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Problems of limit analysis of orthothropic and non-homogeneous plates

Probleme der Grenzlasttheorie von orthotropen und nicht homogenen Platten

Problemas relativos à teoria da carga limite de placas ortotrópicas e não-homogéneas

Problèmes de la théorie de la charge limite des plaques orthotropes et non-homogènes

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Our attention in Poland, particularly of my co-workers and myself, was drawn in the last few years on problems of the limit analysis of plates.

Experience shows that about 70 to 75 % of all structural elements in civil and structural engineering are plates. So we thought it useful to devote more attention to questions connected with the theory and design of plates.

Our research work has been extended to anisotropic plates of the «single» and «double» type of anisotropy. Such «double» (or «laminar») anisotropy is, as a rule, executed in practice. This means that, in such cases, the orthotropy coefficient $\varkappa$ has two different values in accordance with the fact that «doubly» orthotropic slabs have other mechanical anisotropic properties in their «upper» layer (e.g. over the supports), and other still in their «lower» layer (e.g. in the middle of the span).

For practical purposes specially important are slabs which are orthogonally anisotropic or, as they are called, orthotropic.

In this way, we succeeded in obtaining some new solutions for the ultimate load carrying capacity of plates of various shapes, for various boundary conditions, and for various loads, [1; 2; 3; 6; 7], as for instance, for an orthotropic elliptic plate loaded by a concentrated force at an arbitrary point [2b; 3c].

This theory is now being generalized to flat slabs and to orthotropic shells — however, at present only to shells of simple types, like cylindrical orthotropic shells and axisymmetrical shells.

Recently the problems of limit analysis of orthotropic plates have been approached in a somewhat more general way, namely by also taking their non-homogeneity into consideration. As a matter of fact, nearly all practically executed plates are not only orthotropic, but — at the same time — non-homogeneous. Indeed, if e. g., the percentage of the reinforcement in the middle of the span is different from the percentage near the supports of the plate, we just vary the mechanical properties of the plate; this means that these properties are functions of the coordinates of the point considered or, in other words, the plate is non-homogeneous.

In order to establish such a general theory we start from the fundamental equations for plastically non-homogeneous bodies. The foundations of this theory have been recently developed and published in several papers [3a,d; 6a,b].

The theoretical results thus obtained were experimentally verified. A good agreement was found between the theoretical results and experimental data obtained [5].

In addition, graphs and tables were computed in order to facilitate the task of designing orthotropic plates of various shapes and various boundary conditions, from the point of view of their ultimate load carrying capacity (a suitable margin of safety being, obviously, taken into account).

We hope to be able to introduce this kind of design on a comparatively wide scale into current engineering practice, the methods of limit analysis having been accepted by our official specifications and codes.

Recently we have tackled the problem of limit states of circular plates with an eccentric hole as a problem of the theory of plasticity. (The case of a circular plate of concentric annular shape is, of course, one of the possible limit cases of this more general scheme).

However, we have found, that the general solutions for this type of plates is — so far — not known for the range of elastic deformations. That is why we have first solved the elastic problem [4] and only afterwards approached the plastic problem.

This analysis is based on the application of a special type of conformal mapping, the so-called transformation of inversion (or transformation by inverted radii).

The middle surface of the plate is assumed to be a plane of the complex variable $z = x + iy$. Every point of this plane is mapped into a point $Z = X + iY = f(x + iy)$ of a corresponding complex variable plane by the use of the analytic function

$$Z = \frac{k^2}{\bar{z} + h}, \qquad \begin{matrix} \text{where} & z = x + iy \\ \text{and} & \bar{z} = x - iy \end{matrix}$$

are conjugate complex variables.

Thus we map, for instance, the eccentric annulus on a concentric annulus (Fig. 1), the semi-infinite plane with a circular hole (Fig. 2) being a limit case of the general scheme and its solutions being contained in the former as a special case (the parameters of the transformation being chosen in such a way as to make the outer circle degenerate into a straight line).

The use of this method makes it possible to reduce the more complicated problem to a simpler one for which the boundary conditions can easily be formulated.

The elastic problem was solved for different boundary conditions, for instance, for the edges (both interior and exterior) clamped, for the edges (interior and exterior) alternatively clamped and free, and so on,

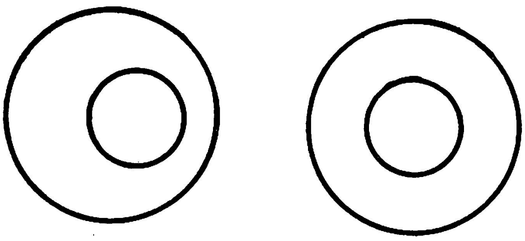

FIG. 1

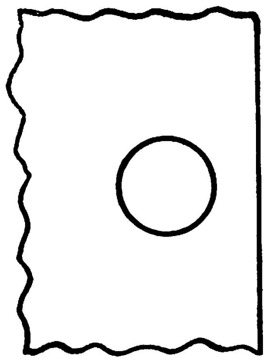

FIG. 2

and at the same time, for different loading schemes, as, for instance, for a uniformly distributed transverse load, q = const., or a continuously variable load, q = q (X, Y), or, finally, for a concentrated load at any point (Green's function).

Thus, starting from an analogous transformation, we are now investigating the problem of the ultimate load carrying capacity of plates of eccentric annular shapes with different boundary conditions and subjected to different loading schemes. These problems are treated as those of limit equilibrium of the theory of plasticity, with the introduction of a suitably formulated yield condition (1).

Of course, problems of the theory of plasticity are essentially non--linear because of the non-linearity of the basic equations. However, it is possible to give the solution of the discussed problem in a comparatively simple form.

We think the methods of limit analysis and design of anisotropic and non-homogeneous plates and shells are very important both from the theoretical point of view, as well as for practical purposes; we consider them to indicate the right way to give our structures the desired degree of safety.

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(1) The solution presents certain analogies with the plane problem of the eccentric ring, which, for the elastic-plastic range, was treated by the author in another paper (1957).

REFERENCES

1. M. KWIECINSKI — *Solution of continuous orthotropic plates according to the limit design theory* (in Polish), Zesz. Nauk. Pol. Warsz. Budown. No. 6, 1956, pp. 93-110.
2. D. NIEPOSTYN, — a) *Limit design of rectangular orthotropic plates.* (in Polish), Zesz. Nauk. Pol. Warsz., Budown. No. 6, 1956, pp. 61-92; b) *Spiral and elliptic yield patterns in plates subjected to concentrated loads* (in Polish); c) *Ultimate load carrying capacity of non-homogeneous rectangular plates* (in Polish), Rozpr. Inz.; d) *The limit analysis of an orthotropic circular cylinder*, 9th Intern. Congr. Appl. Mech., Brussels 1956, and Arch. Mech. Stos., 1956, No. 4, pp. 565-580.
3. W. OLSZAK — a) *Thick-walled reinforced concrete pipes; Proposals for increasing their statical efficiency.* Final Report, 2nd Intern. Congr. Bridge Struct. Eng., 1936; b *Limit analysis of orthotropic plates* (in Polish), Budown. Przem. 1953, pp. 254-265; c) *The problem of orthotropy in the limit analysis of plates* (in Polish), Arch. Mech. Stos., 1953, No. 3, pp. 329-350; *Sur la théorie de la charge limite des plaques orthotropes*, Bull. Acad. Pol. Sci., Cl. IV, 1954, No. 1, pp. 3-8; d) *On the fundamentals of the theory of non-homogeneous elastic-plastic bodies* (I), Arch Mech. Stos., 1954, No. 3, pp. 493-532; Bull. Acad. Pol. Sci., Cl. IV, 1955, No 2, pp. 45-49. (II) Arch. Mech. Stos., 1954, No. 4, pp. 639-656, Bull. Acad. Pol. Sci., Cl. IV, 1954, No. 3, pp. 111-117; *The plane problem of the theory of plastic flow of non-homogeneous bodies*, Bull. Acad. Pol. Sci., Cl. IV, 1955, No. 3, pp. 119-124; *On a classification of non-homogeneous elastic-plastic bodies*, Bull. Acad. Pol. Sci., Cl. IV, 1956, No. 1, pp. 29-35.
4. W. OLSZAK and Z. MRÓZ — *The method of inversion in the theory of plates*, Intern. Assoc. Bridge Struct. Eng., Publ. vol. XVI, (1956).
5. W. OLSZAK and A. SAWCZUK — *Experimental verification of the limit analysis of plates*, Rozpr. Inz. 1955, No. 2, pp. 179-253; Bull. Acad. Pol. Sci., Cl. IV, 1955, pp. 195-200.
6. W. OLSZAK and W. URBANOWSKI — a) *Orthotropy and non-homogeneity in the theory of plasticity* (in Polish), Arch. Mech. Stos., 1956, No. 1, pp. 85-110; b) *The plastic potential and the generalized distortion energy in the theory of non-homogeneous anisotropic elastic-plastic bodies*, 9th Intern. Congr. Appl. Mech., Brussels 1956, and Arch. Mech. Stos., 1956, No. 4, pp. 671-694; c *Thick-walled elastic-plastic non-homogeneous cylinder* (in Polish), Arch. Mech. Stos., 1956, No. 3, pp. 315-336; d) *Thick walled elastic-plastic spherical non-homogeneous shell* (in Polish), Rozpr. Inz., 1956, No. 1, pp. 25-41.
7. A. SAWCZUK — a) *Linear transformation as applied in the theory of limit design of plates* (in Polish), Zesz. Nauk. Pol. Warsz., Budown. No. 3, 1955; b) *Problems of ultimate load carrying capacity of tensioned and bent structural elements* (in Polish), Rozpr. Inz. 1955, No. 2, pp. 255-284; c *On practical applications of the limit analysis of plates* (in Polish), Arch. Inz. Lad., 1956, No. 1, pp. 139--183; d) *Some problems of simultaneous operation of concentrated and continuous loads in the theory of limit design of plates* (in Polish), Zesz. Nauk. Pol. Warsz. Budown. 1956, No. 6, pp. 41-59; e) *Some problems of load carrying capacity of orthotropic and non-homogeneous plates*, 9th Intern. Congr. Appl. Mech., Brussels 1956, and Arch. Mech. Stos. 1956, No. 4, pp. 549-564.

SUMMARY

The importance of a general theory of the ultimate load carrying capacity of anisotropic and, at the same time, non-homogeneous plates is stressed, for, as a rule, nearly all practically executed plates are both anisotropic and non homogeneous. Research work on the fundamentals of such a general theory, its results and their practical applications are discussed.

ZUSAMMENFASSUNG

Die wissenschaftliche und praktische Bedeutung einer allgemeinen Grenzlasttheorie von anisotropen und dabei gleichzeitig nicht homogenen Platten wird dargelegt; in der Tat werden fast alle Platten sowohl als anisotrope als auch als nicht homogene Konstruktionselemente praktisch ausgeführt. Arbeiten über die Grundlagen einer derart allgemeinen Theorie, ihre Resultate und deren praktische Anwendungsmöglichkeiten werden besprochen.

RESUMO

Põe-se em evidência a importância de uma teoria geral da carga limite das placas anisotrópicas e, simultâneamente, não homogéneas; com efeito, quase todas as placas executadas na prática são caracterizadas pelas suas anisotropia e não-homogeneidade mecânicas. Discutem-se os trabalhos relativos à elaboração das bases dessa teoria geral, os seus resultados e as suas aplicações práticas.

RÉSUMÉ

On met en évidence l'importance d'une théorie générale de la charge limite des plaques anisotropes et, en même temps, non-homogènes; en effet, presque toute réalisation pratique des plaques est marquée par leur anisotropie et non-homogénéité mécaniques. On discute les travaux portant sur les bases d'une telle théorie générale, ses résultats et leurs applications pratiques.