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Comments on Semi-rigid Connections in Steel Frames

Commentaires sur l'utilisation d'attaches semi-rigides dans les ossatures métalliques

Einige Bemerkungen zu halbsteifen Verbindungen bei Stahlrahmen

L. C. MAUGH

Professor of Civil Engineering, University of Michigan

Notation

P	Total load on the specimen.
E	Modulus of elasticity.
I	Moment of inertia of beam.
a	Assumed length of elastic portion.
b	Assumed length of inelastic portion.
L	$^1/_2$ span of specimen.
d	$^1/_2$ column width.
Δ	Measured vertical displacement at center.
Δ_e	Theoretical displacement at center due to deformation over elastic range.
ϕ	Angle change over one-half of inelastic range (assumed at face of column).
M	Bending moment at face of column.
Ψ	Slope of $M-\phi$ curve (assumed constant over working range).
M_{ab}, M_{ba}	End moments.
M_{Fab}, M_{Fba}	Fixed-end moments for $\Psi = \infty$.
M'_{Fab}, M'_{Fba}	Fixed-end moments for $\Psi \neq \infty$.
l	Span length of any beam.
K	$\dfrac{EI}{l}$.
A	$1 + \dfrac{3K}{\Psi}$.
C_1, C_2	Coefficients of slope deflection equations.
θ_a, θ_b	End rotations.

Test Procedure for Measuring Beam Connection Properties

The following laboratory procedures and interpretation of the data has been found to be convenient and sufficiently accurate for measuring the rotational restraint of beam connections.

1. The elastic portion a of a typical test specimen as shown in Fig. 1 is assumed to extend within three inches of the edge of the connection. Strain
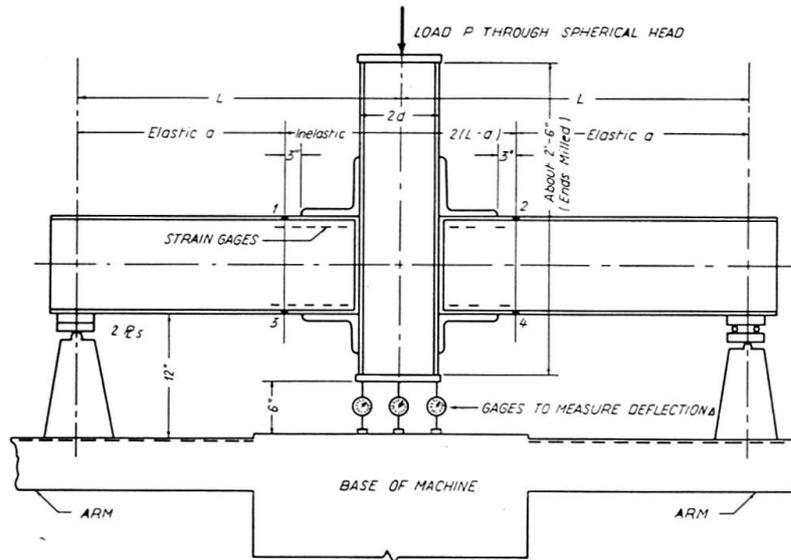

Fig. 1. Arrangement of Specimens in Testing Machine.

gage measurements have shown that between this section and the face of the column, the strain distribution across any transverse section is non-linear. This region is called inelastic although such a description is open to question.

2. The resultant angle changes ϕ, Fig. 2, for the inelastic range is assumed to occur at each face of the column for purposes of reference.

3. A laboratory test specimen as shown in Fig. 1 is therefore divided into two elastic zones a and inelastic zones $L-a$ which contain the connecting elements and column section.

4. The specimen is loaded as shown in Fig. 1 and the only measurement needed besides the central load P is the displacement Δ at the center of the span. This displacement can be measured with ordinary dial gages.

5. The numerical value of the rotation ϕ for the inelastic zone can now be determined by substracting the calculated displacement Δ_e at the center due to the strain in the elastic portions a from the total measured deflection Δ. Thus, from Fig. 2, the following relations can be established.

$$\phi(L-d) = \Delta - \Delta_e = \Delta - \frac{P a^3}{6 E I}$$

or

$$\phi = \frac{\Delta - \dfrac{P a^3}{6 E I}}{L-d}, \tag{1}$$

where

P = total load on the specimen,

E = modulus of elasticity,

I = moment of inertia,

a = length of elastic portion,

$L = {}^1\!/_2$ span of specimen,
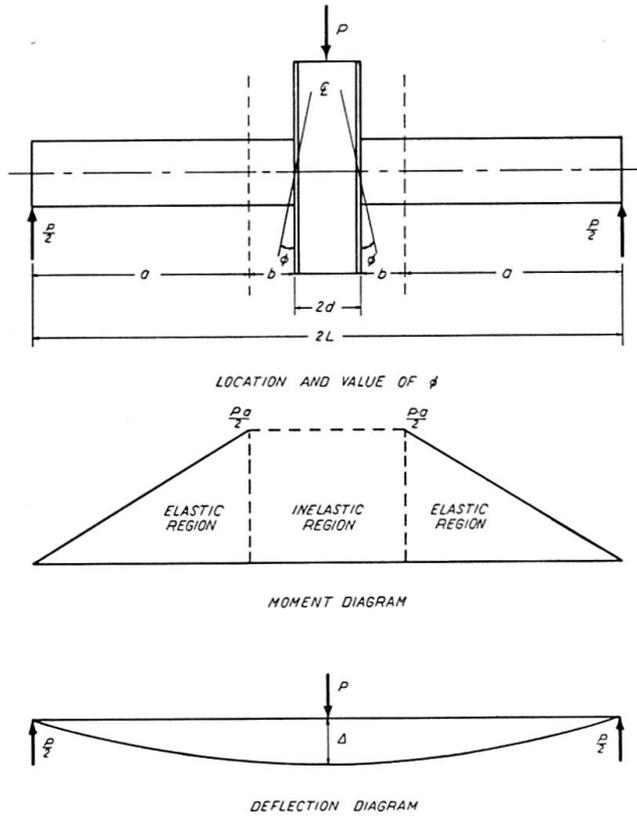
$d = {}^1\!/_2$ width of column.

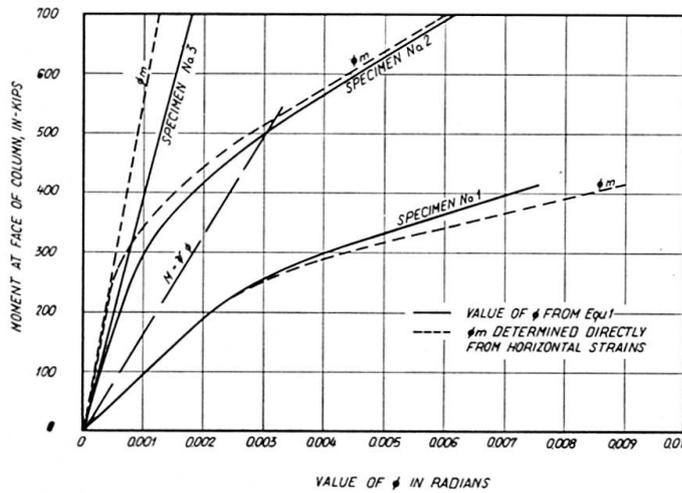Fig. 2. Location of Angle ϕ and Displacement $\varDelta$.

Fig. 3. $M - \phi$ Diagram Determined from Displacement $\varDelta$.

6. The values of ϕ that are determined from the measured value of $\varDelta$ by means of Eq. (1), when plotted as abscissae against the moment at the face of the column as ordinate, provide typical moment-rotation (M, ϕ) curves as shown in Fig. 3. In the diagrams are also shown corresponding rotations ϕ (see broken lines) which are obtained from the horizontal movement between two reference points that were established in each flange at the edges of the inelastic zone. These gage distances are shown by points 1, 2, 3, and 4 in Fig. 1. The sum of the horizontal displacements between points 1 and 2 and between 3 and 4 divided by the vertical distance between the points was used to check the value of ϕ.

Results of Typical Tests

The test procedure as described above was used on three different specimens but, due to lack of space, only one (Fig. 4) is shown. An initial load was applied through a movable head with a spherical bearing and removed several times before the final measurements were made.
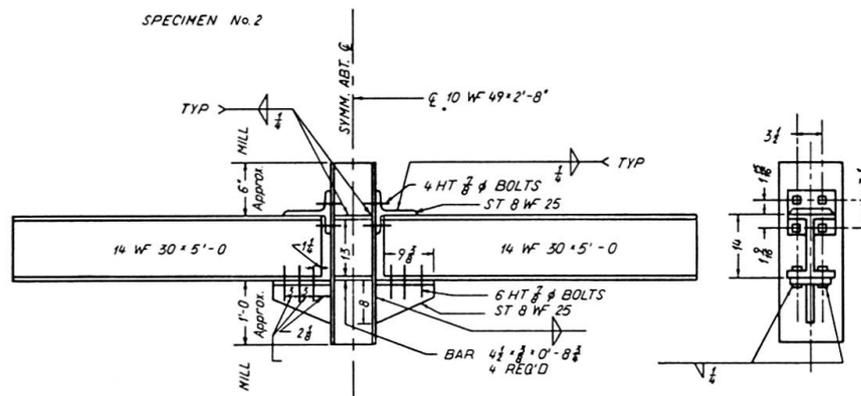
Fig. 4. Connection Details of Specimen No. 2.

The load-deflection curves for each specimen were drawn from the measured values of $\varDelta$. From such diagrams the angle change ϕ in the inelastic zone was calculated for each specimen by means of Eq. (1). The values of ϕ are shown in Fig. 3. The magnitude of the inelastic portions a is 46, 43, and 46 inches for specimens 1, 2, and 3, respectively. L is equal to 60 inches and d is 5 inches for all specimens.

Use of M, ϕ Curves in Design

If the actual M, ϕ curves in Fig. 3 are approximated by a straight line the moment M can be expressed in terms of the rotation ϕ by the relation

$$M = \Psi \phi,$$

where Ψ is the slope of the M, ϕ diagram. The quantity M/Ψ is therefore equivalent to $\dfrac{M\,dx}{EI}$ in a beam and can be treated as such in the calculations. In any frame where the steel girders support a reinforced concrete floor, the actual flexural rigidity EI of the beam is uncertain. However, when only steel members of constant cross-section are considered in determining the beam coefficients, the following assumptions are recommended for design calculations:

a) Consider the beam as a member with constant EI except at the ends where a concentrated angle change of M/Ψ occurs.

b) When the connection stiffness Ψ is the same for both ends of the beam, the coefficients 4 and 2, and the fixed-end moments M_{Fab} and M_{Fba} in the slope-deflection equations

$$M_{ab} = \frac{EI}{l}(4\,\theta_a + 2\,\theta_b) + M_{Fab}, \qquad (3\,\text{a})$$

$$M_{ba} = \frac{EI}{l}(2\,\theta_a + 4\,\theta_b) + M_{Fba} \qquad (3\,\text{b})$$

can be replaced by

$$M_{ab} = \frac{EI}{l}(C_1\,\theta_a + C_2\,\theta_b) + M'_{Fab}, \qquad (4\,\text{a})$$

$$M_{ba} = \frac{EI}{l}(C_2\,\theta_a + C_1\,\theta_b) + M'_{Fba}, \qquad (4\,\text{b})$$

in which, assuming that $\Psi_a = \Psi_b = \Psi$

$$C_1 = \frac{12\,A}{4\,A^2 - 1}, \qquad (5\,\text{a})$$

$$C_2 = \frac{6}{4\,A^2 - 1}, \qquad (5\,\text{b})$$

where $$A = 1 + \frac{3\,EI}{l\,\Psi} = 1 + \frac{3\,K}{\Psi}, \qquad (5\,\text{c})$$

$$K = \frac{EI}{l},$$

in which l = distance center to center of columns.

Also, $$M'_{Fab} = \tfrac{1}{6}[M_{Fab}(2\,C_1 - C_2) + M_{Fba}(2\,C_2 - C_1)], \qquad (6\,\text{a})$$
$$M'_{Fba} = \tfrac{1}{6}[M_{Fab}(2\,C_2 - C_1) + M_{Fba}(2\,C_1 - C_2)], \qquad (6\,\text{b})$$

where M_{Fab} and M_{Fba} are the usual fixed-end moments in Eqs. (3 a) and (3 b) that is for Ψ equals infinity and A equal to one. For a symmetrical loading such that

$$M_{Fab} = -M_{Fba},$$

then $$M'_{Fab} = \tfrac{1}{6}[M_{Fab}(2\,C_1 - C_2 - 2\,C_2 + C_1)]$$

or $$M'_{Fab} = \tfrac{1}{2}(C_1 - C_2)\,M_{Fab}.$$

Important Features of Semi-Rigid Connections

As variations in the coefficients C_1, C_2, M'_{Fab}, and M'_{Fba} are important factors in a structural design, it is interesting to note that a particular end connection may provide considerable restraint for a beam with a small K/Ψ value, but relatively little if the beam has a large K/Ψ value. The variation of the coefficients C_1 and C_2 with respect to K/Ψ are shown in Fig. 5. A disturbing feature
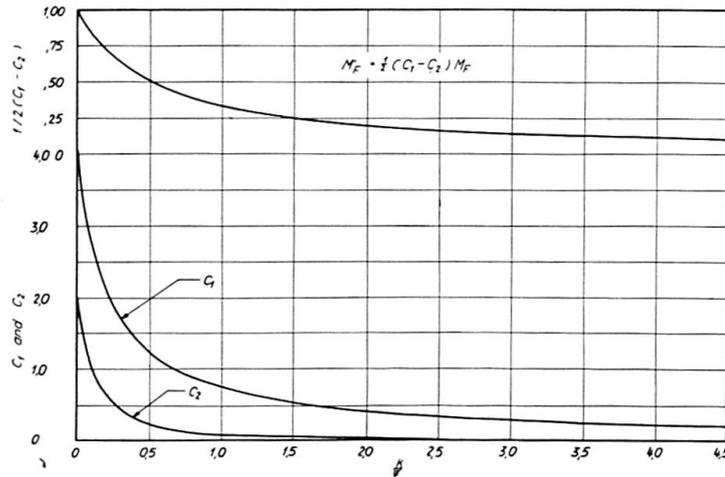

Fig. 5. Values of C_1, C_2 and M'_F.

of these diagrams is the rapid change that may occur in the values of C_1, C_2, and M'_F for small changes in K/Ψ. The changes in the fixed-end moments M'_F are indicated in Fig. 5 for a uniform load over the entire span. It is apparent that the fixed-end moments may change rapidly for even small changes in the K/Ψ values.

If a semi-rigid connection such as in specimen 3 (Fig. 3) is used instead of a rigid connection then a constant value of Ψ of 385×10^6 in-lbs is obtained from the slope of the M, ϕ curve. When this connection is used on a 12 WF 36 beam of 17 feet length the value of K/Ψ is

$$\frac{K}{\Psi} = \frac{EI}{l} = \frac{29 \cdot 10^6 \cdot 280.8}{17 \cdot 12 \cdot 385 \cdot 10^6} = 0.104 \,.$$

From Fig. 5 or from Eqs. (5 a, 5 b, 5 c) we obtain,

$$C_1 = 2.68\,, \qquad C_2 = 1.02\,,$$

$$M'_F = \tfrac{1}{2}(2.68 - 1.02)\,M_F = (0.83)\left(\frac{wL^2}{12}\right) = 0.069\,wL^2\,.$$

Summary

In this paper the importance of considering the deformation of beam connections in the design of steel frames has been emphasized. A laboratory procedure for determining the M, ϕ diagram for any type of beam connection has been discussed.

Analytical methods for incorporating the properties of the connections into the slope-deflection equations are presented. It has been shown that the stiffness of the beam and the magnitude of the end couples may be modified considerably by the rotational restraint factor Ψ of the connections. Therefore the actual beam coefficients and fixed-end moments, for the particular beam and connection, should be determined from test results and used in the structural analysis.

Résumé

L'auteur montre qu'il est important de tenir compte des déformations des attaches des traverses dans le calcul et l'étude des cadres métalliques. Pour des attaches quelconques, l'auteur présente une méthode qui permet de déterminer le diagramme de M en fonction de ϕ, au laboratoire.

L'auteur indique des méthodes analytiques permettant de tenir compte des propriétés des attaches dans les équations de la méthode des déformations. Il montre aussi que le facteur Ψ de l'attache (angle d'inclinaison de la courbe $M - \varphi$) influence fortement la rigidité et les moments d'encastrement total de la traverse. Les coefficients caractéristiques et les moments d'encastrement total de la traverse, nécessaires au calcul du système, devraient donc être déterminés par des essais pour la poutre considérée et le type d'attache utilisé.

Zusammenfassung

In diesem Beitrag wird die Bedeutung der Berücksichtigung der Nachgiebigkeit von Trägerverbindungen im Entwurf von Stahlrahmen betont. Für die Bestimmung des M-ϕ-Diagramms für jeden möglichen Trägeranschluß wird ein Laboratoriumsverfahren dargestellt.

Die Eigenschaften der Stöße werden dann analytisch in den Gleichungen der Deformationsmethode berücksichtigt. Es zeigt sich, daß der Einspannungsgrad die Trägersteifigkeit und die Größe der Endmomente wesentlich beeinflußt. Somit sollten die tatsächlichen Trägerbeiwerte und die Volleinspannmomente für einen besonderen Träger und seinem Anschluß aus einem Versuch bestimmt und dann in die Tragwerksberechnung eingesetzt werden.

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