a fair agreement with reality. Accordingly, the elastic response depends on the age of the material, whereas the creep effects are determined by the time dependence of the loading programme. So, e. g., for uniaxial stress we have, [1],

$$\epsilon_p(t) = \frac{1}{E(t)} p(t) - \int_{\tau_0}^{t} p(\tau) \frac{\partial}{\partial \tau} \left[\frac{1}{E(\tau)} + \eta(t,\tau) \right] d\tau, \tag{2.2}$$

where τ denotes the age of the material (counted from a certain conventional instant), t being reserved for the current time coordinate; $\eta(t,\tau)$ stands for the creep term and we assume that the load has been applied at the instant $t=\tau_0$. Let us, in addition, introduce $\nu_1(\tau)$ for Poisson's ratio for the elastic state depending on the age of the concrete, whereas $\nu_2(t)$ stands for the coefficient of transverse expansion for rheological deformations.

For the analysis we should consider that for technical applications bound reinforced columns may be provided with initial prestress of the spirals. Such a mode of design and execution results in an increase of the load-carrying capacity of the column and presents also some other advantages in the working stage of the element, [3]. Consequently, let us assume that, for $t=\tau_0$, a lateral pressure $p_1(t)$ has been applied on the core. Further on, at the instant $t=\tau_1$, a vertical load σ_z starts acting; this is considered to be constant, $\sigma_z = P/\pi a^2$, and to induce an additional lateral pressure $p_2(t)$ on the core.

Now if we account for elastic as well as rheological effects, we have to replace the relations (1.1) by

$$\begin{aligned} \epsilon_p(t) &= \frac{p_1(t)+p_2(t)}{E(t)}[1-\nu_1(t)] - \int_{\tau_0}^{t} p_1(\tau)\frac{\partial}{\partial t}\Phi(t,\tau)\,d\tau \\ &\quad - \int_{\tau_1}^{t} p_2(\tau)\frac{\partial}{\partial \tau}\Phi(t,\tau)\,d\tau - \sigma_z\frac{\nu_1(\tau_1)}{E(\tau_1)} - \sigma_z\,\eta(t,\tau_1)\,\nu_2(t,\tau_1), \end{aligned} \tag{2.3}$$

$$\begin{aligned} \epsilon_z(t) &= -2\frac{p_1(t)+p_2(t)}{E(t)}\nu_1(t) + 2\int_{\tau_0}^{t} p_1(t)\frac{\partial}{\partial \tau}\Psi(t,\tau)\,d\tau \\ &\quad + 2\int_{\tau_1}^{t} p_2(\tau)\frac{\partial}{\partial \tau}\Psi(t,\tau)\,d\tau + \frac{\sigma_z}{E(\tau_1)} + \sigma_z\,\eta(t,\tau_1), \end{aligned} \tag{2.4}$$

with

$$\Phi(t,\tau) = \frac{1-\nu_1(\tau)}{E(\tau)} + \eta(t,\tau) - \nu_2(t,\tau)\,\eta(t,\tau),$$

$$\Psi(t,\tau) = \frac{\nu_1(\tau)}{E(\tau)} + \nu_2(t,\tau)\,\eta(t,\tau).$$

3. The binding is assumed to be linearly elastic (up to the yield limit Q or rupture ζQ) and its rheological deformations are disregarded when compared with those of concrete. Therefore, the stress-strain relation for steel is simply

$$\epsilon_s(t) = \frac{1}{E_1}\sigma_s(t). \tag{3.1}$$

The ration of the area of binding F_s to that of the core F_c will be denoted by λ. So we have

$$\lambda = \frac{F_s}{F_c} = \frac{2\pi a g}{\pi a^2} = 2\frac{g}{a}. \tag{3.2}$$

We have to determine the quantities $p(t)$, $\epsilon_p(t)$, $\epsilon_z(t)$, $\epsilon_s(t)$, $\sigma_s(t)$ from (2.3), (2.4), (2.5). For this, the two additional required relations are provided by the equilibrium and compatibility conditions:

$$\left.\begin{aligned} F_s\sigma_s(t) + F_c p(t) &= 0, \quad (3.3)\\ \epsilon_s(t) &= \epsilon\, p(t) + \epsilon_0, \quad (3.4)\end{aligned}\right\} \text{ for } r = a,$$

where ϵ_0 denotes the initial strain in the steel jacket necessary to produce the prestress σ_0.

Now the procedure to adopt is the following one: we replace the left hand side of (2.3) by the expressions following from (3.3) and (3.4) and we obtain integral equations for the unknown functions $p_1(t)$ and $p_2(t)$.

4. For the creep function we introduce $\eta(t,\tau) = \eta_0[1 - e^{-\gamma(t-\tau)}]$. For simplicity it is assumed that $\nu_1(t,\tau) = \nu_2(t,\tau) = \nu = \text{const}(t,\tau)$. Then we obtain, for $p_1(t)$, the Volterra integral equation

$$-\frac{2p_1(t)}{\lambda E_s} - \frac{\sigma_0}{E_s} = \frac{p_s(t)}{E}(1-\nu) - \int_{\tau_0}^{t} p_1(\tau)\frac{\partial}{\partial\tau}(1-\nu)\,\eta_0[1 - e^{-\gamma(t-\tau)}]\,d\tau \tag{4.1}$$

(we consider the time interval $t_0 \leqq t \leqq t_1$) and an analogous one for $p_2(t)$.

After solution and computation of the constants, we readily find the expressions for the stress and strain components in the bound element. If, for matured concrete, we introduce $E(t) = E = \text{const}(t)$, all these components may be expressed in closed form by combinations of elementary functions. They are graphically represented in Fig. 1 and 2.

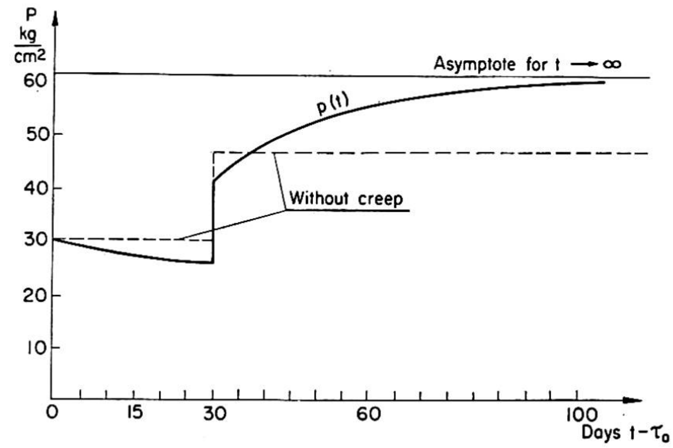

Fig. 1.

Of course, all the above results hold also for elements without prestress. It simply suffices to put everywhere $p_1(\tau_0)=0$, especially also in the final results.

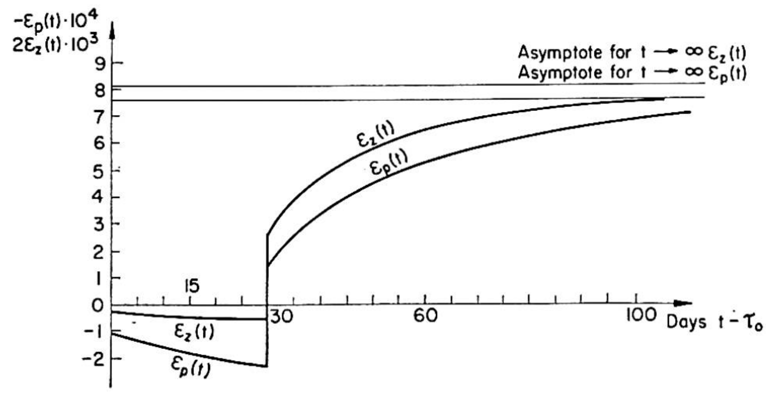

Fig. 2.

To illustrate the results, an example has been computed, with the following data for the core and steel: $E=2.10^5$ kg/cm^2, $\nu=0.286$, $\eta_0=2/E$, $\gamma=0.03$, $\lambda=0.04$, $\sigma_z=300$ kg/cm^2, $\tau_1-\tau_0=30$ days, $E_s=2.1\cdot10^6$ kg/cm^2. Fig. 1 shows the variability in time of $p(t)$, Fig. 2, the variability of the $\epsilon_z(t)$ and $\epsilon_p(t)$. The diagrams hold for $p_1(\tau_0)=30$ kg/cm^2. It is clearly seen that the influence of the rheological phenomena on stresses and strains is extremely pronounced. In responsible structural elements, this should be taken into account.

5. This time-dependent evolution of deformations and the "migration" of interior forces induces, of course, also a time-variation of the load-carrying capacity of the element and of its corresponding factor of safety.

Let us now find the functional coefficients $\alpha_{(\mathrm{I}a)}$, $\alpha_{(\mathrm{I}b)}$ and $\alpha_{(\mathrm{II})}$, characterizing the increase of the load-carrying capacity of the column due to the binding. These coefficients show how many times this load-carrying capacity increases when compared with a column without binding. The label denotes the destruction type of the structure. Thus (I) corresponds to the case when the load-carrying capacity of the core is exhausted, whereas (II) is related to the case when the load-carrying capacity of the binding is exhausted; (Ia) concerns the destruction of the core with undamaged binding, and (Ib) the destruction of the core due to the attainment of the yield limit of the coat (then the transverse pressure of the binding vanishes and the three-dimensional state is destroyed, and causes the destruction of the core which has now become free to deform transversally). This scheme was advanced by A. M. Freudenthal ([2], pp. 62—66)[1]). Considering now the conditions in

[1]) Rheological phenomena were not taken into account. To facilitate comparison of the results, similar notations are, wherever possible, used in what follows. The argument is also explained in a similar manner.

which a bound element subject to the action of a compressive force attains its ultimate strength, we can express the coefficients α of increase of the limit load.

For conglomerates such as concrete, the simplest mathematical expression determining Mohr's limit envelope may be expressed thus

$$\frac{\sigma_z}{R} = b + c\frac{p}{R}, \tag{5.1}$$

where R stands for the compressive strength of concrete, and b and c denote constants. Then the coefficients have the form

$$\alpha_{(\mathrm{I}a)}(t) = b + \frac{c\,p(t)}{R}, \tag{5.2}$$

$$\alpha_{(\mathrm{I}b)}(t) = b + \frac{1}{2}c\lambda\frac{\zeta Q}{R}, \tag{5.3}$$

$$\alpha_{(\mathrm{II})}(t) = \frac{\zeta Q \lambda \sigma_z}{2\,p(t)\,R}. \tag{5.4}$$

The values b and c should be taken from experiment. The corresponding results available in literature differ considerably. For the numerical examples computed here, we adopted $b \approx 1$, $c \approx 3.33$.

Fig. 3 shows the corresponding diagrams for a prestress $p_1(\tau_0) = 30$ kg/cm², Fig. 4 represents the same diagrams for a smaller prestress, $p_1(\tau_0) = 10$ kg/cm².

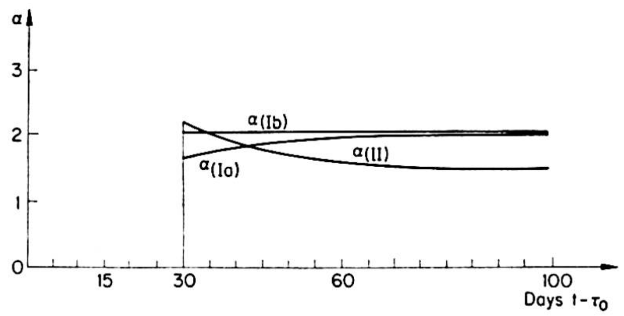

Fig. 3.

The Figs. 1—4 are very instructive. They show that the redistribution and transfer of interior forces is a function of time: the time-variable creep (longitudinal and transverse) of the core forces the steel to a gradually increasing action. The effect of this transfer is important and essential: the load-carrying capacity of the core gradually increases (because of the increase of the lateral pressure in the combined state of stress), whereas the carrying capacity of the steel binding gradually decreases. From Figs. 3 and 4 it may be seen that the coefficients $\alpha_{(\mathrm{I}a)}$ and $\alpha_{(\mathrm{II})}$ (which characterize this load-carrying capacity) are, because of the rheological effects, themselves, respectively, monotonically increasing and monotonically decreasing functions of time, and so are the corresponding factors of safety, which likewise vary monotonically in time.

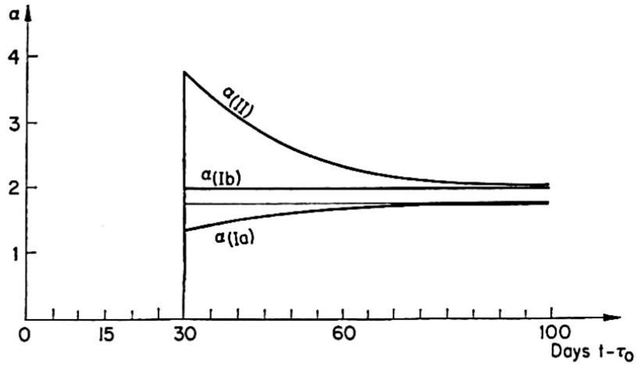

Fig. 4.

It can be observed that the action of creep increases the efficacy of the prestress because, by increasing the lateral pressure, the load-carrying capacity of the core is augmented (with a corresponding reduction of the load-carrying capacity of the steel coating. There is, however, no quantitative correspondence between these phenomena).

In the second example, the creep has an advantageous influence on the structure, because it tends to equalize the coefficients $\alpha_{(Ia)}$, $\alpha_{(Ib)}$, and $\alpha_{(II)}$. In the first example (Figs. 1—3), this influence is apparently less advantageous (always the smallest of the coefficients is decisive). However, let us recall our assumption concerning the properties of binding: this has been supposed to be perfectly elastic up to the yield limit Q (or ultimate strength ζQ) and not to exhibit creep.

In practice, there are considerable deviations from this rule, because the binding passes, before rupture, through a phase of plastic strains. As a result, the stress in the binding does not increase in such a degree as would follow from the assumption of linear elasticity. The coefficient $\alpha_{(II)}$ does not fall, therefore, to the value such as would follow from the theoretical assumptions.

For every bound column there exists an optimum prestress. On the other hand, it is quite possible that, under unfavourable circumstances, the element may undergo collapse under longtime loading. The value of the corresponding loads may readily be computed.

Therefore, in important structures, the influence of creep on their behaviour should be investigated in an individual manner, depending on the material constants and the percentage of binding. This will enable us to select the most advantageous value of prestress.

Further corrections to the above theory, due to the non-linearity and anisotropy of the core, are discussed separately.

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Summary

The redistribution of interior forces in bound compressed reinforced concrete elements is analyzed. The variation of stresses and strains, due to the creep of the concrete core, is expressed as a function of time, as well as the variation of the load-carrying capacity of such elements and of their real factors of safety.

Résumé

Le mémoire présente l'analyse de la redistribution des efforts intérieurs dans des éléments en béton armé frettés. La variation des contraintes et des déformations, due au fluage du noyau en béton, est exprimée en fonction du temps, ainsi que la variation de la capacité portante de tels éléments et de leur coefficient de sécurité.

Zusammenfassung

Die Umlagerung der inneren Kräfte in spiralumschnürten Stahlbetonelementen wird untersucht. Infolge der Kriecherscheinungen des Betonkernes werden der Spannungs- und Formänderungszustand als Funktionen der Zeit ermittelt. Ebenso wird die charakteristische Abhängigkeit der Tragfähigkeit derartiger Konstruktionselemente und deren tatsächlichen Sicherheitsfaktors als Zeitfunktion bestimmt.

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