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## **IIb1**

### **A Research on the application of the theory of orthotropic plates to steel highway bridges**

### **Untersuchung über die Anwendung der Theorie der orthotropen Platte auf Stahlbrücken**

### **Estudo da aplicação da teoria das placas ortotrópicas às pontes de estrada metálicas**

### **Recherches sur l'application de la théorie des plaques orthotropiques aux ponts-route métalliques**

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#### **1. Introduction.**

In the design of steel highway bridges, the main girder, floor beam, stringer and slab are calculated individually as independent members in the conventional design method, but actually these members resist the load as co-operating members and operate together. Therefore, the stress calculated by the conventional method differs considerably from the stress measured from the loading test. On the other hand, the calculation by the theory of continuous plate or grillage beam contains the co-operation action between each beam and slab and has been discussed in various ways as an effective method. The theory of the orthotropic plate is one of these methods and is very effective. In this paper the results of the application of the theory of the orthotropic plate to existing bridges are compared with those obtained by stress measurement and conventional calculation.

#### **2. Strain measurement, discussion of results and notations.**

The strain was picked up by electric resistance wire strain gages and measured by the Baldwin SR-4 Strain Indicator and SR-4 Switching and Balancing Unit. In the calculation of the stress, Young's modulus

of steel and reinforced concrete were assumed as 2,100,000 kg/cm<sup>2</sup> and 210,000 kg/cm<sup>2</sup> respectively. The theoretical stress was calculated by the conventional method (briefly, method A), by the theory of orthotropic plate introduced by M. T. Huber (briefly, method B), by the theory of isotropic continuous plate supported by elastic beam (briefly, method C) and by the theory of grillage beam (briefly, method D).

The result of the measured value is given in the form of stress ratio and deflection ratio. These ratios are obtained by (measured value)/(calculated value).

Ratio  $\infty$  means that the calculated value corresponding to the measured value is zero.

In discussing the result of the measurement in the case of the stringer of the floor system of truss bridges and the result in the case of the main beam of composite beam bridges, the ratio of the load on each stringer or main beam to the total load (briefly, load ratio) was adopted besides the stress ratio. This ratio was calculated as follows:

$$\text{load ratio} = \frac{\text{stress of a stringer or main beam}}{\text{total of the stress of each stringer or main beam}}$$

In this calculation, the measured stresses and calculated stresses by methods A, B, and C were used as the stress in the denominator and numerator.

The following notations are used.

$EI_s$ : The flexural rigidity of the stringer whose effective width of the compression flange is  $\lambda_s$ .

$EI_f$ : The flexural rigidity of the floor beam only in the case of through plate girder bridges.

$EI_c$ : The flexural rigidity of the load distributing cross beam whose effective width of the compression flange is  $\lambda$  in the case of model composite grillage beam bridge.

$\lambda_s$ : The interval of the stringers.

$\lambda_f$ : The interval of the floor beams.

$\lambda$ : The interval of the main beams.

$D_x$ : The flexural rigidity of the orthotropic plate in the direction of x, that is, the bridge axis.

$D_y$ : The flexural rigidity of the orthotropic plate in the direction of y, that is, perpendicular to the bridge axis.

In the calculation by the theory of the orthotropic plate,  $H^2 = D_x \cdot D_y$  and  $\nu = 0$  were assumed.

### 3. Application to the through plate girder bridge.

The Ōtanigawa bridge is a through girder bridge as shown in Fig. 1. In the design of such a through girder bridge, the slab, stringer, floor beam and main girder have been calculated individually as independent members. One method of analysing the stringer is to assume the slab as a one direction continuous rectangular plate simply supported by two

opposite floor beams and elastically supported by stringers. Now, if this bridge is regarded as an orthotropic rectangular plate with two opposite edges simply supported at the abutments and the other two edges elastically supported by the main girders, a rational calculation will be carried out at once for all members.

In this case,  $EI_s/\lambda_s$  is adopted for  $D_x$ . Next, the question is how to determine  $D_y$ . Once more,  $EI_f/\lambda_f$  is adopted for  $D_y$ . As a matter of

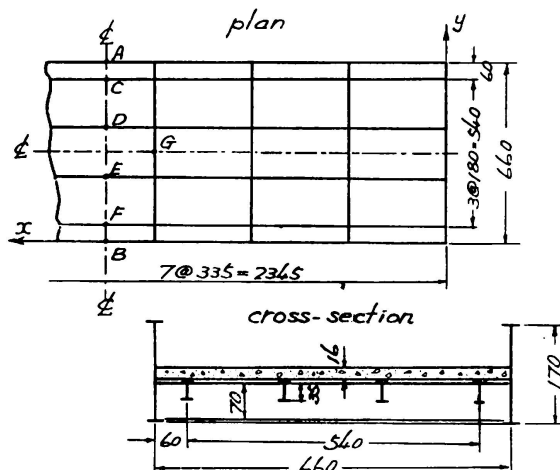


FIG. 1. Plan and cross section of the Otanigawa Bridge

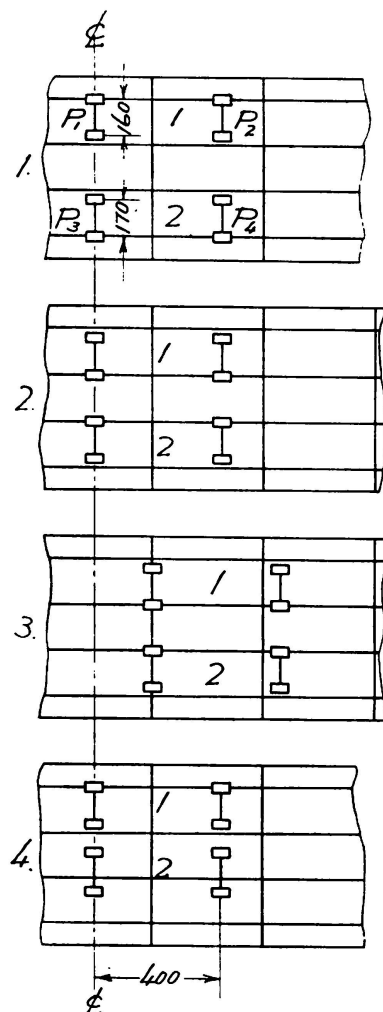


FIG. 2. State of loading on the floor system of the Otanigawa Bridge

1. Dump car  $P_1=6.36t$ ,  $P_2=2.12t$   
2. Truck  $P_3=5.84t$ ,  $P_4=1.40t$

fact, the change of value  $D_y$  produces no remarkable changes in the bending moment. Thus,  $D_x/D_y$  for the orthotropic plate is 0.71.

Both test loads were placed at different positions as shown in Fig. 2 and the stress ratio at sections A, B, C, E, F and G are shown in Table 1. The theory of the orthotropic plate gives better results than the conventional method, particularly so in the case of the stringer.

Furthermore, the following facts appear:

1) When there is a load on the floor beam or on the neighbouring stringer, the bending moment in the observed stringer is positive by method B, and is negative or zero by method A.



TABLE 1  
*Stress Ratio of Each Member of the Otanigawa Bridge (%)*

State of Loading	1		2		3		4	
	A	B	A	B	A	B	A	B
Method of calculation ... ..	A	B	A	B	A	B	A	B
Main Beam, section A... ..	52	62	60	71	54	58	60	71
Main Beam, section B... ..	58	68	60	71	50	54	51	64
Stringer, section C... ..	37	107	38	95	$\infty$	112	47	105
Stringer, section E... ..	60	103	63	104	—	—	52	107
Stringer, section F... ..	38	102	42	109	$\infty$	127	290	118
Cross Beam, section G... ..	40	52	46	65	27	64	48	71

2) By method A, the stress varies according to loading state 2 or 3 in Fig. 2, while it is almost constant by method B.

The measured values show that the theory of the orthotropic plate is more rational. From this theoretical and experimental analysis, it becomes clear that the theory of the orthotropic plate is available to the calculation of through plate girder highway bridges.

Furthermore, the method for the determination of  $D_y$  and the boundary condition about the main girder is a problem to be studied, and by an accurate research of this problem, better results will be obtained.

#### 4. Application to the floor system of truss bridges.

The design of the stringer is one of the important problems in the design of the floor system of a truss bridge. In the conventional calculation, the stringer being considered as a beam (simple or continuous) and the cooperative action between the flexible stringer and slab not taken into account accurately, the measured stress is considerably smaller than the conventionally calculated value. Furthermore, because the composite action between the stringer and the slab by the slab connector and the co-operation between the stringer and the members of the truss are disregarded, this tendency becomes more remarkable. In this paper, the result observed on the stringer of the floor system of the pony-truss of Ryogoku bridge is discussed and compared with the calculated result.

This bridge is shown in Fig. 3, and a 14.8 t truck is used as the test load. The loading line of the rear wheel is at the center of the span of the stringer and the states of loading are shown in Fig. 4. It is most ideal, but difficult, to analyse the slab, stringer and floor beams as

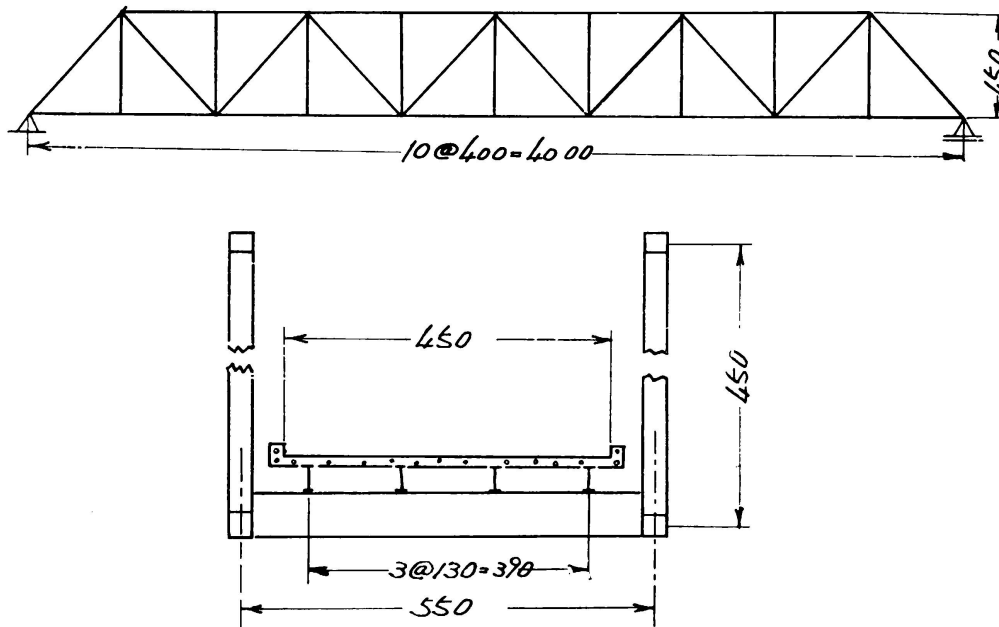


FIG. 3. Pony truss bridge, the floor system of which was under test

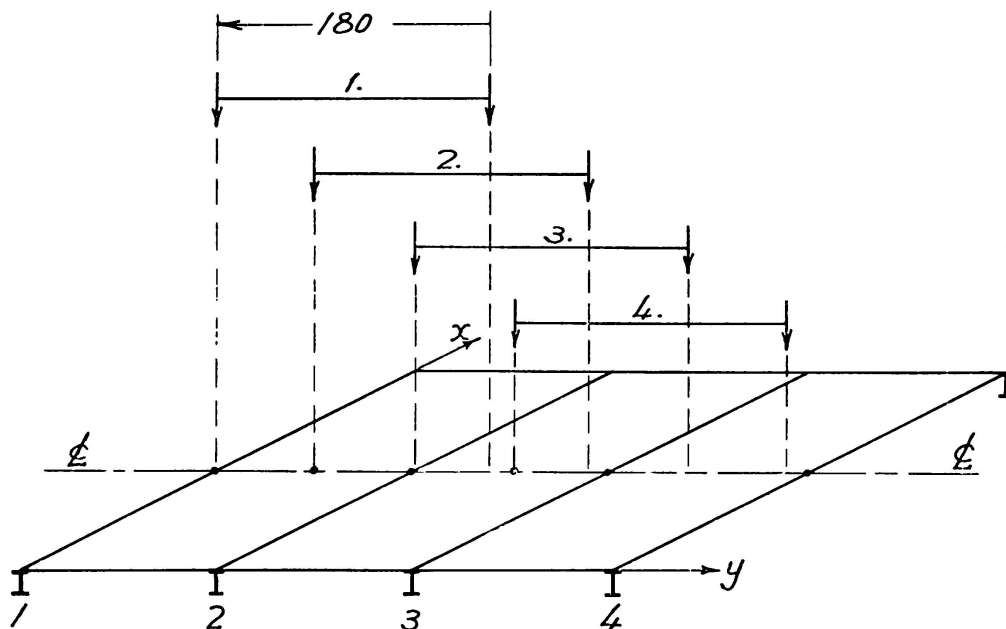


FIG. 4. State of loading of the stringer of the floor system

one body in the calculation of this floor system and the determination of the boundary condition is not as easy as in the above mentioned through plate girder bridge. Here, the theory of the orthotropic plate with two opposite edges simply supported by floor beams and the other two

opposite edges free or elastically supported by outside stringers is applied. As the bending moment of the stringer is almost equal whether the edges are free or elastically supported, the analysis for the case of the free edge is mentioned here.  $D_x$  is determined in the same way as in the case of Ōtanigawa bridge and the flexural rigidity of the slab is adopted as  $D_y$ , and  $D_x/D_y = 5.9$  is obtained.

TABLE 2

*Stress Ratio of the Stringer of the Pony-Truss of the Ryogoku Bridge (%)*

State of Loading	1		2		3		4	
Method of calculation ... ..	A	B	A	B	A	B	A	B
Stringer 1 at its midspan-section	23	64	31	65	∞	70	∞	75
Stringer 2 » » » »	34	55	31	55	22	59	33	66
Stringer 3 » » » »	42	73	23	56	36	60	33	57
Stringer 4 » » » »	∞	62	∞	78	31	58	23	66

TABLE 3

*Load Ratio of the Stringer of the Truss of the Ryogoku Bridge*

State of Loading	1				2				3				4			
Method of calculation ... ..	M	A	B	C	M	A	B	C	M	A	B	C	M	A	B	C
Stringer 1 ... ..	42	56	40	48	27	31	27	28	17	0	15	17	10	0	9	7
Stringer 2 ... ..	30	26	33	30	29	29	29	30	29	47	31	36	20	22	22	20
Stringer 3 ... ..	22	18	19	17	30	40	30	29	30	29	30	33	29	27	31	26
Stringer 4 ... ..	6	0	8	5	14	0	14	13	24	24	24	14	41	51	38	47
Total ... ..	100															

NOTE: Method of calculation M means that the measured stresses are used as the stresses in the denominator and numerator of the load ratio.

The stress ratio by methods A and B for the loading state shown in Fig. 4 is shown in Table 2. The load ratio of each stringer is shown in Table 3. In these cases the stress ratio obtained by method A is

extremely small, its mean value being about 30 %, but the stress ratio obtained by method B is about 60 %, and the ratio of 40 % remains unknown. One of the reasons is that the co-operation between the slab, stringer and lower chord is disregarded. Next, the load ratio calculated by method B agrees very well with the load ratio obtained from the measured stress and its results are better than the results of method C.

### 5. Application to composite beam bridges.

The application to the side span of Ryogoku bridge is described hereunder. This bridge is formed by three composite beams ( $l = 18.0$  m) as shown in Fig. 5. A 14.5 t truck is used as test load and the state of loading is shown in Fig. 6. The center of the span is loaded by the rear wheels.  $D_x$  and  $D_y$  are calculated as in the previous case and  $D_x/D_y$  was 130 in this composite beam bridge. The stress ratio and load ratio of each main beam are shown in Tables 4 and 5.

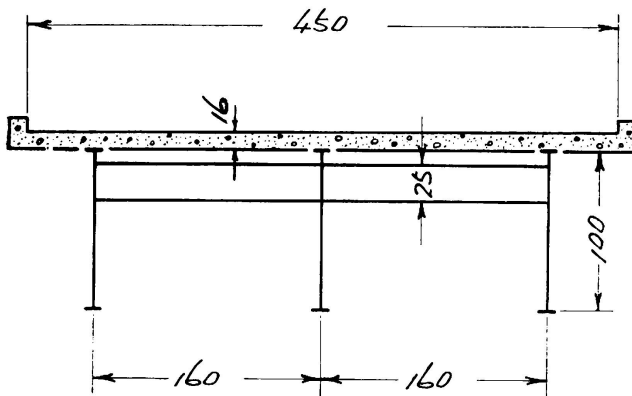


FIG. 5. Cross section of composite beam bridge

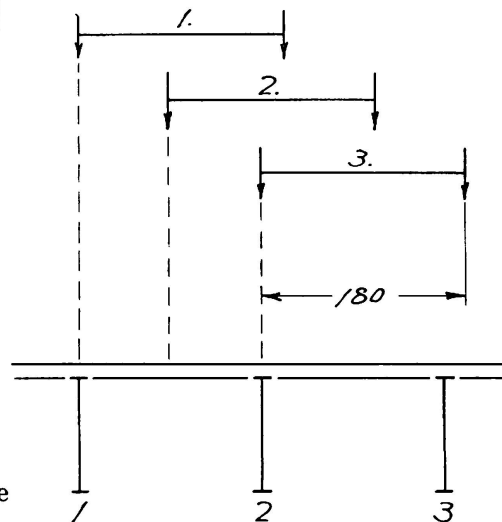


FIG. 6. State of loading on composite beam bridge at mid-span

In the most important case, when one wheel is placed on the side main girder, the stress ratio by method A is not uniform, and the results of method B are better. Next, the stress ratio of the span bending moment of the slab  $M_y$  under one of the rear wheels in the case of loading state 2 in Fig. 6 (rear wheel is at the center of the span of the slab), is 92 % by method B and 42 % by the theory of the continuous plate which is provided by the Japanese Specification of Steel Highway Bridges.

### 6. Application to model composite grillage beam bridge.

The application of the grillage beam theory to bridges has been studied recently, but this solution becomes very complicated when the

TABLE 4

*Stress Ratio of the Composite Beam of the Ryogoku Bridge (%)*

State of Loading	1		2		3	
Method of Calculation... ..	A	B	A	B	A	B
Beam 1 at its midspan-section ...	64	81	69	53	∞	73
Beam 2 » » » »	53	65	46	57	50	68
Beam 3 » » » »	340	72	69	62	69	85

TABLE 5

*Load Ratio of the Composite Beam of the Ryogoku Bridge (%)*

State of Loading	1			2			3		
Method of calculation ... ..	M	A	B	M	A	B	M	A	B
Main Beam 1 ... ..	42	50	38	29	25	31	24	0	26
Main Beam 2 ... ..	31	44	35	35	44	35	32	50	35
Main Beam 3 ... ..	27	6	27	36	31	34	44	50	39
Total ... ..	100								

NOTE: The meaning of M is the same as in the case of Table 3.

grid and slab are composed and the effect of the continuity of the slab or of the torsion of the girder is contained in the theory of the grillage beam or when the number of main girders and load distributing cross beams increases greatly. On the contrary, by the theory of the orthotropic plate all cases are dealt with as plates. A model of the grillage beam bridge as shown in Fig. 7 was tested.  $D_x$  is determined as before, but for this model in which there is only one cross beam, the determination of  $D_y$  in the direction of the cross beam is not easy. In this case,  $D_y = EI_f/\lambda$  was adopted. Thus  $D_x/D_y = 2.15$  was obtained. The deflection ratio and stress ratio at mid-span and quarter-span points of each beam when loads of 4.9 t are placed simultaneously at the center of main girders B and D, are shown in Table 6 which also contains the values obtained by method D. In short, the value obtained by method B agrees with the result of method D, and even in the case of a single cross beam, a satisfactory result is obtained by the application of the orthotropic plate.

Furthermore, the more accurate the determination of  $D_y$ ,  $H$ ,  $\nu$ , etc., the better the result obtained will be.

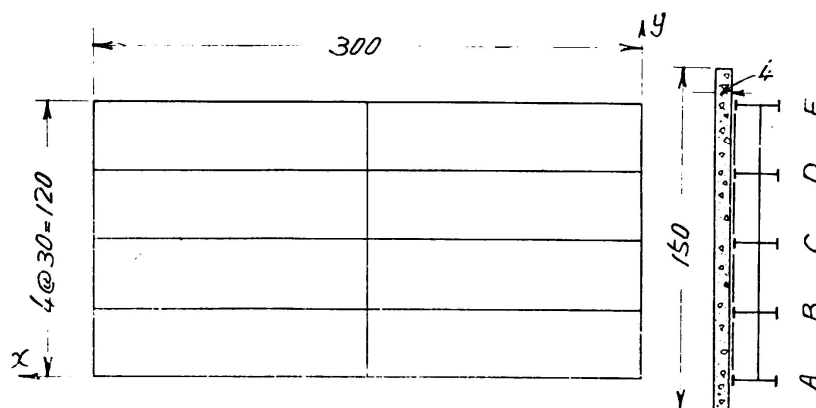


FIG. 7. Model composite grillage beam bridge

TABLE 6

*Stress Ratio and Deflection Ratio of Model Composite Grillage Beam Bridge (%)*

Measured Section ... ..	Stress Ratio				Deflection Ratio			
	$\frac{1}{2}$		$\frac{1}{4}$		$\frac{1}{2}$		$\frac{1}{4}$	
Method of calculation ... ..	B	D	B	D	B	D	B	D
Main Beam A ... ..	81	87	70	95	88	96	83	89
Main Beam B ... ..	80	87	68	70	95	93	91	87
Main Beam C ... ..	85	85	81	82	96	92	91	86

## 7. Conclusion.

By comparing the results measured in various kinds of bridges with the theoretical value obtained by the application of the theory of the orthotropic plate, it becomes obvious that this theory is a suitable method for the calculation of bridges. It is one of the merits of this method that this analysis of a plate is applied simultaneously to the calculation of the main girder, floor beam, stringer, slab, etc.

The method for the determination of  $D_x$ ,  $D_y$ ,  $H$ ,  $\nu$  and the boundary condition stated in this paper is believed to be a proper practical method, but there are many problems to be studied in detail. If the determination of these values and the boundary conditions improves, better results will

be obtained. This will be done by a model test of the beam and girder bridge.

In the analysis by the theory of the orthotropic plate, the differential equation of the 4th order of M. T. Huber was used, but this equation is only effective in the case of the orthotropic plate itself, and can not be applied accurately to the beam and girder bridge structure with slab. Strictly speaking, the differential equation of the 8th order of K. Trenks must be used. However, the result of the application of the differential equation by M. T. Huber shows that the theory is very effective in the analysis of girders and floor systems.

This experimental research was done in the elastic range of small stress, but it can be extended to the whole elastic range. Whether the theory of the orthotropic plate can be applied to the co-operation of each member beyond the elastic range or not must be studied in the future.

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#### SUMMARY

The results of the stress measurement of a through plate girder bridge, stringers of the floor system of a truss bridge, main beams of a composite beam bridge and main beams of a model composite grillage beam bridge are compared with those obtained by the conventional method and by the theory of the orthotropic plate introduced by M. T. Huber. In conclusion, it becomes obvious that this theory is very effective in the analysis of girder and floor system of bridge structures.

#### ZUSAMMENFASSUNG

Die Ergebnisse von Spannungsmessungen an einfache Balkenbrücken mit unterliegende Fahrbahn, an den Längsträgern der Fahrbahnroste von einfache Fachwerkbrücken, an Hauptträgern von Brücken mit Verbundträgern und an Hauptträgern bei Modellbrücken mit Trägerrost wurden mit denjenigen Werten verglichen, welche man in üblicher Weise und mit Hilfe der Theorie der orthotropen Platte erhält, wie sie von M. T. Huber entwickelt wurde. Es zeigt sich dabei in eindeutiger Weise, dass diese Theorie für die Berechnung von Trägern und Platten bei Brückenbauten sehr wertvoll ist.

## RESUMO

Comparam-se os resultados obtidos medindo as tensões numa ponte de alma cheia, nas longarinas do tabuleiro de uma ponte triangulada, nas vigas principais de uma ponte mixta betão/aço e nas vigas principais de um modelo reduzido de ponte mixta com viga reticulada, com os resultados obtidos pelo cálculo convencional e pela teoria da placa ortotrópica da autoria de M. T. Huber.

Em conclusão, torna-se evidente que a referida teoria permite obter resultados muito satisfatórios quando do estudo das vigas e dos tabuleiros das pontes.

## RÉSUMÉ

L'auteur compare les résultats obtenus par la mesure des contraintes dans les ponts-poutre à âme pleine, dans les longerons du tablier d'un pont triangulé, dans les poutres principales d'un pont mixte béton/acier et dans les poutres principales d'un modèle réduit de pont mixte à poutre en treillis avec ceux obtenus par le calcul conventionnel et par la théorie de la plaque orthotropique due à M. T. Huber.

En conclusion, il semble évident que cette théorie permet d'obtenir des résultats très satisfaisants lors de l'étude des poutres et des tabliers des ponts.



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