

**Zeitschrift:** IABSE congress report = Rapport du congrès AIPC = IVBH  
Kongressbericht

**Band:** 6 (1960)

**Artikel:** Fatigue design and endurance of metal structures

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**DOI:** <https://doi.org/10.5169/seals-6941>

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## **Fatigue Design and Endurance of Metal Structures**

*Calcul de la résistance à la fatigue des ouvrages métalliques*

*Ermüdungsberechnung und Dauerfestigkeit von Metallbauten*

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### **1. Introduction**

While the basic mechanisms of fatigue-crack-initiation and propagation in relatively simple metals under simple repeatedly applied stress-fields of constant amplitude are, at present, fairly well understood [1], this understanding has, so far, not been matched by significant improvements in the procedures of fatigue design of metallic structures. This is due to the fact that increasing complexity of structural action is accompanied not only by decreasing knowledge of the actually applied stress-intensities but also by increasingly significant residual stress-fields of unknown and changing intensity which interfere with the correlation between the results of analysis and of simple specimen tests on the one hand, and the results of full-scale structural tests and fatigue performance under services conditions on the other. This interference, which is necessarily the more pronounced the higher the intensity of the residual stress-field compared to that of the repeatedly applied stress-field, may invalidate the design significance of fatigue-test results obtained on material test-specimens or on model-specimens of structural connections in which such residual stresses do not exist; for instance, recent tests on welded structural parts have shown [2] a considerable (one order of magnitude) reduction of the low-stress fatigue life of welded joints by the imposition of a tensile residual stress-field of an intensity approaching the yield-limit. Thus, unless residual stress-fields are of an intensity much lower than the intensity of the repeatedly applied stresses, fatigue performance can only be estimated on the basis of full-scale structural assembly tests provided, moreover, that testing conditions can be made representative of service conditions.

This last provision limits the design significance of the conventional, constant-amplitude fatigue-tests of material specimens and model-joints also under conditions of low residual stresses because of the rather wide discrepancy between the testing conditions and the variable stress-amplitudes encountered in service in the majority of engineering structures. Thus, in spite of the fact that the basic physical phenomenon of fatigue in the metal is the obvious underlying cause of the engineering fatigue phenomenon in the metal structure, quantitative correlation of small scale tests with fatigue performance or with fatigue tests of complex structural assemblies is not to be expected.

This does not imply, however, that the results of small-specimen tests are of no significance in respect to fatigue design. On the contrary, such tests provide the only possibility for the sufficiently large test replication necessary for an analysis of scatter as well as for a reliable determination of the trends of those basic phenomena which, even without quantitative correlation with the fatigue performance of the structure, may significantly affect the general approach to fatigue testing and fatigue design. The high cost, difficulty, and duration of full-scale fatigue tests on structural assemblies preclude investigations requiring more than a very small number of such tests, which are thus relegated to the role of ultimate design-check-tests. The manner in which such tests should be performed so as to provide significant information can, however, be decided only on the strength of test-results concerning the basic fatigue mechanism that are obtainable from small-specimen tests alone. For instance, recent results of such tests suggest that some basic change in the conventional approach to fatigue design for and fatigue testing at the endurance limit may be necessary.

## 2. The "Endurance Limit"

The approach to fatigue design of the majority of steel structures has been considerably simplified by the reliance on the existence of an "endurance limit", a limiting stress-amplitude below which stresses may be repeated any number of times without assumedly ever causing fatigue cracking or fatigue fracture. Hence if in a structure all repeated stress-amplitudes are kept below this limit, fatigue design becomes unnecessary. However, the assumption that in real structures or structural parts subject to dynamic loading stress-amplitudes will never exceed a relatively low limiting value, such as the endurance limit, can usually not be realized, unless the sections are heavily overdesigned or the loading conditions almost stationary, as in the case of steady-state vibrations in machine parts. However, even in such parts during conditions of start-up and shut-down the frequencies usually pass through one or several resonance ranges during which a small number of severe stress-amplitudes is imposed. In highway- and in railroad-bridges the imposed stress-amplitudes vary considerably, both because of variation in weight of the traffic elements,

as well as because of the variation of the dynamic load-effects, particularly in the short-span structural parts. Thus, even the most careful design for endurance cannot prevent the occurrence of a certain number of stress-cycles exceeding the design limit. It is generally assumed that these cycles are of no significance, provided they are sufficiently below the yield-point and do not by themselves produce fatigue damage.

Recent results of fatigue tests under variable stress-amplitudes, however, have shown that a reliable endurance limit exists only under conditions of constant-stress-amplitude tests. It is significantly reduced even by a very small number of stress-amplitudes substantially exceeding this limit. The tests performed were to test the following proposition: if the conventional endurance limit retains its physical significance under random sequences of variable stress-amplitudes including amplitudes below that limit, these latter can produce no damage. Hence the results of fatigue tests under randomized variable stress-amplitudes, representing some well-defined stress-spectra, could not be affected if those amplitudes are left out; therefore, after compensating for the number of missing cycles, the recorded fatigue life should be independent of whether the stress-cycles below the endurance limit have or have not actually been applied.

Randomized spectrum tests based on exponential spectra replaced by six discrete stress-levels and extending below the endurance limit were planned so that for the same slope of the spectrum in semi-logarithmic representation (a) all stress-amplitudes were applied, (b) the lowest stress-amplitude was left out, and (c) the two lowest stress-amplitudes were left out.

The following are the details of the test performed on a high-speed rotating bending random fatigue testing machine of special design [3]: SAE 4340 aircraft steel specimens of  $\frac{3}{16}$  in. dia. were heat-treated to  $\sigma_u = 140,000$  psi ultimate strength, with (estimated) constant amplitude endurance limit of at least  $\sigma_E = 75,000$  psi to 80,000 psi. The exponential stress-amplitude-spectra are represented by six discrete stress-amplitudes between  $\pm 0.35 \sigma_u = \pm 49,000$  psi and  $\pm 0.85 \sigma_u = \pm 119,000$  psi, with intervals of  $0.10 \sigma_u = 14,000$  psi. Thus at least the two lowest amplitudes are below the conventional endurance limit, the other four above it. Three stress-spectra of different severity (slope) were selected and fatigue tests performed for each spectrum using (a) all six, (b) five and (c) four stress-amplitudes, with twenty replications in each test series.

The following are the over-all frequencies of occurrence, in percent, of the different stress-amplitudes in the applied test-spectra, together with the compensating factors for the five- and four-level tests [multipliers  $1/(1 - p_1)$  and  $1/(1 - p_1 - p_2)$  for comparison with the total number of stress-amplitudes in the six level tests under the assumption that the stress-amplitudes below  $\pm 75,000$  psi have no damaging effect], the observed fatigue lives (modes of the distributions) and the fatigue lives compensated for the not applied stress-amplitudes:



*Spectrum A (most severe)*

± 49,000 psi	$p_1 = 82.200$	—	— %
± 63,000 psi	$p_2 = 14.560$	82.200	— %
± 77,000 psi	$p_3 = 2.664$	14.560	82.200 %
± 91,000 psi	$p_4 = 0.458$	2.664	14.560 %
± 105,000 psi	$p_5 = 0.100$	0.458	2.664 %
± 119,000 psi	$p_6 = 0.018$	0.118	0.576 %
Compensating factor	1.0	5.60	30.86
Observed life (mode)			
in $10^6$ cycles	1.55	0.267	0.168
Compensated life (mode)			
in $10^6$ cycles	1.55	1.50	5.18

*Spectrum B (medium severe)*

± 49,000 psi	$p_1 = 90.000$	—	— %
± 63,000 psi	$p_2 = 9.000$	90.000	— %
± 77,000 psi	$p_3 = 0.900$	9.000	90.000 %
± 91,000 psi	$p_4 = 0.090$	0.900	9.000 %
± 105,000 psi	$p_5 = 0.009$	0.090	0.900 %
± 119,000 psi	$p_6 = 0.001$	0.010	0.100 %
Compensating factor	1.0	10	100
Observed fatigue life (mode)			
in $10^6$ cycles	3.48	0.497	0.235
Compensated life (mode)			
in $10^6$ cycles	3.48	4.97	23.50

*Spectrum C (least severe)*

± 49,000 psi	$p_1 = 96.840$	—	— %
± 63,000 psi	$p_2 = 3.060$	96.84	— %
± 77,000 psi	$p_3 = 0.097$	3.06	96.84 %
± 91,000 psi	$p_4 = 0.0029$	0.097	3.06 %
± 105,000 psi	$p_5 = 0.0001$	0.0029	0.097 %
± 119,000 psi	$p_6 = 0.000003$	0.0001	0.003 %
Compensating factor	1.0	31.65	1000
Observed fatigue life			
(mode) in $10^6$ cycles	70.000	2.18	0.33
Compensated life (mode)			
in $10^6$ cycles	70.0	69.0	33.00

The results illustrate the damaging effect of stress-amplitudes below the conventional endurance limit. The compensated fatigue life in the four-level tests in which the applied stress-amplitudes are all above the conventional endurance limit, the two lowest stress-amplitudes having been left out, is 3 to 5 times as long as when the two stress-amplitude below the endurance limit are actually applied. It can be assumed that the difference is probably due to the damaging effect of the low stress-amplitudes brought about by their interaction with the high stress-amplitudes. Since the elimination of the lowest stress-amplitude alone produces no significant change in fatigue life, this stress-amplitude is undoubtedly below the true endurance limit. Hence the level of the endurance limit under spectrum loading must be below the higher of the two lowest stress-amplitudes applied. It appears therefore justified to assume that the conventional endurance limit of 75,000 psi has been reduced by the variable stress-amplitude sequences to between 49,000 and 63,000 psi. This demonstrates the lack of design significance of the conventional endurance limit in the presence of variable stress-amplitudes of which even a very small number is above this limit.

While the tests have been performed on a high-strength steel, there is no reason to assume that the same phenomenon will not be observed in medium-strength structural steels or in any other metal with a well-defined endurance limit. Variable-stress-amplitude fatigue tests on a weldable structural steel are now being performed to check this assumption.

### 3. Fatigue Testing for Endurance Limit

In the light of the above results it appears doubtful whether any real purpose is served by conventional constant-amplitude testing to establish an "endurance limit" that has little significance in terms of fatigue performance under variable load-amplitudes. It would seem much more expedient to devise load-spectra characteristic of various structural types and to establish fatigue lives of small specimens and of model-connections under randomized sequences of load-amplitudes derived from those spectra. Thus the conventional *S-N*-diagrams would be replaced by relations between the two principal characteristics of the stress-spectrum, the stress-amplitude-range and the slope of the spectrum in a straight-line representation (for instance semi-logarithmic representation of spectra of exponential type) versus fatigue life, one of the characteristics being considered as a parameter, the other as a variable. Since the adoption of such an approach would require extensive use of counting strain-gages on different type of structures in operation to determine characteristic stress-spectra, as well as the replacement of most of the conventional constant-amplitude fatigue testing equipment by newly developed variable-amplitude equipment, many years would elapse before such a drastic change

of approach could be realized. In the meantime, however, some modification of the conventional fatigue-testing procedures for endurance appears to be necessary in order to correct the results obtained by constant-stress-amplitude tests for possible stress-interaction-effects due to a small number of excessive stress-amplitudes below the yield-limit, but significantly above the conventional endurance limit.

If, as is customary for practical reasons, the "endurance limit" is identified with the stress-amplitude  $S_0$  that can be applied more than  $10^7$  times without producing fatigue failure, the intermittent application of a total of some  $10^4$  stress-amplitude which, by continually repeated application would produce failure after some  $10^5$  cycles, would seem to represent conditions under which an observed significant reduction of  $S_0$  for  $N = 10^7$  cycles or of  $N$  for the conventional value of  $S_0$  could be safely attributed to stress-interaction. The direct fatigue effect of the  $10^4$  cycles of overstress, at a cycle ratio of  $10^4/10^5 = 0.1$ , could hardly be very significant by itself. In order to approximate operating conditions as far as this is possible with conventional constant-amplitude fatigue testing apparatus the overstress-cycles should be applied intermittently, avoiding, however, application within the first 10 percent of expected life to permit development of the typical low-strain fatigue microstructure rather than of the high-strain work-hardened structure. Thus, application of three over-stress sequences of about 3000—3500 cycles, each at about 10, 25 and 40 percent of expected life, would appear to provide a reasonable compromise between the theoretically desirable randomized application of very short sequences and the practical limitations of conventional fatigue testing. If interaction effects are at all significant it is quite likely that under the conventional "endurance limit"  $S_0$  the second or third application of this overstress-sequence will produce fatigue failure.

While the procedure suggested above is rather arbitrary, it is assumed that it will provide a more reliable "endurance limit" in tests of specimens, model-connections as well as full-scale structural assemblies on which to base design considerations, than can be obtained in constant-amplitude tests, and it is proposed to redefine the concept of the "endurance limit" in terms of this or a similar two-level testing procedure.

### References

1. A. M. FREUDENTHAL, "Fatigue", Handbook of Physics, vol. 6, Springer, Berlin, 1958, p. 596—603.
2. V. I. TRUFYAKOV, *Avtomaticheskaya Svarka* (1956) No. 5; Proceedings, General Motors Conference on Residual Stresses and Fatigue, Detroit, 1958.
3. A. M. FREUDENTHAL, *Proc. Am. Soc. Test. Mat.*, vol. 53 (1953), p. 896.

### Summary

Since the fatigue damage produced by stress-amplitudes below the conventional endurance limit in the presence of a small number of stress-amplitudes exceeding this limit has been demonstrated in variable-amplitude tests, the conventional endurance limit can not be considered to provide significant design information. In the absence of facilities for extensive variable stress-amplitude testing, compromise testing procedures using an intermittent over-stress-amplitude to bring out the damaging stress-interaction effects with the low stress-amplitude may provide a temporary solution of the problem of testing for a "true", that is interaction-free, endurance limit.

### Résumé

Depuis que des essais effectués sous des amplitudes variables ont mis en évidence des dégradations par fatigue sous des contraintes d'amplitude inférieure à la résistance classique à la fatigue accompagnées d'un nombre réduit de dépassements de cette limite, la résistance normale à la fatigue ne peut plus être considérée comme une base effective de calcul. D'autre part, il n'est pas aisé d'effectuer des essais étendus sous des contraintes d'amplitude variable; il faut donc envisager de recourir à un compromis en faisant intervenir des contraintes intermittentes dépassant la limite de fatigue, afin de faire apparaître les dégradations dues à l'interaction de ces contraintes avec celles inférieures à la limite de fatigue. Les résultats ainsi obtenus peuvent fournir une solution transitoire pour le problème du contrôle de la «vraie», résistance à la fatigue, c'est-à-dire exempt des effets d'interaction.

### Zusammenfassung

Seitdem Ermüdungsschäden infolge von Spannungsamplituden unterhalb der herkömmlichen Dauerfestigkeit bei Auftreten einer kleinen Anzahl von Überschreitungen dieser Grenze in Versuchen mit variablen Amplituden nachgewiesen wurden, kann die normale Dauerfestigkeit nicht mehr als maßgebende Berechnungsgrundlage betrachtet werden. Da es nicht leicht ist, ausgedehnte Versuche mit variablen Spannungsamplituden durchzuführen, kann ein Kompromißvorgehen angewendet werden zum Erreichen der Ermüdungsschäden infolge der Zusammenwirkung von wechselnden Spannungsamplituden, und zwar durch die Unterbrechung von Spannungsamplituden an oder unterhalb der Wechselfestigkeit mit einzelnen «Überspannungsamplituden». Die Resultate daraus können eine Übergangslösung für das Problem der Prüfung der «wahren», d. h. von Wechselwirkungen freien, Dauerfestigkeit ergeben.

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