

Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH
Kongressbericht

Band: 5 (1956)

Rubrik: Free discussion

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 21.02.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Discussão livre

Discussion libre

Free Discussion

Freie Diskussion

Leere Seite
Blank page
Page vide

V 1

The use of high tensile steel as reinforcement of concrete

Der hochwertige Stahl als Bewehrung des Eisenbetons

Emprego de aço de alta resistência nas armaduras para betão

Emploi de l'acier à haute résistance dans les armatures à béton

DR. ENG. K. HAJNAL-KÓNYI

London

The importance of the use of high tensile steel as reinforcement of concrete is now internationally recognised. Out of 9 papers in Section Va, 6 are devoted to this question. These 6 papers originate from 5 different countries. France is represented by 2 contributions, Austria, Great Britain, Germany, Hungary, each by one. The papers deal with a great variety of types of steel regarding both shape and quality.

I wish to refer first to the contribution by M. LAZARD who reports on comparative tests on good quality mild steel and Torsteel respectively in beams of unusually large size. His conclusions should therefore be particularly convincing. May I quote the last sentence of his summary (p. 761) :

«Use of Tor 40 reinforcement bars for working stresses of 20 kg/mm² (= 28,450 psi) seems absolutely justified for bridge beams submitted to aggressive fumes of steam railway engine smoke» (1).

This conclusion is of very great importance since there are still many engineers who believe that the best way to avoid corrosion is to use plain mild steel bars and limit the stress to 1400 kg/cm² (20,000 psi), or even less. This view is convincingly refuted in Mr. SZÉPE's paper (p. 851) :

«It has to be stated that the older regulations which try to achieve freedom from cracks in reinforced concrete or crack control by limiting the tensile stress in the steel and in the concrete, have gone the wrong way».

(1) In his verbal contribution at the Congress M. LAZARD has stated that the stress in Torsteel 40 actually adopted by the French Railways is now 21 kg/mm² (= 29,870 psi).

The real cause of corrosion is not the occurrence of transverse cracks but lack of cover and porosity of the concrete. The paper by Messrs. ROBINSON and PELTIER, which is the continuation of a paper submitted by M. ROBINSON in Cambridge 4 years ago, seems to go to the root of the defects commonly known in every country. It is obvious from this research that corrosion cannot be prevented by a limitation of tensile stresses and is bound to occur under certain conditions even if the stresses are nil or compressive. In his verbal contribution at the Congress M. ROBINSON has referred to tests in which corrosion occurred first as a consequence of the porosity of concrete and cracks were a secondary effect caused by corrosion.

Whilst considerable effort is being made in various countries to restrict the width of cracks, it would appear that another important aspect of the admission of high tensile stresses has not received sufficient attention.

The serviceability of a structure depends not only on cracking but probably even more on deformation. With mild steel at working stresses of the order of 1400 kg/cm^2 (20,000 psi) deformations are rarely relevant. When we come into the range of working stresses of 1800 to 2500 kg/cm^2 (25,600 to 37,000 psi) as usual with steels of what I would call medium quality i. e. with a yield point or proof stress of the order of 40 to 50 kg/mm^2 (57,000 to 71,000 psi) conditions become more critical, but in the range of working stresses of 3500 to 4500 kg/cm^2 (50,000 to 64,000 psi) as adopted in Austria for Torsteel 60 and bi-steel (p. 748) respectively, the deformation must be the primary consideration of the designer, particularly the deformation under sustained loading [1] [2]. Since the modulus of elasticity of these special steels is not higher than that of mild steel, it is obvious that the deformation of structures with working stresses of this order may be 2 to 3 times greater than the deformation of structures with mild steel. Other conditions being equal, the substitution of a great number of small size bars for a small number of large size bars improves the crack control but has very little influence on the deflection under sustained loading. Consequently such high stresses must be restricted to exceptional cases where the depth/ span ratio is very favourable. It is interesting to note that based on his tests on beams reinforced with Torsteel 60 and 80 respectively which were otherwise similar to the beams discussed on his paper (Va 4) ⁽²⁾. M. LAZARD has come to the conclusion that «the practical interest of the Torsteels 60 and 80 is slight».

M. SAILLARD has included in his paper two tables (pp 837-8) in which the steel stresses are set out as a function of the diameter of the bars and of the maximum crack widths which can be tolerated. I think that a similar table or graph is necessary to limit the deformation of structures so that the permissible steel stress becomes a function of the slenderness ratio of the member in which the steel is used, of the percentage of reinforcement and of the modulus of elasticity of the concrete.

It may be seen from M. SAILLARD'S table on p. 837 that the stresses in plain round bars are very much restricted by the crack widths. In order

⁽²⁾ Ratio effective depth/span of the order of 1/9.

to use higher stresses, bars with protrusions must be adopted. Torsteel is one solution but according to M. SAILLARD (p. 820) the optimum crack control cannot be achieved with bars having only 2 or even 4 helical nibs. It is clear from the paper by Professor RÜSCH, Figs. 2 & 3, (pp. 795-6) that deformed bars are more efficient for controlling the crack widths. His proposition to standardize the pull-out tests is very good, but I should like to suggest two modifications:

1 - Professor RÜSCH suggests that prisms without any transverse reinforcement should be used (p. 795). From my experience, which is common with that of many other research workers, prisms without transverse reinforcement are bound to burst at a very small movement of the bar which is to be pulled out.

This tendency of bursting the concrete has also been pointed out by M. SAILLARD (p. 820). Indeed, the better the bond, i. e. the more efficient the bar, the greater is the tendency of bursting. A pull-out test which ends by the splitting of the concrete is, in my opinion, most unsatisfactory. The bar should be either pulled out or fractured but the specimen must not burst.

It is, of course, essential that the bursting of the concrete should be prevented in structures and for this reason transverse reinforcement must be provided wherever deformed bars end. Pull-out tests on prisms without transverse reinforcement would therefore not be representative of the correct use of deformed bars.

2 - My second point of disagreement with Professor RÜSCH is the magnitude of the slip at the unloaded end at which the bond stresses should be compared. Professor RÜSCH has suggested a slip of 0.1 mm (p. 796). Admittedly, the magnitude of the slip at which stresses are compared is as arbitrary as the elongation of cold worked bars to which the proof stress is related. However, since we can tolerate crack widths of even 0.3 mm in certain cases, I do not see the reason for adopting a slip of only 0.1 mm in general. I suggest that the pull-out tests should be so standardized that stresses are compared at slips of 0.1, 0.25 and 0.5 mm. This would give a much better idea of the performance of the bar since it would show to what extent the resistance of the bar against being pulled out is increased with increasing slip. With plain bars, the increase is nil, with Torsteel and square twisted bars it is very small, but with deformed bars of suitable shape it is considerable.

Finally, I wish to refer to Professor TORROJA's remark about «the most appropriate types of bar for reinforced concrete» (p. 703).

Regarding the shape I think that the provisions of ASTM [3] should be either adopted or taken as a basis of further research if this is deemed to be necessary. The American Standard is based on very comprehensive research and this should not be ignored.

Regarding the quality of the bars, I am of the opinion that cold worked deformed bars in which sharp edges are avoided and which are not overworked so as to become brittle at low temperatures are preferable to bars having a natural yield point. The lack of a definite yield point is a great advantage from the point of view of the mode of failure, as may be seen from the results of M. LAZARD (p. 761, Conclusions, 4 th. paragraph) and of M. LEWIS (pp 771, 779).

REFERENCES

1. WASHA, GEORGE W. — *Plastic Flow of Thin Reinforced Concrete Slabs*. Journal of the American Concrete Institute, Vol. 19 No. 3, Nov. 1947 p. 237.
2. WASHA, GEORGE W. and FLUCK, P. G. — *Effect of Compressive Reinforcement on the Plastic Flow of Reinforced Concrete Beams*. Journal of the American Concrete Institute, Vol. 24 No. 2, Oct. 1952 p. 89.
3. ASTM — *A 305-53 T*. Tentative Specifications for Minimum Requirements for the Deformations of Deformed Steel Bars for Concrete Reinforcement.

SUMMARY

It is submitted that the limiting factor for the admission of high working stresses in the tensile reinforcement is the deformation of the structure and that the bond strength of deformed bars should be determined on prisms with transverse reinforcement at slips of the unloaded end of 0.1, 0.25 and 0.50 mm respectively.

ZUSAMMENFASSUNG

Es wird vorgeschlagen, dass die zulässigen Zugspannungen in der Bewehrung durch die Formänderung der Bauteile beschränkt werden müssen und dass die Haftfestigkeit von Rippenstählen an Prismen mit Bügelbewehrung bei Gleitwegen des freien Endes von 0,1, 0,25 und 0,50 mm bestimmt werden soll.

RESUMO

O autor é da opinião de que o emprego de tensões de trabalho elevadas nas armaduras de tracção é limitado pela deformação da estrutura e de que a força de aderência de armaduras deformadas deveria determinar-se em prismas armados transversalmente para escorregamentos medidos na extremidade de 0,1, 0,25 e 0,50 mm.

RÉSUMÉ

L'auteur est d'avis que l'emploi de contraintes de travail élevées dans les armatures de traction est limité par la déformation des structures et que la force d'adhérence des armatures déformées devrait être déterminée sur des prismes armés transversalement pour des glissements mesurés à l'extrémité libre de 0,1, 0,25 et 0,50 mm.

V 2

Verbesserung der elastischen Eigenschaften der kaltverformten Stähle

Melhoramento da elasticidade das armaduras deformadas a frio

Amélioration de l'élasticité des armatures déformées à froid

Improvement of the elasticity of cold worked deformed reinforcement bars

DR. G. DE KAZINCZY
Stockholm

Die Rissbreite im Stahlbeton hängt — bei einem Betonstahl mit guter Haftung — unter gleichen Umständen, von der totalen Verlängerung des Stabes ab. Kaltverformte Betonstähle werden vorwiegend durch Verwinden hergestellt, oder wird die Verformung durch Verwinden abgeschlossen. Durch diese Behandlung verliert aber der Stahl seine ursprüngliche ausgeprägte Streckgrenze und bleibende Verlängerungen treten schon bei geringen Spannungen auf. Die Rissbreite ist also bei gleicher Spannung bei verwundenen Stäben grösser, als bei warmgewalzten.

Der Verfasser hat nachgewiesen, dass durch eine mässige Reckung nach dem Verwinden die Elastizitätsgrenze erhöht und dadurch die Rissbreite im Beton verringert werden kann. Die Reckung kann bei Spannbeton auch während des Spannens der Einlagen durchgeführt werden.

Im Bild 1 zeigt A die Arbeitslinie eines warmgewalzten Stahles. Durch eine Kaltverformung, die aus Reckung und nachfolgender Verwindung bestand, wurde Linie B erhalten. Verwindung allein gibt einen ähnlichen Verlauf, wie Linie B. Stab C wurde genau im selben Mass bearbeitet wie B, jedoch zuerst verwunden und nachher gereckt. Die Streckgrenze liegt um 15-30 % höher als bei B. Ein legierter Stahl mit höherer ursprünglicher Streckgrenze kann durch Verwindung die selbe 0,2-Grenze erreichen wie B, in dem Falle verläuft die Arbeitslinie wie D.

Man sieht, dass die Verlängerung und so die Rissbreiten, die mit W_m proportional sind, bei nur-*verwundenem* Stahl grösser sind.

Im Bild 2 ist die Durchbiegungs-Linie von zwei Stahlbetonbalken zu sehen. Der Balken B ist mit *verwundenem* und der Balken C mit *nachher gerecktem* Stahl bewehrt. Die Durchbiegung vor dem Bruch ist bei Balken C grösser, als bei B. Die Risse im Balken C (gestrichelte Linie) sind schmaler und beginnen nur nach Durchschreiten des Punktes x sich stark zu öffnen, so bleibt die «Warnung» nicht aus.

Es hat sich weiters gezeigt, dass die abschliessende Reckung nicht nur die Elastizitätsgrenze und die 0,2-Grenze erhöht, sondern auch die

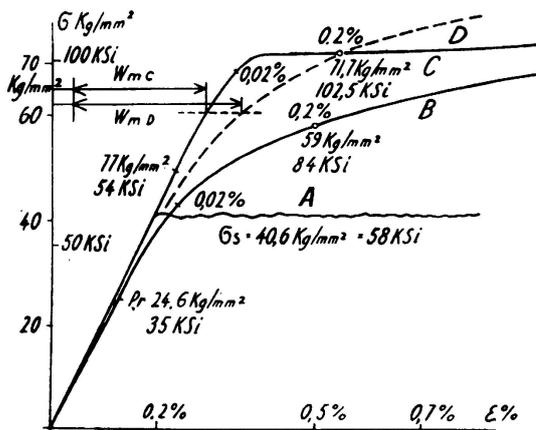


BILD 1

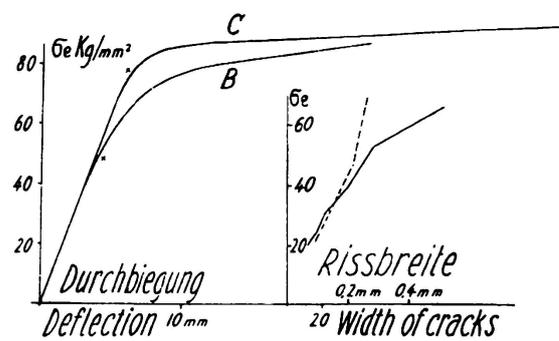


BILD 2

Kerbempfindlichkeit (Sprödbrüchigkeit) gewisser Stahlsorten (z. B. Thomasstahl) günstig beeinflusst, so dass dieses Verfahren bei gerippten Stäben besonders günstig ist.

Aus legiertem Stahl mit $\sigma_s = 60 \text{ kg/mm}^2$ kann man nach diesem Verfahren einen sehr geeigneten Spannbetonstahl auch mit Durchmesser $\phi 16-25 \text{ mm}$ mit etwa $100-110 \text{ kg/mm}^2$ 0,2-Grenze herstellen, der höhere Elastizität- und Kriechgrenze besitzt, als die bekannten kaltbearbeiteten Stähle.

ZUSAMMENFASSUNG

Die Elastizitätsgrenze des kaltverwundenen Betonstahles kann durch eine abschliessende, mässige Reckung bedeutend erhöht werden. Die Rissbreite im Stahlbeton wird dadurch bei der selben Belastung geringer und höhere Spannungen können zugelassen werden.

RESUMO

O limite de elasticidade das armaduras para betão, deformadas a frio, pode ser consideravelmente aumentado por meio de estiramento. Para cargas idênticas a largura das fissuras diminui podendo-se portanto admitir tensões mais elevadas.

S U M M A R Y

The yield stress of cold worked deformed reinforcement bars for concrete can be considerably increased through stretching. For identical loads the width of the cracks is smaller, thus allowing higher permissible stresses.

R É S U M É

La limite d'élasticité des armatures pour béton déformées à froid peut être considérablement augmentée au moyen d'un étirage. A charge égale la largeur des fissures est plus faible, ce qui permet d'admettre des contraintes plus élevées.

Leere Seite
Blank page
Page vide

V 3

Efficacité des armatures de torsion

Efficiency of torsion reinforcement bars

Wirksamkeit von Torsionsarmierungen

Efeito das armaduras de torsão

ANDRÉ PADUART

Professeur à l'Université Libre

Bruxelles

Il est curieux de constater que, dans l'étude des constructions en béton armé, l'attention se porte avant tout sur la résistance à la flexion et, dans une mesure moindre, sur le problème de l'effort tranchant. Quant à la sollicitation par torsion, il est bien rare de trouver dans la littérature technique un article qui lui soit consacré.

Aussi est-ce avec beaucoup d'intérêt que nous avons pris connaissance du mémoire V a 6 du Prof. Dr. Ing. E. Rausch, consacré aux armatures de torsion dans les constructions en béton armé. Il faut savoir gré à cet auteur d'avoir, en quelques pages, fait le point de la question et d'avoir rassemblé les formules nécessaires au calcul soit des armatures en hélice, soit des grillages.

L'auteur étudie le rôle de ces armatures à partir de la *force tangentielle* K (Tangentialkraft), qui agit en tout point de la surface latérale de la pièce, et il procède par décomposition vectorielle de cette force suivant des directions à 45° et suivant l'axe longitudinal.

Nous estimons cependant que cette façon de procéder ne fait pas suffisamment ressortir la différence d'efficacité des hélices et des grillages et, pour notre part, nous préférons raisonner à partir des *contraintes tangentielles*.

Le moment de torsion fait naître en chaque point un état double de tension composé de deux contraintes principales, égales et opposées, agissant sur des facettes à 45° . Si l'on place une armature normalement à la facette sollicitée par la traction, cette hélice intervient dès le début de la mise en charge et elle empêche l'ouverture des fissures microscopiques du béton. En écrivant que l'hélice doit reprendre la somme des tractions qui agissent sur le béton qui l'entoure, on retrouve la formule (2) du Prof. Rausch.

Si l'on remplace l'armature hélicoïdale par un grillage composé d'étriers et de barres longitudinales, la théorie de l'élasticité montre que ces armatures ne reprennent aucune contrainte normale de sorte que leur efficacité est nulle aussi longtemps que la pièce ne s'est pas fissurée.

Dès que la traction limite du béton est atteinte, la pièce se découpe en hélices de béton comprimé qui assurent à elles seules la résistance à la torsion. Les armatures ne participent donc pas directement à cette résistance; leur rôle devient de second plan et consiste à assurer en tout point l'équilibre des hélices comprimées: la poussée latérale résultant de la courbure des hélices est à reprendre par les étriers et la poussée axiale doit être absorbée par des barres longitudinales.

La traction des barres longitudinales produit une augmentation de la distance séparant deux sections transversales voisines. Pour que les hélices de béton puissent entrer en compression, il est nécessaire que ces sections tournent l'une par rapport à l'autre d'une quantité qui n'est pas négligeable. Les fissures s'ouvriront donc avant que les armatures orthogonales puissent intervenir efficacement.

Il est cependant un cas particulier où les étriers suffisent pour empêcher la fissuration de la pièce: c'est lorsque celle-ci est sollicitée en outre par une compression longitudinale d'origine extérieure (précontrainte ou poteau) suffisante pour équilibrer la composante longitudinale des hélices de béton, c'est-à-dire au moins égale à $\frac{M}{r}$, r étant le rayon

moyen de la section transversale. Dans ce cas, la rotation relative des sections dont il était question ci-dessus ne se produit pas; de plus, il est inutile d'ajouter les barres longitudinales.

En conclusion de cette note, nous constatons donc:

- que les armatures hélicoïdales empêchent la fissuration.
- que les grillages n'entrent en jeu qu'après l'apparition de fissures importantes.
- que dans le cas où la pièce subit une compression longitudinale au moins égale à M/r , les étriers suffisent pour empêcher l'ouverture des fissures capillaires.

R É S U M É

En basant l'étude des armatures de torsion dans les constructions en béton armé sur la contrainte tangentielle et non sur la force tangentielle comme le fait le Prof. Dr. Ing. E. Rausch, dans le mémoire Va 6, il est facile de comparer l'efficacité des types différents de ferrailage utilisés. On constate ainsi que les armatures hélicoïdales empêchent la fissuration, tandis que les grillages d'armatures croisées n'entrent en jeu qu'après l'apparition de fissures importantes sauf si la pièce subit une compression longitudinale suffisante.

S U M M A R Y

If the design of torsion reinforcement in concrete structures is based upon shear stress and not shear force as Prof. Dr. Eng. E. Rausch does in his paper Va 6, it becomes easy to compare the efficiency of the

different types of reinforcement. It is thus seen that helical reinforcement bars avoid cracking while grill shaped reinforcement networks only actuate after larger crack formation except in cases where the element is submitted to a sufficiently large longitudinal compression.

ZUSAMMENFASSUNG

Gründet man die Untersuchung von Torsionsarmierungen des Eisenbetons auf die Tangentialspannung anstelle der Tangentialkraft, wie durch Prof. Dr. Ing. E. Rausch in der Abhandlung Va 6. gezeigt wird, so ist es einfach, die Wirksamkeit der verschiedenen Armierungstypen zu vergleichen. Man stellt dann fest, dass die Spiralarmierungen die Rissebildung verhindern, während die Netzarmierungen aus Bügeln und Längseisen erst nach dem Auftreten von deutlich sichtbaren Rissen wirksam werden. Eine Ausnahme liegt vor, wenn der Bauteil eine genügend grosse, längs gerichtete Druckspannung erleidet.

RESUMO

Baseando o estudo das armaduras de torção das estruturas de betão armado na tensão tangencial e não no esforço tangencial, como faz o Prof. Dr. Eng.º E. Rausch na sua contribuição Va 6, torna-se fácil comparar o efeito das diferentes formas de armaduras utilizadas. Verifica-se assim que as armaduras helicoidais impedem a fissuração ao passo que as armaduras cruzadas em forma de rede só actuam depois da formação de fissuras importantes, salvo quando o elemento é submetido a uma compressão longitudinal suficiente.

Leere Seite
Blank page
Page vide

V 4

Zur Drillbewehrung im Stahlbetonbau

Diskussion

As armaduras de torsão nas construções de betão armado

Discussão

Les armatures de torsion dans les constructions en béton armé

Discussion

Torsion reinforcement bars in reinforced concrete structures

Discussion

PROF. DR. ING. ERNST RAUSCH

Kettwig (Ruhr)

Zu den Bemerkungen von Herrn Prof. Paduart nehme ich wie folgt Stellung:

Es ist richtig, dass die Spiralbewehrung bei Belastung von Anfang an mitwirkt, da die Bewehrung in Richtung der Hauptzugspannungen

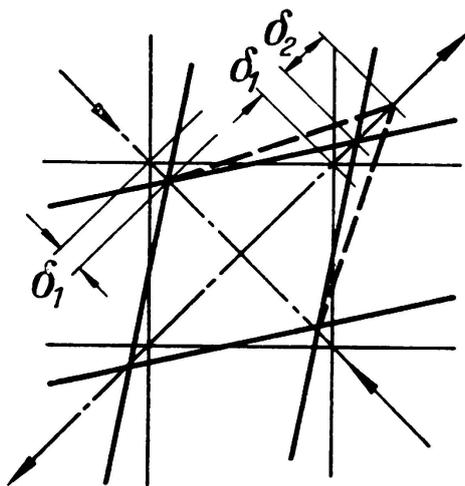


ABB. 1. Dehnungen der Bewehrungen vor der Rissbildung

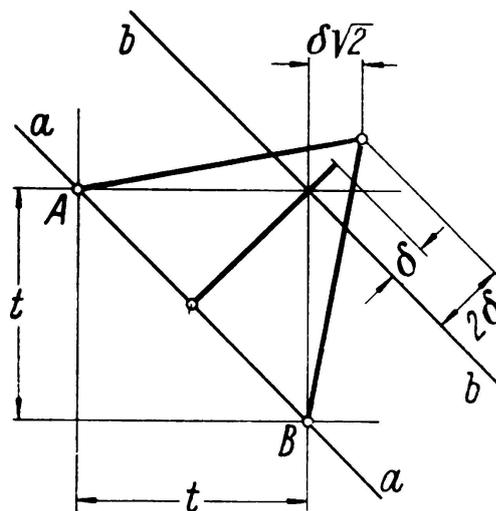


ABB. 2. Dehnungen der Bewehrungen nach der Rissbildung

liegt, die Netzbewehrung sich dagegen zunächst spannungsfrei rhombusartig verformt (ε_1 in Abb. 1). Vor Erreichung der Betonzugfestigkeit

wird jedoch die Dehnung in der Zugrichtung relativ grösser (δ_2 in Abb. 1). An dieser zusätzlichen Dehnung nimmt auch die Netzbewehrung teil, sodass auch sie in Spannung kommt, wenn auch weniger als die Spiralbewehrung. – Eine erhebliche Verzögerung der Rissbildung dürfte nach

bisherigen Versuchen auch bei Spiralbewehrung nicht zu erwarten sein.

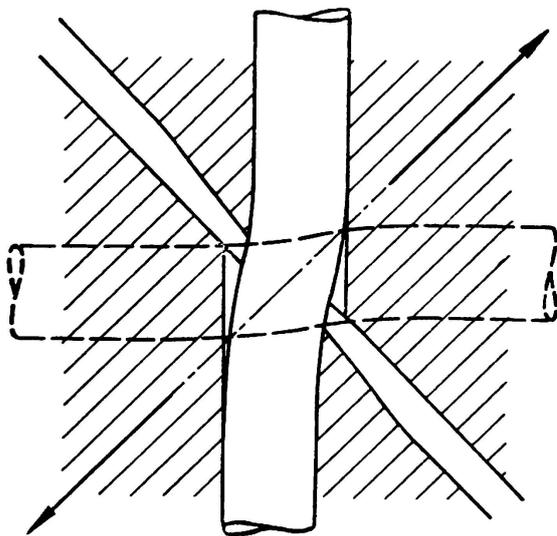


ABB. 3. Verformung der Netzbewehrung am Schrägriss

Nach dem Auftreten feiner Risse unter 45° bis zur Vollbelastung (Stahlspannung σ_s in den Spiralen ebenso, wie in den Stäben der Netzbewehrung) dehnen sich die Bewehrungen zwischen zwei Schrägrissen a und b nach Abb. 2. Die Rissweite wird also bei Netzbewehrung doppelt so gross wie bei Spiralbewehrung. Die Verkürzung der Beton-Druckdiagonale AB (Abb. 2) verursacht eine zusätzliche Dehnung in der Zugrichtung, die jedoch schätzungsweise weniger als 10 % von 2δ beträgt.

Wieweit die grössere Dehnung der Netzbewehrung und örtliche Verformungen nach Abb. 3 die Tragfähigkeit beeinflussen, ist durch

Versuche noch nicht genügend geklärt, weitere Versuche wären erwünscht. Die Netzbewehrung ist aber jedenfalls weniger wirksam als die Spiralbewehrung, ähnlich, wie die Bügel bei der Schubsicherung eines auf Biegung mit Querkraft beanspruchten Balkens weniger wirksam sind als die Schrägeisen.

Für die Bauausführung kommt m. E. trotzdem hauptsächlich die Netzbewehrung in Betracht, aus den auf Seite 786 des Vorberichtes bereits genannten Gründen, und da die Konstruktion, zeichnerische Darstellung und räumliche Biegung einer Spiralbewehrung der Praxis nicht zumutbar sind. Sie behindert ferner die Verlegung sonstiger Bewehrungen (Biegebewehrung).

Wenn auf den Stahlbeton-Hohlzylinder nach Abb. 4 ausser einem Drillmoment M auch eine achsiale Druckkraft N einwirkt (Drilling mit achsialer Vorspannung ⁽¹⁾), vgl. Abb. 6), dann entsteht an einer beliebigen

⁽¹⁾ Rausch: «Drilling (Torsion), Schub und Scheren im Stahlbetonbau», 3. Auflage 1953, im Vertrieb Deutscher Ingenieur-Verlag, Düsseldorf

Stelle des Mantels der mit Hilfe des Mohr'schen Spannungskreises in Abb. 5 dargestellte Spannungszustand mit der Hauptzugspannung

$\sigma_1 = -\sigma/2 + \sqrt{(\sigma/2)^2 + \tau^2}$. Auf eine Umfangsstrecke t wirken die Tangentialkraft $K = M t/(2F)$ und die achsiale Druckkraft $L = N t/U$ ($U = \text{Umfang}$), demnach $\tau = K/(td)$ und $\sigma = L/(td)$. Es ist dann $Z_1 = \sigma_1 td = L/2 + \sqrt{(L/2)^2 + K^2}$ und die Schrägzugkraft $Z_s = \sigma_1 d \cdot (t \cos \gamma) = Z_1 \cos \gamma$ (vgl. Abb. 6) für die Bemessung eines zur Umfangsstrecke t gehörenden Stabes der Spiralbewehrung. Wie man sieht, ist sie geringer als ohne Achsialkraft und die Spiralen verlaufen flacher. Für den Winkel γ ergibt sich aus Abb. 5: $\text{tg } \gamma = \tau/\sigma_1 = K/Z_1$. Bei Netzbewehrung erhält man zur Strecke t waagerecht (Ringe) wie lotrecht (Längseisen) $Z_w = Z_1 = Z_l$, also ein waagerecht und lotrecht gleich starkes Bewehrungsnetz, da ein solches Netz beliebig gerichtete Zugspannungen in gleicher Weise aufnehmen kann. Für einen Schrägriss nach Abb. 7 von der Länge l ist $Z_s' = \sigma_1 d$; Zugkraft für waagerechte Bügel: $Z_s' \sin \gamma$, je Längeneinheit: $Z_s' \cdot \sin \gamma / \sin \gamma = \sigma_1 d$, für die Länge t dementsprechend: $Z_w = \sigma_1 t d = Z_1$; ebenso für die lotrechten Längseisen (\cos statt \sin).

Die von Herrn Paduart erwähnte zusätzliche Achsialkraft M/r bedeutet Gleichsetzung der Achsialkraft L mit der Tangentialkraft K ($L = K$). Für diesen Fall erhält man $Z_1 = \text{ca. } 0,62 K$; $\text{tg } \gamma = \text{ca. } 1,6$; $\gamma = \text{ca. } 58^\circ$; $\cos \gamma = \text{ca. } 0,53$; für die Bemessung der Spiralbewehrung demnach: $Z_s = \text{ca. } 0,33 K$ (statt $\text{ca. } 0,70 K$ ohne Achsialkraft), für die Netzbewehrung: $Z_w = Z_1 = \text{ca. } 0,62 K$ (statt K ohne Achsialkraft).

Durch Anbringung der achsialen Druckkraft $L = K$ werden also die Längseisen der Netzbewehrung nicht entbehrlich, obwohl die Druckkraft L ebenso gross ist wie die ohne Achsialkraft ermittelte Zugkraft K der Längseisen. Dieser scheinbare Widerspruch besteht nicht, wenn man bedenkt, dass wir bei Drillung ohne Längskraft von Zugrissen unter 45° ausgehen, bei Drillung mit Längskraft jedoch von flachen geneigten Rissen (andere Kraftzerlegung).

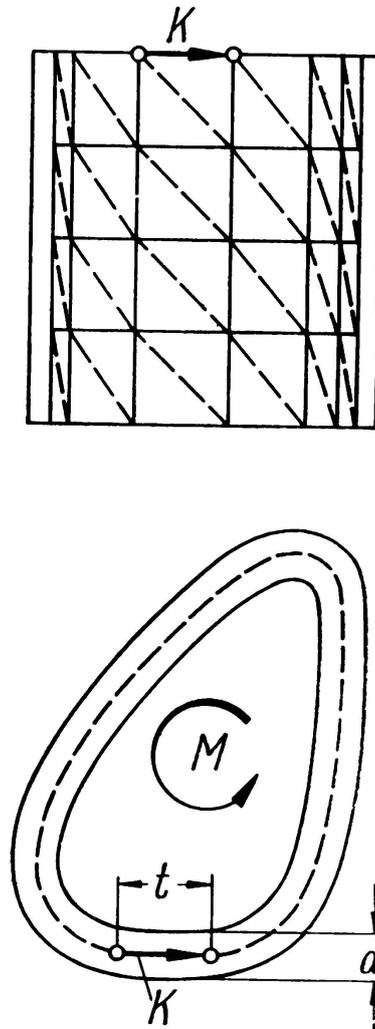


ABB. 4. Auf Drillung (Torsion) beanspruchter Stahlbeton-Hohlzylinder

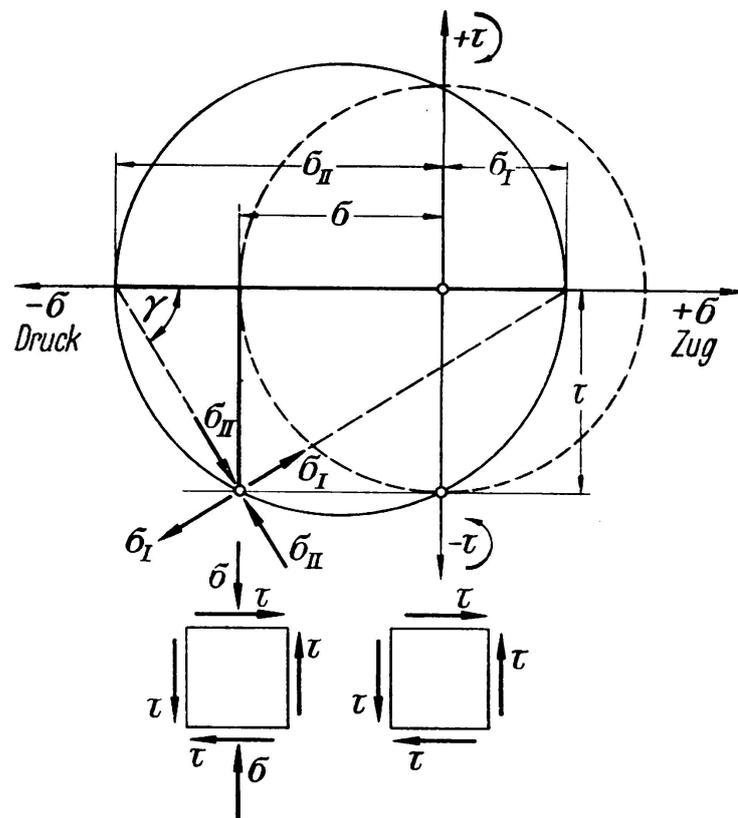


ABB. 5. Mohr'sche Spannungskreise für Drillung (Torsion) allein (gestrichelt) und Drillung mit Achsialdruck (ausgezogen)

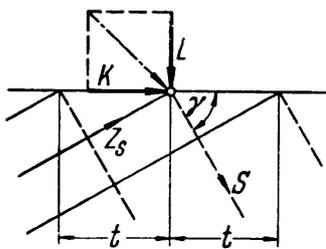


ABB. 6. Kräftespiel für Spiralbewehrung bei Drillung (Torsion) mit Achsialdruck

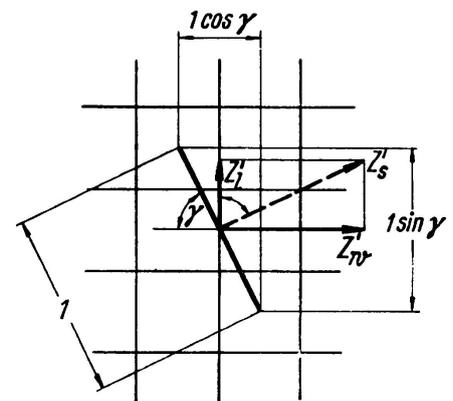
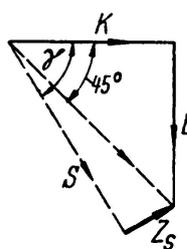


ABB. 7. Kräftespiel für Netzbewehrung bei Drillung (Torsion) mit Achsialdruck

ZUSAMMENFASSUNG

1. Vor der Rissbildung beteiligt sich auch die Netzbewehrung an der Beanspruchung, jedoch weniger als die Spiralbewehrung; eine erhebliche Verzögerung der Rissbildung ist auch bei Spiralbewehrung nicht zu erwarten.

2. Nach erfolgter Rissbildung öffnen sich die Risse bei Netzbewehrung etwa zweimal so stark wie bei Spiralbewehrung, sie ist weniger wirksam, ähnlich, wie Bügel zur Schubsicherung bei Biegung weniger wirksam sind als Schrägeisen.
3. Für die Bauausführung kommt trotz dieser Nachteile hauptsächlich die Netzbewehrung in Betracht.
4. Bei zusätzlicher achsialer Druckkraft (Vorspannung) sind die Längseisen der Netzbewehrung nicht entbehrlich, es ist ein nach beiden Richtungen gleich starkes Bewehrungsnetz erforderlich, jedoch mit geringerem Eisenquerschnitt.

RESUMO

1. Antes do aparecimento de fissuras as armaduras em forma de rede suportam também uma parte da solicitação, embora menor do que a suportada pelas armaduras helicoidais; estas no entanto não trazem uma diminuição sensível da fissuração.
2. Com armaduras em forma de rede as fissuras são cerca de duas vezes mais importantes do que com armaduras helicoidais; o efeito relativo dos dois tipos de armaduras pode ser comparado ao dos estribos e dos ferros oblíquos no caso da flexão.
3. Na prática, no entanto, apesar destes inconvenientes, consideram-se principalmente as armaduras em forma de rede.
4. No caso de esforços de compressão axiais (Betão preesforçado) os ferros longitudinais são indispensáveis, o que torna então necessário prever redes com armaduras igualmente importantes em ambas as direcções, sendo no entanto menor a secção de aço.

RÉSUMÉ

1. Avant l'apparition de fissures les armatures en forme de grillage supportent aussi une part de l'effort, cette part étant néanmoins inférieure à celle prise par les armatures hélicoïdales; celles-ci n'entraînent cependant pas une diminution sensible de la fissuration.
2. Dans le cas d'armatures en forme de grillage les fissures sont environ deux fois plus importantes que dans le cas d'armatures hélicoïdales; l'effet relatif des deux types d'armatures peut être comparé à celui des étriers et des bârres obliques dans le cas de la flexion.
3. Dans la pratique, néanmoins, malgré ces inconvénients, on considère surtout les armatures en forme de grillage.
4. En présence d'efforts de compression axiaux (Béton précontraint) les barres longitudinales sont indispensables, ce qui rend donc nécessaire de prévoir des grillages aux armatures également importantes dans les deux sens, la section d'acier étant toutefois moindre.

SUMMARY

1. Before the formation of cracks, grill-shaped reinforcement networks also carry part of the load, although in a smaller proportion than helical ones; these, nevertheless, do not noticeably improve crack formation.
2. With grill-shaped reinforcement networks cracks are approximately twice as large as with helical ones; the relative effect of both types of reinforcement can be compared to that of stirrups and bent up bars in the case of flexural loading.
3. In spite of all those drawbacks, grill-shaped reinforcement networks are currently considered in practice.
4. When there are axial compression loads (Prestressed concrete), longitudinal reinforcement becomes indispensable and it is then necessary to have grill networks with equally important reinforcement in both directions; the necessary reinforcement area is nevertheless smaller.

V 5

Remarks about the design of reinforced concrete beams

Zur Bemessung von Stahlbetonbalken

Observações acerca do dimensionamento de vigas de betão armado

Remarques sur le calcul de poutres en béton armé

FERRY BORGES

Research Engineer

ARGA E LIMA

Assistant Engineer

Laboratório Nacional de Engenharia Civil

Lisbon

1. Introduction

The remarks in this paper are the result of preliminary studies carried out with a view to revising the Portuguese reinforced concrete code and introducing into it clauses which make it possible to take full advantage of the strength of the steels available at present.

As the ruin of reinforced concrete beams may be caused by excessive cracking, deformation or rupture, some considerations about the behaviour of reinforced concrete in relation to these three features are presented.

Mention is also made of the results obtained by tests on some concrete beams reinforced with mild steel and plain twisted steel ⁽¹⁾. These tests were carried out with a view to permitting the manufacture in Portugal of steels hardened by twisting. By means of tests the indispensable experimental basis was sought for a better understanding of the existing bibliography.

Simple bending tests were carried out on 14 beams with a span of 2.8 m. The beams were 20 cm wide and 30 cm deep. The percentages of the reinforcement were 0.47 % (8 ϕ $\frac{1}{4}$ "), 0.70 % (3 ϕ $\frac{1}{2}$ ") and 1.59 % (3 ϕ $\frac{3}{4}$ "). The loads were applied at a quarter span.

Half the beams were reinforced with mild steel (minimum yield stress of 24 kg mm⁻²) and the other half with twisted steel (minimum conventional stress of proportionality at 0.2 % of 42 kg mm⁻²).

⁽¹⁾ Mr. Augusto Tavares de Castro, Assistant Engineer of the L. N. E. C., was in charge of the tests.

Fig. 1 shows one view of the arrangement for testing the beams.

The Laboratório Nacional de Engenharia Civil has obtained steels from various origins and with different mechanical properties, on which tests are to be carried out with a view to confirming the conclusions presented.

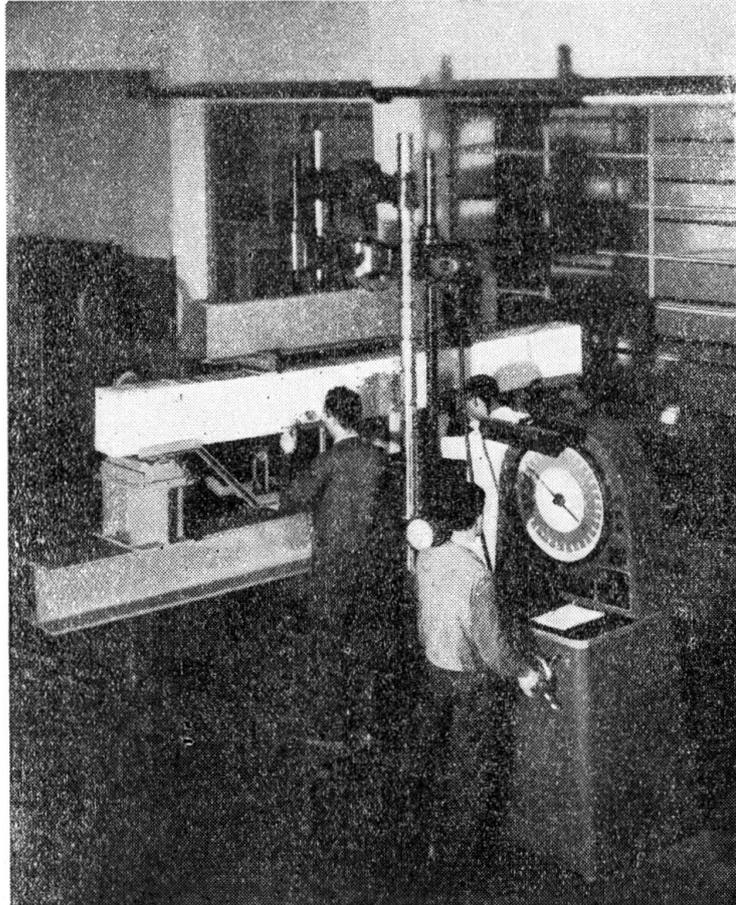


FIG. 1. Test of a beam

2. Cracking

Figs. 2 and 3 show some results in relation to cracking of beams observed during these tests. Fig. 2 gives the widest crack observed as a function of the stresses in the steel, calculated in accordance with the classic design method for reinforced concrete ($m = 15$). As an analysis of this figure shows, in the mild steel beams, the width of the widest cracks remained below 0.25 mm up to stresses in the neighbourhood of the yield stresses for the steel.

In the twisted steel beams, the width of the cracks of 0.25 mm corresponds to stresses in the steel lying between 30 and 40 kg mm⁻².

In fig. 3 the width of the cracks was plotted against the percentages of the ultimate bending moment. The analysis of this figure also shows that at 50 percent of the bending moments for twisted steel the widest cracks are 0.25 mm whilst those for mild steel beams are about half this value. When the comparison is made in this way, the cracks in the twisted steel beams are seen to be greater than those in the mild steel ones, but it must be remembered that for the same percentage of the

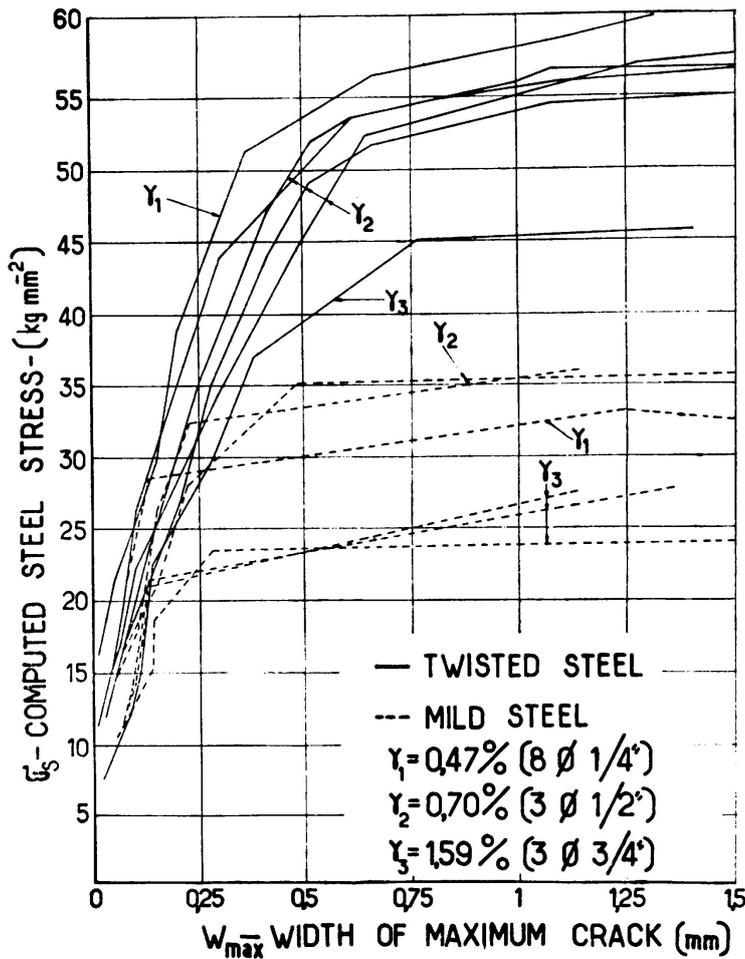


FIG. 2. Width of maximum crack in function of the computed steel stress

ultimate bending moments the stresses in the twisted steel are much greater than in the mild steel.

To interpret the results obtained in relation to cracking the method given by Rüsçh ⁽¹⁾ was followed.

⁽¹⁾ Rüsçh, H. — Der Zusammenhang zwischen Rissbildung und Haftfestigkeit unter besonderer Berücksichtigung der Anwendung hoher Stahlspannungen—Preliminary Publication, Fifth Congress — International Association for Bridge and Structural Engineering, Lisbon, 1956.

Fig. 4 shows a diagram in which the mean distances between the cracks, a , are taken as ordinates and as abscissae the values of a parameter H defined by the expression:

$$H = \frac{\tau_{0.1}}{0.1 \sigma_{cc}} \frac{u}{A_{tc}}$$

where

$\tau_{0.1}$ = the mean adherence stress obtained in a pull-out test, as described in the publication mentioned and for a displacement of 0.1 mm at the free end of the bar.

$0.1 \sigma_{cc}$ = the estimate of the tensile strength of concrete, which is equal to one tenth of the cube strength.

u = perimeter of the cross section of the reinforcement.

A_{tc} = the area of the tensile zone of the cross section of the beam, calculated by the classical method of design of reinforced concrete.

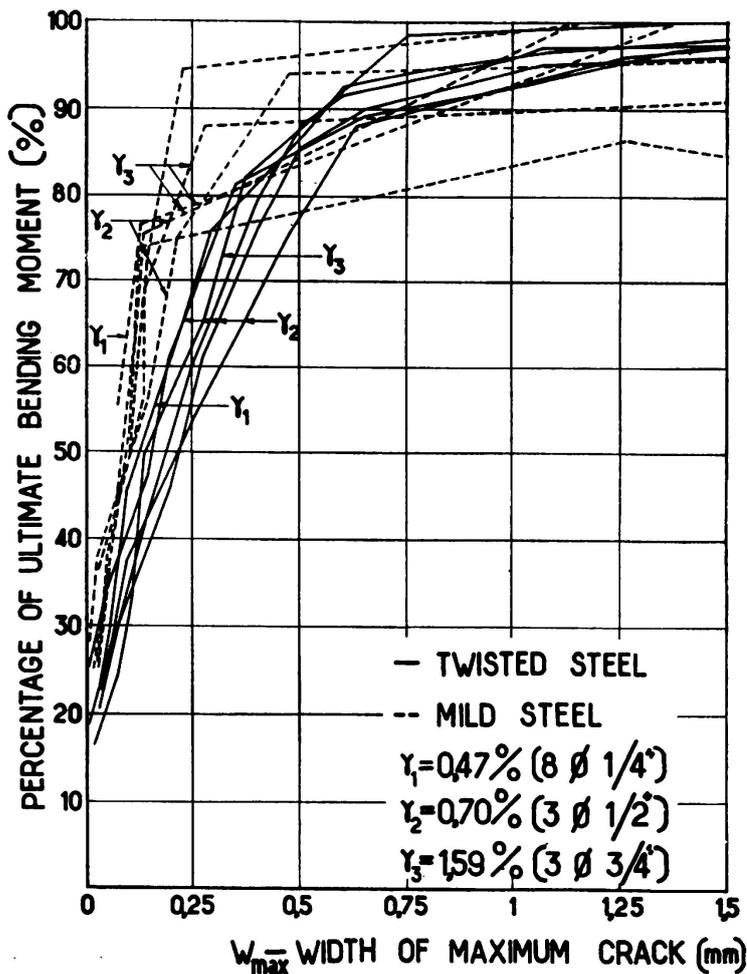


FIG. 3. Width of maximum crack in function of the percentage of ultimate bending moment

As the same quality concrete was used in all the beams, σ_{cc} was given a mean value of 250 kg cm^{-2} . For $\tau_{0.1}$ also a mean value of 25 kg cm^{-2} was taken for both mild and twisted steel. The tests showed that the adherence for twisted steel was greater than that of the mild steel for

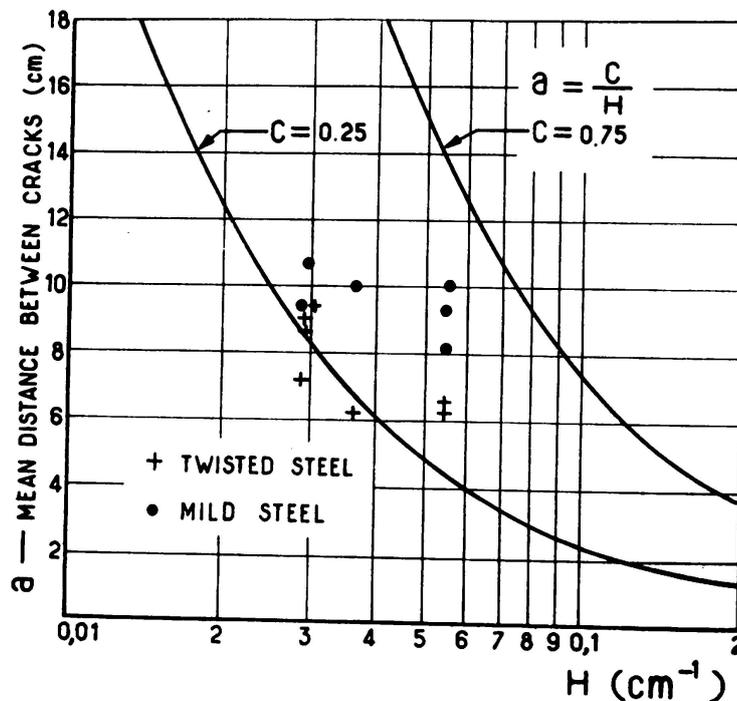


FIG. 4. Interpretation of cracking results (Rüsch's analysis)

high values of slip (inspite of the twisted steel not having fillets). For a slip of 0.1 mm the difference between the two steels was not appreciable and therefore the same value was adopted for the two types.

The analysis of fig. 4 shows that the results come out approximately between the curves given by Rüsch, the twisted steel beams showing less spacing between cracks than the mild steel ones, which is proof of greater adherence. On the other hand smaller distances between cracks do not correspond to greater values of H which is not surprising, for, as the author referred to says, for high values of H the expression deduced by Kuuskoski, $a = \frac{c}{H}$, no longer has significant value. It can also be seen in Rüsch's diagram that for values of $H > 0.05$, a no longer decreases.

To check if the results confirm the hypothesis that the maximum width of the crack, W_{max} , is practically proportional to the mean distance, a, and to the stress in the steel, σ_s , the diagram of fig. 5 was plotted, taking as ordinates the values $K = \frac{W_{max} E}{a \sigma_s}$ and as absissae the values of the stresses calculated in the steel.

The analysis of this diagram shows that roughly for stresses at which the number of cracks is stable, K is found to be between 1 and 2,

which agrees with Rüsç's results. It is to be noted, however, that the scatter of the values is relatively high and cannot be only considered as random, as the greater ratios between the areas and the perimeters of the reinforcements correspond to great values of K . This fact is also seen in fig. 2, where for the same stresses wider cracks were observed in beams having larger diameter bars. This variation, however, is small for the beams tested.

The analysis of the relation between the greatest width and the mean width of the cracks shows that the relation is approximately cons-

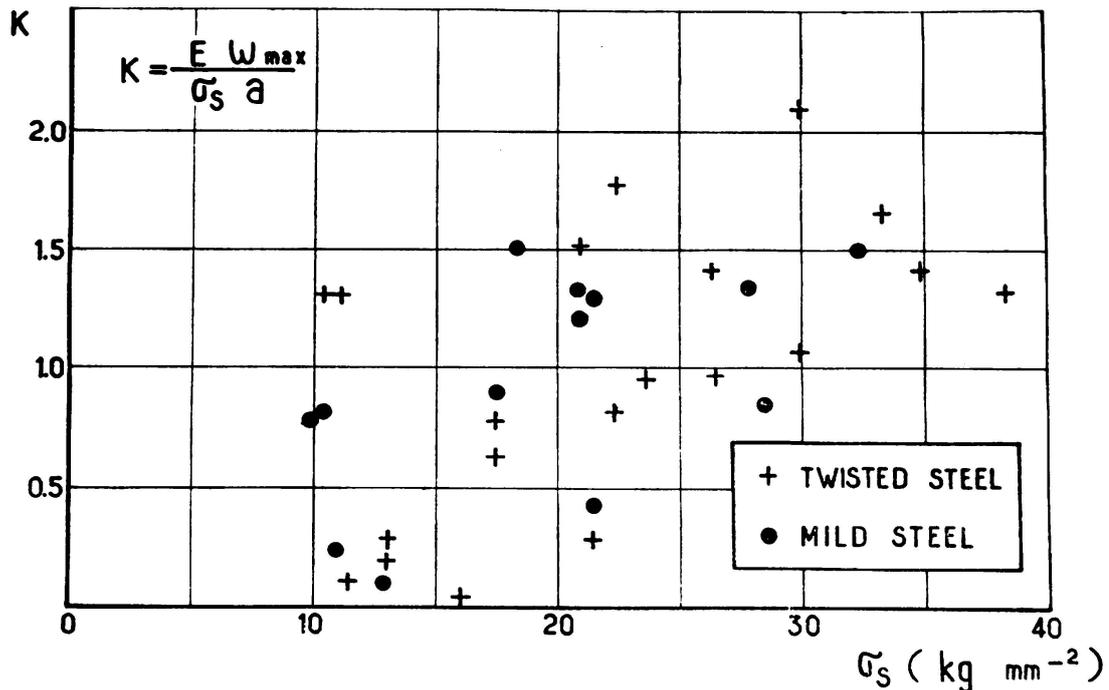


FIG. 5. Relation between spacing and maximum width of cracks

tant for the range of stresses in the steels concerned and for the different types of beams. The values obtained lie between 1.5 and 2.0.

Rüsç's diagram for the relation between a and H shows that for low values of H , $H < 0.05$, the relation given by Kuuskoski holds and therefore the spacing between cracks increases as H decreases.

The conclusion, in accordance with the above remarks, is that for a given maximum steel stress, σ_s , in order to limit the maximum width of cracks, there must be a maximum permissible spacing of the cracks. It was also seen that for low values of H the spacing was influenced by

the quocients $\frac{\tau_{0.1}}{\sigma_{cc}}$ and $\frac{u}{A_{t_c}}$. Hence the limitation of spacing should be made for the different values of $\frac{\tau_{0.1}}{\sigma_{cc}}$ and by the limitation of the quocient $\frac{u}{A_{t_c}}$.

For practical purposes it would be helpful to substitute a more easily calculable value for the relation $\frac{u}{\Delta t_c}$. It can be shown that the expression given above $a = \frac{C}{\Pi}$ with $C = 0.50$ is equivalent to the following $a = 0.0075 \frac{\sigma_{cc}}{\tau_{0.1}} \frac{d}{\gamma}$

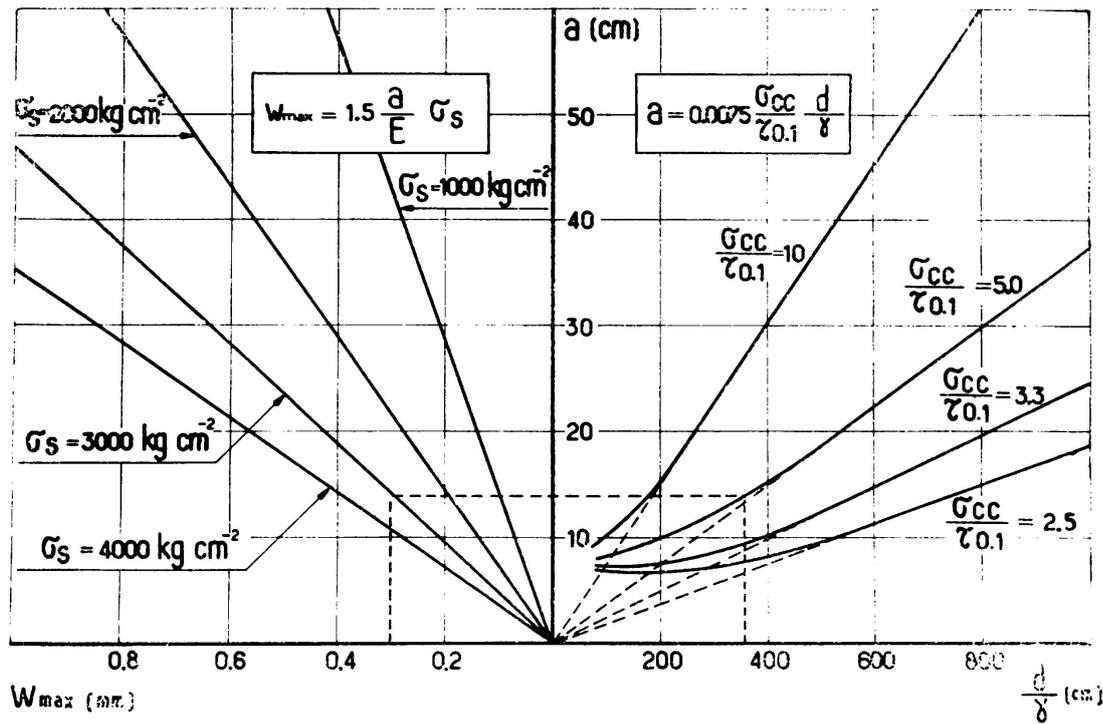


FIG. 6. Diagram connecting maximum width of cracks with $\frac{d}{\gamma}$

where

d = diameter of reinforcing bars

γ = percentage of reinforcement in relation to the area $b h$
 (b = width of the web of the beam, h = effective depth).

When establishing the last expression it was assumed that A_{t_c} could be approximately calculated by the expression, $A_{t_c} = 0.6 b h$.

For the spacing to be less than a given limit it is necessary for the values of $\frac{d}{\gamma}$ (quocient of the diameter of the bars and the percentage of the reinforcement) not to exceed certain limits.

A diagram in which the values of W_{max} , a and $\frac{d}{\gamma}$ respectively are marked is plotted in fig. 6.

The lines plotted correspond to different values of σ_s and $\frac{\sigma_{cc}}{\tau_{0.1}}$. It is easy to determine by this diagram the value of $\frac{d}{\gamma}$ which should not be exceeded in order to limit the maximum crack width. Taking, for example, $W_{max} = 0.3$ mm and $\sigma_s = 3000$ kg cm⁻², a becomes 14 cm and for $\frac{\sigma_{cc}}{\tau_{0.1}} = 5.0$, $\frac{d}{\gamma}$ becomes approximately 400 cm.

The limitation of the values of $\frac{d}{\gamma}$ does not, from the cons-

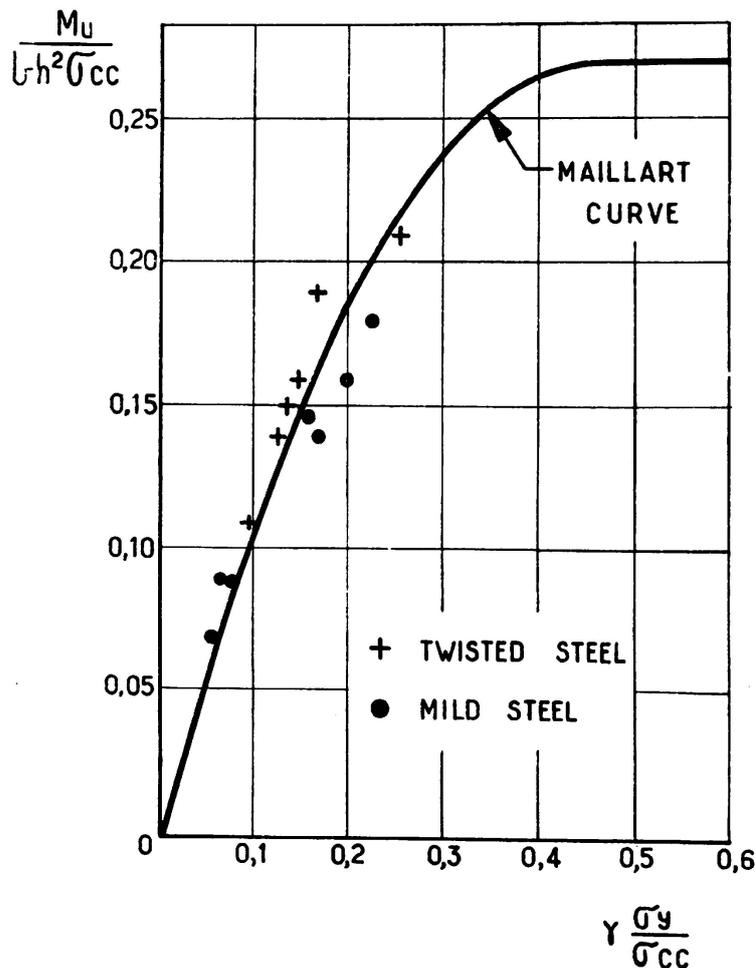


FIG. 7. Analysis of the ultimate bending moments

truction point of view, impose large restrictions and it would be a simple and efficient method to limit the width of cracks, especially when high stresses are adopted for the steel.

3. Deformability and ultimate strength

The analysis of the results obtained showed that the ultimate bending moments, M_u , reached were in agreement with those derived by present day theories.

In fig. 7 the values of $\frac{M u}{b h^2 \sigma_{cc}}$ were taken as ordinates and those of $\gamma \frac{\sigma_y}{\sigma_{cc}}$ as abscissae where σ_y is the yield stress of the steel.

As was to be expected, agreement was found between the experimental results and the curve established by Maillart, the variations being explainable by the randomness of the mechanical properties of the materials.

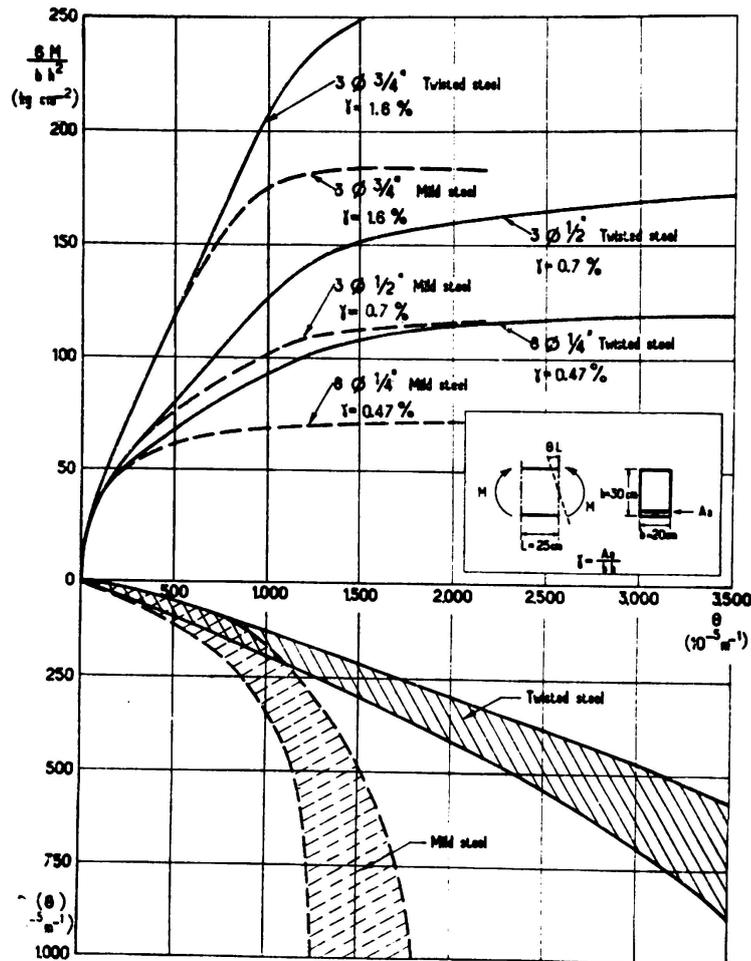


FIG. 8. Analysis of the deformability of the beams

The diagram of fig. 8 was plotted to analyse the deformability of the beams, taking as ordinates the values of $\frac{6M}{bh^2}$ (M = applied bending moment) and as abscissae the mean values of the rotation per unit of length. These rotations were measured in each beam between 6 cross sections, 25 cm apart from each other.

As in each beam various measurements of rotations were made and as various beams were tested it was possible to calculate the standard deviations of the rotations and plot them against their mean values, as can be seen in the bottom part of fig. 8. The analysis of this diagram shows that for small deformations the rotations have a coefficient of variation

which is practically constant and equal for beams with mild steel and twisted steel. As the rotations increase, a considerable increase in their scatter is seen for the mild steel beams, whilst the coefficient of variation for the twisted steel remained constant even for high values of rotations.

The scatter of the deformability depends to a great degree on the beam dimensions, a factor which cannot be ignored when studying this scatter.

The results given show that for reinforced structures with mild steel it would be difficult to foresee the deformation for advanced stages, near the rupture, because the values are very scattered. This does not happen in the case of high tensile steel, as the coefficient of variation remains constant for large deformations and these can consequently be foreseen with satisfactory precision.

4. Conclusions

The considerations presented are an attempt to demonstrate that with current cases it is possible to make full use of the capacity of the available steels without being limited by phenomena of cracking.

To avoid excessive cracking limits have to be given to the relation $\frac{d}{\gamma}$ between the diameter of the bars and the percentage of reinforcement. The limits will have to be established principally as a function of the working stresses adopted for the steel and the degree of adherence between steel and concrete.

In structures not needing to be watertight or which do not have to stand severe corrosive ambient conditions, the limitations to be established for $\frac{d}{\gamma}$ do not restrict the planning of structures very much.

It seems to be advisable to introduce clauses relative to the limitations of these values into the codes.

As far as cracking is concerned it would be of value to proceed with experiments principally with a view to studying the influence of adherence for high values of $\frac{d}{\gamma}$.

With regard to ultimate strength, the existing theories are perfectly satisfactory and it is not considered necessary to seek further experimental confirmation. It seems however that there would be advantages in studies on the relation between the randomness of the mechanical properties and that of the behaviour of the beams. These studies would contribute towards improving the probabilistic criteria for defining the safety of reinforced concrete structures and would be of interest not only in relation to ultimate strength but particularly in relation to deformation.

SUMMARY

Some results of bending tests of concrete beams reinforced with normal and twisted steel are presented. Interpretation is made of cracking, deformability and rupture.

As for cracking it is verified that, in order to prevent too wide cracks, it is necessary to limit the ratio between the diameter of the bars and the percentage of reinforcement.

Concerning deformability, the analysis of dispersions that it was possible to make, has shown that the dispersions were considerably different for the beams reinforced with mild and twisted steel.

Finally, regarding rupture, the results obtained agree with those foreseen by the existing theories.

It seems of interest, nevertheless, to carry out studies that, taking into account the randomness of the mechanical properties of the materials, allow to foresee the statistic behaviour, not only in relation to rupture but particularly in relation to deformation.

RESUMO

Os resultados obtidos pelo ensaio à flexão de algumas vigas de betão, armadas com aço normal e aço torcido liso, são interpretados em relação à fendilhação, deformabilidade e rotura.

Em relação à fendilhação verifica-se que, para evitar largura excessiva das fendas, se torna necessário limitar a relação entre o diâmetro dos varões e a percentagem de armadura.

Quanto à deformabilidade, a análise que foi possível fazer relativamente a dispersões mostrou que estas eram bastantes diferentes para as vigas com aço macio e aço torcido.

Finalmente, no que se refere à rotura, os resultados obtidos concordaram com os previstos por teorias existentes.

Considera-se, no entanto, que haveria interesse em realizar estudos que, entrando em consideração com a aleatoriedade das propriedades mecânicas dos materiais, permitissem prever probabilisticamente o comportamento não só em relação à rotura mas particularmente em relação à deformação.

RÉSUMÉ

On présente les résultats obtenus lors de l'essai à la flexion de quelques poutres en béton, armées avec de l'acier normal et de l'acier torsadé, et l'on fait leur interprétation du point de vue de la fissuration, de la déformabilité et de la rupture.

En ce qui concerne la fissuration, on constate que, pour éviter une largeur excessive des fissures, il faut limiter le rapport du diamètre des ronds au pourcentage de renforcement.

Pour la déformabilité, l'analyse que l'on a pu faire par rapport aux dispersions a montré que celles-ci sont bien différentes pour les poutres armées d'acier normal et d'acier torsadé.

Finalement, pour la rupture, les résultats des essais sont en accord avec les résultats fournis par les théories existantes.

On considère que, tenant compte du caractère aléatoire des propriétés mécaniques des matériaux, il serait toutefois intéressant de faire des études pour la prévision probabiliste du comportement, en ce qui concerne non seulement la rupture mais surtout aussi la déformabilité.

ZUSAMMENFASSUNG

In der vorliegenden Arbeit werden die Ergebnisse der Biegeversuche einiger Stahlbetonbalken (mit Normal- und gedriltem Stahl bewehrt) in bezug auf Rissbildung, Verformung und Bruch dargestellt.

Zur Frage der Rissbildung ist festzustellen, dass der Quotient zwischen Durchmesser der Rundstäbe und Bewehrungsgehalt begrenzt werden muss, um eine unzulässige Breite der Risse zu vermeiden.

Soweit es in der Verformungsuntersuchung möglich war, eine Analyse der Streuungen durchzuführen, zeigte sich, dass diese Streuungen für Normal- und gedrilten Stahl ziemlich unterschiedlich sind.

Was den Bruch anbetrifft, stimmten die Ergebnisse der Versuche mit den auf Grund der laufenden Theorien zu erwartenden Werte überein.

Es wäre jedoch interessant, weitere Versuche durchzuführen, welche, unter Berücksichtigung der Streuung mechanischer Eigenschaften der Baustoffe, eine Wahrscheinlichkeitsvoraussage des Verhaltens der Balken in bezug auf Bruch und ganz besonders auf Verformung ermöglichen.

V 6

Problems of limit analysis of orthotropic and non-homogeneous plates

Probleme der Grenzlasttheorie von orthotropen und nicht homogenen Platten

Problemas relativos à teoria da carga limite de placas ortotrópicas e não-homogêneas

Problèmes de la théorie de la charge limite des plaques orthotropes et non-homogènes

PROF. W. OLSZAK

D. Sc., Member of the Polish Academy of Sciences

Warsaw

Our attention in Poland, particularly of my co-workers and myself, was drawn in the last few years on problems of the limit analysis of plates.

Experience shows that about 70 to 75 % of all structural elements in civil and structural engineering are plates. So we thought it useful to devote more attention to questions connected with the theory and design of plates.

Our research work has been extended to anisotropic plates of the «single» and «double» type of anisotropy. Such «double» (or «laminar») anisotropy is, as a rule, executed in practice. This means that, in such cases, the orthotropy coefficient ν has two different values in accordance with the fact that «doubly» orthotropic slabs have other mechanical anisotropic properties in their «upper» layer (e.g. over the supports), and other still in their «lower» layer (e.g. in the middle of the span).

For practical purposes specially important are slabs which are orthogonally anisotropic or, as they are called, orthotropic.

In this way, we succeeded in obtaining some new solutions for the ultimate load carrying capacity of plates of various shapes, for various boundary conditions, and for various loads, [1; 2; 3; 6; 7], as for instance, for an orthotropic elliptic plate loaded by a concentrated force at an arbitrary point [2b; 3c].

This theory is now being generalized to flat slabs and to orthotropic shells — however, at present only to shells of simple types, like cylindrical orthotropic shells and axisymmetrical shells.

Recently the problems of limit analysis of orthotropic plates have been approached in a somewhat more general way, namely by also taking their non-homogeneity into consideration. As a matter of fact, nearly all practically executed plates are not only orthotropic, but — at the same time — non-homogeneous. Indeed, if e. g., the percentage of the reinforcement in the middle of the span is different from the percentage near the supports of the plate, we just vary the mechanical properties of the plate; this means that these properties are functions of the coordinates of the point considered or, in other words, the plate is non-homogeneous.

In order to establish such a general theory we start from the fundamental equations for plastically non-homogeneous bodies. The foundations of this theory have been recently developed and published in several papers [3a,d; 6a,b].

The theoretical results thus obtained were experimentally verified. A good agreement was found between the theoretical results and experimental data obtained [5].

In addition, graphs and tables were computed in order to facilitate the task of designing orthotropic plates of various shapes and various boundary conditions, from the point of view of their ultimate load carrying capacity (a suitable margin of safety being, obviously, taken into account).

We hope to be able to introduce this kind of design on a comparatively wide scale into current engineering practice, the methods of limit analysis having been accepted by our official specifications and codes.

Recently we have tackled the problem of limit states of circular plates with an eccentric hole as a problem of the theory of plasticity. (The case of a circular plate of concentric annular shape is, of course, one of the possible limit cases of this more general scheme).

However, we have found, that the general solutions for this type of plates is — so far — not known for the range of elastic deformations. That is why we have first solved the elastic problem [4] and only afterwards approached the plastic problem.

This analysis is based on the application of a special type of conformal mapping, the so-called transformation of inversion (or transformation by inverted radii).

The middle surface of the plate is assumed to be a plane of the complex variable $z = x + iy$. Every point of this plane is mapped into a point $Z = X + iY = f(x + iy)$ of a corresponding complex variable plane by the use of the analytic function

$$Z = \frac{k^2}{\bar{z} + h}, \quad \text{where } z = x + iy \\ \text{and } \bar{z} = x - iy$$

are conjugate complex variables.

Thus we map, for instance, the eccentric annulus on a concentric annulus (Fig. 1), the semi-infinite plane with a circular hole (Fig. 2) being a limit case of the general scheme and its solutions being contained

in the former as a special case (the parameters of the transformation being chosen in such a way as to make the outer circle degenerate into a straight line).

The use of this method makes it possible to reduce the more complicated problem to a simpler one for which the boundary conditions can easily be formulated.

The elastic problem was solved for different boundary conditions, for instance, for the edges (both interior and exterior) clamped, for the edges (interior and exterior) alternatively clamped and free, and so on,

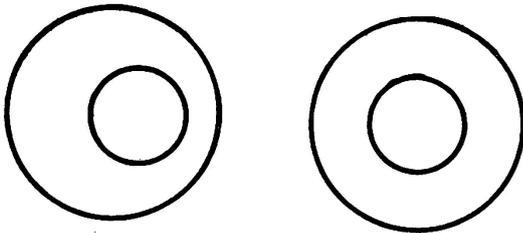


FIG. 1

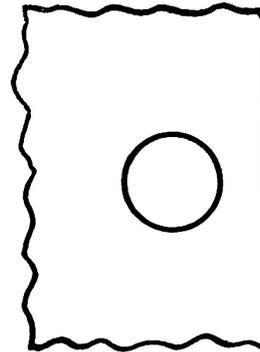


FIG. 2

and at the same time, for different loading schemes, as, for instance, for a uniformly distributed transverse load, $q = \text{const.}$, or a continuously variable load, $q = q(X, Y)$, or, finally, for a concentrated load at any point (Green's function).

Thus, starting from an analogous transformation, we are now investigating the problem of the ultimate load carrying capacity of plates of eccentric annular shapes with different boundary conditions and subjected to different loading schemes. These problems are treated as those of limit equilibrium of the theory of plasticity, with the introduction of a suitably formulated yield condition ⁽¹⁾.

Of course, problems of the theory of plasticity are essentially non-linear because of the non-linearity of the basic equations. However, it is possible to give the solution of the discussed problem in a comparatively simple form.

We think the methods of limit analysis and design of anisotropic and non-homogeneous plates and shells are very important both from the theoretical point of view, as well as for practical purposes; we consider them to indicate the right way to give our structures the desired degree of safety.

⁽¹⁾ The solution presents certain analogies with the plane problem of the eccentric ring, which, for the elastic-plastic range, was treated by the author in another paper (1957).

REFERENCES

1. M. KWIECINSKI — *Solution of continuous orthotropic plates according to the limit design theory* (in Polish), Zesz. Nauk. Pol. Warsz. Budown. No. 6, 1956, pp. 93-110.
2. D. NIEPOSTYN, — a) *Limit design of rectangular orthotropic plates*. (in Polish), Zesz. Nauk. Pol. Warsz., Budown. No. 6, 1956, pp. 61-92; b) *Spiral and elliptic yield patterns in plates subjected to concentrated loads* (in Polish); c) *Ultimate load carrying capacity of non-homogeneous rectangular plates* (in Polish), Rozpr. Inz.; d) *The limit analysis of an orthotropic circular cylinder*, 9th Intern. Congr. Appl. Mech., Brussels 1956, and Arch. Mech. Stos., 1956, No. 4, pp. 565-580.
3. W. OLSZAK — a) *Thick-walled reinforced concrete pipes; Proposals for increasing their statical efficiency*. Final Report, 2nd Intern. Congr. Bridge Struct. Eng., 1936; b) *Limit analysis of orthotropic plates* (in Polish), Budown. Przem. 1953, pp. 254-265; c) *The problem of orthotropy in the limit analysis of plates* (in Polish), Arch. Mech. Stos., 1953, No. 3, pp. 329-350; *Sur la théorie de la charge limite des plaques orthotropes*, Bull. Acad. Pol. Sci., Cl. iv, 1954, No. 1, pp. 3-8; d) *On the fundamentals of the theory of non-homogeneous elastic-plastic bodies (I)*, Arch. Mech. Stos., 1954, No. 3, pp. 493-532; Bull. Acad. Pol. Sci., Cl. iv, 1955, No. 2, pp. 45-49. (II) Arch. Mech. Stos., 1954, No. 4, pp. 639-656, Bull. Acad. Pol. Sci., Cl. iv, 1954, No. 3, pp. 111-117; *The plane problem of the theory of plastic flow of non-homogeneous bodies*, Bull. Acad. Pol. Sci., Cl. iv, 1955, No. 3, pp. 119-124; *On a classification of non-homogeneous elastic-plastic bodies*, Bull. Acad. Pol. Sci., Cl. iv, 1956, No. 1, pp. 29-35.
4. W. OLSZAK and Z. MRÓZ — *The method of inversion in the theory of plates*, Intern. Assoc. Bridge Struct. Eng., Publ. vol. xvi, (1956).
5. W. OLSZAK and A. SAWCZUK — *Experimental verification of the limit analysis of plates*, Rozpr. Inz. 1955, No. 2, pp. 179-253; Bull. Acad. Pol. Sci., Cl. iv, 1955, pp. 195-200.
6. W. OLSZAK and W. URBANOWSKI — a) *Orthotropy and non-homogeneity in the theory of plasticity* (in Polish), Arch. Mech. Stos., 1956, No. 1, pp. 85-110; b) *The plastic potential and the generalized distortion energy in the theory of non-homogeneous anisotropic elastic-plastic bodies*, 9th Intern. Congr. Appl. Mech., Brussels 1956, and Arch. Mech. Stos., 1956, No. 4, pp. 671-694; c) *Thick-walled elastic-plastic non-homogeneous cylinder* (in Polish), Arch. Mech. Stos., 1956, No. 3, pp. 315-336; d) *Thick walled elastic-plastic spherical non-homogeneous shell* (in Polish), Rozpr. Inz., 1956, No. 1, pp. 25-41.
7. A. SAWCZUK — a) *Linear transformation as applied in the theory of limit design of plates* (in Polish), Zesz. Nauk. Pol. Warsz., Budown. No. 3, 1955; b) *Problems of ultimate load carrying capacity of tensioned and bent structural elements* (in Polish), Rozpr. Inz. 1955, No. 2, pp. 255-284; c) *On practical applications of the limit analysis of plates* (in Polish), Arch. Inz. Lad., 1956, No. 1, pp. 139-183; d) *Some problems of simultaneous operation of concentrated and continuous loads in the theory of limit design of plates* (in Polish), Zesz. Nauk. Pol. Warsz. Budown. 1956, No. 6, pp. 41-59; e) *Some problems of load carrying capacity of orthotropic and non-homogeneous plates*, 9th Intern. Congr. Appl. Mech., Brussels 1956, and Arch. Mech. Stos. 1956, No. 4, pp. 549-564.

SUMMARY

The importance of a general theory of the ultimate load carrying capacity of anisotropic and, at the same time, non-homogeneous plates is stressed, for, as a rule, nearly all practically executed plates are both anisotropic and non homogeneous. Research work on the fundamentals of such a general theory, its results and their practical applications are discussed.

ZUSAMMENFASSUNG

Die wissenschaftliche und praktische Bedeutung einer allgemeinen Grenzlasttheorie von anisotropen und dabei gleichzeitig nicht homogenen Platten wird dargelegt; in der Tat werden fast alle Platten sowohl als anisotrope als auch als nicht homogene Konstruktionselemente praktisch ausgeführt. Arbeiten über die Grundlagen einer derart allgemeinen Theorie, ihre Resultate und deren praktische Anwendungsmöglichkeiten werden besprochen.

R E S U M O

Põe-se em evidência a importância de uma teoria geral da carga limite das placas anisotrópicas e, simultaneamente, não homogêneas; com efeito, quase todas as placas executadas na prática são caracterizadas pelas suas anisotropia e não-homogeneidade mecânicas. Discutem-se os trabalhos relativos à elaboração das bases dessa teoria geral, os seus resultados e as suas aplicações práticas.

R É S U M É

On met en évidence l'importance d'une théorie générale de la charge limite des plaques anisotropes et, en même temps, non-homogènes; en effet, presque toute réalisation pratique des plaques est marquée par leur anisotropie et non-homogénéité mécaniques. On discute les travaux portant sur les bases d'une telle théorie générale, ses résultats et leurs applications pratiques.

Leere Seite
Blank page
Page vide

V 7

Der Einfluss von Einzellasten auf die Festigkeit der Biegedruckzone

Influência das cargas concentradas na resistência da zona de compressão em flexão

Influence des charges concentrées sur la résistance de la zone comprimée en flexion

Influence of concentrated loads on the resistance of the compression zone in bending

PROF. DR. - ING. H. RÜSCH

Technische Hochschule

München

Bei der Prüfung von Balken durch 2 symmetrische Einzellasten entsteht in der Nähe der Lasteintragungsstelle eine spürbare Hebung der neutralen Achse. In Bild 1 kann man diese Hebung deutlich an dem Verlauf der oberen Enden der Risse erkennen. Nachstehend soll eine

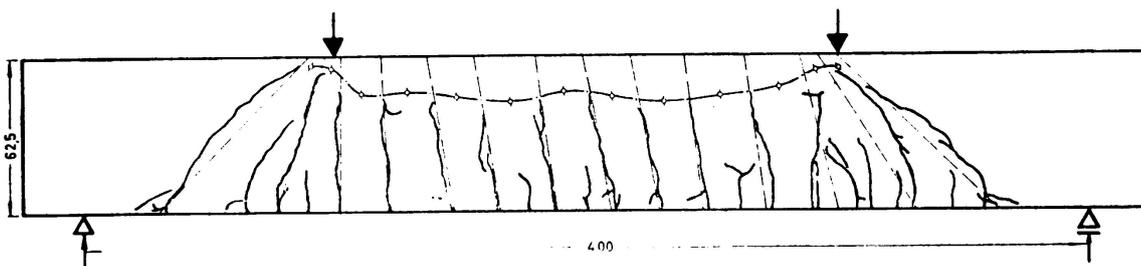


BILD 1

Erklärung für die Ursache dieser merkwürdigen Erscheinung versucht werden.

In dem zwischen den beiden Einzellasten liegenden Bereich wird ein solcher Balken durch ein konstantes Moment beansprucht. Deshalb verlaufen hier die Risse annähernd parallel. Unter dieser Voraussetzung kann man die Höhe der neutralen Achse mit Hilfe der auf die Längen-

einheit bezogenen Stauchung ε_b des Betons am Druckrand und der bezogenen Dehnung des Stahles ε_e in der Zugzone bestimmen. Es ist dann

$$x = \frac{\varepsilon_b}{\varepsilon_b + \varepsilon_e} h = \frac{h}{1 + \frac{\varepsilon_e}{\varepsilon_b}} \quad (\text{vergl. Bild 2})$$

Unter der Wirkung der Einzellast entsteht im Schubbereich ein rascher Übergang der senkrechten Biegerisse zu schrägliegenden Schubrissen. In diesem Bereich darf man die Lage der neutralen Achse nicht

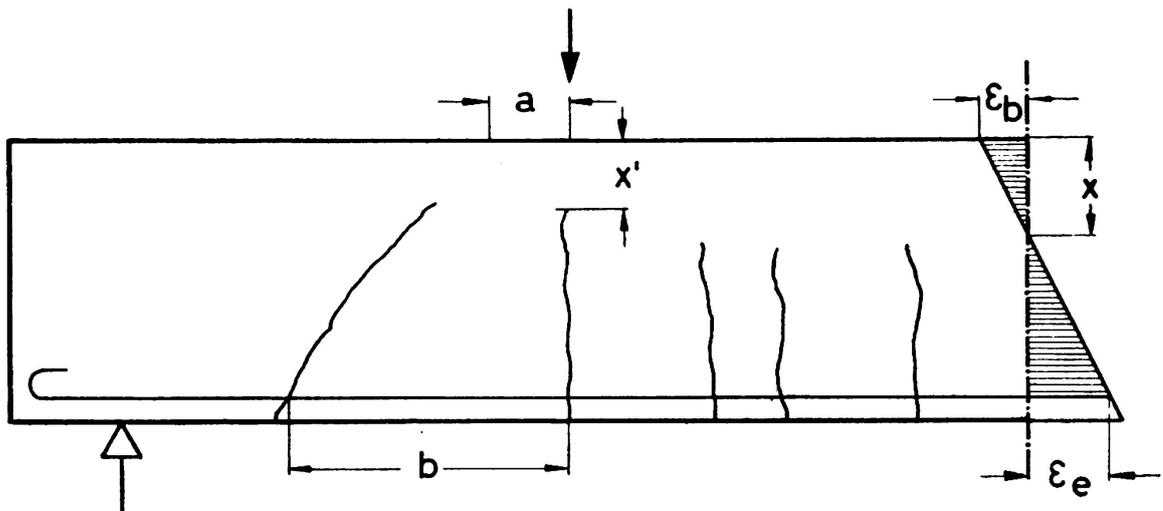


BILD 2

mehr aus den bezogenen Dehnungen bestimmen, sondern muss berücksichtigen, dass die zwischen den Rissen liegenden Balkenelemente keilförmig sind (vergl. Bild 2). Die Lage der neutralen Achse ergibt sich dann aus der Gleichung

$$x' = \frac{a \varepsilon_b}{a \varepsilon_b + b \varepsilon_e} h = \frac{1}{1 + \frac{b}{a} \frac{\varepsilon_e}{\varepsilon_b}} h = \frac{h}{1 + k \frac{\varepsilon_e}{\varepsilon_b}}$$

Der in der letzten Form dieser Gleichung erscheinende Faktor $k = \frac{b}{a}$ hängt von der Neigung der Risse ab. Um ihn zu bestimmen, braucht man nur dem Rissbild die Abstände a und b zu entnehmen (vergl. Bild 3). Mit Hilfe von k kann man den ganzen Verlauf der neutralen Achse berechnen. Die Bilder 1 und 3 zeigen das Ergebnis einer solchen Untersuchung. In Feldmitte und an der Stelle der Einzellast sorgfältig durchgeführte Dehnungsmessungen für einen Belastungsgrad, der etwa 75 % der Bruchlast entspricht, haben die Richtigkeit dieser Werte bestätigt. Es zeigte sich dabei, dass die Risse nicht ganz bis an die gemessene Lage der neutralen Achse heranreichten.

Sehr interessante Ergebnisse liefert eine Untersuchung der Abhängigkeit des k-Wertes vom Belastungsgrad. Bis zum Beginn der Rissbildung ist $k = 1$, wächst aber darüber mit steigender Last rasch an.

Dadurch entsteht die schon beschriebene Krümmung der neutralen Achse. In der Nähe der Bruchlast kann man aber wieder ein Strecken der neutralen Achse beobachten. Die Ursache hierfür kann nicht in einer Veränderung des Beiwertes k gesehen werden. Man muss vielmehr annehmen, dass die Bruchstauchung ε_b unter der Einzellast wesentlich grösser werden kann als in Balkenmitte und zwar wegen der Querpressung, welche die Belastungsplatten in der Druckzone erzeugen. Es entsteht dadurch ein zweiachsiger Spannungszustand, der ein Ausbrechen des Betons verhindert und auf diese Weise ein ähnliches Anwachsen der Verformung erlaubt, wie man es in einer umschnürten Säule beobachten kann.

Trotz dieser Erscheinung wird in der Nähe des Bruches die Druckzone unter der Einzellast immer noch kleiner sein als in Balkenmitte. Ausser-

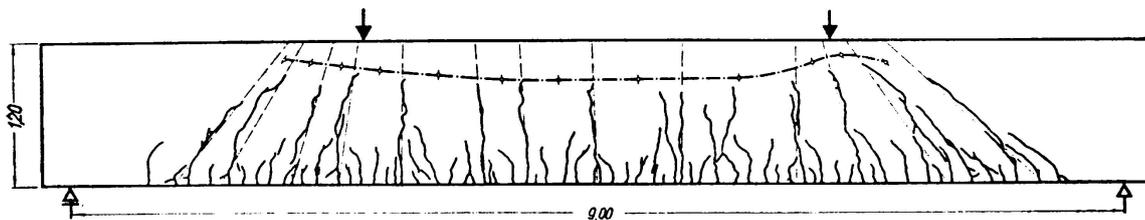


BILD 3

dem erhält die Druckzone an dieser Stelle noch eine erhebliche Beanspruchung durch die Querkraft. Daraus erklärt sich, dass der Bruch bei solchen Balkenversuchen meist unmittelbar unter der Einzellast erfolgt, obwohl die beschriebene Querpressung eine spürbare Erhöhung der Festigkeit der Druckzone herbeiführen kann.

ZUSAMMENFASSUNG

Wenn ein Stahlbetonbalken durch Einzellasten beansprucht wird, beobachtet man eine spürbare Anhebung der neutralen Achse in der Umgebung der Einzellasten. Für diese Erscheinung wird eine Erklärung gegeben.

RESUMO

O autor explica a razão pela qual, numa viga de betão armado solicitada por uma carga concentrada o eixo neutro sofre, na vizinhança da carga, um deslocamento sensível na direcção da fibra superior.

RÉSUMÉ

L'auteur explique la raison pour laquelle, dans une poutre de béton armé soumise à une charge concentrée, l'on observe un soulèvement de l'axe neutre au voisinage de la charge.

SUMMARY

The author explains the reason why, in a reinforced concrete beam submitted to a concentrated load, the neutral axis is moved upwards in the neighbourhood of the load.

Leere Seite
Blank page
Page vide

V 8

The safety of prestressed concrete as affected by creep and fatigue

Discussion

Die Sicherheit beim vorgespannten Beton unter Berücksichtigung des Kriechens und der Materialermüdung

Diskussion

Efeito da fluência e da fadiga sobre a segurança das estruturas de betão preesforçado

Discussão

Influence du fluage et de la fatigue sur la sécurité du béton précontraint

Discussion

A. J. HARRIS

B. Sc (Eng.) M. I. C. E.

Consulting Engineer

London

Professor Freudenthal's remarks on the safety of prestressed concrete as affected by creep and fatigue are of a speculative nature.

Now speculation is of value in inverse proportion to the number of facts available.

It so happens that there is a large body of experimental facts concerning creep of concrete, cracking load and fatigue strength of concrete beams in existence; these facts have been built up continuously over the last 30 years and more. Speculation on these subjects which takes no account of these facts is impermissible.

In particular, Professor Freudenthal suggests that a prestress of more than 0.25 fc will result in loss of security against cracking. It is common practice to prestress to much higher stress values than this; if Professor Freudenthal believes that such stress values are harmful to security against cracking he must prove it, either by quoting tests which have already been carried out (and we have noted that there are many such) or by arranging tests himself.

The divergence between fact and speculation is even more startling when we consider Professor Freudenthal's remarks on fatigue. An appre-

ciable number of fatigue tests have been carried out, very few have caused fatigue failure and their general import seems to be that the fatigue strength of prestressed concrete relative to its static rupture strength is higher than that of any other structural material. It may be noted in passing that it is rare that there is any appreciable variation in stress in the anchorage zones of prestressing tendons.

S U M M A R Y

Professor Freudenthal's statement on the safety of prestressed concrete as affected by creep and fatigue is indefensible.

His remarks on rupture and working loads are particularly startling and are contrary to all experimental results obtained so far.

ZUSAMMENFASSUNG

Die von Professor Freudenthal vertretene Auffassung betreffend der Sicherheit von vorgespanntem Beton im Zusammenhang mit dem Kriechen und oft wiederholter Beanspruchung ist nicht haltbar.

Im Besonderen sind die Bemerkungen über die Bruchsicherheit und die Arbeitsfestigkeit erstaunlich und widersprechen jeden bisherigen Versuchsergebnissen.

R E S U M O

A comunicação do Professor Freudenthal acerca do efeito da fluência e da fadiga sobre a segurança das estruturas de betão preesforçado não é defensável.

As observações apresentadas acerca das cargas de rotura e das cargas de serviço são particularmente surpreendentes e contradizem todos os resultados experimentais obtidos até hoje.

R É S U M É

La communication du Professeur Freudenthal concernant l'influence du fluage et de la fatigue sur la sécurité du béton précontraint n'est pas défendable.

Les observations présentées sur les charges de rupture et de service sont particulièrement surprenantes et contredisent tous les résultats expérimentaux obtenus jusqu'à présent.

V 9

Risques de fissuration et de rupture dans le béton précontraint

Danger of crack formation and collapse in prestressed concrete

Risse - und Bruchrisiko in vorgespannten Beton

Risco de fissuração e rotura do betão preesforçado

R. LÉVI

Directeur à la S. N. C. F.

Paris

Je voudrais seulement présenter une observation que m'a suggérée la lecture des rapports préliminaires et l'audition des exposés.

A l'origine du béton précontraint, on s'est beaucoup préoccupé du risque de fissuration. On reconnaît maintenant que l'apparition de fissures fines n'est pas suffisante pour produire des désordres et laisse subsister une large marge de déformation avant la rupture. Il semble donc que l'on puisse reprendre la détermination des règles de sécurité en reportant presque uniquement l'attention sur le risque de ruine.

Il y a quelques années, j'avais constaté qu'on arrivait à des résultats à peu près équivalents en cherchant

- pour la ruine, une probabilité de 1/100 000
- pour la fissuration occasionnelle dans les conditions les plus défavorables, une probabilité de 1/1 000.

Mais cette dernière considération introduit parfois des restrictions que j'estime maintenant exagérées. Il semblerait possible d'admettre pour la fissuration occasionnelle une probabilité de l'ordre de 1/10, ce qui permettrait souvent des diminutions du prix de revient sans qu'il en résulte un accroissement véritable du risque.

R É S U M É

Tandis que l'on s'était, tout d'abord, beaucoup préoccupé du risque de fissuration, il est actuellement admis qu'entre l'apparition de fissures fines et la rupture il existe une grande marge de déformation. Les règles

de sécurité pourraient donc être définies presque uniquement à partir du risque de rupture, en cherchant pour celle-ci une probabilité de 1/100.000. Pour la fissuration occasionnelle on peut rechercher une probabilité de 1/10.

S U M M A R Y

While at first the danger of crack formation was considered as most important, it is now admitted that between the formation of fine cracks and collapse lies a wide margin of deformation. Safety rules could therefore be defined almost exclusively from the risk of collapse aiming at a probability of 1/100,000. For the occasional crack formation a probability of 1/10 could be aimed at.

ZUSAMMENFASSUNG

Während früher der Rissegefahr besondere Beachtung geschenkt wurde, erkennt man heute, dass zwischen dem Auftreten von feinen Rissen und dem Bruchzustand eine grosse Deformationsspanne liegt. Es scheint daher zu genügen, die Sicherheit auf den Bruchzustand allein zu definieren. (Wahrscheinlichkeit 1:100.000). Für die Rissebildung dürfte eine Wahrscheinlichkeit von 1:10 zulässig sein.

R E S U M O

Ao passo que anteriormente se considerava o risco de fissuração como muito importante, admite-se actualmente que entre a aparição das fissuras finas e a rotura existe uma grande margem de deformação. As regras de segurança poder-se-iam portanto definir considerando quase unicamente o risco de rotura, procurando para esta uma probabilidade de 1/100.000. Para a fissuração ocasional pode procurar-se uma probabilidade de 1/10.

V 10

Methode de calcul de hourdis de ponts en béton précontraint

Method of calculation of prestressed concrete bridge slabs

**Berechnungsmethode von Brückenträgern aus
vorgespanntem Beton**

Método de cálculo de lages para pontes de betão preesforçado

Y. GUYON

Paris

Cette méthode de calcul dérive des principes faisant l'objet de la communication V. c. 2. de la Publication Préliminaire, et des considérations suivantes:

1. – La plupart des règlements interdisent la prise en compte de la résistance à la traction. Ceci peut être justifié pour une poutre (défaut local), mais non pour une dalle, une fissure ne pouvant s'ouvrir sans mettre en jeu la résistance à la traction des bandes adjacentes.
C'est donc la résistance moyenne, et non la résistance minimum qui compte.
De ce fait, la probabilité d'une fissure sous une contrainte égale à environ $1/3$ de la résistance moyenne à la traction ne dépasse pas l'ordre de grandeur tolérable.
2. – Au cas où cette fissure se produirait, elle resterait très fine et le câble ne peut être en danger si elle n'atteint ou ne dépasse pas le niveau de celui-ci.
3. – Si ces fissures se produisaient au centre, il en résulterait une modification du fonctionnement, avec augmentation des moments sur appuis.

L'expérience montre que sous une augmentation de charges, l'épaisseur de la fissure n'augmente pas au centre; elle ne devient visible que lorsque l'appui se trouve à son tour sur le point de se fissurer par suite de cette augmentation de moment.

4. – Dès que cette double fissuration, non dangereuse, se produirait (au centre et sur appui), un nouveau système porteur serait mis en jeu. Ce sont les bielles de compression dans le sens de la portée, issues de la charge et prenant appui sur les bords, que mentionne la communication précitée.
5. – La charge à laquelle la dalle peut résister sans fissuration préjudiciable (et souvent sans fissuration visible, tout au moins à l'oeil nu) grâce à la résistance de ce nouveau système porteur est notablement supérieure à celle correspondant à la fin de la phase 3. (Dans nos essais elle a atteint le double, sous une charge concentrée au milieu).
6. – Il reste encore une réserve de résistance très importante entre la fin de la phase 5 et la rupture (coefficient de l'ordre de 3 dans nos essais).

Nous dimensionnons nos dalles de façon à nous placer au début de la phase 4, dans le cas où les probabilités joueraient d'une façon défavorable dans la phase 1 et en conséquence dans les phases 2 et 3;

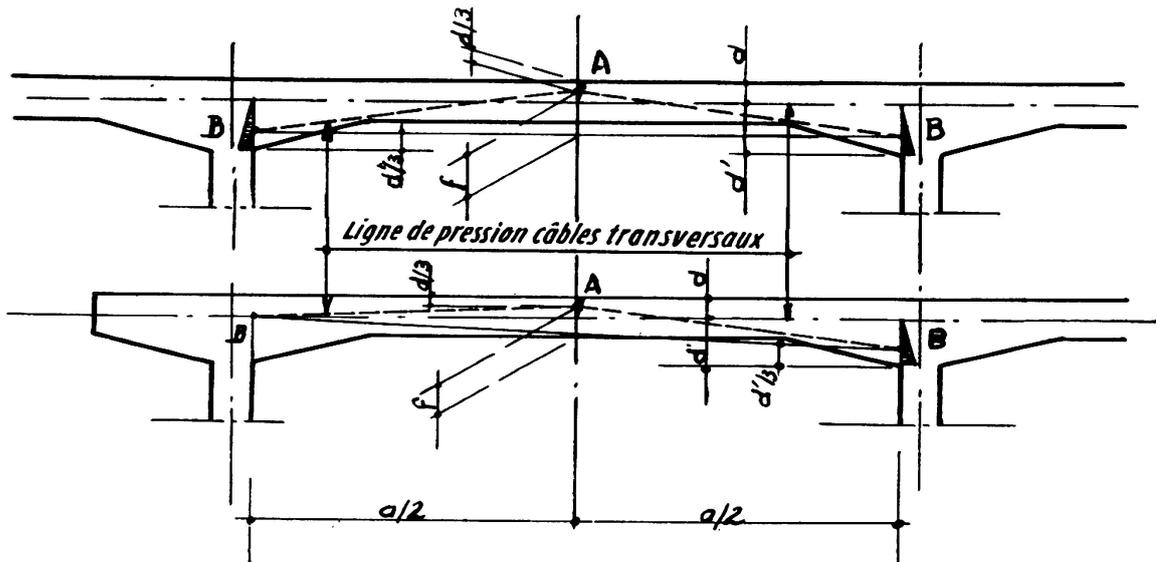


FIG. 1

c'est-à-dire que nous supposons que les bielles 4 commencent à fonctionner.

Nous admettons conventionnellement que la résistance à la traction est nulle et que, dans cette hypothèse la fissure atteint le niveau du câble.

Nous admettons d'autre part que le système de bielles, soustendu par la précontrainte, équilibre les moments qui se produiraient dans la dalle articulée de mêmes portées que la dalle.

La fig. 1 représente les axes de bielles admis: (a) dans le cas d'une dalle courante (points de passage sur appuis au niveau du 1/3 inférieur du diagramme triangulaire de contrainte atteignant le câble); (b) dans

le cas d'une dalle de rive (point de passage du câble). Ces schémas donnent la flèche f de la ligne de pression; d et d' étant les distances du câble aux parois: supérieure au centre, inférieure à l'appui, $f = \frac{2}{3} (d + d')$ dans le cas a; $f = \frac{2}{3} d + \frac{1}{3} d'$ dans le cas b.

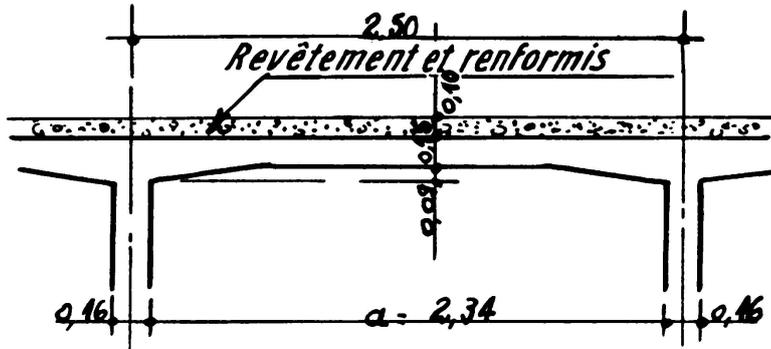
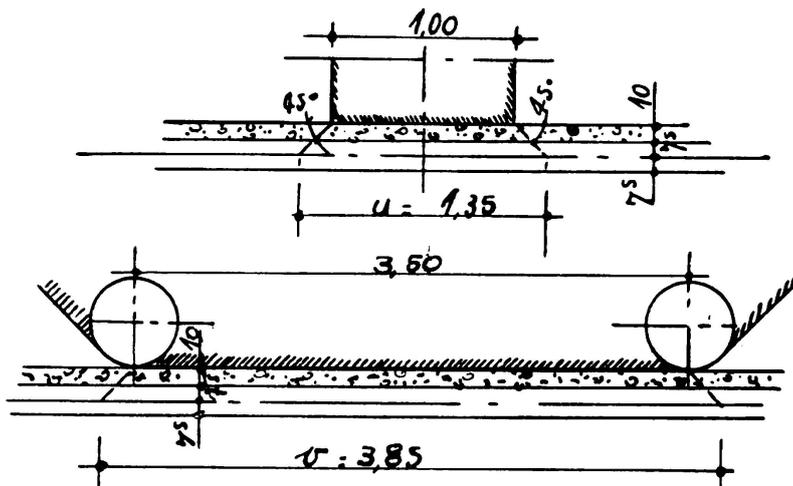


FIG. 2

Connaissant par les abaques usuels le moment M par unité de longueur, l'effort F de précontrainte nécessaire par unité de longueur est $F = \frac{M}{f}$. D'autre part pour que la contrainte admissible R du béton ne soit pas dépassée il faut que $\frac{F}{d} < \frac{R}{2}$.

EXEMPLE. — Panneau courant de pont suivant fig. 2. —

Entr'axe poutres 2,50 m; entr'axe entretoises 9,60 m; Charges permanentes par m² (hourdis et renformis): 0,580 t/m². Surcharges S sui-



$$u = 1,00 + (2 \times 0,10 + 0,15) = 1,35$$

$$v = 3,50 + (2 \times 0,10 + 0,15) = 3,85$$

FIG. 3

vant fig. 3: $S = 40$ t. majorations pour effet dynamique comprises, appliquée sur un rectangle (chenille) de $1,00 \times 3,50$ m.

$$\text{Flèche } f \text{ (pour une dalle courante): } f = \frac{2}{3} (0,075 + 0,075 + 0,09) = 0,16 \text{ m.}$$

$$\text{Moments: Portées nettes: } a = 2,34 \text{ m. } b = 9,36 \text{ m. } \frac{a}{b} = 0,25$$

Côtés u et v du rectangle chargé (sur le feuillet moyen) $u = 1,35$ m.
 $v = 3,85$ m.

Calcul par les abaques PIGEAUD: M_1 et M_2 désignant les moments par unité de largeur dans le sens de la portée et dans le sens du grand axe (coefficient de POISSON négligé).

Sous charges permanentes ($a \times b = 21,90$ m²).

$$P = 21,9 \times 0,580 = 12,7 \text{ t; } M_1 = 12,7 \times 0,030 = 0,381 \text{ t } M_2 = 0.$$

Sous surcharges ($S = 40$ t.).

$$M_1 = 40 \times 0,090 = 3,6 \text{ t } M_2 = 40 \times 0,012 = 0,48 \text{ t}$$

$$\text{Total: } M_1 = 0,381 + 3,6 = 3,981 \text{ t } M_2 = 0,48 \text{ t}$$

Du fait de la flexion longitudinale du tablier, le hourdis est soumis à une compression longitudinale telle que la décompression due au moment M_2 n'est généralement pas dangereuse. Nous vérifierons uniquement la résistance sous le moment M_1 .

$$\text{Effort de précontrainte } F \text{ nécessaire: } \frac{M}{f} = \frac{3,981}{0,16} = 24,9 \text{ t/m. lin.}$$

$$\text{Contrainte } n \text{ du béton (} d = 7,5 \text{ cm) } n = \frac{24,900 \times 2}{100 \times 7,5} = 66,4 \text{ k/cm}^2.$$

Dans le cas où la charge peut se trouver sur un panneau de rive, $f = \frac{2}{3} \times 0,075 + \frac{1}{3} \times 0,065 = 0,105$.

$$F = 37,9 \text{ t/m. lin. et } n = 100 \text{ k/cm}^2$$

REMARQUE

Nous vérifions ainsi que, même si le fonctionnement supposé, qui admet la fissuration, se produisait, il resterait une large sécurité vis à vis de la fissuration préjudiciable.

R É S U M É

Méthode de calcul de hourdis tenant compte de la possibilité d'un deuxième fonctionnement élastique.

S U M M A R Y

Method of calculation of a slab taking into account the possibility of a second elastic behaviour.

ZUSAMMENFASSUNG

Berechnungsmethode von Brückenträgern unter Berücksichtigung der Möglichkeit eines weiteren elastischen Verhaltens.

R E S U M O

Método para o cálculo de lages entrando em conta com a possibilidade de existência de um segundo comportamento elástico.

Leere Seite
Blank page
Page vide