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Die Berechnung eines Tragwerks kann auch etwa durch Einführung eines leichter zu überblickenden Ersatztragwerks erleichtert werden. Ferner sei festgehalten, daß in Sonderfällen auch der Modellversuch die eigentliche Berechnung ersetzen oder ergänzen kann.

3. Bei der Ausführung unserer Tragwerke hat der Konstrukteur immer daran zu denken, daß normalerweise die Bauten, die wir heute herstellen, auch späteren Generationen noch ihre Dienste leisten müssen und deshalb einem langdauernden Betrieb unter veränderlichen Bedingungen unterworfen sein werden. Je besser wir diese wirklichen Arbeitsbedingungen in bezug auf statische und dynamische Belastungen, auf räumliche Zusammenhänge und räumliche Kraftwirkungen, aber auch in bezug auf das Langzeitverhalten der Baustoffe beim Entwurf und bei der Berechnung der Tragwerke berücksichtigen, um so zuverlässiger erfüllen wir unsere Aufgabe als konstruierende Ingenieure.

General Report

Theme Ia. The properties of Materials

When the subject of discussion at a Congress of the International Association for Bridge and Structural Engineering is the determinant mechanical properties of materials, it seems advisable to state, first of all, the point of view we have adopted and hence the purpose of the discussion.

In this connection, I would venture to quote the remarks made by Professor WEIBULL[1]), of Stockholm, which may also serve as a directive for us:

"There are two quite different lines of attacking fatigue problems: the phenomenological and the metallographical. The first one is the line of the designer who wants to know what happens; the second one, that of the metallographer, who wants to know why it happens. As a link between the two, you will find the third man, the tester of materials.

All these categories have different opinions as to the way of designing the experiments and of conducting the testing. The metallographer, for instance, may think — I have met this statement quite recently — that the basic physical facts of fatigue should be clarified before the organisation of any large scale testing. The designer, on the other hand, wants urgently many more facts about fundamental questions than are available today, and it is definitely impossible for him to wait for the many years that it will certainly

[1]) W. WEIBULL: The Statistical Aspect of Fatigue Failures and its Consequences. Fatigue and fracture of metals, edited by W. M. Murray, Mass. Inst. Techn., 1952.

by a double logarithm, we obtain the linear relationship

$$\log\log\frac{\sigma_{0Z}-\sigma_{aW}}{\sigma_W-\sigma_{aW}} = r\log i+\mu_0.$$

Let us compare these two expressions by considering, for example, the results of a series of tests carried out on test-pieces, in which a hole was bored (diameter of hole 4 mm, total width 30 mm) made of 44 steel manufactured by the Société de Roll (Fig. 1) *). As is evident from Fig. 2 *), the two expressions show good agreement in the range studied from $n=30\cdot 10^3$ to more than $n=10\cdot 10^6$; the unavoidable scatter, which is particularly high in the case of small stresses, made it impossible to determine which of the expressions gave the best reproduction of the trend of the Wöhler curve. If I give preference to expression 1, it is because it can be accurately applied to other phenomena of long duration and is simpler in its form and in its application than expression 2.

It is not only by conducting and developing further series comprising a large number of tests, it is also, in all probability, by means of a "two-stage test", understood in the sense of the cumulation of partial damage in each stage, that it will be found possible to decide which is the true law of fatigue. It is known that the Palmgren-Miner theorem does not hold good for the two stage test; this test comprises, in a first loading stage σ_1 (rupture after n_1 cycles), Δn_1 cycles followed, in the second stage σ_2 (n_2), by Δn_2 cycles to failure. When the stresses are of the same kind in both stages (for example, alternating stresses or a constant mean stress σ_m), we obtain, by putting

$$\xi_1=\frac{\Delta n_1}{n_1}, \qquad \xi_2=\frac{\Delta n_2}{n_2},$$

for $\sigma_1<\sigma_2, \quad n_1>n_2:\ \xi_1+\xi_2>1$

and for $\sigma_1>\sigma_2, \quad n_1<n_2:\ \xi_1+\xi_2<1$

whereas the Palmgren-Miner theorem indicates that $\xi_1+\xi_2=1$. Consequently, the damage curve $S-\xi$ cannot be a straight line [2]).

We can form an idea of the trend of this curve by means of the following considerations; let us suppose that the damage e of the material increases in proportion to the number of cycles [3]),

$$e=\alpha\,\xi.$$

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*) See Figures in German text.

[2]) N. M. NEWMARK: A Review of Cumulative Damage in Fatigue. Fatigue and fracture of metals, edited by W. M. Murray, Mass. Inst. Techn., 1952.

[3]) This hypotheses seems to be justified since, under constant stress, the cracks are propagated in proportion to the number of cycles; see W. WEIBULL: Size effects on fatigue crack initiation and propagation in aluminium sheet specimens subjected to stresses of nearly constant amplitude. F.F.A., Flygtekniska Försöksanstalten, Meddelande 86, Stockholm 1960.

take to obtain well-founded theories which will be of any use to him. There is no need, I think, to point out that both of the ways have to be trod and that progress in one field may have quite important influences on the proceedings in the other."

Such are the reflections of Mr. WEIBULL and I should like to draw the attention to the great value of his work concerning the critical consideration and the interpretation of the results of fatigue tests.

Our point of view is thus clearly determined; we are obliged to ascertain the behaviour of materials because it governs the dimensioning of structures and we are compelled to acquire this basic knowledge by proceeding in a phenomenological manner as long as the underlying physical causes of the fundamental phenomena remain concealed from us. We would like to point out that co-operation between the physicist and the expert in testing materials will, of course, always be welcome.

In order to illustrate these reflections, let us consider the relationship which connects the resistance of a metal to alternating stresses σ_W with the number of cycles n which causes rupture of the test-piece or of the part. This curve, known as Wöhler's curve, describing the resistance to alternating stresses, may, in principle, be expressed in various ways, as I have already pointed out in the "Preliminary Publication". I suggested a relationship having the form

$$\sigma_W = \frac{\sigma_{0Z} + f_W \sigma_{aW}}{1 + f_W}. \tag{1}$$

The resistance sought appears in this expression as the weighted average of two limiting values: the static breaking strength σ_{0Z} and the asymptotic fatigue limit σ_{aW}.

The "weight" is given by the endurance function $f_W(n)$

$$f_W = \frac{\sigma_{0Z} - \sigma_W}{\sigma_W - \sigma_{aW}} \tag{1a}$$

the logarithm of which varies linearly with the value $i = \log n$; and we therefore have

$$\lambda = \log f_W = \log \frac{\sigma_{0Z} - \sigma_W}{\sigma_W - \sigma_{aW}} = p\,i + \lambda_0$$

and

$$f_W = a^\lambda = f_{0W}\, n^p. \tag{1b}$$

On the basis of a suggestion made by Mr. EPREMIAN and using the Gaussian curve of errors, Mr. WEIBULL[1]) derived another expression which may be written in the form

$$\sigma_W = \sigma_{aW} + (\sigma_{0Z} - \sigma_{aW})\, e^{-m_0 i^r}; \tag{2}$$

Since it concerns an increasingly damaged test-piece, for which we assume that the capacity of resistance w decreases linearly,

$$w = 1 - \beta\,\xi,$$

the relative damage may be written

$$S = \frac{e}{w} = \frac{\alpha\,\xi}{1-\beta\,\xi}. \tag{3}$$

Failure occurs for $\xi = 1$ and we have

$$S = 1 = \frac{\alpha}{1-\beta}$$

and hence

$$\alpha + \beta = 1.$$

Let us divide the numerator and the denominator in Eq. (3) by α and put

$$a = \frac{1}{\alpha}, \qquad b = \frac{\beta}{\alpha}.$$

We can then write

$$S = \frac{\xi}{a - b\,\xi}$$

or since $a = 1 + b$,

$$S = \frac{\xi}{1 + b\,(1-\xi)}. \tag{3a}$$

In actual fact, the two-stage test does not enable us to determine the damage function b directly; we can only obtain relative damage curves, with reference to an arbitrarily assumed curve for the first loading stage σ_1. However, the results of a few preliminary tests with alternating stresses are in good agreement with the calculated values if we take as the damage function b, the fatigue function f_W expressed by Eq. (1a), namely

$$b = f_W = \frac{\sigma_{0Z} - \sigma_W}{\sigma_W - \sigma_{aW}} \tag{4}$$

or

$$S_W = \frac{\xi}{1 + f_W\,(1-\xi)}. \tag{4a}$$

Let us apply, in a two-stage test, the loading stage σ_1 comprising $\Delta\,n_1 = \xi_1\,n_1$ cycles followed by the stage σ_2; we shall have at our disposal in this last-mentioned stage $\Delta\,n_2 = \xi_2\,n_1$ cycles corresponding to the field of damage extending from $S = S_1$ to $S = 1$ and hence

$$\xi_2 = \frac{1-\xi_1}{1+\varphi\,\xi_1}, \tag{5}$$

with (Fig. 3) *)

$$\varphi = \frac{f_2 - f_1}{1 + f_1} \tag{5a}$$

In Fig. 4 *), the results of two series of tests are compared with these

*) See Figures in German text.

calculated values; in the experiments conducted by Mr. WILKINS[4]) we have $\sigma_1 < \sigma_2$, $f_1 > f_2$, $\xi_1 + \xi_2 > 1$, whereas in my own tests, carried out on de Roll steel test-pieces with a hole drilled in them, we have the opposite case, namely $\xi_1 + \xi_2 < 1$. We should point out explicitly that the relationship $b = f_W$ (Eq. (4)) only holds good for alternating stresses ($\sigma_{max} = -\sigma_{min}$, $\sigma_m = 0$) and that for other types of stress, with $\sigma_m \neq 0$, the correlation between b and f_W would have to be determined by means of similar tests.

Tests on specimens should reveal the basic relationships which govern the behaviour of materials subjected to stresses of long duration. It would be desirable that these laboratory researches should be complemented by large-scale tests on structural members or on complete structures; this would make it possible to control the application, to the dimensioning of structures, of the results obtained in the laboratory and ensure their accuracy. It is in this connection that the importance of the paper by Prof. MICHALOS and his collaborators is to be found.

Mr. FREUDENTHAL supplemented his paper in the "Preliminary Publication" and replied to the comments I had made in my general report; it will be observed that our respective points of view have not yet been reconciled. The problem raised by Mr. FREUDENTHAL is fundamental and it remains very much a matter of current interest; I feel sure that a solution will be found, which will also be satisfactory for the civil engineer, even though it may require a lengthy and exacting experimental investigation.

When tests are undertaken, the difficulty lies in the unavoidable scatter of the results. It is only by means of series comprising a large number of tests that it becomes possible to obtain thoroughly reliable average values and to determine the magnitude of the probable deviations; this often require more time than is available for the particular case in question. It is consequently of considerable interest for mathematical statistics to show us how to make valid use of limited series of tests. The paper by Mr. DAVIN deals with certain basic aspects of this problem.

Theme Ib. Development of Methods of Calculation

In order to establish numerical methods of calculation for the purpose of structural design, it is possible, in principle, to adopt one or other of two procedures, both of which are illustrated by contributions to the discussion.

In the first place, we may transform — by means of relationships making it possible to integrate numerically (calculation of a surface area by means of Simpson's rule, equation of the funicular polygon, introduction of develop-

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[4]) E. W. C. WILKINS: Cumulative damage in fatigue. Colloquium on fatigue, Stockholm 1955, Berlin-Göttingen-Heidelberg 1956.

ments in series) or to differentiate — differential equations into systems of equations written at a finite number of points in the field of integration. It is a process of successive integration (calculation of a surface area by means of Simpson's rule) that is employed by Mr. GILG; he starts from the derived function of the highest order occurring in the differential equation in order to evaluate the derived functions of a lower order. It should be noted, however, that the equations may become unmanageable when there are a large number of intervals.

MM. NASH and HO employed a development in series in order to establish a system of equations; they studied a fixed circular plate on an elastic foundation.

The efficacy of numerical procedures of this kind should be determined in accordance with the following criterion: the obtaining of the greatest possible accuracy with the least possible number of equations.

A second method of procedure consists in introducing, when studying a structure, a "system of replacement" which is easy to examine. It is in this manner that, in elementary statics, the graphical calculation of a simple beam rests on the analogy between the bending moment of the beam and the equilibrium figure of a loaded wire. MOHR extended this analogy by applying to the study of an elastic line. In his paper (to be published in "Publications"), Mr. ASPLUND describes another analogy, namely that which exists between the calculation of thin shells and the calculation of spatial lattices.

Mr. KOLOUŠEK, whose studies on vibrations are well known, considers the vibrations in continuous structures, formed of successive or identical elements. For this purpose he employs another process of numerical investigation, namely that of successive approximations.

The application of electronic computers to the study of civil engineering structures, an application illustrated by the paper given by Mr. MEHMEL and by that of Mr. MASSONNET and his co-workers, only increases the importance of efficient and accurate methods of calculation; an additional requirement must be satisfied; the programming must be easy.

In certain cases, tests on a model may be used to replace or supplement calculation; the paper by Mr. BERIO showed an application of this procedure, which is mainly suitable for complicated structures that are not amenable to a method of calculation which is both simple and sufficiently accurate.

Mr. OLSZAK and Mr. DRUCKER (for the latter, see volume 21 of "Publications"), in their papers, considered limiting states. These methods of investigation, however attractive they may be from the theoretical point of view, are not yet generally accepted as a basis for structural design.

Mr. WIERZBICKI and MM. MASSONNET and MOENAERT supplemented their papers published in the "Preliminary Publication".

Conclusions

1. The contributions to the first working session of the Congress clearly emphasised the tendency, already widespread at the present time and constantly increasing, towards basing structural design, not only on the results of laboratory tests of short duration,but also on the actual behaviour of the materials subjected to variable stresses, during prolonged periods of service. The phenomena in question are of long duration and relate mainly to problems of fatigue, of resistance to prolonged stresses, and of relaxation, shrinkage and creep.

In the present state of our knowledge, it is not yet possible to provide a final solution to these problems on the basis of a physical interpretation of the phenomena. Consequently, the civil engineer is obliged, for the moment, to proceed in a phenomenological manner, while at the same time hoping that co-operation between the physicists and those concerned with the testing of materials will continue.

The discussion during the Congress indicated, however, in principle, the course to be followed, as well as the general tendency and the interpretation of the tests to be carried out in the future; these researches, which are urgently necessary, will make it possible to verify existing laws, to correct them where necessary, and to extent them.

2. The development of methods of calculation will be mainly characterised by the extension of numerical procedures; structural design is, in fact, a problem of an essentially numerical character. The conditions of equilibrium and of deformation, which govern play of forces in structures and often lead to differential equations, can be expressed by means of systems of simple equations which can be solved numerically. A method of investigation is suitable when it enables the problem raised to be solved without much trouble, but in a reliable manner, that is to say, with sufficient accuracy. A rigorous analytical solution is only applicable provided the basic hypotheses are really satisfied. With the introduction of electronic computers, it is more necessary than ever to have reliable numerical methods.

The introduction of a system of replacement, which is easier to consider, may also facilitate structural design. In certain special cases, a test on a model may replace or complement the actual study.

3. The engineer responsible for the design and execution of a structure must never lose sight of the fact that structures built at the present time should normally last for several generations and will consequently be subjected, during a prolonged period of service, to variable conditions. We shall always fulfil our tasks as engineers and constructors more amply by taking into account, to an increasing extent, in the design and study of structures, these conditions of actual service, concerning both static and dynamic loads, spatial effects and the play of forces in space, as well as the behaviour of materials in course of time.