Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 5 (1956)

Artikel: A rheological model for concrete

Autor: Torroja, E. / Paez Balanca, A.

DOI: https://doi.org/10.5169/seals-5972

I a 8

A rheological model for concrete

Ein rheologisches Modell für Beton

Um modelo reológico para betão

Un modèle rhéologique pour béton

E. TORROJA — *Prof. Dr. h. c.*
A. PAEZ BALACA — *Ingeniero de Caminos*
Madrid

Reinforced concrete, and especially prestressed concrete, technology, requires every day a more detailed knowledge of strain phenomena, or more generally, of the rheological behaviour of the material.

The non-elastic deformation of concrete is so important that in many calculations and projects it is essential to take it into account. This requires the availability of mathematical expressions capable of describing with sufficient accuracy and generality the laws relating the non-elastic deformations and their causes.

Unfortunately the ultimate and detailed causes of this type of phenomena such as their physical, of physico-chemical origin are not yet known. Besides, the technological laws obtained by various research workers are too varied to be used as a basis for a general theory.

Both shrinkage and non-elastic deformations — and even elastic deformation itself — depend on so many variable factors that, for the present, it is practically impossible to determine the influence of each of them. Each factor affects the others, and the resulting complexity makes it very difficult, if not altogether impossible to obtain general results by means of straightforward technological research on concrete test specimens, whatever the number of tests.

This is emphasized if it is realised that the results are influenced at least by the following factors: the chemical and physical composition of the cement, the water/cement ratio, the proportioning of aggregate, the shape of the pebbles, their petrographic composition, the type of curing and conservation, the ambient humidity at various times of the year, the temperature, the shape and size of the test specimens, the intensity of loading applied during the life history of the concrete related to the age of the material at the time of loading, and to the other variables. It is clear that the interaction and mutual influence of all these variables so complicates the phenomenon that it is almost impossible both to collect enough experimental data to cover all the very complicated phenomena, and then to disentangle this multiplicity of experimental data, into an ordered picture.

If the basic cause of these phenomena is to be found in the cement/ /water paste, it will be logical to look into what happens within the paste, avoiding at least, all the additional factors arising from the presence of the aggregate. The lack of homogeneity and the slowness caused by diffusion phenomena within the test piece can be avoided by experimenting on the smallest possible specimens. Indeed, attempt must be made to achieve a detailed discovery of the essential phenomena affecting the compounds of the set cement by making full use of the powerful research techniques available in physics and physico-chemistry.

Only after the rheological laws of paste are well known, as functions of the cement/water ratio, age, and hygrometric and thermic history, does there seem to be any possibility of investigating sucessfully the more complex case of normal sized mortar and concrete test specimens.

A great deal has been achieved in this initial stage of research. Yet what remains to elucidate is so much and the path of further progress so tortuous and intricate that there is always danger of never achieving final sucess, due to lack of perseverance. It is possible, also, that due to the exceptionally rapid growth of technology, this ultimate success may only be obtained when constructional methods are orientated towards new materials, thus causing this type of research to lose interest.

On the other hand, in its present stage of development, constructional technique, although continuously setting itself a higher standard of accuracy, is not so advanced in this respect that it can afford to ignore any attempt to establish approximate general laws, however rough this first approximation may be.

Consequently, fortified by the courage of my own ignorance, I make bold to present this rheological theory. It must be taken as a pastime, without other value than to serve to emphasize the merit of other papers.

In accordance with experimental results, it may be assumed that in a short-time compression test to destruction the strain δ increases more rapidly that the stress T. It may also be assumed that the divergence from the hookean law becomes greater as stresses approach their failing value, until finaly, just before failure strain increases indefinitely without an increment of stress. If these experimental facts are admitted it is easy to establish a general law for the phenomenon. It suffices to measure off along the ordinates the relative stresses: $\sigma = T/R$ (i. e., the ratio of the actual stress T to the maximum failing stress R of the material), and along the abscissae the relative strain $\varepsilon = \delta/\Delta$ (i. e., the ratio of the actual strain to the strain corresponding to the maximum stress R, mentioned previously). In the resulting diagram the values obtained by various research workers can be aligned (with no important scattering) along a parabola, whose degree is between 1.8 and 2.5 (fig. 1).

For the moment, to simplify the expressions, let it be assumed that this parabola is of degree 2, so that the law may be written thus:

$$1 - \varepsilon = (1 - \sigma)^{\frac{1}{2}} \tag{1}$$

In this expresion rupture corresponds to the values $\sigma = 1$ and $\varepsilon = 1$. Hence the corresponding stress/strain relations, according to Hooke's law, will be given by

$$\varepsilon_E = \frac{\sigma}{2}$$

and the non Hookean strain will be

$$\varepsilon_n = \varepsilon - \varepsilon_E = [1 - (1 - \sigma)^{\frac{1}{2}}] - \frac{\sigma}{2} = \frac{1}{2}(1 - \sqrt{1 - \sigma})^2 \tag{2}$$

From the few experimental data available it appears that this type of strain remains constant when the duration of the test is reduced (this reduces the breaking strain as well), so it can be extrapolated until the test duration tends to zero, provided inertia phenomena are neglected, which is merely a theoretical consideration.

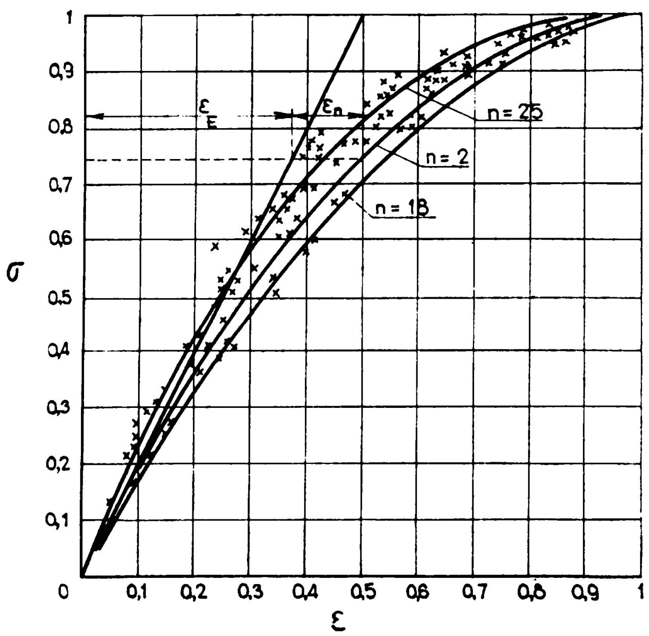


FIG. 1

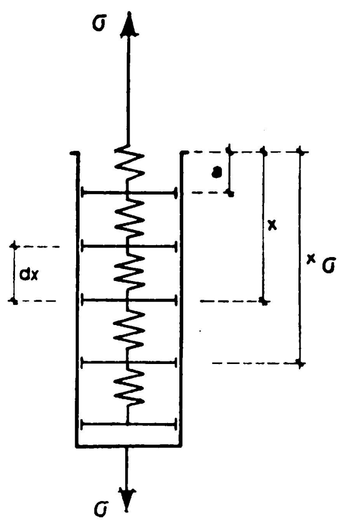


FIG. 2

Besides this, when the load is removed a residual strain, or permanent set, remains, which increases steeply for increasing applied relative stresses. All this leads one to suppose that this non Hookean type of strain may be expressed as a function of frictional forces, which vary with the stress.

Use will now be made of a physical model, but only with the purpose of making it easier to work out the mathematical treatment. Let us think of a basic unit (fig. 2) consisting of a cylinder, inside which there are a number of discs. Each of these discs has a different friction coefficient with the cylinder wall. The discs are connected one to another by means of equal springs. If a force σ is applied to the mechanism, as shown on fig. 2, the first spring will be subject to a tensile force

$$(\sigma_m)_1 = \sigma - r_1$$

where r_1 is the frictional force acting on the first disc. Behind the nth disc the tensile force will be

$$(\sigma_m)_n = \sigma - \sum_{i=1}^{n} r_i$$

In order to adopt the continuity of change required by differential calculus, it may by supposed that the frictional forces operating on each disc, as well as the lenght of the springs, tend to zero, so that the force on the spring, at a distance x_1 from the top will be

$$\sigma_m = \sigma - \int_0^{x_1} r(x)\, dx$$

If for the sake of generality, it is is supposed that a number of discs at the top of the cylinder have no friction, then the previous expression will become

$$(\sigma_m)_{x_1} = \sigma - \int_a^{x_1} r(x)\, dx$$

where $r(x)\, dx$, is the elementary frictional force operating on the differential disc at any depth x, so that $a < x < x_1$

If we suppose, to express the frictional force function, the simplest law:

$$r(x) = bx^{\nu}$$

Then the former expression becomes

$$\sigma_m = \sigma - \int_a^{x_1} bx^{\nu}\, dx = \sigma - \frac{b}{\nu+1} x_1^{\nu+1} + \frac{b}{\nu+1} a^{\nu+1}$$

and its value vanishes for

$$x_1 = x_\sigma = \left(\frac{\nu+1}{b} \sigma + a^{\nu+1} \right)^{\frac{1}{\nu+1}}$$

I. e., under the action of the force σ only the springs situated up to a depth x_1, less than x_σ, experience any strain or movement from their original position. In these circumstances the total strain of the system is:

$$
\begin{aligned}
\varepsilon &= \frac{1}{E} a\sigma + \int_a^{x_\sigma} \left[\sigma - \int_0^{x_1} b x^\nu \, dx \right] \frac{dx_1}{E} = \\
&= \frac{b}{(\nu+2)E} \left(\frac{\nu+1}{b} \sigma + a^{\nu+1} \right)^{\frac{\nu+2}{\nu+1}} - \frac{b}{E} \frac{a^{\nu+2}}{\nu+2}
\end{aligned}
\tag{3}
$$

If the sum of all the differential frictional forces is equated to the maximum failing stress $\sigma = 1$. Then:

$$
\int_a^\infty b x^\nu \, dx = 1 \qquad\qquad b = -\frac{\nu+1}{a^{\nu+1}}
$$

Let the further condition be imposed that $\varepsilon = 1$ for $\sigma = 1$; this gives on simplification

$$
\varepsilon = 1 - (1-\sigma)^{\frac{\nu+2}{\nu+1}}
$$

i. e.,

$$
1 - \varepsilon = (1-\sigma)^n
$$

This agrees with expression (1) if $n = 1/2$, so that the following values are obtained:

$$
E = 1 \qquad b = a = \frac{1}{2} \qquad \nu = 3
$$

The Hookean strain,

$$
\varepsilon_E = \frac{\sigma}{E} a = \frac{\sigma}{2}
$$

is pictured by the set of springs and discs without friction, whilst the non Hookean deformation, given by

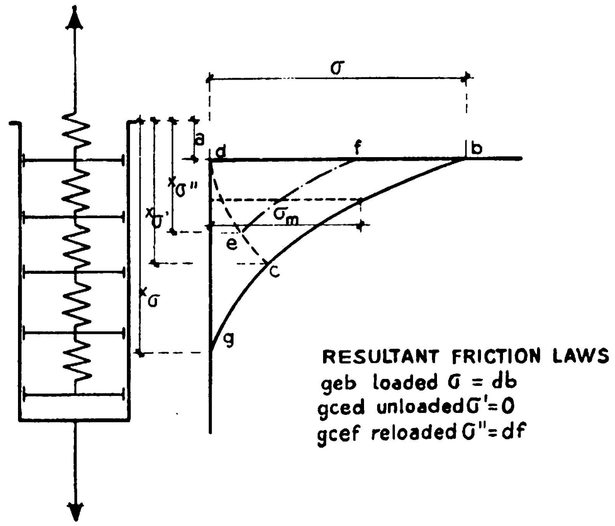
$$
\varepsilon_n = \varepsilon - \varepsilon_E = 1 - (1-\sigma)^{\frac{1}{2}} - \frac{\sigma}{2}
\tag{4}
$$

is produced by the system of springs and disc submitted to friction.

This correspondence would have no particular interest, if it were not that the proposed model makes it also possible to describe the behaviour of concrete under succesive loading and unloading cycles.

For in fact, if after attaining a load σ this is progressively removed, until finally no load is applied, after having passed through intermediate loadings σ', the springs tend to return to their initial positions. Now the frictional forces act in the opposite sense, and the length x_σ, (fig. 3) over which, the recovery motion of the discs is produced, overcoming these frictional reactions is given by the equation:

$$\sigma - \int_a^{x_{\sigma'}} r_x \, dx = \sigma' + \int_a^{x_{\sigma'}} r_x \, dx \tag{5}$$



This gives a new value for x

$$x_{\sigma'} = [4 - 2(\sigma - \sigma')]^{\frac{1}{2}} - \frac{1}{2} \tag{6}$$

FIG. 3

Hence the final strain, including the hookan strains $\frac{\sigma'}{2}$ will be

$$\varepsilon' = \frac{\sigma'}{2} + \int_a^{x_{\sigma'}} [\sigma' + \int_a^x r_x \, dx] \, dx + \int_{x_{\sigma'}}^{x_\sigma} [\sigma - \int_a^x r_x \, dx] \, dx \tag{7}$$

The first term is the hookean strain, the second term is the strain between a and $x_{\sigma'}$ and the third term the strain between $x_{\sigma'}$ and x_σ.

After integrating and simplifying this reads

$$\varepsilon' = 2\left(1 - \frac{\sigma - \sigma'}{2}\right)^{\frac{1}{2}} - 1 - (1 - \sigma)^{\frac{1}{2}} \tag{8}$$

When the load is totally removed, there remains a residual strain or permanent set:

$$\varepsilon'_0 = 2\left(1 - \frac{\sigma}{2}\right)^{\frac{1}{2}} - 1 - (1 - \sigma)^{\frac{1}{2}} \tag{9}$$

This corresponds to a value of x given by $x_{\sigma'} = o = (4 - 2\sigma)^{-\frac{1}{2}} - \frac{1}{2}$.

If after being unloaded, the model is again loaded, for increasing values of $\sigma'' < \sigma$ the position of the first disc which is not displaced will be at a depth $x_{\sigma''}$, defined by the condition

$$\sigma'' - \int_o^{x_{\sigma''}} r_x \, dx = \sigma' + \int_o^{x_{\sigma''}} r_x \, dx \tag{10}$$

From this $x_{\sigma''}$ is given explicitly by

$$x_{\sigma''} = [4 - 2(\sigma'' - \sigma')]^{-\frac{1}{2}} - \frac{1}{2} \tag{11}$$

In order to obtain the total strain ε'' produced by this reload process, it is useful to suppose the model divided in three ranges. In the first, corresponding to depths limited by x_σ and $x_{\sigma'}$ the stress at depth x, where $x_\sigma > x > x_{\sigma'}$ is:

$$\sigma_m = \sigma - \int_a^x r_x \, dx$$

Whilst if $x_{\sigma'} > x > x_{\sigma''}$, the stress becomes

$$\sigma_m = \sigma' + \int_a^x r_x \, dx$$

and if $x_{\sigma''} > x > a$

$$\sigma_m = \sigma'' - \int_a^x r_x \, dx$$

The total strain ε'' will be the sum of deformations due to these three ranges of stress, together with the hookean strain produced by the load σ'' finally applied. Hence according to (3):

$$\int_{x_{\sigma'}}^{x_\sigma} [\sigma - \int_a^x r_x \, dx] \, dx + \int_{x_{\sigma''}}^{x_{\sigma'}} [\sigma' + \int_a^x r_x \, dx] \, dx + \int_a^{x_{\sigma''}} [\sigma'' - \int_a^x r_x \, dx] \, dx + \frac{\sigma''}{2} \tag{12}$$

Working out the integrals and simplifying, this becomes

$$\varepsilon'' = 1 + [4 - 2(\sigma - \sigma')]^{\frac{1}{2}} - [4 - 2(\sigma'' - \sigma')]^{\frac{1}{2}} - (1 - \sigma)^{\frac{1}{2}} \tag{13}$$

If the load $\sigma' = o$ is made zero (i. e, the device is entirely unloaded before reloading), 13. becomes

$$\varepsilon''_{\sigma' = o} = 1 + (4 - 2\sigma)^{\frac{1}{2}} - (4 - 2\sigma'')^{\frac{1}{2}} - (1 - \sigma)^{\frac{1}{2}} \tag{14}$$

If during the second loading up cycle, the load surpasses the value reached in the first cycle, the stress-strain diagram, in this reloading cycle, is the same as if the load were applied for the first time when, $\sigma'' > \sigma$

If the diagrams $\sigma = f(\varepsilon)$ resulting from these expressions are expressed graphically, fig. 4 is obtained. This shows narrow hysteresis cycles in the unloading and reloading processes. The mean slopes of these repeated diagrams increase slightly as the total stress increases. When stresses are low this slope coincides with the initial elasticity modulus ($E=1/2$). All this agrees closely with experimental data.

This same imaginary device may also serve to explain creep phenomenae, to which concrete under long periods of compressive stress is subjected.

In fact, so far, data is still very incomplete as to how this plastic flow varies in terms of the applied stress. But available information indicates that for the same intervals of time and equal increments of relative stress, the greater the relative stress, the greater the increment of creep. In the absence of greater detail of information, it appears advisable to assume that creep is the same as for short time strains, when the duration of loading tends to zero. If this were not so, it would only be necessary to change the law of variation of friction, as adopted previously. Though actually lacking experimental justification, it seems better to accept the earlier law of variation.

Evidently it is now necessary to take into account the viscosity. Consequently the unit previously adopted will be completed by adding

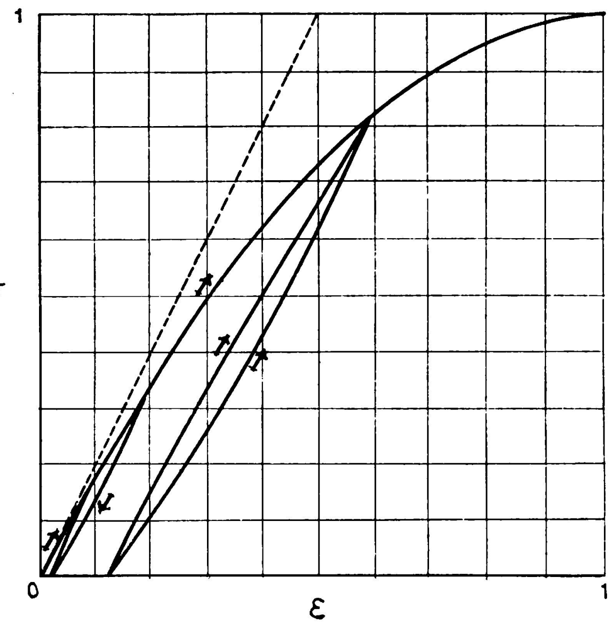


FIG. 4

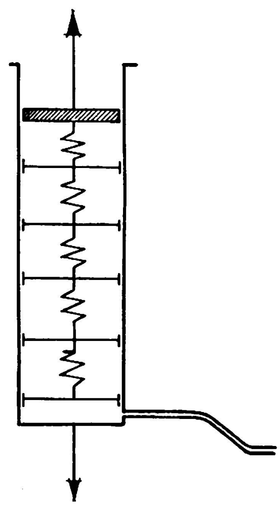


FIG. 5

to it a piston without friction. Its movement will force the liquid out of the cylinder through one orifice. This will define one single viscosity constant for the unit. This unit will be connected in series with the earlier creepless system so that the new deformations that will now occur in the course of time will be added to those already recorded previously.

Let σ_v be the part of the load σ that actuates on the viscous liquid (here again inertia forces will be neglected). Then the behaviour of an inelastic creepless unit, such as that already considered, with variable friction and subject to a double deformation coefficient (i. e., the spring is assumed. for the sake of convenience in later operations to be twice as flexible) implies the following result, obtained from (2) or (4) :

$$\varepsilon_L = [1 - \sqrt{1 - (\sigma - \sigma_v)}]^2 \tag{15}$$

Hence

$$\sigma - \sigma_v = 2\sqrt{\varepsilon_L} - \varepsilon_L \tag{16}$$

where $\sigma_v = (2\eta)\dfrac{d\varepsilon}{dt}$

Here t is the variable referring to the loading time and (2η) is a viscosity constant (1).

Hence the following differential equation is obtained:

$$\sigma = 2\sqrt{\varepsilon_L} + \varepsilon_L + (2\eta)\frac{d\varepsilon_L}{dt} \tag{17}$$

Its solution is

$$e^{-\frac{t}{2\eta}\sqrt{1-\sigma}} = \frac{\sqrt{\varepsilon_L}(1+\sqrt{1-\sigma}) - \sigma}{\sqrt{\varepsilon_L}(1+\sqrt{1-\sigma}) + \sigma} \times \left(\frac{\sigma}{\varepsilon_L - 2\sqrt{\varepsilon_L} - \sigma}\right)^{\sqrt{1-\sigma}} \tag{18}$$

when applying the limiting condition that $\varepsilon_L = o$ when $t = o$.

This expression can also be written in the form

$$e^{\frac{t}{2\eta}} = \frac{\left(\dfrac{1+\zeta-\sqrt{\varepsilon_L}}{1+\zeta}\right)^{\frac{1}{\zeta}+1}}{\left(\dfrac{1-\zeta-\sqrt{\varepsilon_L}}{1-\zeta}\right)^{\frac{1}{\zeta}-1}} \tag{19}$$

where $\zeta = \sqrt{1-\sigma}$
and so, for $t = o$. there corresponds the value $\varepsilon_L = o$.

If σ is assumed to be constant, there is a value of ε_L and hence, according to (16), also a value of $(\sigma - \sigma_v)$, which correspond to each value of t. Fig. 6 illustrates graphically this relationship.

There are four distinct cases that may be considered when the initially applied force σ_1 varies. Let t_1 be the duration of action of the first load σ_1, and let σ_2 be the load subsequently applied.

(1) Following general practice (2η) is taken, instead of η Normally the dimensions of viscosity are (velocity) $\times$ (stress $\times$ length) $-1 =$ (stress $\times$ time) -1. But since in our case stress is relative stress, and η is the inverse of the viscosity, the dimension of η is time.

According to the expressions already established, the deformation ε_1 obtained after this first period t_1 will satisfy the condition.

$$e^{\frac{t_1}{2\eta}} = \frac{\left(\frac{1+\zeta_1-\sqrt{\varepsilon_1}}{1+\zeta_1}\right)^{\frac{1}{\zeta_1}+1}}{\left(\frac{1-\zeta_1-\sqrt{\varepsilon_1}}{1-\zeta_1}\right)^{\frac{1}{\zeta_1}-1}} \qquad \text{with } \zeta_1 = \sqrt{1-\sigma_1}$$

First case $\sigma^2 > \sigma_1$

In this case the deformation law will follow the above general law, with no other modification than that consequent upon the change of boundary conditions. For now, instead of $\varepsilon_L = o$ when $t = o$, the operative condition is that $\varepsilon_L = \varepsilon_1$ when $t = t_1$. Substituting the new value of the integrating constant, the following expression results:

$$e^{\frac{t-t_1}{2\eta}} = \frac{\left(\frac{1+\zeta_2-\sqrt{\varepsilon_L}}{1+\zeta_2-\sqrt{\varepsilon_1}}\right)^{\frac{1}{\zeta_2}+1}}{\left(\frac{1-\zeta_2-\sqrt{\varepsilon_L}}{1-\zeta_2-\sqrt{\varepsilon_1}}\right)^{\frac{1}{\zeta_2}-1}} \tag{21}$$

In this $\zeta_2 = \sqrt{1-\sigma_2}$

It is evident that (19) is a particular case of (21), for which $\varepsilon_1 = o$ $t_1 = o$.

Second case $\sigma_1 > \sigma_2 > 2\sqrt{\varepsilon_1} - \varepsilon_1$

The new loading σ_2 applied after the first loading period is less than σ_1, but greater than the loading $\sigma_1 - \sigma_{v_1} = 2\sqrt{\varepsilon_1} - \varepsilon_1$ supported by the springs of this unit. Strains continue to increase, though at smaller rate, as if tending more rapidly to a position of equilibrium. Equation (21) can also be applied without modification in this case.

Third case $\sigma_2 = 2\sqrt{\varepsilon_1} - \varepsilon_1$

If after the first loading process, with loading σ_1 and duration t_1, the load is reduced to the value

$$\sigma_2 = 2\sqrt{\varepsilon_1} - \varepsilon_1 = \sigma_1 - \sigma_{v_1} \tag{22}$$

and this is kept constant, the system will suffer no modification. The springs and discs will be kept in equilibrium with the new loading so the viscous damping mechanism will exert no force, and the whole system will remain statically balanced, without experiencing any further creep.

In this manner the system attains its limiting stability, and ε_1 remains constant throughout time.

Fourth case $\sigma_2 < 2\sqrt{\varepsilon_1} - \varepsilon_1 < \sigma_1$

In contrast with previous cases, the new deformations in this case are decreasing. The new applied loading σ_2 is less than the loading $\sigma_1 - \sigma_{v_1} = 2\sqrt{\varepsilon_1} - \varepsilon_1$ acting on the first spring at the end of the first period t_1. Hence the piston, and the whole creep system begins to move back towards a new position of equilibrium. The previously established

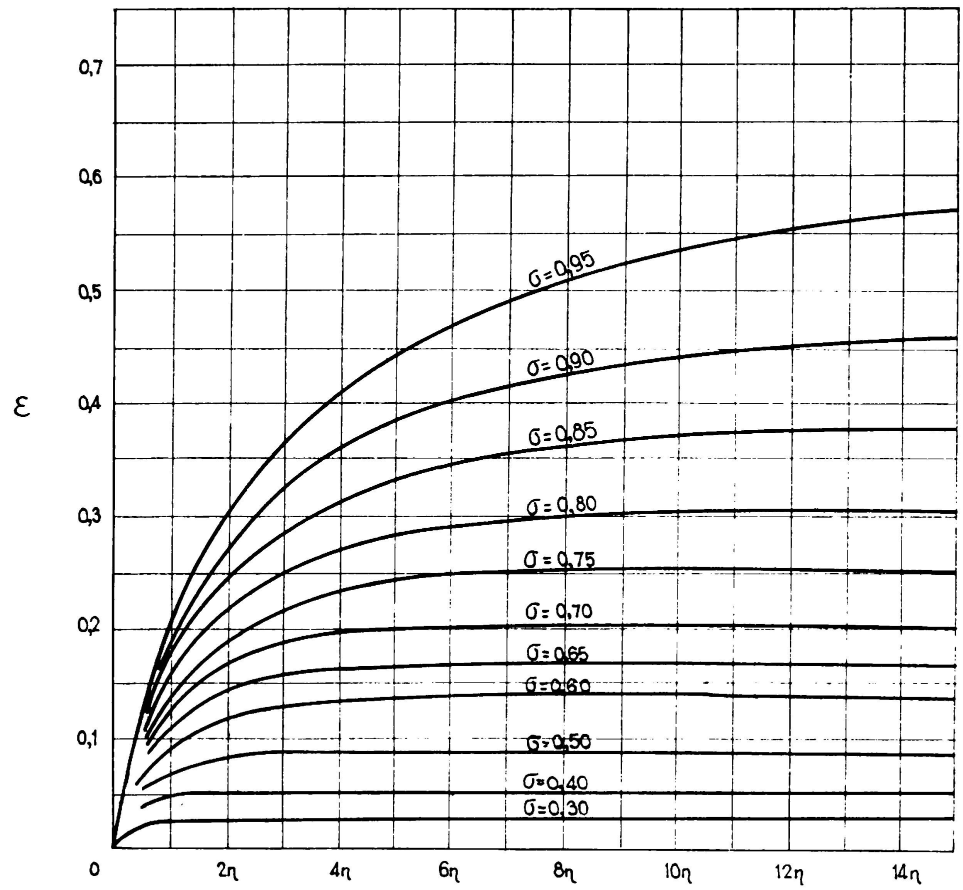


FIG. 6

formula are not longer operative since the friction forces of the various discs now act in the opposite sense. The law of deformation defined by (15) which defines the movement of the discs is not applicable. It must be substituted by expression (8): this determines the elastic deformation $\sigma/2$ and a factor of 2 is applied to the whole expression, in the same manner as was done to obtain expression (15) from (4).

Hence the law of deformation for the springs and discs in this case is given by

$$\varepsilon_L = 4\sqrt{1 - \frac{1}{2}[(\sigma_1 - \sigma_{v_1}) - (\sigma_1 - \sigma_v)]} - 2\sqrt{1 - (\sigma_1 - \sigma_{v_1})} - 2(\sigma_1 - \sigma_v) \tag{23}$$

That is to say, the loading $\sigma_1 - \sigma_v$ acting on the springs and discs at any time, is given by

$$\sigma_1-\sigma_v = 2-2\sqrt{1-(\sigma_1-\sigma_{v_1})}-2\sqrt{2}\sqrt{2-2\sqrt{1-(\sigma_1-\sigma_{v_1})}-\varepsilon_L-(\sigma_1-\sigma_{v_1})}-\varepsilon_L \tag{24}$$

The loading acting on the piston which operates on viscous liquid is

$$\sigma_v = -(2\eta)\frac{d\varepsilon_L}{dt}$$

The negative sign describes the change in the sense of movement. Since deformation decreases the increments $d\varepsilon_L$ are negative for positive values of σ_v.

If this value of σ_v is substituted in expression (24), the differential equation defining the movement of the creep system is obtained, thus:

$$2-2\sqrt{1-(\sigma_1-\sigma_{v_1})}-2\sqrt{2}\sqrt{2-2\sqrt{1-(\sigma_1-\sigma_{v_1})}-\varepsilon_L-(\sigma_1-\sigma_{v_1})} - \varepsilon_L + 2\eta\frac{d\varepsilon_L}{dt} = \sigma_1 \tag{25}$$

The solution of this equation is:

$$\frac{t}{2\eta}+C = l_n\left[\varepsilon_1-\varepsilon_L-2\sqrt{2(\varepsilon_1-\varepsilon_L)}+2-C_1\right]+\sqrt{\frac{2}{C_1}}\, l_n\frac{\sqrt{\varepsilon_1-\varepsilon_L}-\sqrt{2}-\sqrt{C_1}}{\sqrt{\varepsilon_1-\varepsilon_L}-\sqrt{2}+\sqrt{C_1}} \tag{26}$$

The integration constant C is determined by the condition that when $t = t_1$, $\varepsilon_L = \varepsilon_1$ and once this value is introduced, expression 26 becomes:

$$e^{\frac{t-t_1}{2\eta}} = \frac{\left(1-\frac{\sqrt{\varepsilon_1-\varepsilon_L}}{\sqrt{2}+\sqrt{C_1}}\right)^{\sqrt{\frac{2}{C_1}}+1}}{\left(1-\frac{\sqrt{\varepsilon_1-\varepsilon_L}}{\sqrt{2}-\sqrt{C_1}}\right)^{\sqrt{\frac{2}{C_1}}-1}} \tag{27}$$

where $C_1 = 2-2\sqrt{\varepsilon_1}+\varepsilon_1+\sigma_0 < 2$

If $\sigma_2 = o$ in these expressions the law is obtained which defines the delayed strain, after removing the loading σ_1. It is supposed, for this law to operate, that the loading σ_1 had been applied over a period t_1, and that at the end of this period the maximum deformation was ε_1. A part of this deformation is recovered in the course of time. This provides the well known type of phenomenon called delayed elasticity. The curve obtained by correlating the change of deformation, (after removing the loading) against time, tends asymptotically towards a position of stability. The equation of this asymptote is

$$\varepsilon_\infty = \varepsilon_1 - (\sqrt{2}-\sqrt{C_1})^2 \tag{28}$$

Fig. 7 shows the diagramatic representation of this delayed deformation phenomenon. Fig. 8 shows curves corresponding to various periods of time during which the loading has been removed.

All these laws correspond to actual observed phenomena affecting concrete subjected to simple compression. But if the results shall have real values, it will be necessary to assume not one creep system, but several of them, acting under at least two distinct viscosities.

For it an imaginary basic model be supposed to consist of the hookean unit, or spring whose deformation is ε_E, connected, in series

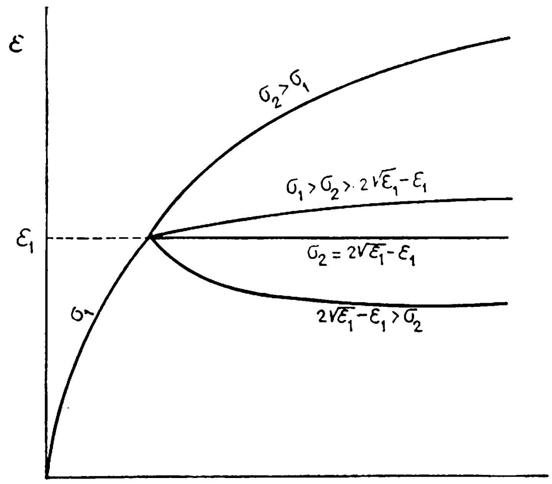


FIG. 7

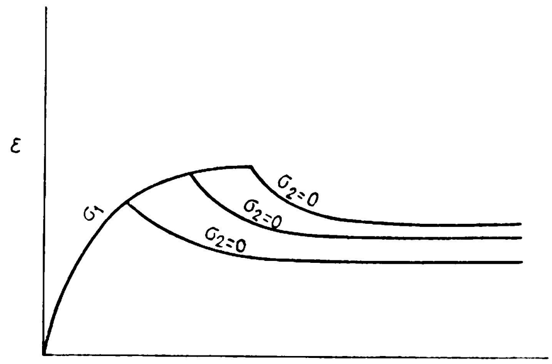


FIG. 8

with an inelastic creepless unit, whose deformation is ε_n, and this again connected in series with 4.2 creep systems with viscosity $\tau_1 = 7$ minutes, which in turn are serially connected with 9.6 systems, whose viscosity is $\tau_1 = 2$ years (t in years), then the curves obtained experimentally by Glanville ([2]) are reproduced with sufficient accuracy.

It is possible that more numerous and homogenous tests may make it advisable to change the values of the numerical coeficients just mentioned. But it seems, nevertheless, that the schematic arrangements proposed above can describe fairly well this type of phenomena, as we know them at present, and as described by the average laws. Of course, all these numerical factors must depend upon many other variables and should describe laws that are even more complex and difficult to handle.

There are two further possibilities which it is interesting to investigate in relation to the basic model previously proposed as a means of interpreting rheological phenomena.

The first of these possibilities is that failure corresponds, in the inelastic creepless unit (ε_n), to the total sliding of the movable part of the mechanism i. e., because all frictional force has been overcome

---

([2]) W. H. Glanville. Dept. of Scientific and Industrial Research. Technical Papers, N° 12 and 21, London.

when $\sigma = 1$ $\varepsilon_n = 1$. But in the case of a creep system, ultimate failure occurs under a loading which is smaller the longer the period during which a load close to the failing load is kept applied. Thus, for exemple, if a load is kept applied during three hours, failure will occur after that period if the magnitude of the load is .88 of the load that will cause failure in a short duration test. If the load applied is .86 of the short duration failing load, then the time that will ellapse before failure will be almost indefinite. All this coincides with the experimental results of Shank. These figures may be modified by varying the number of discs of the creep systems.

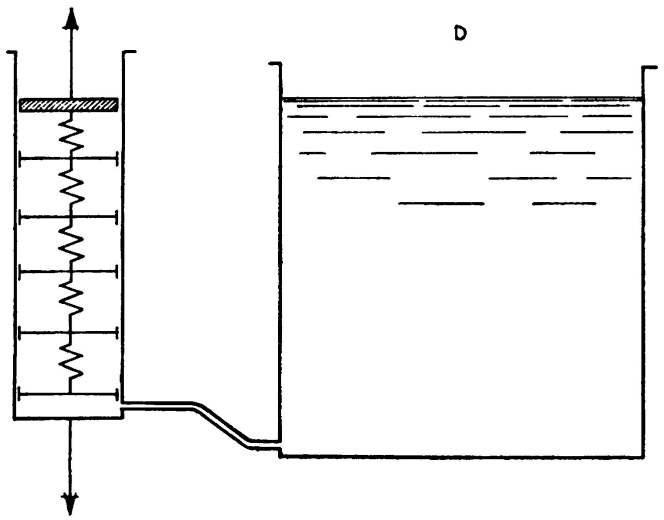


FIG. 9

Secondly, shrinkage may be described by assuming that, in the creep systems, the viscous liquid of each unit (fig. 9) moves into a reservoir D, whose liquid level corresponds to the hygrometric conditions of the ambient. When the level goes down in the reservoir a suction force would actuate on the piston, and consequently a shortening takes place. This is a movement that progressively decreases with time and is only partly recovered.

Furthermore, if the actual test piece is supposed to consist of a given number of basic models, such as the one previously described, coupled in parallel (fig. 10), so that the liquid has to pass from one system to another to reach the reservoir which describes the humidity (3), it follows that the shortening in a given time is smaller, the greater the number of basic models in parallel. That is to say, this shortening will be less, the greater the transversal dimensions of the test piece, as in fact is the case.

This analogy will even explain the strange phenomenon observed by Duke and Davis, according to which when evaporation and compressive loading operate simultaneously contraction is greather than when these two influences actuate in succession.

For in the latter case the delay in shrinkage due to the viscosity is greater in the internally situated units than in the outer ones (i. e., in those close to the reservoir, which represent the surrounding conditions). Thus compressive stress is greater in the centre than on the external layers. And as the relative stress/strain diagram is a curve (with strains

(3) So that the vertical movement of the piston will not exert an additional hydrostatic suction or compression (due to difference in the level between the liquid and the piston, produced by the vertical movement of the latter). it will be supposed that the group of mechanisms is placed on the same horizontal plane, and that the reservoirs are sufficiently ample to render negligible the variation of level due to the displaced liquid.

becoming larger than those proportional to the stress, when the latter increases) it follows that the average strain is also greater than the sum of the strains due to each cause (shrinkage and loading) separately. This will coincide with the account of the phenomenon as advanced by Pickett.

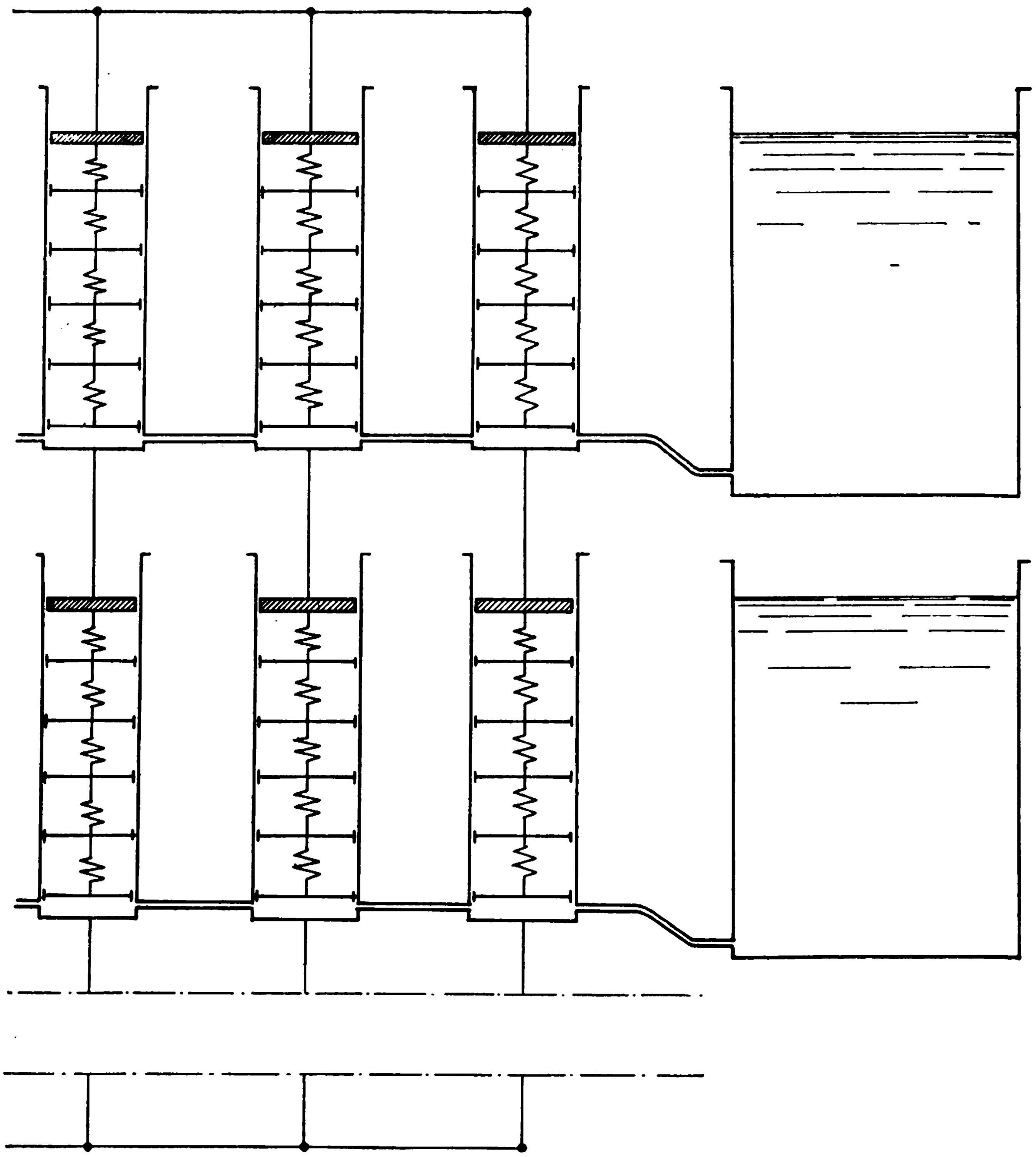


FIG. 10

SUMMARY

The rheological behaviour of reinforced concrete depends upon the rheological behaviour of the cement/water paste. This behaviour is related in a complex manner with elastic deformation, residual strain or permanent set, creep, shrinkage, swelling, etc. However, it appears that it can be expressed by general analytical laws, which in turn may be made

to correspond to the behaviour of a combination of mechanisms, connected in series and in parallel. These consist of differential elastic elements, coupled to a piston operating in a viscous fluid, subject to variable frictional forces. Different types of behaviour may thus be represented by simply varying the operating conditions of the mechanisms.

The measure of approximation obtained between these theoretical results and the experimental facts is encouraging. When experimental results become more accurate the theoretical pattern will have to be readjusted (using the same basic pattern), correcting the free parameters or the applied laws.

As more complex behaviours are considered, it will become increasingly difficult to narrow the correspondence between theoretical pattern and experimental results: however, the particular case of concrete may be investigated for the benefit of those engaged in this branch of research.

It does not seem likely that these patterns and laws will be of great use to the physicist looking for the intrinsic causes of these deformations in the micro-mechanical composition of the paste. But they may prove useful pro tem to the engineer who is only concerned with having available approximate laws with which to manage the practical results of experiment, and who is anxious to predetermine roughly the magnitude of deformations that will occur in reinforced concrete structures.

ZUSAMMENFASSUNG

Das rheologische Verhalten des erhärteten Betons hängt von demjenigen der Wasser/Zement Paste ab. Es ist durch verwickelte Verhältnisse bedingt, deren Ursprung in den elastischen und bleibenden Verformungen, Kriechen und Schwinden, Quellungen u. a. Umständen begründet ist. Trotzdem scheint es möglich, das rheologische Verhalten des Betons durch allgemeine Gesetze von analytischem Charakter auszudrücken, die ihrerseits in dem Verhalten eines entsprechend kombinierten Mechanismus dargestellt werden können. Dieser besteht aus einer Serie von elastischen Elementen, welche teils in Serie und teils parallel miteinander verbunden und an einen Kolben angeschlossen sind, der sich in einer zähflüssigen Masse bewegt und veränderlichen Reibungskräften ausgesetzt ist.

Die mit dem erwähnten Prüfmechanismus festgestellten Ergebnisse stimmen sehr gut mit den bei praktischen Messungen erhaltenen Resultaten überein. Eine Verbesserung der praktischen Messmethoden infolge technischer Fortschritte in dieser Richtung lässt sich ohne Veränderung des Grundprinzips auf das theoretische Prüfgerät anwenden, indem die Teile, welche die Konstanten darstellen, oder die angewandten Gesetze, entsprechend geändert werden.

Für die physikalische Forschung, welche die inneren Gründe der Betonverformung in der mikro-mechanischen Zusammensetzung der Zementpaste untersucht, dürfte der neue Prüfmechanismus kaum verwendbar sein. Dagegen wird derselbe dem Ingenieur, der aufgrund annähernder Werte die Grösse der möglichen Verformungen in Betontragwerken vorausbestimmen will, gute Dienste leisten.

RESUMO

O comportamento reológico do betão armado depende do comportamento da massa água-cimento. Este está relacionado de uma maneira complexa com os fenómenos de deformação elástica, de tensões residuais ou deformações permanentes, de deformações lentas, de contracção, de aumento de volume, etc. Parece no entanto ser possível conseguir exprimi-lo por leis analíticas gerais que, por sua vez, se podem representar pelo comportamento de um conjunto de dispositivos mecânicos, ligados em série e em paralelo. Esses dispositivos são constituídos por elementos diferenciais elásticos, acoplados a um êmbolo que se desloca num meio viscoso, submetidos a forças de atrito variáveis. Podem-se assim reproduzir diversos tipos de comportamentos, pela simples variação das condições de funcionamento dos dispositivos.

O grau de aproximação dos valores teóricos obtidos por este processo em relação aos obtidos experimentalmente é muito animador. Quando a precisão dos resultados experimentais aumentar será necessário adaptar de novo essa disposição teórica (partindo do mesmo sistema de base) corrigindo, ou os parâmetros livres, ou as leis aplicadas.

À medida que se considerarão comportamentos mais complexos tornar-se-á mais difícil aproximar a forma teórica dos resultados experimentais: o caso particular do betão pode no entanto ser investigado o que beneficiará aqueles que se dedicam a pesquisas relacionadas com esse material.

Não parece muito provável que essas disposições e essas leis venham a ser de grande utilidade para os físicos que procuram as razões intrínsecas dessas deformações na composição micro-mecânica da massa água-cimento. Podem, no entanto, ser úteis ao engenheiro que pretende ùnicamente ter à sua disposição leis aproximadas que lhe permitam utilizar os resultados práticos das experiências e que procura prever a ordem de grandeza das deformações nas estruturas em betão armado.

RÉSUMÉ

Le comportement rhéologique du béton armé dépend de celui de l'ensemble eau-ciment. Ce comportement est en rapport, par l'intermédiaire de relations complexes, avec les phénomènes de déformation élastique, de tensions résiduelles ou de déformations permanentes, de déformation lente, de retrait, de gonflement, etc.. Il semble néanmoins qu'il soit possible d'arriver à l'exprimer par des lois générales, de caractère analytique, qui peuvent à leur tour être représentées par le comportement d'un ensemble de dispositifs mécaniques reliés en série et en parallèle. Ces dispositifs se composent d'élements élastiques différentiels, accouplés à un piston se déplaçant dans un milieu visqueux, soumis à des forces de frottement variables. Différents types de comportements peuvent ainsi être reproduits en variant simplement les conditions de fonctionnement de ces dispositifs.

Le degré d'approximation des valeurs théoriques obtenues par ce procédé par rapport à celles obtenues expérimentalement est encourageant. Lorsque les résultats expérimentaux deviendront plus précis, il conviendra de ré-adapter cette disposition théorique (à partir du même système fondamental) en corrigeant soit les paramètres libres, soit les lois appliquées.

A mesure que l'on considère des comportements de plus en plus complexes il deviendra plus difficile de rapprocher la forme théorique des résultats expérimentaux: le cas particulier du béton peut néanmoins être étudié en détail ce qui peut rendre service à ceux qui s'occupent de recherches concernant ce matériau.

Il ne semble guère probable que ces dispositions et ces lois puissent être grandement utiles aux physiciens qui recherchent les raisons intrinsèques de ces déformations dans la composition micro-mécanique du complexe eau-ciment. Elles peuvent néanmoins être utiles à l'ingénieur qui cherche uniquement à avoir à sa disposition des lois approximatives qui lui permettent de manier les résultats pratiques des expériences, et qui tient à prévoir, l'ordre de grandeur approximatif des déformations dans les structures en béton armé.