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On the Lateral Buckling of Multi-Story Building Frames with Shear Bracing

Sur le flambage latéral des portiques étagés multiples munis de contreventements (shear bracing)

Über das seitliche Ausknicken eines mehrstöckigen Gebäudes mit Windverbänden (shear bracing)

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Introduction

With the trends which seem to be developing in the architectural design of tall buildings, it is likely that questions of general instability will assume greater importance than they have had in the past. In particular, the lurching, sidesway or translational mode of buckling of tall buildings may demand greater attention than it has been given in the past.

In the past, skeleton-type tall building frames have been sheathed by rather substantial walls or wall panels of masonry construction and it is quite likely that these were sufficiently stiff to brace the frame against a lurching mode of buckling. In place of the masonry envelope which, in the past, obviated or at least minimized the necessity for considering the translational mode of buckling, the present architectural trend seems to be toward the use of light and often prefabricated panels having considerably reduced shear stiffness and hence much less effective in bracing the frame against buckling in a sidesway mode. Calculations made upon some recently designed bulding frames for the sidesway mode show that the equivalent or effective column length may be as much as three stories. This is far from the one story assumption which, in the past, has been a convenient and apparently adequate basis of design.

Unbraced symmetrical frames under symmetrical loads may buckle in either the symmetrical mode, which does not involve translation of the joints, or in the anti-symmetrical mode involving lateral displacement or lurching.

However, by considering the limiting cases of unbraced frames with infinitely stiff girders and with infinitely flexible girders, it can be shown that the lurching mode will always occur at lower loads than the symmetrical case. Unbraced unsymmetrical frames will, in general, buckle in a mode involving some lateral displacement of the joints.

When bracing is provided against lateral displacement, either in the form of shear panels or supplementary bracing members, the critical loads for the lurching mode of buckling are, naturally, increased over the corresponding loads for the unbraced frame. As the stiffness of this bracing is increased continuously, the critical loads for the lurching mode will increase until, in the symmetrical case, these loads become greater than those associated with a mode which does not involve translation of the joints.

The present paper contains some results which have been obtained during a preliminary and exploratory study of the general problem.

Limiting Cases

It is of interest to determine the stiffness required in the lateral bracing to preclude the lurching mode of buckling. For our immediate purpose it is sufficient to consider a single column and, in order to establish the requirement under the most severe condition, we shall first take the case corresponding to infinitely rigid girders.

We consider a single story and we assume that the bracing force is applied in a horizontal direction at the top of the column. This corresponds essentially to a situation in which the bracing is in the form of a broad shear-resistant panel in the plane of buckling, the panel being attached to the frame only at its upper and lower edges. The equivalent arrangement is indicated in Fig. 1. The critical load of a single column in the lurching mode may be determined

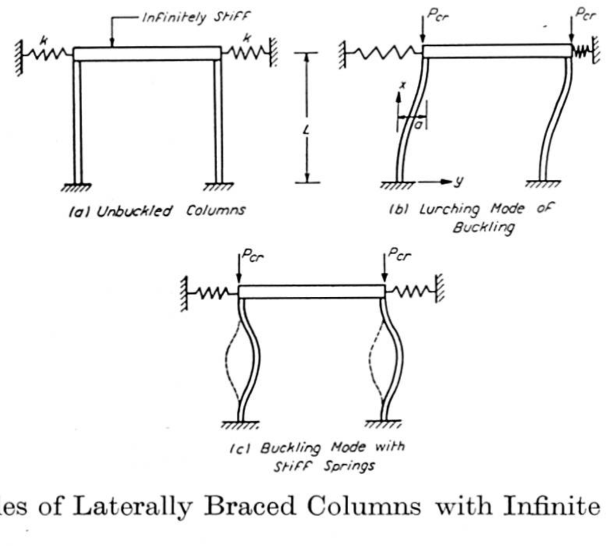

Fig. 1. Buckling Modes of Laterally Braced Columns with Infinite Rotational Restraint.

by the energy method. We take the deflections of the column as

$$y = \frac{a}{2}\left(1 - \cos\frac{\pi x}{L}\right),$$

where a is the arbitrary amplitude. Setting the bending strain energy of the column plus the extensional strain energy of the spring equal to the work done by the critical load during the buckling process leads to the stability criterion. This may be written in the form

$$\frac{P_{cr}}{P_e} = 1 + \frac{8\,k\,L}{\pi^2 P_e}, \tag{1}$$

where P_{cr} = critical load
P_e = Euler load $= \pi^2 E I / L^2$
k = spring rate of bracing

and E is the appropriate modulus.

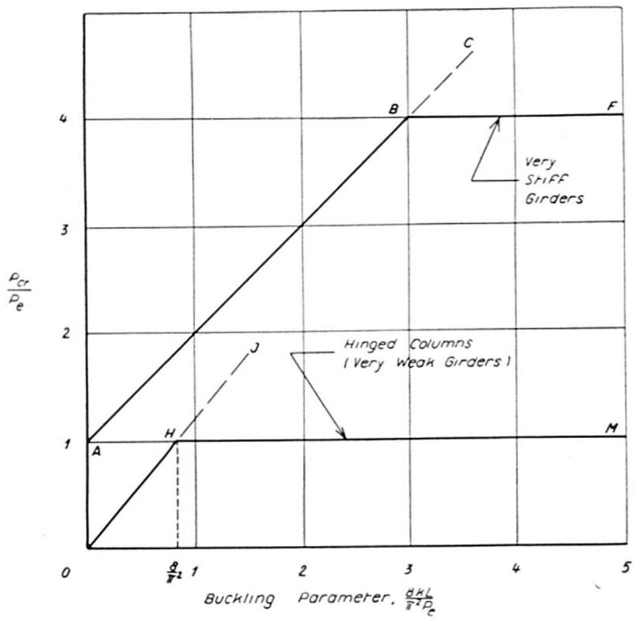

Fig. 2. Relation of Critical Load to Buckling Parameter for Limiting Cases.

Eq. (1) is plotted in Fig. 2 as the line ABC. It would appear from Eq. (1) that the critical load will increase without limit as the stiffness, k, of the lateral bracing is increased. This, however, does not follow since, under any circumstances if the column is not braced at intermediate points, the critical load cannot exceed the magnitude corresponding to the buckling mode shown in Fig. 1c. The criterion for the latter mode is shown in Fig. 2 as the line BF. It is thus seen that, in the case of very stiff girders, the column will buckle in the lurching mode when the buckling parameter has a value less than three, and will buckle without lateral displacement of its ends when the value of the buckling parameter exceeds three. That is to say, the column which is restrained by very stiff girders will not buckle in the lurching mode if

$$k > \frac{3\,\pi^2}{8\,L} P_e. \tag{2}$$

The complete criterion for the case of very stiff girders is represented in Fig. 2 as the curve ABF.

The foregoing analysis was developed for an individual column. However, certain conclusions can be drawn for the case of several columns in a given story and for complete frames. Clearly, for the collection of columns in any one story of a frame with very stiff girders laterally supported by ideal shear panels which are connected at the top and bottom of the story, the required stiffness of the lateral supports is the sum of the stiffnesses required for each of the columns, provided that each element of lateral stiffness is directly available to each column. In particular, a lurching or translational mode cannot develop in that story if

$$\sum k > \frac{3\,\pi^2}{8\,L} \sum P_e \qquad \text{(very stiff girders)} \tag{3}$$

provided that the condition on availability is satisfied. One may infer further that if the lateral support is an *ideal* (but not necessarily uniform) shear beam for the entire height of the building frame and attached only at the top and bottom of each story, the lurching mode of buckling will not develop if the total stiffness at each story satisfies Inequality (3).

In the foregoing analysis, we have considered the limiting case of a frame with very stiff girders. For comparison, we may consider the lower limiting case of a frame having girders with negligible flexural stiffness. For simplicity, we shall assume that each story segment of the multi-story column under consideration is precisely in the same state relative to the possibility of buckling; that is, the relation between axial load on each story segment and its critical load is such that no story segment either tends to support or to be supported by its neighbouring segments. This is equivalent to assuming that each story segment may be treated as hinged at each end.

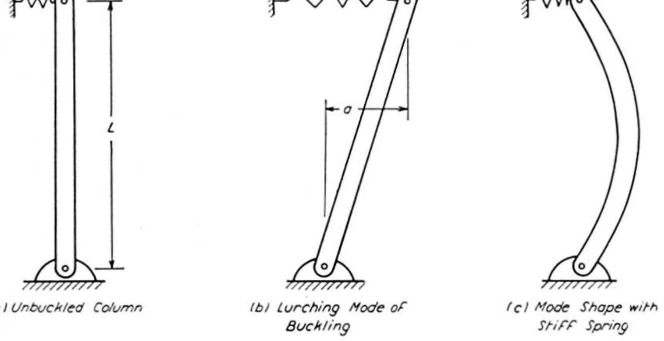

Fig. 3. Buckling Modes of a Laterally Supported Column with Negligible Rotational Restraint.

In the lurching mode of buckling shown in Fig. 3b, no bending occurs and it can be shown that the stability criterion for this mode is

$$P_{cr} = k\,L$$

or

$$\frac{P_{cr}}{P_e} = \frac{k\,L}{P_e}. \tag{4}$$

Eq. (4) is plotted in Fig. 2 as the line OHJ. As in the case of extremely stiff girders, this equation implies that the critical load increases with increasing stiffness of the lateral support. However, when the critical load reaches or exceeds the magnitude of the Euler load computed with the appropriate modulus, the column will buckle in the mode shown in Fig. 3c. Thus, the line $P_{cr}/P_e = 1$ is an upper limit to the buckling strength of the column and is shown as AHM in Fig. 2. Hence, the complete criterion for the case of very flexible girders is represented in that figure as the curve OHM.

As in the previous case, the stiffness which is required in the lateral supports for the collection of columns in any one story of a frame is the sum of the stiffnesses required for each column. In particular, a lurching mode cannot develop in that story if the spring rates of the lateral supports are such that

$$\sum k > \frac{1}{L} \sum P_e \qquad \text{(hinged columns)} \tag{5}$$

provided again that each element of lateral stiffness is directly available to each column.

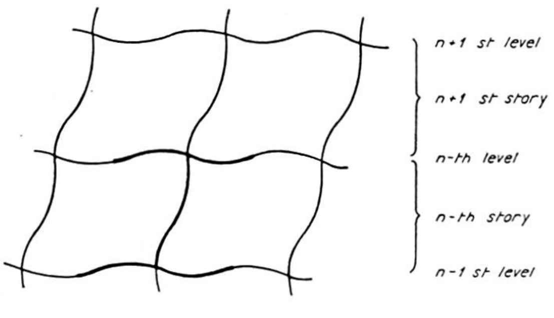

Fig. 4. Portion of Frame in Lurching Mode.

If the premises or assumptions upon which the foregoing analyses were based valid, the critical loads for each column segment in an actual frame would fall in the region MHOABF of Fig. 2. However, while the curve ABF is a reliable upper bound on the critical loads, it cannot be said that the curve OHM is an equally reliable lower bound. The assumption of very stiff girders in the first case prevented propagation of buckling deformation from story to story without violating continuity. An analogous uncoupling was assumed in the second case so that the column segments could again be treated individually. Instead of merely assuming that the girders are infinitely weak in flexure, the second case corresponds to assuming that the column segments are hinged at the joints. In real structures, however, the columns are continuous and the segments cannot be treated on this individual basis. Curve OHM is not an entirely dependable lower bound on the critical loads.

In the case of frames with infinitely stiff girders, we were able to establish a value for the stiffness of the lateral bracing at which bifurcation of the buckling modes is possible. When, in such frames, the stiffness of the bracing is less than this value, the frame will buckle in the lurching mode; and when the stiffness of the bracing exceeds this value, the stories will tend to buckle in a "symmetrical" mode not involving translation. Although the development postulated infinitely stiff girders, this case is of more than academic interest since the results which have been obtained might serve as a basis for approximate design in cases where the girders are relatively, but somewhat less than infinitely, stiff. Furthermore, it is clear that the critical or bifurcation value of the shear stiffness, $k = 3\pi^2 P_e/8L$, for the case of infinitely stiff girders is also an upper bound to the critical value of the *stiffness* for a case in which the girders are less than infinitely stiff. Thus, when the stiffness of the shear bracing exceeds the stated value, $3\pi^2 P_e/8L$, at each story but the girders are less than infinitely rigid, the frame will tend to buckle in a "symmetrical" mode and the lurching mode generally will not have to be considered.

One additional point is in order and may be discussed at this time. The *line* OA includes all frames, broadly speaking, for which no lateral bracing is provided. To neglect any appreciable lateral bracing which actually may exist is to restrict the design to the *line* OA when, in fact, the design may lie anywhere in the area MHOABF. In such cases, if general instability is a consideration, the design may be seriously penalized as a result of neglecting the lateral bracing.

General Method of Analysis

For more accurate determination of the critical loads in cases of moderate stiffness of the girders, a more comprehensive approach must be employed. The generalized slope deflection theory may be taken as the basis for this approach. We consider a single column, continuous through the entire height of the frame and rigidly connected to the intersecting girders at each story. We may think of this column as one of the two columns of a symmetrical plane frame, and the results will be as exact as one wishes. The results will be equally exact if the column is one of a set of identical, identically loaded and identically restrained columns of a multi-bay frame; and thus, without further generalization or refinement, this approach may be used in determining approximately the critical loads of such a frame.

The displacements and loads acting upon a column segment are shown in Fig. 5. The bending moments at the top and bottom of this column segment are

$$\begin{aligned} M^C_{n,n-1} &= -K^C_n\left[A_n\,\theta_n + B_n\,\theta_{n-1} - (A_n+B_n)\frac{y_n}{L_n}\right], \\ M^C_{n-1,n} &= -K^C_n\left[A_n\,\theta_{n-1} + B_n\,\theta_n - (A_n+B_n)\frac{y_n}{L_n}\right], \end{aligned} \tag{6}$$

where
$$\begin{aligned} K^C_n &= E\,I/L_n, \\ A_n &= \frac{\sin p\,L - p\,L\cos p\,L}{\frac{2}{p\,L}(1-\cos p\,L) - \sin p\,L}, \\ B_n &= \frac{p\,L - \sin p\,L}{\frac{2}{p\,L}(1-\cos p\,L) - \sin p\,L}, \\ p\,L &= \pi\sqrt{\rho}, \\ \rho &= \frac{P}{\pi^2\,E\,I/L^2_n}. \end{aligned}$$

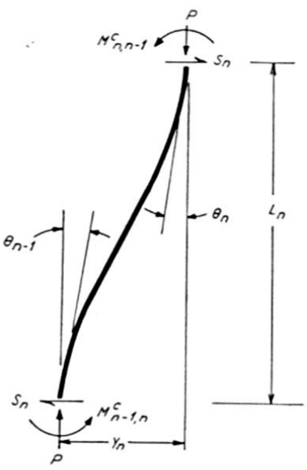

Fig. 5. Column Buckled in Lurching Mode.

Combining pertinent expressions for column moments with expressions for girder moments into a rotational equilibrium equation for a typical joint yields

$$\begin{aligned} K^C_n B_n\,\theta_{n-1} + [K^C_n A_n + K^C_{n+1} A_{n+1} + 6\,(1+\delta)\,K^G_n]\,\theta_n + K^C_{n+1}\,B_{n+1}\,\theta_{n+1} \\ - K^C_n(A_n+B_n)\frac{y_n}{L_n} - K^C_{n+1}(A_{n+1}+B_{n+1})\frac{y_{n+1}}{L_{n+1}} = 0, \end{aligned} \tag{7}$$

where K^G_n is the stiffness of the girder at the top of the n-th story column, the $n+1$st story is above this girder and

$$\delta = \begin{cases} 0 \text{ when number of bays} = 1 \\ 1 \text{ when number of bays is large.} \end{cases}$$

Taking moments about one end of the column shown in Fig. 5 leads to an expression for the transverse shear

$$S_n = \frac{1}{L_n}K^C_n(A_n+B_n)\left(2\frac{y_n}{L_n} - \theta_n - \theta_{n-1}\right) - P_n\frac{y_n}{L_n}. \tag{8}$$

The shearing force in the lateral bracing system may be taken as

$$H_n = k_n y_n \tag{9}$$

where k_n is the spring rate of the shear panel at the n-th story.

Since the total shear at each story must be zero, or

$$S_n + H_n = 0,$$

the substitution of Eqs. (8) and (9) yields

$$K_n^C (A_n + B_n)(\theta_n + \theta_{n-1}) - [2 K_n^C (A_n + B_n) - P_n L_n + k_n L_n^2] \frac{y_n}{L_n} = 0. \tag{10}$$

Eliminating the y's in Eq. (7) by means of Eq. (10) yields the recursion formula

$$\begin{aligned} &K_n^C (B_n - C_n F_n) \theta_{n-1} + [K_n^C A_n + K_{n+1}^C A_{n+1} - K_n^C C_n F_n \\ &\quad - K_{n+1}^C C_{n+1} F_{n+1} + 6(1+\delta) K_n^G] \theta_n + K_{n+1}^C (B_{n+1} - C_{n+1} F_{n+1}) \theta_{n+1} = 0, \end{aligned} \tag{11}$$

where
$$C_n = A_n + B_n,$$
$$D_n = k_n L_n^2 / K_n^C C_n = \pi^2 k_n L_n / P_e C_n,$$
$$F_n = \frac{1}{2 - \frac{\pi^2 \rho_n}{C_n} + D_n}.$$

It may be noted that the dimensionless term D_n should be computed with the true value of $K_n^C = E I / L_n$, but all other K's may be taken as *relative* values.

Eqs. (7) and (10) or Eq. (11), written for each story, are a homogeneous set of linear algebraic equations in the lateral and angular displacements of the joints. These equations, together with the boundary conditions at the base form an eigenvalue problem in which the appropriate multiple of a prescribed pattern of column loads may be treated as the eigenvalue to be determined. There will be a number of such multiples or eigenvalues which satisfy the equilibrium equations and boundary conditions. However, only the lowest of the non-zero eigenvalues is of engineering interest.

In the case of low shear stiffness of the bracing, the frame will buckle in a lurching mode. When sufficient shear bracing is provided, the critical loads for the lurching mode may be greater than those for a "symmetrical" mode which does not involve translation of the joints. This mode involves a different bending configuration of the girders and leads to the single recursion formula

$$K_n^C B_n \theta_{n-1} + [K_n^C A_n + K_{n+1}^C A_{n+1} + 2(1+\delta) K_n^G] \theta_n + K_{n+1}^C B_{n+1} \theta_{n+1} = 0, \tag{12}$$

when the symmetrical mode is being investigated. Eq. (12) is to be written for every story. The lowest eigenvalue for this set defines the critical loads for the symmetrical mode, and comparison with the results for the lurching mode will show whether the frame will buckle with or without lurching.

Remarks on Method of Solution

Because of the highly transcendental manner in which the loads enter into the equations, ordinarily it is not feasible to extract the eigenvalues directly from the sets of equations. In rare cases, the coefficients of the displacements may have the regular character which would permit solution by difference equation methods. In other cases, a small adjustment of these coefficients may put these equations in regular form and thus permit at least an approximate solution by difference equation methods.

In the usual case, the most practical method for either desk or electronic computer will be a trial-and-error procedure in which the magnitudes of the loads are assumed. The corresponding values of A_n and B_n are then substituted into Eqs. (7) and (10) or (11) or into Eq. (12) and it is determined whether or not these equations and the base condition can be satisfied.

The solution of the equations for trial values of the loads can be obtained by any of several techniques. For example, we observe that Eq. (11) for the top level contains two unknowns, the rotation at that level and at the next lower level. We may solve this equation for the rotation at the top level in terms of the rotation at the next lower level. We use this result to eliminate the topmost rotation from the next lower equation and solve this equation for the second rotation in terms of the third from the top. We proceed in this manner, eliminating unknowns in the successive equations down to θ_2 in terms of θ_1. The rotation, θ_0, at the base is defined by a stated boundary condition and we may therefore dispose of θ_0 as an unknown in the equation for the first level above the base. With the sequential substitution and elimination of rotations, Eq. (11) for the first level above the base becomes homogeneous in θ_1. Now, the left-hand side of Eq. (11) is in fact equal to the external moment required to maintain equilibrium at the joint. Therefore, in view of the homogeneous form, if the coefficient of θ_1 vanishes, the loads form an eigenvalue set; if the coefficient is positive the frame is stable in the configuration which has been developed; if the coefficient is negative, the frame is unstable in this configuration.

Other procedures are, of course, available for effecting a solution of the set of equations. However, space does not permit a more general discussion at this time.

It may be remarked that, if an electronic computer is available, it becomes feasible to handle the exact problem of a multi-story frame having several spans. To formulate the larger problem, Eqs. (7), (10), (11) and (12) are generalized in a straightforward manner to include different rotations at each joint of each level. The resulting equations can be solved, with the aid of the computer, by relatively simple partitioning of the set and an external moment can again be computed as a criterion of stability or instability for a trial set of loads.

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Summary

The effect of shear bracing upon the critical loads of a multi-story building frame is discussed and formulas are presented for the critical loads of column segments in the two limiting cases of infinite girder bending stiffness and negligible girder stiffness. Equations for a more comprehensive theory founded upon the slope-deflection method are presented and comments are made upon methods of solution.

Résumé

L'auteur décrit tout d'abord l'influence des voiles de contreventement sur les charges critiques d'un portique étagé multiple. Pour les deux cas limites — rigidité de la traverse infinie et pratiquement négligeable —, l'auteur indique les formules permettant de déterminer les charges critiques des éléments de montants. De plus, il présente des équations découlant d'une théorie plus complète, basée sur la méthode des déformations et il commente quelques procédés de résolution de ces équations.

Zusammenfassung

Zunächst wird der Einfluß von schubfesten Tafeln auf die kritischen Lasten eines mehrstöckigen Rahmens besprochen. Für die beiden Grenzfälle des Trägers mit unendlich großer und mit vernachlässigbarer Steifigkeit werden die Formeln für die kritischen Lasten von Stützenabschnitten angegeben. Ebenso wurden Gleichungen für eine umfassendere Theorie, basierend auf der Deformationsmethode, dargestellt und dazu einige Lösungsmethoden besprochen.