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Methods of Safety Analysis of Highway Bridges

Méthodes pour l'étude de la sécurité des ponts-routes

Methoden für die Untersuchung der Sicherheit von Straßenbrücken

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1. Introduction

The interpretation of the safety factor used in the design of engineering structures as a multiplier to compensate for the expected variations of the acting loads and of the carrying capacity of the considered structure, as well as for unavoidable shortcomings in design and construction is gradually gaining admission into design specifications, mainly in the form of "load-factors". There is, however, considerable reluctance to accept the consequence of this re-interpretation and to admit, at least for important structures, "safety analysis" as an integral part of the structural analysis, based on the relation between safety factor and probability of structural failure or probability of functional unserviceability, depending on the definition of the critical condition with respect to which the "safety" has to be established. Most likely this is due to the reluctance of the profession to discuss the possibility of structural failure and its probability with the same rational detachment as that of other possible types of accidents. It is not generally understood that the introduction of the probability of structural failure into the discussion of structural safety does not imply an unjustified attempt to lower the present safety standards, but simply expresses the recognition of the fact that no other rational measure of such safety exists and that, therefore, the level of structural safety implicit in the working stresses or load factors of current conventional specifications is actually unknown.

Since all assumptions of design parameters are based on extrapolations to values so unlikely that they are far beyond the range of actual observation, the risk of failure (or of unserviceability) due to this inherent uncertainty in the assumptions can never be completely eliminated; it may, however, be reduced to an "acceptable" very low level, the numerical specification of which removes the associated safety factor from the realm of irrational guess-work to that of rational probability analysis. The fact that, for instance, a "minimum" strength value defined as the smallest value obtained in samples of 4 specimens, or a "maximum" load value defined as the largest value obtained in 99 observations are associated, respectively, with a chance of one in five of the strength not being attained or of one in a hundred of the load being exceeded makes it impossible to deal with the concept of structural safety on any other than on a probability basis. Methods of safety analysis for simple structures established on this basis have been proposed by various investigators, including the present author [1]. However, most of these methods not only introduce a number of restrictive assumptions, but also disregard the effect of the service life of the structure on its probability of failure and on the associated safety factor. The present approach attempts to eliminate those shortcomings, which particularly affect the safety analysis of structures subject to moving composite loading, of which highway bridges represent the most important example.

2. The Safety Factor as a Statistical Variable

One feature of the new approach to safety analysis is the consideration of the safety factor ν as a statistical variable rather than as a definite number. The central problem in safety analysis is therefore the development of the frequency-distribution $p(\nu)$ or cumulative distribution $P(\nu) = \int_0^\nu p(\nu)\,d\nu$ from the distribution functions, $p_2(S)$, of the load-intensity S over the range of operating conditions, and, $p_1(R)$, of the carrying capacity or "resistance" R of the structure over the range of variation of the relevant material properties and the effects of geometry of the structural parts and connections.

Defining the safety factor as the ratio

$$\nu = R/S \tag{2.1}$$

the distribution $p(\nu)$ is defined as the distribution of a quotient of two statistical variables R and S. Considering that

$$R = \nu S \quad \text{and} \quad dR/d\nu = S, \tag{2.2}$$

the distribution

$$p_1(\nu S)\,p_2(S)\,(dR/d\nu)\,d\nu\,dS = p_1(\nu S)\,p_2(S)\,S\,d\nu\,dS. \tag{2.3}$$

is the joint distribution $p(\nu, S)$ of the quotient ν and the variable S. The "marginal" distribution of (2.3), obtained by integrating over $0 \leqq S < \infty$ is the distribution of the quotient ν alone

$$p(\nu) = \int_0^\infty S\, p_1(\nu S)\, p_2(S)\, dS \tag{2.4}$$

and the associated cumulative distribution

$$P(\nu) = \int_0^\nu p(\nu]\, d\nu = \int_0^\nu \int_0^\infty S\, p_1(\nu S)\, p_2(S)\, dS\, d\nu. \tag{2.5}$$

Changing the order of the integration in eq. (2.5) the expression is obtained [2]

$$P(\nu) = \int_0^\infty P_1(\nu S)\, p_2(S)\, dS = \int_0^1 P_1(\nu S)\, dP_2 \tag{2.6}$$

which represents the relation between $P(\nu)$ and the known distribution functions of R and S. The probability $P(1)$ of a value $\nu < 1$ or of $R < S$ expresses directly the probability of failure P_F.

Eq. (2.6) can be evaluated either directly or by numerical integration, depending on the assumed form of the functions $P_1(R)$ and $P_2(S)$. Thus of the asymptotic extremal distributions [3] which have been found to reproduce strength test results and observations of high load intensities quite well

$$P_1(R) = 1 - \exp[-(R/\tilde{R})^{\alpha_R}], \tag{2.7}$$

with $R > 0$, $P_1(\tilde{R}) = (1 - 1/e)$ at the modal value $\tilde{R}$, and $\alpha_R = \pi/\sigma(\ln R)\sqrt{6}$, where σ denotes the standard deviation, represents the (asymptotic) distribution of smallest values R, and

$$P_2(S) = \exp[-(S/\tilde{S})^{-\alpha_S}] \tag{2.8}$$

with $S > 0$, $P_2(S) = 1/e$ at the modal value $\tilde{S}$ and $\alpha_S = \pi/\sigma(\ln S)\sqrt{6}$, represents the (asymptotic) distribution of largest values S. Introducing the auxiliary variable $P_2(S) = y$ and the ratio between the modal values $\nu_0 = \tilde{R}/\tilde{S}$, eq. (2.6) is transformed into

$$P(\nu) = \int_0^1 \left\{1 - \exp\left[-\left(\frac{\nu S}{\tilde{R}}\right)^{\alpha_R}\right]\right\} dP_2 = 1 - \int_0^1 \exp\left[-\left(\frac{\nu}{\nu_0}\right)^{\alpha_R} (-\ln y)^{-\alpha_R/\alpha_S}\right] dy. \tag{2.9}$$

The integral on the right-hand side of eq. (2.9) must be numerically evaluated to obtain the probability function $P(\nu)$ and the probability density of the safety factor $p(\nu) = dP(\nu)/d\nu$. The probability of failure $P_F = P(\nu)$ for $\nu = 1$ is, according to eq. (2.9),

$$P_F = P(1) = 1 - \int_0^1 \exp[(-\nu_0)^{-\alpha_R} (-\ln y)^{-\alpha_R/\alpha_S}]\, dy \tag{2.10}$$

and is thus directly related to the ratio ν_0, which might be considered a "central" safety factor based on the modes $\tilde{R}$ and $\tilde{S}$ of the distributions $p_1(R)$ and $p_2(S)$, and the exponents α_R and α_S, that are inversely proportional to the standard deviations $\sigma(\ln R)$ and $\sigma(\ln S)$, respectively. The probability of failure P_F refers to a single application of the (statistically variable) load intensity S to any of a large number of nominally identical structures or structural parts of (statistically variable) resistance R.

Fig. 1 presents the results of the numerical evaluation of eq. (2.10) for several ratios $\sigma_S/\tilde{S}$ and $\sigma_R/\tilde{R}$ which clearly illustrates the dependence of the probability of failure on the selection of ν_0 and the range of variation of S and R [4]. The fact that the values of ν_0 associated with low probabilities of failure are much higher than the conventional safety factors is due to the selected design basis of the modal or most likely values $\tilde{S}$ and $\tilde{R}$ rather than the conventional "maximum" and "minimum" values.

If the structure is designed for a constant maximum load $S_{max} = \tilde{S} = S$

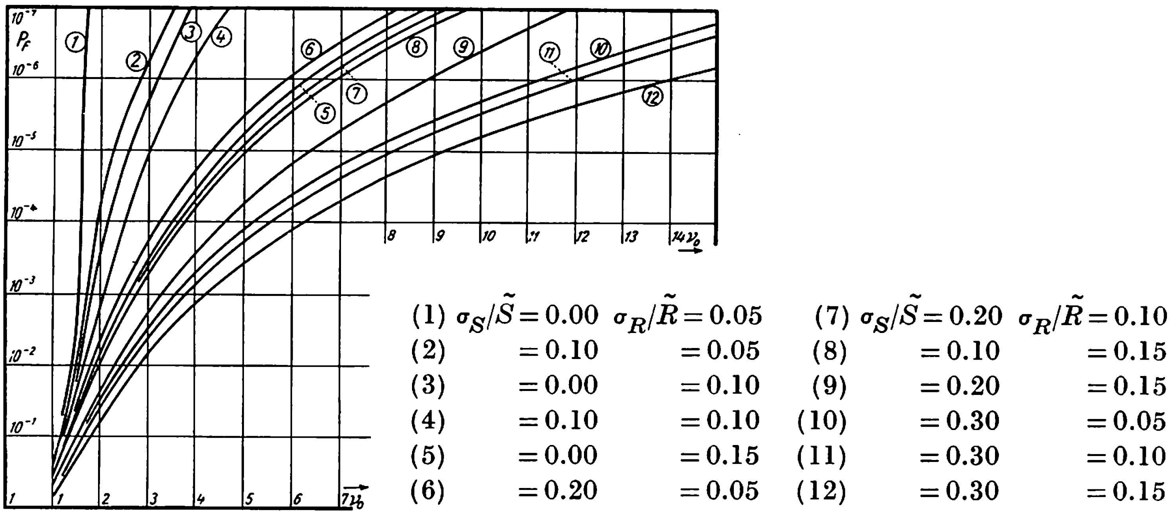

Fig. 1 Relation of "central" safety factor $\nu_0 = \tilde{R}/\tilde{S}$ and probability of failure P_F. (extremal distributions).

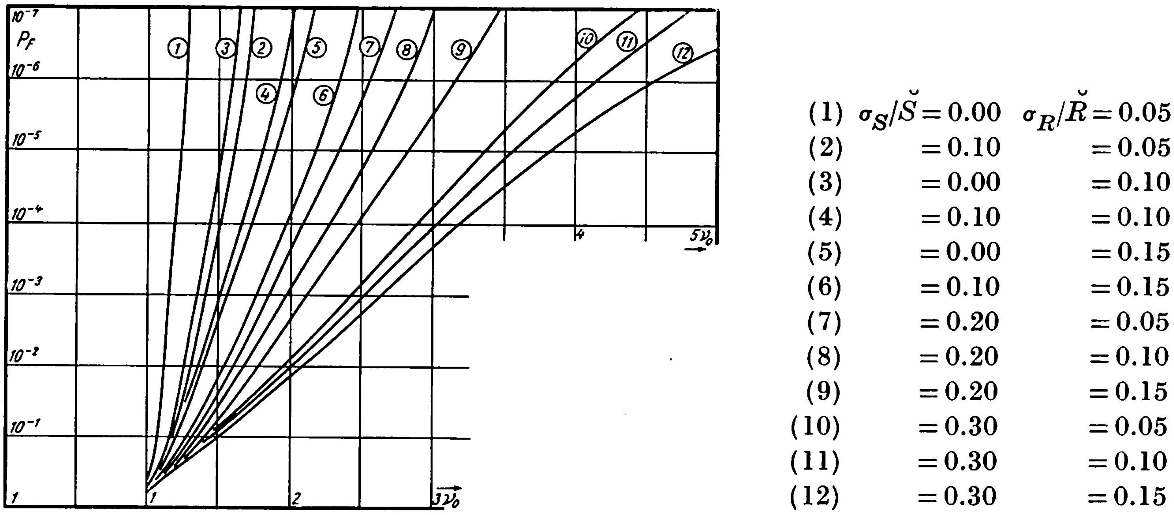

Fig. 2. Relation of "central" safety factor $\nu_0 = \breve{R}/\breve{S}$ and probability of failure P_F. (log-normal distributions.)

rather than for a statistically variable load-intensity, $\alpha_S \to \infty$; therefore according to eq. (2.10)

$$P_F = P(1) = 1 - \exp(-\nu_0)^{-\alpha_R}, \tag{2.11}$$

where $\nu_0 = \tilde{R}/S_{max}$.

A particularly simple distribution of ν arises when both R and S are logarithmic-normally distributed

$$p_1(R) = \frac{\log e}{\delta_R R \sqrt{2\pi}} \exp\left[-\frac{(\log R - \overline{\log R})^2}{2\delta_R^2}\right] \tag{2.12}$$

with $\overline{\log R} = \log \breve{R}$ where $\breve{R}$ is the median of R at $P_1(\breve{R}) = 0.5$, and $\delta_R = \sigma(\log R) = \sqrt{0.434 \log[1 + v_R^2]}$, where $v_R = (\sigma_R/\bar{R})$ is the coefficient of variation of R, and

$$p_2(S) = \frac{\log e}{\delta_S S \sqrt{2\pi}} \exp\left[-\frac{(\log S - \overline{\log S})^2}{2\delta_S^2}\right. \tag{2.13}$$

with $\overline{\log S} = \log \breve{S}$, where $\breve{S}$ is the median of S at $P_2(\breve{S}) = 0.5$, and $\delta_S = \sigma(\log S) = \sqrt{0.434 \log(1 + v_S^2)}$, where $v_S = (\sigma_S/\bar{S})$ is the coefficient of variation of S. Writing eq. (2.1) in the form

$$\log \nu = \log R - \log S \tag{2.1a}$$

it is clear that when $(\log R)$ and $(\log S)$ are normally distributed their difference $\log \nu$ is also normally distributed with mean $\log \breve{R} - \log \breve{S} = \log(\breve{R}/\breve{S}) = \log \nu_0$ and standard deviation $\delta = \sqrt{\delta_R^2 + \delta_S^2}$, where ν_0 represents a central safety factor based on the medians. Introducing the reduced variate $y = \frac{1}{\delta} \log\left(\frac{\nu}{\nu_0}\right)$ the distribution $p(\nu)$ and function $P(\nu)$ can be plotted directly from Normal tables; the reduced variate corresponding to $\log \nu = 0$ or $P_F = P(1)$ is $y_0 = \frac{1}{\delta} \log\left(\frac{1}{\nu_0}\right)$ so that $P_F = P(y_0)$ can be read off. Fig. 2 presents the relations $P_F(\nu_0)$ for various ratios $(\sigma_S/\breve{S})$ and $(\sigma_R/\breve{R})$ [4].

3. The Influence of Service Life

The effect of the anticipated service life of a structure on its safety arises from the dependence on the service life of the distribution of extreme load-intensities produced either by random configurations of composite loads, as in long-span highway bridges, or by the random application of one load-type of statistically variable intensity, as in short-span highway bridges or bridge-elements, airplane wings and other structure subject to gust-loads, or in flood-protection dams. This dependence is most expediently expressed by introducing the concept of the "recurrence period" $T(S)$ of a specified or higher load intensity in relation to the anticipated service life of the structure.

For structures subject to a non-configurational loading of statistically variable intensity S, recurrence periods are derived from the observed distribution function of load intensities $p(S)$ on the basis of the relation

$$T(S) = 1/[1 - P(S)], \tag{3.1}$$

where $T(S)$ is expressed in terms of the number of "observations", i.e., of load applications expected between occurrences of stress-intensities equal to or exceeding S. Since, however, the application of a specific load intensity $\geqq S$ will produce failure only if it coincides with the condition $S > R$ or $\nu < 1$, the "recurrence period" of failure T_F is identical with the recurrence period of values $\nu < 1$ or

$$T_F = T(\nu < 1) = 1/P(1) = 1/P_F, \tag{3.2}$$

expressed in terms of the number of load applications of variable intensity S to any of the large number of nominally identical structures or parts, of which the considered structure is one. Since, in general, $p(S)$ is not derived from all load-observations, but only of observations of "extremal" intensity, such as the highest intensity observed in groups of 10 (highest 10 percent), 100 (highest 1 percent) or 365 (highest, per year, of daily observations), the number of load applications is only that of such "extremal" loads, rather than of all load-intensities.

The large differences in the factors ν_0 required to produce the same values P_F and T_F, arising from the assumptions of extremal or of logarithmic normal distributions of R and S (see figs. 1 and 2) should be considered in the light of the fact that logarithmic-normal distributions, when they are applicable, usually represent *all* observations, while extremal distributions, as their name suggests, represent only the extreme (largest or smallest) observations out of rather large samples of observations. They will, therefore, be characterized by much narrower ranges of variation than comparable logarithmic-normal distributions.

With a recurrence period of failure in terms of number of load applications determined from eq. (3.2) and figs. 1 or 2, the probability of failure $P_F(L)$ during the anticipated service life L of the structure associated with n load applications (which do *not* produce fatigue effects) per unit of life can be expressed by the Poisson distribution

$$P_F(L) = 1 - \exp(-L n/T_F) \tag{3.3}$$

since the probability of failure of any adequately designed structure can obviously be considered a "rare event". Thus the probability of "survival" $l(L)$ during the life of the structure can be expressed by the straightline semi-logarithmic relation

$$\ln l(L) = \ln[1 - P_F(L)] = 2.3026 \log(1 - P_F(L)] = -(n L/T_F). \tag{3.4}$$

Since the population sizes of structures of service life L, to which eq. (3.4) refers, are usually quite limited, it appears that the specification of $P_F(L) \sim 10^{-1}$ will ensure adequate safety. Thus according to eq. (3.4) $2.30\, T_F \sim 22\, n\, L$ where n for bridge-structures depends on the density of the traffic. Assuming that the passage of a single vehicle is equivalent to a single load application, and that an extremal distribution represents the load-intensities of the heaviest five percent of vehicles with a ratio $\sigma_S/\tilde{S} = 0.10$, while the low end of the distribution of the resistance is represented by an extremal distribution with $\sigma_R/\tilde{R} = 0.10$, the relation between P_F and ν_0 can be read off fig. 1. With an assumed medium-heavy traffic density on a highway bridge of 5000 vehicles per day per lane or roughly $N = 2 \times 10^6$ vehicles per year, of which the heaviest one-twentieth only is counted so that $n = 10^5$, the recurrence period of failure will be roughly $10^5 L$ with respect to load application; with $L = 25$ years this is associated with a safety factor based on $\tilde{R}$ and $\tilde{S}$ of $\nu_0 = 4.2$, which would represent the safety-factor to be applied to short-span structures and structural parts of medium-heavily travelled highway bridges the critical load of which is represented by a single heavy vehicle of *modal* weight $\tilde{S}$.

It is interesting to use the safety analysis for comparison of the above safety factor with the factor that would be required to design a medium or long-span girder of a bridge the critical load of which is represented by sequences of r heavy vehicles; their percentage in the total number is, as before, 5 percent.

The probability of occurrence of such sequences can be evaluated on the basis of the theory of runs [5]. The expected number, per year, of runs of length r of heavy vehicles within a long sequence of $n \gg r$ vehicles is given by

$$\lambda(r) \sim N(1-p)^2 p^r, \tag{3.5}$$

where N is the total number of vehicles per year and p the expected percentage of heavy vehicles. With $N = 2 \times 10^6$ and $p = 0.05$

$$\lambda(r) = 1.8 \times 10^6 p^r \tag{3.6}$$

the mean recurrence time of such runs $T(r) = N/\lambda(r) = 1.11\, p^{-r}$. For rather unlikely and therefore not too short runs the probability of exactly x runs of length r is governed by the Poisson distribution, so that

$$p(x) = e^{-\lambda(r)}[\lambda(r)]^x/x! \tag{3.7}$$

Hence the probability of at least one run of such length

$$P(x \geqq 1) = 1 - p(0) = 1 - e^{-\lambda(r)}. \tag{3.8}$$

A bridge span accommodating 6 heavy vehicles on one lane would be critically loaded by a sequence of $r=6$ such vehicles; its recurrence time, under the above assumptions, is $T(6)=71\times 10^6$ which, with $N=2\times 10^6$, represents 35.5 years. The expected number of such runs during a service life of $L=25$ years being $\lambda(r)=\frac{25}{35.5}=0.71$, the chance of at least one such run during this life is as high as $1-e^{-0.71}=0.51$. Hence the full heavy vehicle sequence must be considered as the critical (maximum) design load, although the probability of occurrence of an individual heavy vehicle is only $p=0.05$. The expected load intensity of a single or a very small number of occurrences of a sequence of 6 heavy vehicles is closely enough represented by the mode $\tilde{S}$ of the distribution of single vehicle loads, considering that the average load intensity arising from simultaneous action of r vehicles has a fairly normal distribution and a much narrower range of variation [standard deviation $(\sigma/\sqrt{r})$] than the distribution $p(S)$ of the individual heavy vehicle loads (central limit theorem). Since the maximum load intensity $S_{max}\sim\tilde{S}$ can be expected to occur not more than a few times during the service life ($n\,L<10$), the recurrence period of failure in terms of these repetitions for $P_F(L)\sim 10^{-1}$ as before is roughly $T_F\sim 90$ and therefore $P_F\sim 10^{-2}$, with an associated safety factor for $\sigma_S=0$ and $\sigma_R/R=0.10$ according to fig. 1 of $\nu_0=1.4$.

The comparison of ν_0 for the short-span and medium-span structural parts of the considered highway-bridge suggests that for the same design-load intensity S the specific resistance R provided in the short-span structural parts should be about $\frac{4.2}{1.4}=3$ times higher than that for the medium-span structure. Alternatively, if the resistance analysis is based on uniform values of material resistance the design load intensity of the medium-span structure can be reduced by a factor of 3 in comparison to that of the short span parts, disregarding the effect of impact which may further increase the difference, as well as the fact that the ratio $\sigma_R/\tilde{R}$ is likely to decrease with increasing size of the structural section because of the increasing number of elements making up the section.

For long-span structures sequences of heavy vehicles occupying the whole span will have recurrence times several order of magnitude higher than the anticipated service life. In this case the maximum load is represented by a vehicle sequence shorter than that filling the whole span, with recurrence period of a length comparable to the service life. The average load-intensity is therefore further reduced in a ratio roughly equal to the ratio of sequence length and span, if the relatively small effect of the weight of light vehicles filling the remainder of the span is neglected. Thus, for a variety of spans the decrease of design load intensity with span can be evaluated.

It is implied that the estimated recurrence times of heavy vehicle sequences are determined only by the probabilities of "runs" of such vehicles, independently of their spacing, while in reality the average spacing in free travel, which is a function of speed, may be so high as to increase the length of the critical sequence far beyond the length of the span it would occupy at rest or in congested slow travel [6]. To correct for this effect it would be necessary to consider the probability of occupancy of a span of given length by the r vehicles in the "runs" on the basis of the average number of such vehicles on the span in uncongested travel. However, there are so many non-statistical effects that may cause close spacing ("bunching") such as repairs, traffic congestion, etc., that the probabilities of spacing, derived statistically may considerably overestimate the actual recurrence time.

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Summary

The presented method of safety analysis illustrated by its application to highway bridges demonstrates the significance of the anticipated traffic density and service life as well as that of a uniform probability of failure in the establishment of the basis for designs of balanced safety. They also show that the current practice of specifying design loads or load factors independently of working stresses will, in general, not lead to structures of uniform safety.

Résumé

La méthode de détermination de la sécurité qui est ici exposée, avec application aux ponts-routes, met en évidence l'importance de la densité du trafic à prévoir, de la durée de vie de l'ouvrage, ainsi que d'une probabilité uniforme de rupture, pour l'étude d'un projet impliquant une sécurité homogène.

L'auteur montre également que la pratique courante consistant à fixer des charges de service ou des coefficients de charge indépendants des taux de travail ne permet pas, en général, d'obtenir des ouvrages offrant une sécurié uniforme.

Zusammenfassung

Die dargestellten Methoden zur Bestimmung der Sicherheit mit Anwendung auf Straßenbrücken zeigen die Bedeutung der zu erwartenden Verkehrsdichte und Lebensdauer wie auch diejenige einer gleichmäßigen Bruchwahrscheinlichkeit bei Aufstellung von Bemessungsgrundlagen für Konstruktionen mit ausgeglichener Sicherheit.

Sie zeigen auch, daß die übliche Praxis der Annahme von Normenlasten oder von Lastfaktoren unabhängig von den Materialbeanspruchungen im allgemeinen nicht zu einer Konstruktion gleichmäßiger Sicherheit führt.