

**Zeitschrift:** IABSE congress report = Rapport du congrès AIPC = IVBH  
Kongressbericht

**Band:** 5 (1956)

**Rubrik:** VI. Practice of reinforced and prestressed concrete

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## **VI**

Prática do betão armado e preesforçado

Pratique du béton armé et du béton précontraint

Practice of reinforced and prestressed concrete

Praxis des Eisenbetons und des vorgespannten  
Betons

**Relator Geral**

**Rapporteur Général**

**General Reporting Member**

**Generalreferent**

**PROF. DR. G. WÄSTLUND**

Stockholm

## **VIa**

Realização das construções (Andaimes, cofragens, fabricação e controle de betão, transporte do betão, ligação de elementos prefabricados, observação, controle e conservação das obras)

Réalisation de la construction (Echafaudages, coffrages, fabrication et contrôle du béton, transport du béton, liaison d'éléments préfabriqués, observation contrôle et entretien des ouvrages)

Execution of the structures (formwork, shuttering, placing and control of concrete, transport of concrete, jointing of prefabricated elements, observation, control and maintenance of structures)

Ausführung der Tragwerke (Lehrgerüste, Schalungen, Herstellung und Kontrolle des Betons, Betontransport, Verbindung von vorfabrizierten Elementen, Beobachtung, Kontrolle und Unterhalt der Bauten)

## **VIb**

Aperfeiçoamentos recentes

Développements nouveaux

New developments

Neue Entwicklungen

## **VI**

**Practice of reinforced and prestressed concrete**

**Praxis des Eisenbetons und des vorgespannten Betons**

**Prática do betão armado e preesforçado**

**Pratique du béton armé et du béton précontraint**

**General Report – Generalreferat – Relatório Geral – Rapport Général**

**PROF. DR. G. WÄSTLUND**

**Stockholm**

This subject covers various questions relating to the execution of concrete work, and includes site work, as well as factory work, reinforced concrete as well as prestressed concrete. The papers falling under this head are enumerated below in the order which may be regarded as their logical sequence. Those papers which deal with more or less basic problems are therefore placed at the beginning.

Three authors, Mr. GOSCHY, Mr. CARDOSO, and Mr. BERGFELT, thus study those internal stresses and deformations in concrete structures which are in a high degree influenced by the procedure used in the execution of the structure. Mr. BATE and Mr. PLOWMAN then express in their papers their views on the manufacture of prefabricated concrete members. Mr. ENSKOG describes a method of facilitating and rationalising the design and construction of high reinforced concrete bridge piers by means of sliding forms. Finally, Mr. COURBON presents two prestressed concrete systems and their applications on bridges. — These papers are examined in what follows in the same order as above.

Mr. GOSCHY has made a theoretical study of the redistribution of stresses in a composite system in which one part consists of older concrete (Fertigbetonteil), whereas another part is made of more recently placed concrete (Ortbeton) cast so as to be integrated with the former part. Internal stresses are set up in his case on account of the difference in shrinkage between the older part and the more recent part. Furthermore, when the external load is constant, the internal stresses are redistributed owing to the difference in creep between these two parts of the structures.

Completely analogous conditions are met with in composite systems made up of steel and concrete (e. g. composite beams). Moreover, a

closely similar redistribution of stresses takes place in prestressed concrete structures as a consequence of creep and shrinkage of concrete. The problem dealt with by Mr. GOSCHY can be applied to concrete structures of two types, viz. first, those structures which consist of several parts cast in different stages, e. g. the top and bottom of hollow box sections of bridge arches, cf. Mr. CARDOSO's paper below, and second, those structures which are made up of prefabricated units integrated into monolithic systems.

The redistribution of stresses is chiefly of importance under the normal load conditions, whereas the ultimate loads are less or not at all influenced. Mr. GOSCHY also deals with this problem, where he calculates the ultimate loads on the assumption that the material is perfectly plastic, cf. Fig. 4, so that he can apply the law of addition. Factors of safety in this connection are discussed.

The problems studied by Mr. GOSCHY have previously been treated in the literature. A good review of the European publications dealing with this subject is to be found, together with the author's own contributions, in a paper by H. RÜHLE <sup>(1)</sup>. He refers, for instance, to earlier papers by FREYSSINET (1936), DISCHINGER (1937), CRAEMER, SATTLER, and RÜSCH, cited in chronological order.

American researchers have taken a different line, and two authors represented in «Preliminary Publications» have referred to these works. BÄCKSTRÖM thus refers to a work by MCHENRY (1943), while BERGFELT refers to works by ROY CARLSON (1937) and PICKETT (1946).

It is of interest to compare the European and American works. DISCHINGER and others take as a starting-point the creep function

$$\varphi_t = \varphi_\infty (1 - e^{-t})$$

which implies that the elongation or the contraction after the time  $t$  is

$$\left( \frac{\Delta l}{l} \right)_t = \frac{\sigma}{E} (1 + \varphi_t)$$

where  $E$  is the modulus of elasticity, which can also be a function of the time, and  $\varphi_\infty$  is the asymptotic value of the creep (order of magnitude 2 to 3).

Furthermore, in the European publications, the shrinkage function  $\epsilon_t$  is usually supposed to be affine to the creep function, that is to say,

$$\epsilon_t = \frac{\epsilon_\infty}{\varphi_\infty} \cdot \varphi_t$$

MCHENRY's creep function is much more intricate, see BÄCKSTRÖM's paper, while BERGFELT gives a creep function which is somewhat between DISCHINGER's and MCHENRY's functions. MCHENRY's creep func-

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<sup>(1)</sup> RÜHLE, H.: «Die Ermittlung der zeitabhängigen Eigenspannungen in Verbundkonstruktionen aus Stahlbetonfertigteilen mit Ortbeton». Bauplanung und Bautechnik, No. 10, 1954.

tion comprises a time function which is dependent on several variables, e. g. the dimensions of the concrete body and the composition of the concrete at the time of load application. Moreover, there is one term, the first, which is independent of the age of the concrete. It is highly desirable that these relations, which have been but little studied up to now, should be elucidated as far as possible in further researches.

The variation in the shrinkage function with the time must also be conditioned by the dimensions of the concrete body, but this circumstance has not been taken into account in the above-mentioned shrinkage function.

It is possible that the final states ( $t = \infty$ ) after redistribution of internal stresses are not influenced to any large extent by certain factors, e. g. the dimensions of the concrete body, and that the simpler expressions can therefore be used to calculate the final states, whereas it seems that these expressions can scarcely be employed to compute the variations with the time themselves.

Mr. CARDOSO deals with the secondary stresses which are caused in reinforced concrete bridge arches by the execution of concrete work in several stages, by deformations of formwork, and by shrinkage of concrete. Furthermore, the author describes various methods of counteracting the occurrence of such secondary stresses. In addition, several concrete practical cases are discussed from this point of view. (Unfortunately, since I am not familiar with the language used in this paper, I am not in a position to make any detailed comments on its contents).

Mr. BERGFELT begins his paper with an enumeration of the various factors which cause losses of stress in the steel cables used in prestressed concrete. The author examines the theory of friction losses along a curved cable. After that, he presents the results of measurements which were made on two prestressed concrete bridges designed according to Freyssinet. In this connection, the losses are resolved into the following components:

Losses in the jacks and at the thread angle for attachment to the jacks. (NOTE. The losses are here denoted by  $C$ , but the real loss in stress is  $1 - C$ ).

Friction losses due to the intentional curved line of the cable.

Losses due to the unintentional wave form of the cable and to adhesion along the cable.

Losses after anchoring; influences of creep, shrinkage, and temperature.

The results are reproduced in the author's summary.

It is of special interest to note that the author has attempted to calculate the plastic deformations of one of the bridges under investigation and to compute the deformations due to shrinkage, cf. Fig. 7. For this purpose he used the creep and shrinkage functions given in the formulae (10a, 11a) and (12, 13). The author states that these formulae have formerly been used in a successful manner for the calculation of long-time deformations of the Sandö Bridge, Sweden, but are now at variance with the values observed so far. The measurements are continued.

Mr. BATE has studied the variation in cracking moments and failing moments of small prefabricated prestressed concrete beams, and has investigated the causes of this variation. He found that the variation in question is to be attributed to many causes, viz. first, the same causes that give rise to variations in cube strength, second, the heterogeneous vibration of long beams (thorough in the neighbourhood of the vibrators, but inadequate midway between them), third, the variations in the dimensions of the beams, and finally, some factors of minor importance. Accurate supervision of concrete quality has proved to be essential but it is at least as important to keep the dimensions exact. Accordingly, the author recommends the use of steel moulds, and considers that consolidation of the concrete on the vibration table in the individual mould system is superior to the use of fixed shutter-vibrators on the «long-line» system.

Mr. PLOWMAN deals with the question of how prefabricated concrete structural components for post-tensioned bridge members should be cast so as to ensure high concrete quality. In particular, he examines two properties which high-quality concrete must possess, viz., high strength and homogeneousness, cf. BATE'S paper. According to the author, concrete is usually consolidated by vibration. The author lays special stress on the fact that the vibration must be so intense as to produce in every point of the concrete mass a sufficiently high acceleration, usually at least 2,5g to 3g, depending on the frequency of the vibration and the workability of the mix. Furthermore, the author points out that the vibrations are propagated through the surface layers of the concrete mass without being damped to any appreciable degree, while the damping increases towards the interior. Therefore, we should not be led to believe that the whole concrete mass has been thoroughly vibrated solely for the reason that the concrete surface appears to be compacted. The author then touches on the effect of overvibration, and states that the troubles of overvibration and segregation only result from a badly designed mix. This might be true within some limits. BERGSTRÖM<sup>(2)</sup> considers that a concrete mix is stable under vibration only for a certain definite time. Fig. 5 in the paper by BERGSTRÖM also shows that the effect of comparatively small variations in the grading is remarkable.

Mr. ENSKOG describes the construction of a large railway bridge in North Sweden. The piers of this bridge were the first in Sweden to be built by means of a special method using sliding forms (the «Concretor System»). The lengths of the piers ranged from about 10 to 23 m, and their cross sections were either circular or more intricate, see Fig. 2. A distinguishing characteristic of this sliding form system is the fact that the forms are raised by several hydraulic jacks, and that all jacks can be controlled from a central point. Each jack can also be operated individually, if required. The amount of manual labour needed for raising the forms is reduced to a minimum. It was found that this concreting method saved materials used for temporary auxiliaries, and that the quality, the accuracy of dimensions, and the appearance of the

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<sup>(2)</sup> BERGSTRÖM, SVEN G.: «An Experimental Study of the Relation between the Properties of Fresh and Hardened Concrete». I. A. B. S. E., Vol. XIII.

finished structure were good. This method was therefore soon employed again in the construction of other bridges.

Finally, Mr. COURBON describes two types of prestressed concrete and mentions some bridges in which these types were used. One of these types was invented by CHALOS, and its characteristic feature is the use of very thick, spiral-wound cables, which are provided with massive, but articulated steel cores before they are delivered to the site. The cables are stretched on these cores and fixed at the ends. The cables are embedded in concrete while they are in this stretched state. When the concrete has hardened, the core is detached and removed, with the result that the concrete is subjected to compression. This method has some advantages, namely, the tension in the cable is uniform throughout its length, and all losses involved in the application of the initial tension are eliminated.

Each of the reinforcing cables used in the other type of prestressed concrete consists of a number of separate, small, spiralloid wire ropes. In this case, the initial tension is applied to the cables in an ordinary way and the cables anchored by driving wedges between the cable and the concrete after hardening. The cables can therefore be stretched successively, as the erection of the structure advances. The coefficient of friction is stated to be extremely low, 0.08.

#### S U M M A R Y

This subject covers various questions relating to the execution of concrete work, and includes site work as well as factory work, reinforced concrete as well as prestressed concrete. Long-time deformations of concrete, and in some measure those of steel, play an important part in this connection, and are dealt with in several papers. Secondary stresses or losses in stress due to other causes are also discussed. Various views on the manufacture of prefabricated concrete components are expressed with special reference to their quality characteristics. The use of a new sliding form method for the construction of a bridge is described in one of the papers. Finally, two types of prestressed concrete are presented together with their applications.

#### ZUSAMMENFASSUNG

Dieses Thema umfasst verschiedene Fragen, die sich auf die Ausführung der Betonarbeiten beziehen, wobei es sich um Blauplätze oder Fabriken, um Stahlbeton oder vorgespannten Beton handeln kann. In diesem Zusammenhang spielen die Dauerverformungen des Betons und in gewissem Masse auch diejenigen des Stahls eine wichtige Rolle. Dementsprechend werden sie in mehreren Berichten behandelt. Ausserdem werden sekundäre Spannungen oder durch andere Ursachen bedingte Spannungsverluste erörtert. Die Herstellung von Fertigteilen bildet den Gegenstand einiger Betrachtungen, wobei die Güteeigenschaften dieser Teile besonders berücksichtigt werden. In einem Bericht wird die Ver-

wendung eines neuen Gleitschalungsverfahrens beim Bau einer Brücke dargestellt. Schliesslich werden zwei Arten von vorgespanntem Beton und ihre Anwendungen beschrieben.

### RESUMO

Trata este tema de vários problemas relativos à execução de estruturas de betão armado e betão preesforçado, quer no local da obra quer em oficina. As deformações de grande duração do betão e, até certo ponto, as do aço têm um papel importante nestes problemas o que é posto em evidência em várias contribuições. Discutem-se igualmente os esforços secundários ou perdas de tracção devidos a outras causas. Alguns autores apresentam considerações acerca da produção de elementos de betão prefabricados, principalmente no que respeita às características que determinam a sua qualidade. Uma das contribuições descreve um novo processo de construção utilizando cofragens deslizantes, empregado na execução de uma ponte. Indicam-se, igualmente, dois tipos de betão preesforçado e suas aplicações.

### RÉSUMÉ

Ce thème s'occupe de diverses questions relatives à l'exécution des travaux en béton. Il comprend les travaux sur chantiers de construction ainsi que les travaux effectués en atelier. Il se rapporte au béton armé aussi bien qu'au béton précontraint. Les déformations de longue durée du béton ainsi que, dans une certaine mesure, celles de l'acier jouent un rôle important dans cet ordre d'idées, ce qui ressort de plusieurs rapports. On discute également les efforts secondaires ou les pertes de tension dues à d'autres causes. Quelques auteurs présentent des considérations sur la production des éléments préfabriqués en béton, surtout en ce qui concerne les caractéristiques déterminant leur qualité. Un des rapports contient une description d'un procédé nouveau utilisant des coffrages glissants, procédé dont on a fait usage dans la construction d'un pont. Enfin, on signale deux types de béton précontraint et leurs applications.



## **Vla1**

**The performance of small prestressed concrete units**

**Ueber die Ausführung kleiner Bauteile aus vorgespanntem Beton**

**Comportamento de pequenos elementos de betão preesforçado**

**Comportement des petits éléments en béton précontraint**

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### ***Introduction***

The design and manufacture of prestressed concrete units is now widely practised and conforms to generally accepted procedures, which, in most countries, are governed by specifications, codes of practice or other authoritative recommendations. These specifications and recommendations have been based on experience and research over a number of years and the purpose of this paper is to extend this background of information relating to the performance of small prestressed concrete units with pretensioned steel.

The structural design of these units is usually required to ensure that cracking of the concrete is avoided under normal working loads and that collapse will not occur under abnormal loading. The first requirement is met by limiting the stresses in the materials arising from the prestress and from normal combinations of the prestress and the working loads after making the necessary allowances for losses of prestress. Security against collapse is obtained by adopting an adequate load factor for the ratio of the total failing load to the total working load for the unit. The results obtained in the investigation described in this paper have a direct bearing on the calculation of cracking and failing moments, and provide

some information on variability in behaviour, which may influence the choice of load factor and the technique used in manufacture.

The investigation was carried out in two distinct phases. In the first instance, a number of routine tests were made on several different types of floor joist, in which behaviour under static loading was examined and data were obtained on variations in production. Some of this work was summarised by Masterman [1] in a review of developments in prestressed concrete sponsored by the Ministry of Works; some new and more detailed information on these tests is now given. In the second phase, several series of experiments have been made on lines of enquiry suggested in a reassessment of the previous work, and this information is presented for the first time.

### *Behaviour of the units under load*

Three types of prestressed concrete floor joist were tested; each was 12 ft. 9 in. long with the sectional dimensions given in Fig. 1. Some information on their design and manufacture is shown in Table 1. The

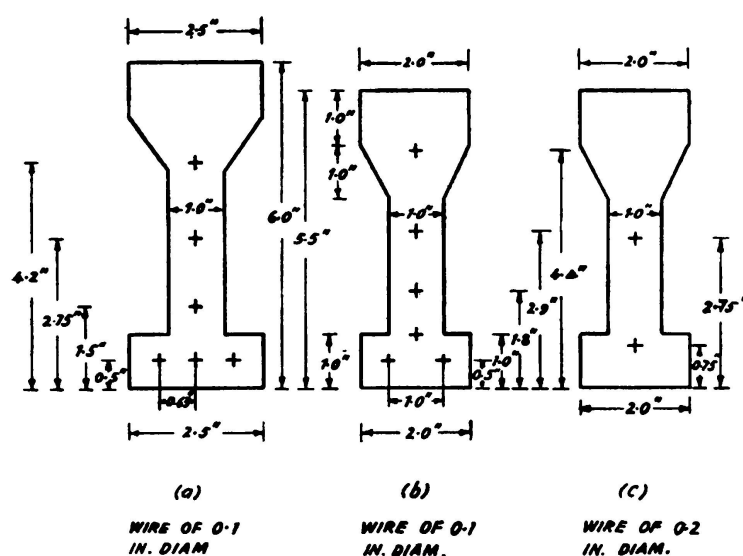


FIG. 1

units were made by the «long-line» system of pretensioning, and release of the wires was effected at between 4 and 6 days after casting. Most of the tests were carried out when the units were between 3 and 9 weeks old, but a few joists were as old as 18 weeks.

The joists were tested on a span of 12 ft. 4 1/2 in. with quarterpoint loading deflections were recorded at mid-span and in some cases the strains in the concrete were measured on gauge lengths of 10 in. or 30 in. at all

stages of loading to failure. Each unit exhibited the characteristic behaviour of prestressed concrete under load. For normal working loads, the relationship between deflection and applied load was linear; at higher loading cracking commenced, and the deflection increased at a disproportionately greater rate with increasing load until failure occurred (as shown in Fig. 2). A summary of the results for the total moment at the appearance of the first cracks and at failure is given in Table 2 together with the strengths of the materials at the time of test.

It was found that the total moment at the appearance of the first cracks could be calculated with reasonable agreement with the experimental results when the data in Table 3 were assumed.

TABLE 1

*Design and manufacturing details for the floor joists*

| Type of joist                                                                                                    | (a)<br>6" × 2 1/2" | (b)<br>5 1/2" × 2" | (c)<br>5 1/2" × 2" |
|------------------------------------------------------------------------------------------------------------------|--------------------|--------------------|--------------------|
| Diam. of wire: in. ... ..                                                                                        | 0.104              | 0.104              | 0.200              |
| Initial stress in the wires: lb. per sq. in. ...                                                                 | 182,000            | 182,000            | 154,000            |
| Mix proportions by weight of rapid-hardening<br>Portland cement and combined sand and<br>gravel aggregate ... .. | 1 to 4 1/2         | 1 to 4 1/2 (2)     | 1 to 4             |
| Water/cement ratio ... ..                                                                                        | 0.42 to<br>0.54    | 0.45 (2)           | 0.37               |
| Estimated maximum precompression in the<br>concrete: lb. per sq. in. ... ..                                      | 1760               | 1940               | 2050               |

(2) Of the 65 joists tested of this type, two were made of concrete with proportions of 1 to 3.5 and a water/cement ratio 0.42 and six were made with concrete with proportions of 1 to 4 and water/cement ratios between 0.41 and 0.45.

TABLE 2

| Type of joist                                                                     | (a)<br>6" × 2 1/2"                                         | (b)<br>5 1/2" × 2"                      | (c)<br>5 1/2" × 2"          |
|-----------------------------------------------------------------------------------|------------------------------------------------------------|-----------------------------------------|-----------------------------|
| Design moment: lb. in. ... ..                                                     | 21,000                                                     | 16,000                                  | 16,000                      |
| Total moment for first cracks: lb. in.                                            |                                                            |                                         |                             |
| Mean ... ..                                                                       | 37,600                                                     | 27,500                                  | 26,700                      |
| Max. ... ..                                                                       | 39,600                                                     | 30,700                                  | 30,500                      |
| Min. ... ..                                                                       | 34,700                                                     | 22,800                                  | 23,500                      |
| Coefficient of variation: per cent ... ..                                         | —                                                          | 6.1                                     | —                           |
| Total moment at failure: lb. in.                                                  |                                                            |                                         |                             |
| Mean ... ..                                                                       | 63,000                                                     | 47,800                                  | 44,600                      |
| Max. ... ..                                                                       | 67,600                                                     | 57,600                                  | 49,300                      |
| Min. ... ..                                                                       | 57,300                                                     | 35,200                                  | 39,300                      |
| Coefficient of variation: per cent ... ..                                         | —                                                          | 9.1                                     | —                           |
| Crushing strength of 6 in. concrete cubes at<br>the time of test: lb. per sq. in. |                                                            |                                         |                             |
| Mean ... ..                                                                       | 7,570                                                      | 7,630                                   | 8,300                       |
| Max. ... ..                                                                       | 9,600                                                      | 11,400                                  | 9,500                       |
| Min. ... ..                                                                       | 6,050                                                      | 5,410                                   | 8,000                       |
| Coefficient of variation: per cent ... ..                                         | —                                                          | 16.3                                    | —                           |
| Tensile strength of steel: lb. per sq. in.                                        |                                                            |                                         |                             |
| Mean ... ..                                                                       | 319,000                                                    | 326,000                                 | 231,000                     |
| Max. ... ..                                                                       | 325,000                                                    | 327,000                                 | 233,000                     |
| Min. ... ..                                                                       | 314,000                                                    | 318,000                                 | 229,000                     |
| Ultimate mode of failure of the joists                                            | Fracture of<br>the wires or<br>crushing of<br>the concrete | Crushing or<br>shear of the<br>concrete | Crushing of<br>the concrete |

TABLE 3

*Data assumed for the calculation of cracking moment for the tests*

|                                                                            |                              |
|----------------------------------------------------------------------------|------------------------------|
| Relaxation of the steel (over-stressed by 5 per cent for 2 minutes) ... .. | 5 per cent of initial stress |
| Modulus of elasticity of the concrete at transfer: lb. per sq. in. ... ..  | $5 \times 10^6$              |
| Shrinkage of the concrete ... ..                                           | $150 \times 10^{-6}$         |
| Creep of the concrete: per lb. per sq. in. ... ..                          | $0.2 \times 10^{-6}$         |
| Bending tensile strength of the concrete: lb. per sq. in. ... ..           | 800                          |

These figures for the creep and shrinkage of the concrete are half those recommended for pretensioning in the «First Report on Prestressed Concrete», of the Institution of Structural Engineers, [2] and

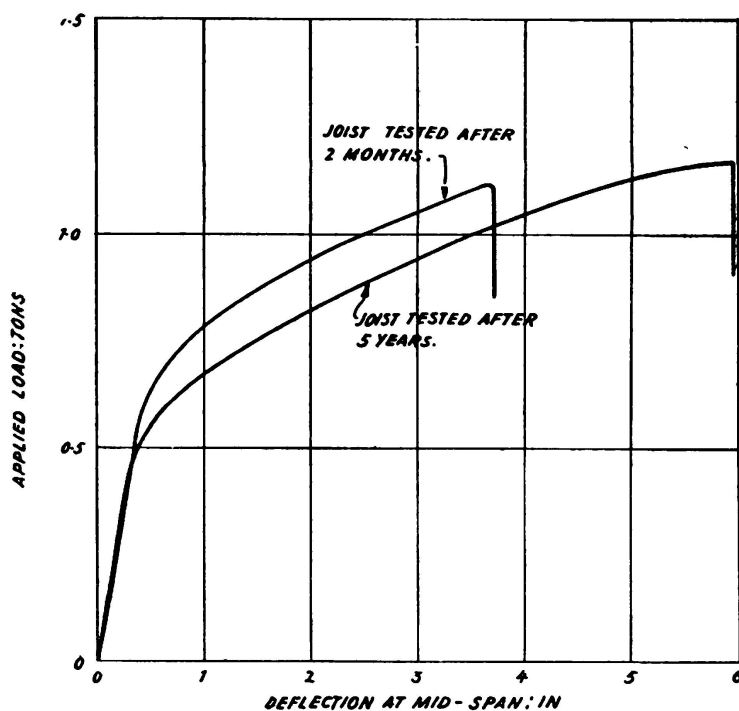


FIG. 2

when the losses calculated from these figures are combined with the losses due to the elastic contraction of the concrete and creep of the steel, a total loss of about 17 per cent is obtained for the joists of the type shown in Fig. 1-b. If these joists remain unloaded throughout their life, the loss of prestress may be expected to increase from 17 per cent to between 25 and 30 per cent, and this was confirmed by tests described later in the paper.

The calculation of the failing moment, which was based on the following assumptions, gave good agreement with the experimental results. It was assumed that. —

- a) the distribution of compressive stress in the concrete was uniform across the compression zone with a value of 0.6 times the cube strength of the concrete;

- b) the wires below the neutral axis were stressed to their tensile strength;
- c) the maximum compressive strain that could be sustained by the concrete was 0.2 per cent (from this assumption the stress in any wires above the neutral axis at failure could be calculated).

This method of calculation which is very similar to that originally proposed by Abeles [3], gave results for the three types of joists in Figs. 1-a, 1-b and 1-c that under-estimated the failing moments by 8 per cent, 3 per cent and 2 per cent respectively on average for each type.

The maximum compressive strains in the concrete recorded at failure varied between 0.21 and 0.39 per cent, but much of this variation developed during the final phases of the tests, as, at 95 per cent of the failing moment, the maximum recorded strains were between 0.18 and 0.22 per cent. The assumption of a value of 0.2 per cent for estimating the stress in any steel in the compression zone at failure would therefore appear reasonable.

#### *Variations in cracking and failing moments*

The results of the tests on the joists of the type shown in Fig. 1-b were sufficient in number to obtain useful information on the likely variations in behaviour of small prestressed concrete units arising from variations in their production. The sixty-five joists tested were drawn from seven different batches each of sixteen joists. In each batch, the joists were cast in timber moulds in groups of four for each concrete mix, and concrete cubes were also cast at the same time as each of the joists for test with the joists. Samples of the steel were also tested. Information was, therefore, available on the cracking and failing moments and on the strength of the materials for each joist tested. The results are to some extent heterogeneous as in a few instances the concrete mix proportions were varied, a few joists were as old as eighteen weeks and from one to sixteen joists were selected from each batch for test. For simplicity, the results have been treated as being drawn from a normal population and the effects of several of the variables in manufacture and test were examined experimentally in further tests which are discussed later.

The coefficients of variation for cracking and failing moment and for cube strength are given in Table 2; the variation in the strength of the steel was very small and is ignored in the examination of these results. Histograms for cracking and failing moment are shown in Fig. 3 and for cube strength in Fig. 4. It may be observed that variation of cracking moment is less than that for failing moment. The coefficient of variation for cube strength of 16.3 per cent is higher than would normally be expected for good factory production, but this may be explained by the variations in the age and mix proportions of the concrete which have already been mentioned.

The regression of cracking moment ( $M_c$ ) on cube strength ( $C_u$ ) was derived and is given by the following equation, which shows a highly significant correlation: —

$$M_c = 0.78 C_u + 21,600 \text{ (lb.in. units)} \quad (1)$$

The form of this expression is such that the whole of the influence of cube strength on the cracking moment can be explained by its influence on the bending tensile strength of the concrete. It must then be assumed

that the bending tensile strength is  $0.084 C_u$ , which is equivalent to the tensile stress resulting from the application of a bending moment of  $0.78 C_u$ .

Two apparently distinct types of failure were experienced in the tests on this particular type of joist: about two-thirds of the joists failed in shear near one of the load points and the remainder by crushing of the concrete near mid-span. A series

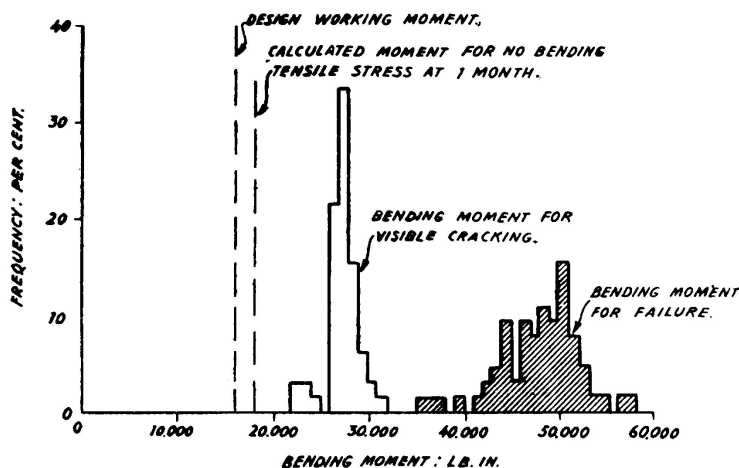


FIG. 3

of subsidiary tests were made on similar joists with central loading on spans of 1 ft., 2 ft., 3 ft., 4 ft., 5 ft., and 7 ft. to determine the effects of the shear conditions on ultimate strength. The results of these tests are shown in Fig. 5, from which it may be seen that the shear conditions did not affect the failing moment when the span was 4 ft. or more, which is equivalent to a span of 8 ft. for quarter-point loading. It would, therefore, appear that in the main tests shear failure only developed at maximum load when crushing of the concrete was imminent, and its appearance in the tests may be neglected in the analysis.

The regression of failing moment ( $M_u$ ) on cube strength is given by the following relationship, which also shows a highly significant correlation:—

$$M_u = 1.69 C_u + 34,900 \text{ (lb. in. units)} \quad (2)$$

from which it may be inferred that the behaviour of the steel has a greater influence on the failing moment than the cube strength of the concrete.

#### *The effect of dimensional errors*

The analysis of variance, from which expressions (1) and (2) were derived, are summarised in Table 4, where it may be observed that the residual variances are high with respect to the total variance. Measu-

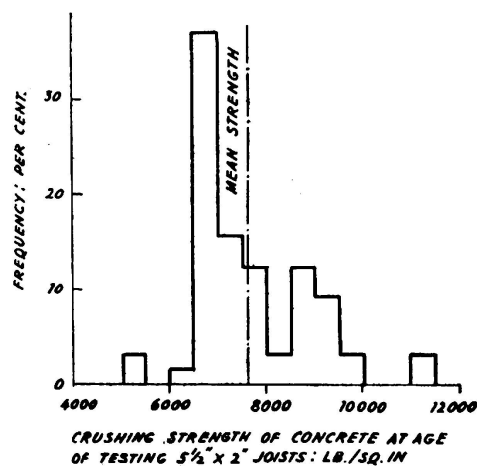


FIG. 4

rements of the dimensions of twelve joists from one batch showed that there were considerable dimensional errors arising from the repeated use of timber moulds. Analysis indicated that the variance in cracking moment and in failing moment for these twelve joists due to these dimensional errors accounted for much of the total variance in each case as shown in Table 4. It is of interest to note that, had the coefficient of variation for cube strength been very much less than the experimental value of 16.3 per cent, it is unlikely that the coefficients of variation for cracking and failing moments could have been less than 5.1 and 8.0 per cent respectively as compared with the experimental values of 6.1 and 9.1 per cent respectively. For the manufacturing conditions under which these joists were produced, greater uniformity could have been obtained by reducing the dimensional errors than by increasing the rigidity of the control of concrete quality. For uniformity in the behaviour of small prestressed concrete units such as these, the use of steel moulds would seem to be essential.

TABLE 4

*Effects of variations in cube strength and dimensional errors  
on variations in cracking and failing moment*

| Series for the examination of                                                                                   | Effects of variations in cube strength | Effects of dimensional errors |
|-----------------------------------------------------------------------------------------------------------------|----------------------------------------|-------------------------------|
| Number of tests in series ... ..                                                                                | 65                                     | 12                            |
| <i>Cracking moment</i>                                                                                          |                                        |                               |
| (3) Variance of cracking moment ... ..                                                                          | 2.8                                    | 2.8                           |
| Variance due to variations in the quality of the concrete as indicated by the strength of concrete cubes ... .. | 0.9                                    |                               |
| Variance due to dimensional errors ... ..                                                                       |                                        | 1.6                           |
| Residual variance... ..                                                                                         | 1.9                                    | 1.2                           |
| <i>Failing moment</i>                                                                                           |                                        |                               |
| Variance of failing moment... ..                                                                                | 19.0                                   | 14.1                          |
| Variance due to variations in the quality of the concrete as indicated by the strength of concrete cubes ... .. | 4.5                                    |                               |
| Variance due to dimensional errors ... ..                                                                       |                                        | 6.5                           |
| Residual variance... ..                                                                                         | 14.5                                   | 7.6                           |

#### *Variations in the quality of materials*

It was not known to what extent the strength of a concrete cube represented the strength of the concrete in a joist and what variation in strength occurred within the length of the joist. A series of concrete specimens each 12 in. long with a section of 3 in. by 3 in. were made

(3) The units in which variance is expressed in the above table are  $(\text{lb. in.})^2 \times 10^6$ .

in moulds which were of about the same size as those used for the joists. These moulds were made for casting 16 beams of 12 ft. in length in groups of four and were sub-divided to produce eleven specimens in each beam mould, that is a total of 176 specimens. The moulds were mounted on the prestressing bed and filled with concrete, which was consolidated with two shutter-vibrators fixed to each group of four beam moulds in

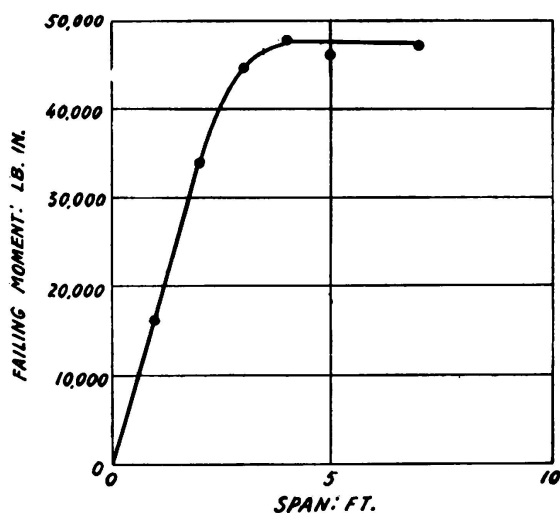


FIG. 5

the same manner as for the joists. At or about the age of 28 days, the specimens were tested as beams to obtain the modulus of rupture and the broken halves were then tested between 3 in. by 3 in. steel plates to obtain the crushing strength. Control specimens each 16 in. long with a section of 4 in. by 4 in. were cast in steel moulds and tested in a similar manner. The results of these tests are shown graphically in Figs. 6-a and 6-b.

Analysis of the results indicated that, apart from the differences between mixes, the most significant influence on modulus of rupture

and crushing strength was a positional effect which may have been related to the position of specimen with respect to the vibrators; specimens at the ends of each beam mould gave low strengths. The coefficients of variation for modulus of rupture and for crushing strength calculated from the mean variances for the specimens in each beam mould were 9.6 and 5.1 per cent respectively, which show that the variation in the strength of the concrete within each joist may have been considerable and would account for part of the residual variances for cracking and failing moments.

The mean values for the modulus of rupture for the control specimens and for those cast on the prestressing bed were 781 and 767 lb. per sq. in. respectively.

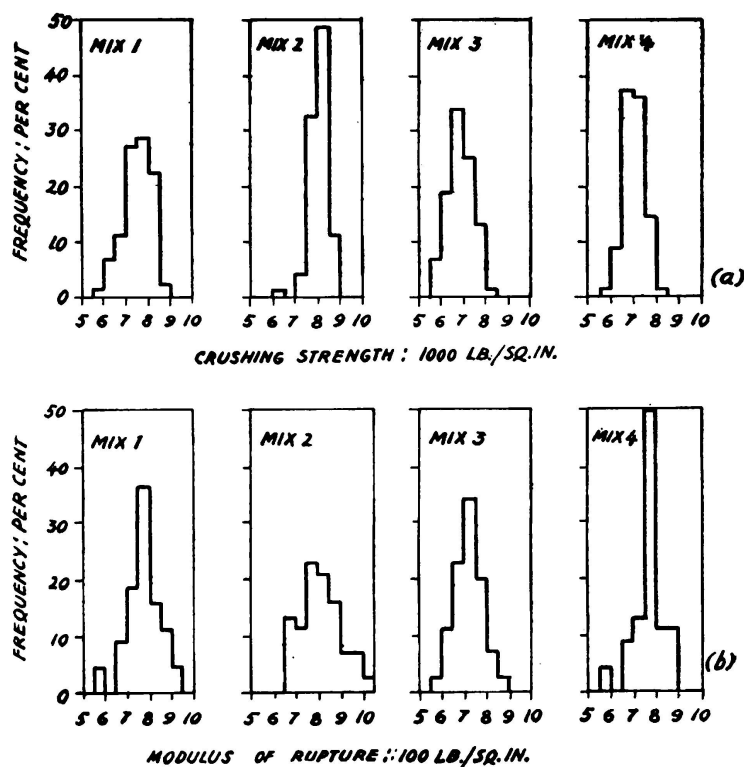


FIG. 6



vely. The corresponding values for crushing strength were 6270 and 7400 lb. per sq. in. respectively. These results suggest that the mean crushing strength of the concrete in the joists may have been higher than that of the concrete cubes; as a result, however, of the variation within the joist, this advantage was not reflected in the results for failing moment, except for one or two results that cannot be explained in any other way.

The variation in the quality of the steel in the sixty-five joists tested was too small to be significant in the examination of variations of cracking and failing moments. To obtain some information on the effects of variations in the strength of the steel, a series of eight joists of the type illustrated in Fig. 1-b were made in pairs in a steel mould by the «individual mould» system of pretensioning. For consolidation of the concrete the mould was clamped to a vibrating table. The strength of the concrete and the steel was deliberately varied. Transfer was effected after 5 days and the joists were tested when they were about 28 days old. The mean results for each pair of joists are given in Table 5 with the strengths of the materials. The average difference between the individual results in each pair was 0.6 per cent and the maximum difference was 1.2 per cent.

TABLE 5  
*Effect of variations in the materials  
on failing moment*

| Tensile strength<br>of steel;<br>lb. per sq. in. | Crushing strength | Failing moment of joists;<br>lb. in. |            |
|--------------------------------------------------|-------------------|--------------------------------------|------------|
|                                                  |                   | Experimental                         | Calculated |
| 286,000                                          | 7,300             | 44,100                               | 43,400     |
| 312,000                                          | 8,530             | 47,500                               | 47,700     |
| 273,000                                          | 6,430             | 40,100                               | 40,400     |
| 326,000                                          | 6,660             | 43,200                               | 43,100     |

The regression equation for failing moment on tensile strength of the steel and cube strength of the concrete was as follows: —

$$M_u = 0.037 t_u + 2.76 C_u + 12,500 \text{ (lb. in. units)} \quad (3)$$

This relationship gave a less significant correlation than was obtained for expressions (1) and (2), the correlation being significant at between the 1 and 2 per cent levels. The failing moments calculated from the regression equation are also shown in Table 5. The residual variance for failing moment of these joists was  $1.70 \times 10^6$  (lb. in. units), which was a very considerable reduction when compared with the residual variances for failing moment in Table 4. It confirms that the use of steel moulds leads to greater uniformity, and also suggests that consoli-

dation of the concrete on the vibration table in the «individual mould» system is superior to the use of fixed shutter-vibrators on the «long-line» system as used in the production of the prestressed concrete joists.

If the failing moment (or the cracking moment) is expressed in terms of the strengths of the materials as in expression (3) in the form: —

$$M_u = X \cdot t_u + Y \cdot C_u + Z \quad (4)$$

then the variance of failing moment can be expressed in terms of the variances of the strengths of the materials and a residual variance: —

$$\sigma^2 M_u = X^2 \sigma_{t_u}^2 + Y^2 \sigma_{C_u}^2 + \sigma_r^2 \quad (5)$$

where,  $\sigma_r^2$  is the residual variance of failing moment resulting from variations in production technique. This analysis of variance could be used to simplify the quality control of the manufacture of prestressed concrete units, which is often checked by loading tests on individual units, by indicating the relative importance of the different causes of variation in the strength of the product. If the regression equation at (3) is used to relate the variance in failing moment with the variances in the strengths of the materials, the following relationship is obtained: —

$$\sigma^2 M_u = 0.00140 \sigma_{t_u}^2 + 7.62 \sigma_{C_u}^2 + 1.70 \times 10^6 \quad (6)$$

The analysis given here is little more than an example of the method as the different strengths of the materials were obtained artificially and not as a result of sampling from production. It could be shown, however, from (6) that variations in the strength of the steel had a smaller effect on the variation of ultimate strength than variations in the strength of the concrete for this method of manufacture. Under well controlled conditions, the coefficient of variation for the steel is unlikely to exceed 3 per cent and the coefficient of variation for the concrete should not be greater than 10 per cent; expression (6) then leads to the finding that the coefficient of variation for failing moment should not exceed 5 per cent. If greater uniformity were required it could most easily be obtained in this case by improving the control of concrete quality. This method of analysis might also be applied to variations in cracking moment; it was not done in this instance as the data are insufficient to give significant results.

#### *The effects of the age of the concrete at transfer*

In the manufacture of the joists in the earlier phases of the investigation, transfer of the prestress to the concrete was effected at between 4 and 6 days after casting, but no information was available to show the effects of earlier or later imposition of the prestress. Several series of joists of the type shown in Fig. 1-b were made in pairs in a steel mould by the «individual mould» system and release of the wires was

effected at different ages. All the joists were tested in the same manner as before, 28 days after casting. The results for one of these series of tests are shown in Fig. 7, where each result for cracking and failing moments represents the mean for two joists. The tensile strength of the steel used in these tests was 286,000 lb. per sq. in.

The age of the concrete at transfer did not have important influence on the observed cracking moment, but this moment was slightly lower when transfer was effected at one day than when it was effected at greater ages. The strength of concrete after one day was between 3,000 and 3,500 lb. per sq. in., which was little more than  $1\frac{1}{2}$  times the maximum precompression in the concrete. Once the cube strength was more than twice the maximum precompression, the age of the concrete at transfer did not appear to affect the cracking moment, and although greater creep losses would have been expected for the earlier ages they were not excessive.

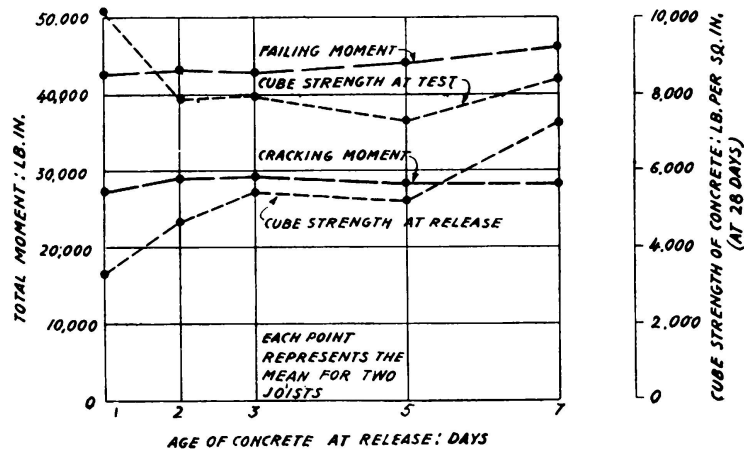


FIG. 7

Within the limits for the age of the concrete at transfer which were examined in these tests, an increase in age at transfer caused a small but distinct increase in the failing moment that was independent of the cube strength of the concrete at the age of test. All the joists in this series failed finally by crushing of the concrete.

#### *The effects of the age of unit at test*

The results of tests on a series of joists of the type shown in Fig 1-b, which were carried out after different periods of storage up to 3 months, were reported by Masterman [1]. Since then further tests have been made on joists from this same series after periods of 1 year and 5 years of storage in the laboratory. The joists were tested in pairs on a clear span of 12 ft. and were supported at each end on brick piers. Uniform loading of dead weights was applied to the joists through a flexible timber platform. The results of the full series of tests are illustrated in Fig. 8.

A single joist from one of the batches included in the series of sixty-five tests was tested at the age of five years under quarter-point loading on a span of 12 ft.  $4\frac{1}{2}$  in. The result of this test is shown in Fig. 2 in comparison with that for a similar joist tested at the age of 2 months.

The results show that the age of the joists has little effect on the failing moment. The cracking moment, however, appears to increase

with age during the first few weeks after stressing, and later decreases after further ageing. The results are insufficient to be conclusive but suggest that the overall loss of prestress for this type of unit may be as much as 30 per cent when they are stored in an unloaded condition for a long period.

### Conclusions

The following conclusions were drawn in relation to the performance of small prestressed concrete units with pretensioned steel as a result of the investigation:—

a) Cracking and failing moments for the joists could be calculated with reasonable accuracy on the basis of simple assumptions;

b) Variations in the behaviour of the units may be much greater than those arising from variations in the qualities of the materials, unless particular care is taken to ensure their dimensional accuracy; in this connection, the use of steel moulds is desirable;

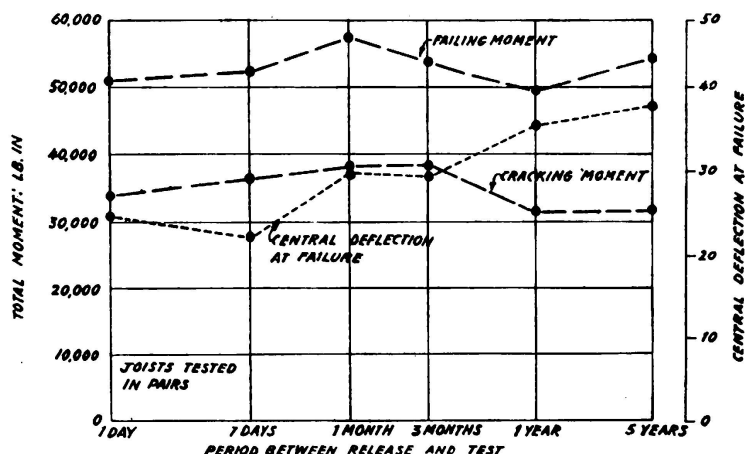


FIG. 8

c) Variations in the cube strength could be related to the consequent variations in the cracking and failing moments;

d) Analysis of the variations in failing moment arising from variations in the strengths of the materials offered a method of facilitating the control of the quality of the product;

e) The age of the concrete at transfer had little effect on subsequent behaviour when the cube strength of the concrete at this stage was greater than twice the maximum precompression in the concrete;

f) The age of the unit at the time of test had little influence on the failing moment, but after 5 years the cracking moment was considerably reduced.

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## ACKNOWLEDGMENTS

The work described in this paper forms part of the research programme of the Building Research Board of the Department of Scientific and Industrial Research, and is published by permission of the Director of Building Research.

## SUMMARY

The paper gives some details of a laboratory investigation of the properties of small prestressed concrete beams, which was based on an examination of their behaviour under static loading. Several different types of unit were tested to failure in routine checks of production on a pilot plant scale. In some of the tests, measurements of the strain in the concrete were made to obtain further information on the process of the development of failure.

One type of unit was tested in sufficient numbers for a study to be made of the sources of variation in behaviour. Significant correlations were established between the strength of the concrete and the bending moment at which cracks appeared and between the strength of the concrete and the bending moment at failure; from these, the effect of variations in the strength of the concrete on the variations in performance was obtained. Experimental examinations of other causes of variations were made to determine the effect on subsequent behaviour of such factors as: —

- I variations in the dimensions of the units;
- II variations in the quality of the concrete arising from casting techniques;
- III variations in the quality of the prestressing steel;
- IV the age of the concrete at the time of transfer;
- V the age of the unit at the time of test.

These sources of variation are discussed in relation to the factory production of prestressed concrete members generally, and a method of examining the different sources of variation is suggested.

## ZUSAMMENFASSUNG

Die Arbeit enthält einige Ausführungen zu einer Laboratoriumsuntersuchung über die Eigenschaften kleiner Balken aus vorgespanntem Beton. Die Untersuchung bezog sich auf das Verhalten bei ruhender Belastung. Verschiedenartige Balkentypen wurden unter laufender Registrierung des Vorgangs bis zum Bruch beansprucht. Bei einzelnen Untersuchungen

Es wurden auch folgende Einflüsse untersucht, um die Verschieden- wurden Spannungsmessungen am Beton vorgenommen, um weitere Aufschlüsse über die Bruchbildung zu erhalten.

Von einem Prototyp wurde eine grössere Anzahl untersucht, um die Ursachen für das verschiedenartige Verhalten ermitteln zu können. Es war möglich, bedeutsame Beziehungen zwischen der Betonfestigkeit und den Biegemomenten bei der Rissebildung und beim Bruch festzustellen; daraus liessen sich die Auswirkungen aus der Aenderung der Betonfestigkeit auf das Verhalten der Bauteile übertragen.

Es wurden auch folgende Einflüsse untersucht, um die Verschiedenartigkeit des Verhaltens zu erfassen:

- I. Aenderung der äussern Abmessungen.
- II. Veränderung der Betonqualität im Zusammenhang mit der Betonherstellung.
- III. Aenderung der Qualität des Vorspannstahls.
- IV. Berücksichtigung des Betonalters beim Aufbringen der Vorspannung.
- V. Berücksichtigung des Alters der Betonbalken zur Zeit der Untersuchung.

Diese Ursachen des unterschiedlichen Verhaltens werden im Zusammenhang mit der fabrikmässigen Herstellung vorgespannter Betonteile besprochen; ferner schlägt der Verfasser ein Verfahren vor, mit welchem die verschiedenen Ursachen der Festigkeitsunterschiede geprüft werden können.

## RESUMO

O autor dá alguns pormenores acerca de um estudo experimental das propriedades de pequenas vigas de betão preesforçado, baseado no exame do seu comportamento sob cargas estáticas. Ensaaiaram-se à rotura vários tipos diferentes de vigas por ocasião de verificações de rotina periódicas de produção de uma fábrica experimental. Em alguns casos mediram-se as deformações do betão de modo a obter pormenores mais completos acerca do modo de rotura.

Um dos tipos de viga foi ensaiado bastantes vezes para se poder elaborar um estudo sobre as causas das diferenças de comportamento que se constatarem. Estabeleceram-se relações interessantes entre a

resistência do betão e o valor do momento flector correspondente à fissuração por um lado, e entre a resistência do betão e o momento flector correspondente à rotura por outro; destas relações deduziu-se o efeito das variações de resistência do betão sobre as irregularidades do comportamento. Efectuou-se um estudo experimental sobre as outras causas de variação, de modo a determinar o efeito dos factores seguintes sobre o comportamento das vigas:

- I Variações das dimensões das vigas;
- II Variações de qualidade do betão provenientes das diferentes técnicas de betonagem;
- III Variações de qualidade do aço das armaduras;
- IV Idade do betão quando da tensão dos cabos;
- V Idade da viga quando do ensaio.

O autor discute estas causas de irregularidade em relação às condições de produção em oficina de vigas em betão preesforçado e sugere um método de estudo para as várias causas.

#### R É S U M É

L'auteur donne quelques détails sur une recherche expérimentale concernant les propriétés de petites poutres en béton précontraint, fondée sur une étude de leur comportement sous des charges statiques. Plusieurs types différents de poutres ont été soumis à des essais de rupture lors de vérifications de routine périodiques de la production d'une usine-pilote. Au cours de certains de ces essais on a mesuré les déformations du béton de manière à étudier de plus près le processus de rupture.

L'un des types de poutre a fait l'objet d'un nombre d'essais suffisamment grand pour permettre d'étudier les causes de différences de comportement que l'on avait observées. L'on a établi des relations intéressantes entre la résistance du béton et le moment fléchissant correspondant à la fissuration d'une part, et entre la résistance du béton et le moment fléchissant correspondant à la rupture d'autre part; ces relations ont permis de déterminer l'effet des variations de résistance du béton sur l'irrégularité du comportement. Les autres causes d'irrégularité ont également été étudiées de manière à déterminer l'effet des facteurs suivants sur le comportement des poutres:

- I Variations des dimensions de la poutre;
- II Variations de la qualité du béton dues à des techniques de bétonnage différentes;
- III Variations de qualité des aciers de l'armature;
- IV L'âge du béton lors de la mise en précontrainte;
- V L'âge de la poutre lors de l'essai.

L'auteur discute les causes de variation par rapport aux conditions de production en atelier de poutres en béton précontraint et propose une méthode d'examen de ces différentes causes.

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## **Vla2**

### **Cuidados a ter na construção das grandes pontes de betão armado**

### **Précautions à prendre au cours de l'exécution des ponts en béton armé de grandes dimensions**

### **Practice of large reinforced concrete bridge construction**

### **Die Berechnung von grossen Brücken in Eisenbeton**

EDGAR CARDOSO

*Prof. do I. S. T.*

Lisboa

#### **1 - INTRODUÇÃO**

Dum modo geral o engenheiro projectista, quer demasiado influenciado pelas operações analíticas ou por processos especiais de cálculo, quer pela sua pouca experiência da construção, dimensiona as obras que concebe sem ter em conta alguns dos importantes esforços que se geram durante as diferentes fases de execução, como se elas fossem por magia construídas de um jacto e só entrassem em funcionamento depois de completamente prontas e descimbradas.

Por outro lado, o empreiteiro, por justificados motivos de interesse material, tem a natural tendência de levar àvante da forma mais económica a construção que lhe foi adjudicada sem se importar com as hipóteses de que o autor do projecto se serviu para base dos seus estudos, para ele até a maior parte das vezes desconhecidas.

Evidentemente que não deverá ser assim.

Logo após algumas horas da primeira betonagem o betão ganha forma própria e começa a ser solicitado. Todos os engenheiros o reconhecem e desde os bancos da escola técnica onde se diplomaram ouviram as razões pelas quais é necessário verificar a estabilidade das construções à medida que estas se vão desenvolvendo, a que não é estranho a variável tempo, pois são, de resto, os próprios regulamentos a exigí-lo.

Todos os constructores sabem também que uma betonagem que não siga um plano devidamente estudado, que um cavalete mais deformável ou uma interrupção dos trabalhos são a causa de uma, por vezes completa, alteração das tensões calculadas.

Ora, se tais factos, de uns e outros conhecidos, não têm importância na maioria das obras correntes, o mesmo não se poderá dizer das construções de grande volume em que, por exemplo, as cargas permanentes provocam estados de tensão bem superiores aos das sobrecargas de serviço ou em que os esforços da retracção diferencial chegam a provocar a própria rotura do material.

Assim, se uma viga contínua for betonada dos apoios para o meio dos vãos poderá acontecer que à medida que a betonagem vai progredindo, vá o cavalete por seu turno sofrendo deformações que levem o betão já endurecido a absorver uma grande parte das cargas que realmente competiram à obra provisória de apoio. Resultará um maior valor para os momentos flectores nos apoios e uma acentuada redução nos vãos relativamente aos que corresponderiam ao cálculo da viga contínua ideal.

Uma outra ordem de betonagem poderá certamente originar alterações em sentido inverso.

Um pilar de grande largura com a betonagem interrompida por algum tempo em certa junta estará sem dúvida predisposto a fissurar no betão mais jovem, perpendicularmente à junta, dado que este contrairá, por efeito de presa, enquanto que o que lhe serviu de apoio já havia contraído na sua maior parte.

Estas considerações são suficientes para só por si justificarem o maior cuidado na elaboração do projecto das grandes obras de betão armado e das disposições que devem ser adoptadas na construção para que exista perfeita correspondência entre a concepção analítica e a materialização dos cálculos. É destes problemas que vamos tentar dar uma ideia através de casos concretos e de que somos responsáveis, total ou parcialmente, pelos defeitos a seguir assinalados.

## 2 - BETONAGEM E O FUNCIONAMENTO PREMATURO DA CONSTRUÇÃO OU DOS SEUS ELEMENTOS CONSTITUTIVOS

Um elemento estrutural de grandes dimensões não pode ser betonado duma só vez e instantaneamente. Há pois que fraccioná-lo, mesmo na hipótese de betonagem contínua.

No caso particular das pontes — mais geralmente o arco e o tabuleiro vigado — duas técnicas principais podem ser usadas:

betonagem por camadas sobrepostas;

betonagem a toda a altura dos elementos resistentes e em fracções do comprimento.

Ainda outro caso se apresenta: o das estruturas formadas pela associação de vários elementos que não podem ser executados simultaneamente.

### 2.1 - Betonagem por camadas

Neste caso, de betonagem por camadas sobrepostas, é necessário garantir que as camadas já endurecidas não entrem em funcionamento

premature, o que será difícil de conseguir ou, melhor ainda, que possam resistir eficazmente aos esforços gerados.

O arco (ou abóboda) executado por camadas sobrepostas (Fig. 1-a) terá que receber as sucessivas camadas sem que as anteriores sejam fechadas. Executando primeiramente o intradorso, depois a alma e finalmente o extradorso, mesmo «corrigido» o arco, de forma alguma poderá trabalhar como um único monólito resistente para o seu próprio peso como se ele fosse de material homogêneo e só entrasse em funcionamento depois de completamente executado. Mesmo que não existisse contracção

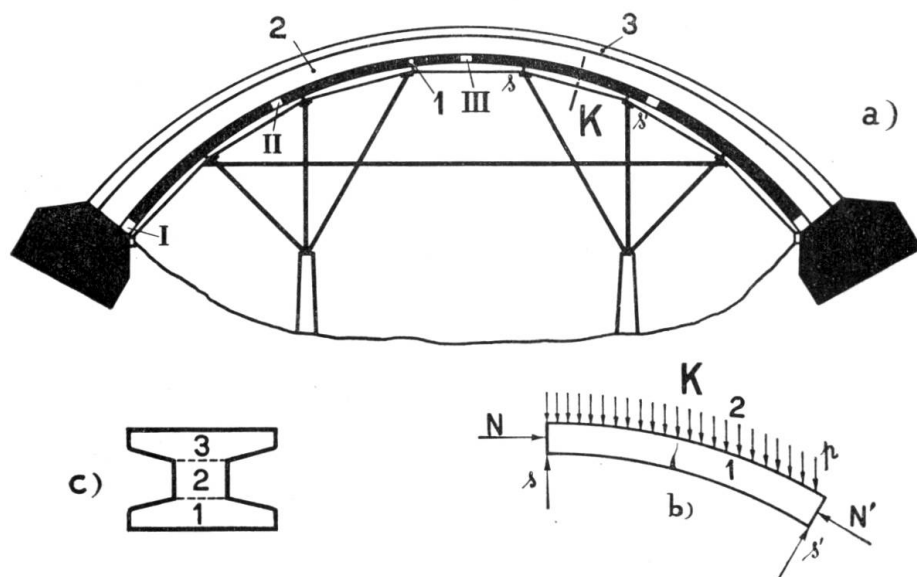


FIG. 1. Esquema dum arco betonado por camadas

diferencial ter-se-ia de admitir que a primeira camada estivesse já solicitada por parte do seu próprio peso e da segunda camada e que estas duas suportariam, em parte, a terceira camada, em condições extremamente difíceis de precisar.

E, se o cimbrio tem apoios suficientemente espaçados haverá que garantir que entre esses apoios ss' (Fig. 1-a) a deformação da cambota não provoque, nas débeis camadas endurecidas, flexões exageradas pela actuação do seu próprio peso e em particular pelo peso do betão fresco.

De facto, se fecharmos as juntas I, II, III... antes de lançarmos o betão da 2.<sup>a</sup> camada ao executar-se esta passará a primeira a ser solicitada pois é de admitir que o cimbrio por ser mais deformável absorva apenas uma pequena fracção da carga do 2.<sup>o</sup> anel. Por melhor que se pense corrigir os esforços do arco, dado que a correcção não poderá provocar o deslizar das camadas, há-de fatalmente haver fortes tensões no intradorso em benefício do extradorso, resultando o imperfeito funcionamento da estrutura. Mas se não executarmos as juntas I, II, III... então, não existindo o impulso parcial N e N' (Fig. 1-b) pode dar-se o caso do troço entre escoras s e s' trabalhar à flexão sob a acção do peso p da camada 2, a ponto de romper por não possuir armaduras adequadas. Ao fugir-se dum perigo poder-se-á cair, portanto, num perigo maior.

Seja agora o caso duma *viga contínua* (Fig. 2).

Ao betonar-se a camada 3 sobre as camadas já endurecidas 1 e 2 estaremos a colocar cargas sobre uma viga resistente que evidentemente só transmitirá os seus efeitos ao cavalete depois de partir, dado que uma viga de grande altura tem deformações desprezíveis relativamente

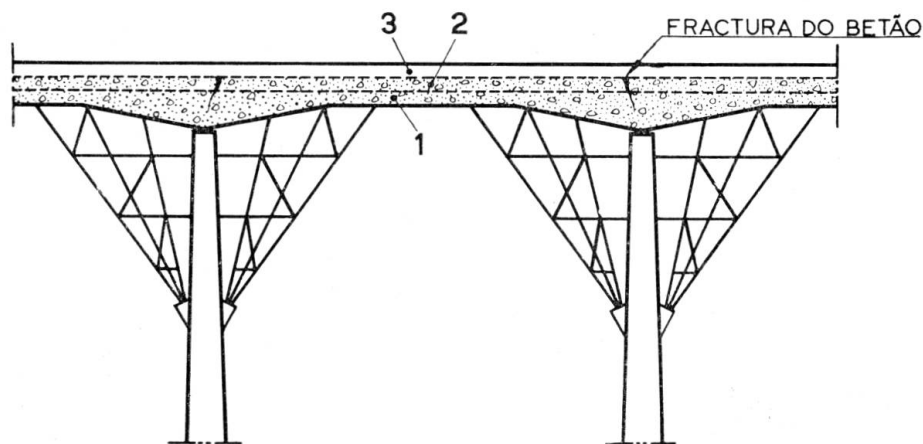


FIG. 2. Viga contínua betonada por camadas

ao cavalete. A meio dos vãos natural é que a viga resista por existirem armaduras suficientes mas nos apoios só não se dará a rotura se nas camadas 1 e 2 existirem fortes armaduras de tracção, o que nem sempre é previsto por quem projecta. Uma destas fendas foi notada na betonagem assim estabelecida no primeiro apoio da ponte do Vale da Ursa, ponte que se representa na Fig. 3. Verificada a deficiência do projecto no que se refere à distribuição das armaduras de tracção nas zonas sobre os apoios e alterado o plano de betonagem de modo a que as camadas de betão envolvessem sempre pelo menos uma fiada de armaduras deixarem de aparecer fendas em todos os outros apoios.

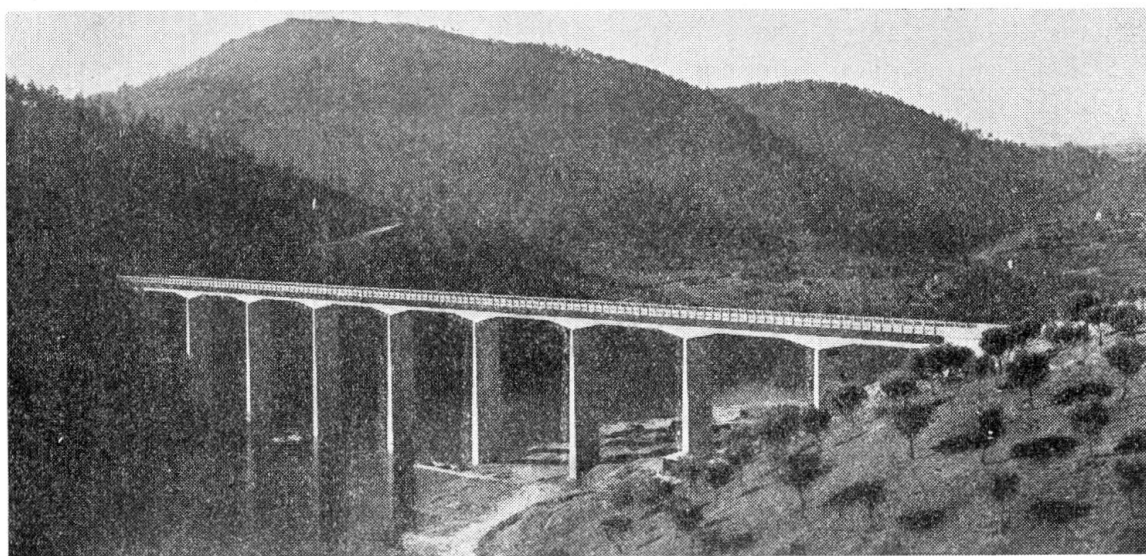


FIG. 3. Ponte do Vale da Ursa na albufeira de Castelo de Bode

## 2.2 – Betonagem a toda a altura dos elementos resistentes

Também quando se utiliza esta técnica de betonagem não são de desprezar os esforços gerados durante a construção. Assim, no caso dum arco, as grandes aduelas, devido aos movimentos do cimbre, funcionam

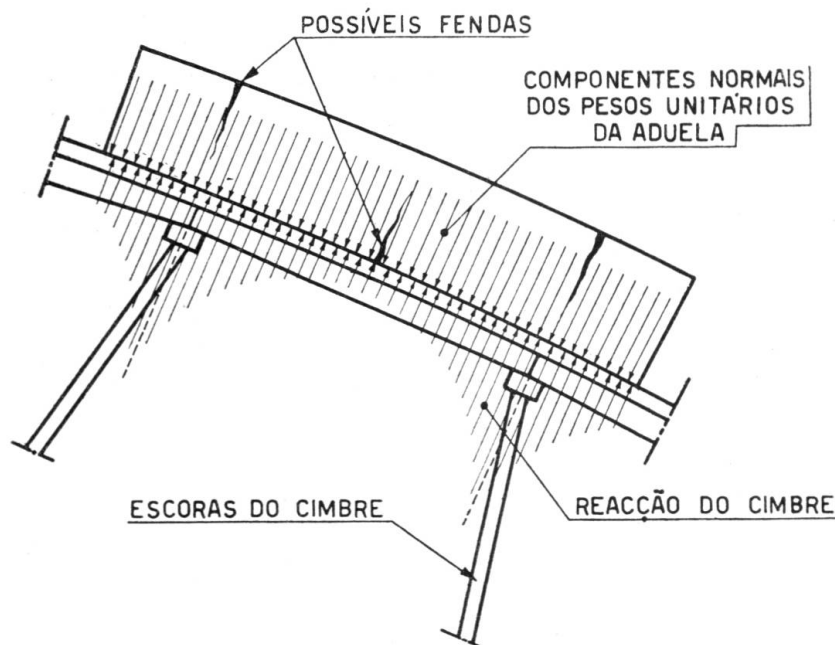


FIG. 4. Esquema duma aduela dum arco betonado a toda a altura das secções

à flexão pelo facto da distribuição das reacções do cimbrio não corresponder ponto por ponto ao peso da aduela. É pois possível a existência de fendas como se indicam na Fig. 4, que desaparecerão quando o arco entrar no seu verdadeiro funcionamento. E se o arco é ôco (Fig. 5-a)

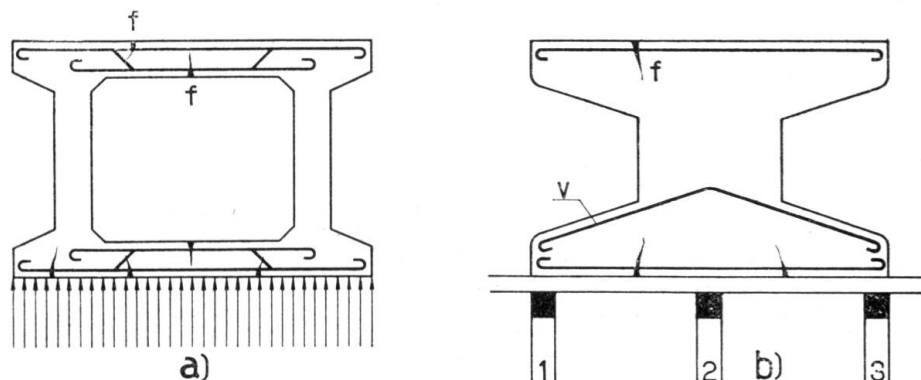


FIG. 5. Esquema das secções dum arco vazado (a) e dum arco aligeirado (b). Localização das possíveis fendas

ou de secção aligeirada em 1, (Fig. 5-b), podem ainda gerar-se esforços na direcção transversal.

Assim, admitindo que a aduela se encontra já executada, pelo facto de não existir impulso que se oponha às forças resultantes do peso e da reacção do cimbrio, os banzos, em especial o inferior, serão fortemente solicitados à flexão. Se não se betona rapidamente toda a aduela ao



executar-se o banzo superior já o banzo inferior estará endurecido e então o peso daquele, transmitindo-se apenas pelas almas ao cimb্রে produzirá neste uma reacção transversal uniforme ou quase uniforme a que o banzo de intradorso do arco terá que resistir por flexão.

Por seu turno o banzo superior, logo a seguir à desmoldagem, ficará solicitado sob a acção do seu próprio peso, com a tendência a romper segundo as linhas *f* indicadas nas Figs. 5-a e 5-b.

Como se verifica pela Fig. 6 — pormenor das aduelas do arco da ponte da Foz do Sousa indicada na Fig. 7 — a secção transversal do arco é do tipo da Fig. 5-b, apresentando os banzos uma saliência de 1,60 m para cada lado da alma. Para garantir a segurança das aduelas contra tal solicitação foi pois necessário prever as armaduras indicadas.

E se no descimbramento se aliviam primeiramente as cambotas 1 e 3 (Fig. 5-b), serão de admitir esforços no banzo inferior em sentido inverso obrigando por seu turno a empregar as armaduras *v*. Retirando em primeiro lugar a cambota 2, sob a alma, maiores tensões de tracção surgirão na face inferior do intradorso.

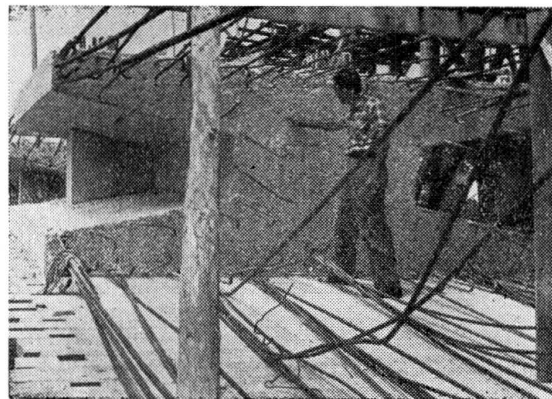


FIG. 6. Pormenor da secção duma aduela do arco da ponte da Foz do Sousa

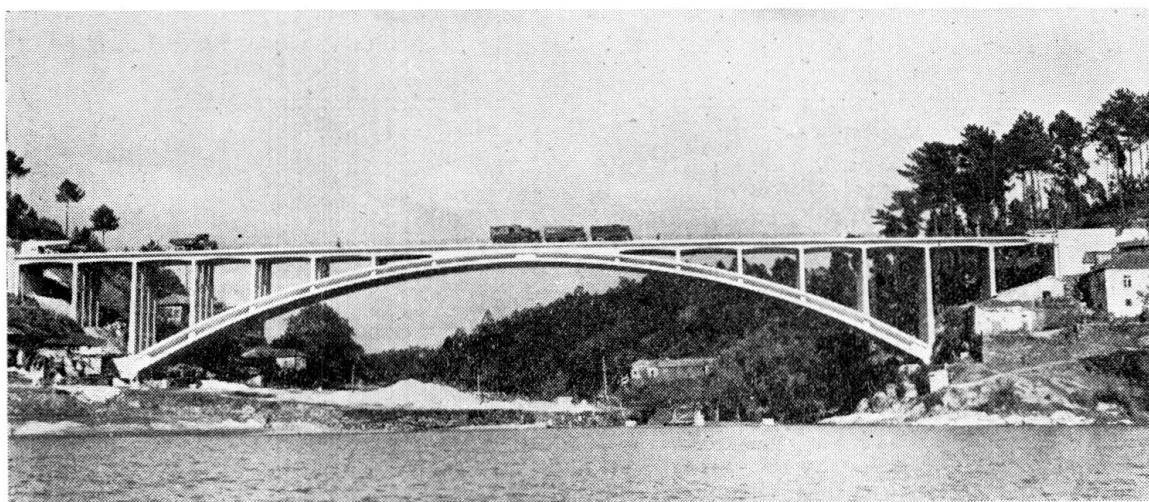


FIG. 7. Ponte da Foz do Sousa — Conjunto

É pois necessário que as secções da estrutura sejam concebidas para resistirem não só ao seu próprio peso mas também que o descimbramento se faça por forma a não obrigar a dispêndio escusado do material para absorver com segurança as tensões acidentalmente desenvolvidas nas diferentes fases de execução.

Procedendo com todos estes cuidados — no projecto e na execução (sem falar no funcionamento longitudinal, por mais conhecido) foi a citada ponte da Foz do Sousa construída sem que se notasse a mais insignificante fenda.

No caso das *vigas contínuas*, com a betonagem por troços a toda a altura, o problema tem ainda alta complexidade.

Para que o funcionamento da viga sob a acção do seu próprio peso corresponda exactamente à hipótese comumente idealizada no projecto torna-se necessário que a viga, uma vez terminada, não esteja já esforçada por deformação dos cavaletes, condição que para ser satisfeita obriga a admitir que a reacção do cavalete seja em cada zona elementar igual ao peso do volume do betão sobre essa zona.

Praticamente tal hipótese só se consegue deixando pequenos intervalos  $i$  Fig. 8 entre os diferentes troços 1, 2, 3, 4... e desligando as próprias armaduras o que evidentemente complica a construção.

O que acontecerá então quando não se deixam esses intervalos e se não desligam ou interrompem as armaduras?

Se executarmos a betonagem a partir dos apoios para o meio vão, sempre para um e outro lado do mesmo pilar, é certo que à medida que

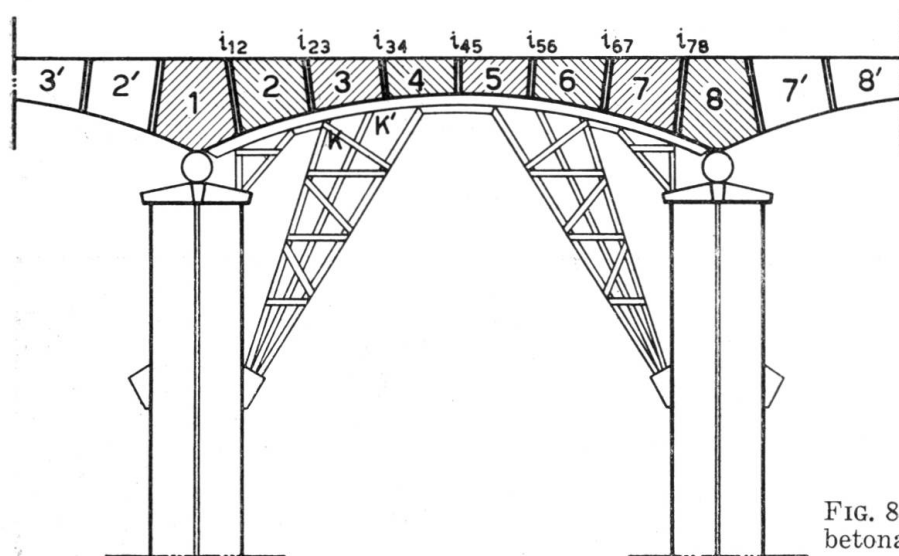


FIG. 8. Viga contínua betonada por aduelas

a betonagem vai progredindo começarão a gerar-se esforços nas duplas consolas sobre cada pilar pois que uma parte do peso do betão informe será absorvido pelo atrito e aderência das juntas de betonagem, pela continuidade dos moldes e do cavalete e das armaduras, em vez de ser integral e localmente transmitido ao cavalete.

Assim, o troço 3, (Fig. 8), deveria descarregar inteiramente sobre a zona  $KK'$  mas estando este troço ligado à parte restante já construída 1 e 2, viria para o cavalete apenas uma fracção do peso 3. Prosseguindo por esta ordem com a betonagem a viga contínua passaria a ter momentos flectores nos apoios consideravelmente maiores do que os correspondentes à viga contínua teórica.

Deixando para o fim as zonas 1 e 8, sobre os pilares, dado que os troços 2 e 7 têm apoios no cavalete consideravelmente mais rígidos que

os centrais 3, 4, 5 e 6, resultará que a viga 2-7, já resistente, entrará em funcionamento de peça simplesmente apoiada antes de existir o betão da viga sobre os pilares ou de este betão estar em condições de suportar esforços. Nesta hipótese, aparecerão apenas tenues momentos flectores sobre os apoios.

Só pelo plano de betonagem adoptado será pois possível prever com precisão qual o estado de tensão com que a viga fica depois de descimbrada.

Do exposto e atendendo ao facto de ser sempre ou quase sempre vantajoso reduzir os esforços nos vãos em prejuízo dos apoios deverá a primeira solução ser a adoptada.

Mas é preferível contar nos cálculos com o momento flector mais conveniente para o vão e para os apoios e estabelecer rótulas provisórias que permitam que a estrutura antes de «fechada» seja isostática. Ao «bloquear» os troços que fazem de rótulas a estrutura passará a hiperestática para as solicitações ulteriores, ficando contudo com esforços devidos à carga permanente perfeitamente conhecidos e com a melhor distribuição.

Citaremos a este respeito dois exemplos.

#### 2. 2. 1 - *Caso da Ponte de Coimbra*

Esta ponte é formada por 5 vãos contínuos com uma articulação num vão extremo, apoiada em pilares intermédios pendulares e com os os vãos extremos ligados monoliticamente a montantes articulados na base e constituindo uma superestrutura do tipo pórtico múltiplo.

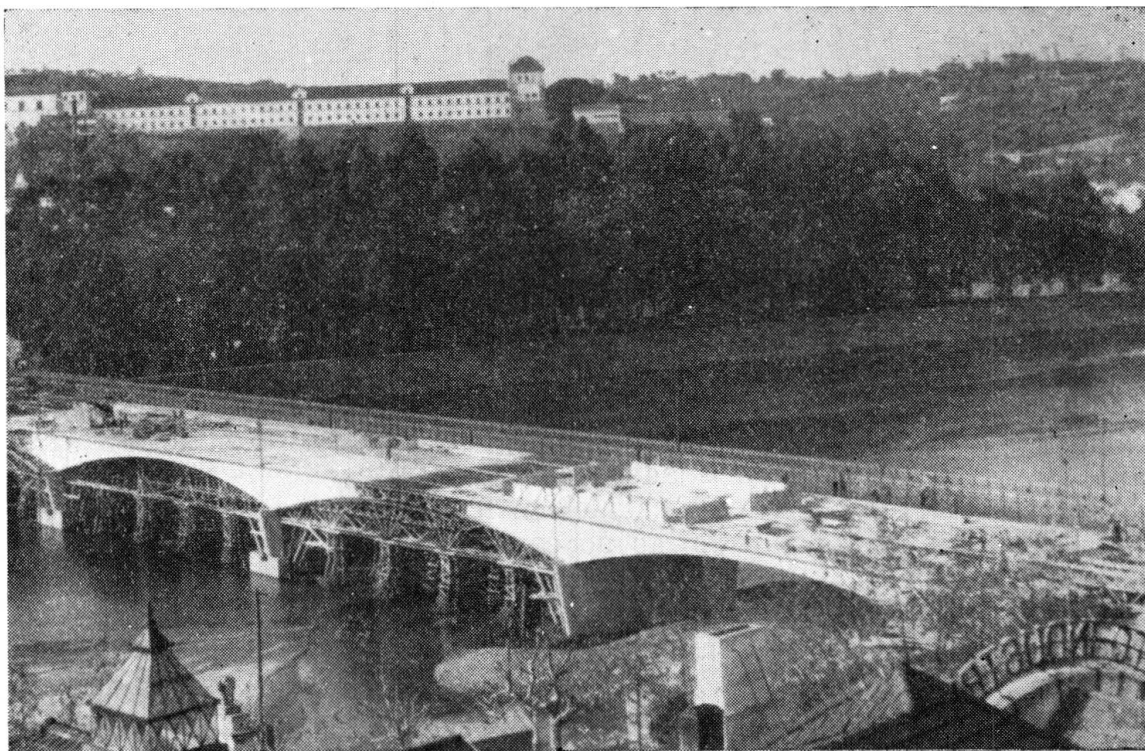


FIG. 9. Ponte de Coimbra — fase de construção — Superestrutura em funcionamento isostático por não estarem fechados os vãos 2.º e 4.º



Para termos a garantia de que a distribuição dos esforços era a desejada e que não se introduziam esforços internos por assentamentos diferenciais das fundações procedeu-se à betonagem do vão central e dos vãos extremos e duma certa parte em consola dos vãos 2.º e 4.º ficando estes por fechar durante tempo suficiente para que se produzissem

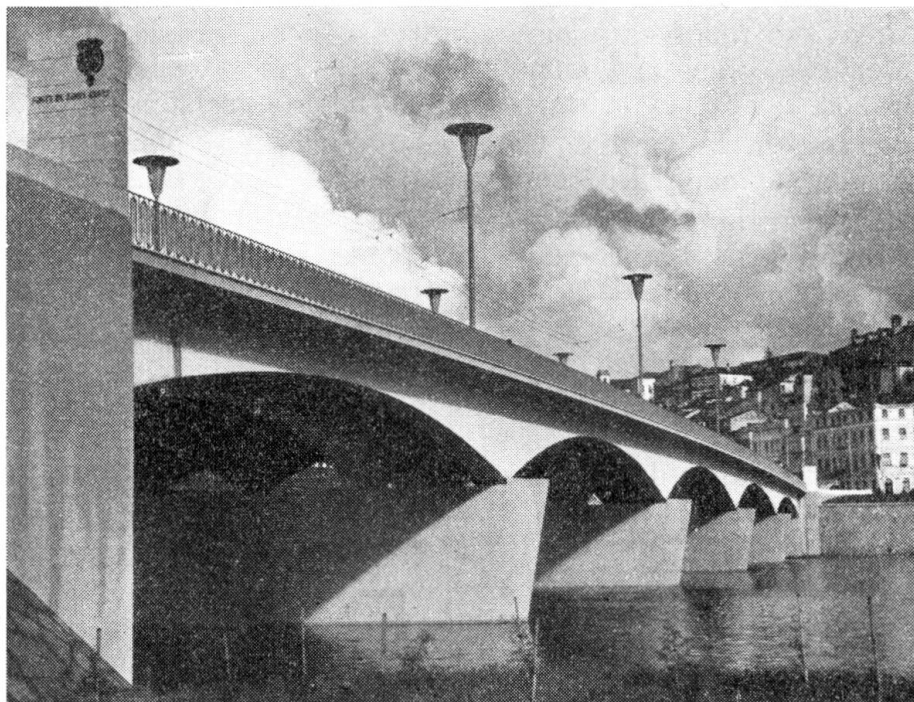


FIG. 10. Ponte de Coimbra (Santa Clara), terminada

os assentamentos dos pilares e se comprovasse por ensaios de carga a indeformabilidade das fundações. Chegou-se mesmo a descimbrar completamente o 1.º vão e a maior parte do 3.º antes de se executarem os pequenos troços centrais de fecho dos vãos 2.º e 4.º, como se vê na Fig. 9.

A correcção dos esforços pode pois fazer-se pela variação do comprimento dos troços em consola, de cargas acidentais colocadas em certas zonas ou com o auxílio de macacos hidráulicos impulsionando pontaletes situados nos vãos ou nas consolas. Utilizou-se esta última técnica para corrigir os esforços com o que se reduziram de cerca de 30 mm as contra-flechas das consolas, como convinha.

Tomando estes cuidados, no projecto e na construção, foi a ponte terminada com pleno êxito. Assinalamos apenas o facto de não termos ligado a importância devida aos deslocamentos durante as diferentes fases da betonagem do que resultou uma razante poligonal curvilínea com ângulos nos topos das consolas e que posteriormente teve que ser corrigido pela pedra de cantaria do focinho da laje dos passeios, correcção que resultou perfeita como se verifica pela Fig. 10.

O estudo das concordâncias dos troços a betonar não pode ser desprezado para que arquitectonicamente nada haja a dizer ou a corrigir. É pois a conclusão que se tira desta última nota.

### 2. 2. 2 – Caso das Pontes do Cávado e do Caldo

Depois da betonagem da ponte do Cávado, na albufeira da Caniçada da Hidro Eléctrica do Cávado, ao serem retirados os taipais laterais das zonas vazadas sobre os pilares, notaram-se umas ligeiras fissuras capilares na parte superior do banzo inferior (Fig. 11). Por essas fissuras se apresentarem em todos os apoios admitiu-se nessa ocasião tratar-se dum efeito sistemático — possivelmente o resultado duma cedência do cavalete antes que o betão dessas zonas tivesse suficiente resistência.

Na betonagem da ponte do Caldo adoptou-se outro plano de betonagem e reforçou-se a armadura da zona onde se notaram as fissuras na ponte do Cávado com a finalidade de evitar a repetição daquelas fissuras.

Efectuado o descimbramento da ponte do Cávado essas fissuras aumentaram um pouco mais de espessura, não se notando contudo outras.

Imediatamente a seguir a uns dias de intenso calor tropical novas fissuras capilares surgiram em *s* e *m* (Fig. 11).

Revistos os cálculos com a maior atenção concluiu-se estarem praticamente exactos para as hipóteses formuladas e que é uso considerar, mas não poderia haver dúvidas que algum erro fundamentalmente importante estava em acção.

Analísado o problema sob uma mais alta especulação analítica e experimental — esta com o auxílio de modelos reduzidos de aglomerado de cortiça, de gesso e de celuloide, planos e espaciais — a par do minucioso exame dos cavaletes e dos planos de betonagem, concluiu-se inequivocamente terem essas fendas origem:

- a) numa importante modificação do diagrama dos momentos devidos à carga permanente, que em vez de ser  $M M'$  passou a ser  $M_1 M'_1$ , (Fig. 11), proveniente da formação de rótulas plásticas na zona vasada dos apoios, devidas ao sistema de betonagem e à deformabilidade dos cavaletes;

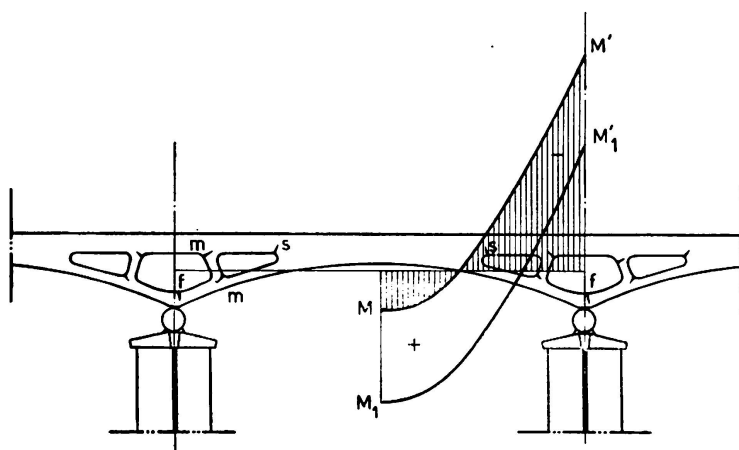


FIG. 11. Esquema duma viga contínua de rótula. Diagramas dos momentos flectores e fissuras

- b) na variação de temperatura da laje do tabuleiro, de menor espessura e inteiramente exposta, relativamente às nervuras das vigas, de maior massa e protegidas, quando sobre ela incidiu o intenso

calor solar de uns dias de verão o que aumentou ainda mais consideravelmente o momento positivo, fenómeno que «grosso-modo» se pode computar de intensidade da ordem de grandeza da carga permanente a avaliar pelas flechas observadas em cantilevers construídos, pelos próprios ensaios em modelo reduzido e pelos cálculos analíticos efectuados;

- c) na retracção do betão (contração de presa) e mais particularmente na retracção diferencial das zonas menos armadas para as mais armadas.

Estas diferentes acções, sendo do mesmo sinal, somam os seus efeitos e dada a pequena percentagem de armaduras dessas zonas (pois o seu funcionamento previsto no cálculo era especialmente de compressão) as fendas deveriam realmente formar-se como se constatou na supracitada análise.

Qual a solução a adoptar para dar à estrutura o estado de tensão inicialmente previsto ou o mais aconselhável?

Depois de aturado e cuidadoso estudo e perante o espanto dos operários das obras, resolvemos cortar alternadamente os vãos os quais apenas ficaram ligados pelas armaduras inferiores. Deixaram pois de existir momentos positivos nos vãos constatando-se nos novos cantilevers formados (em substituição da viga contínua) as deformações correspondentes à eliminação dos momentos positivos.

Uma vez cortado o betão e o ferro dessas zonas e ainda corrigido o estado de solicitação com o auxílio de cargas adicionais nos extremos das consolas, soldaram-se a electrogéneo as armaduras cortadas. Retiradas essas cargas encheram-se de betão as zonas que se havia demolido. Desta forma aumentaram-se de 10 a 15 % os momentos flectores sobre os apoios (aumento que se eliminará para o efeito da variação diferencial de temperatura) o que não apresenta qualquer perigo para as estruturas mas em contra-partida puseram-se em funcionamento óptimo aquelas secções que temporariamente foram esforçadas além do que era permitido.

É pois de concluir que um tabuleiro contínuo merece toda a atenção para que de qualquer modo não haja — por razões construtivas — uma profunda alteração na distribuição dos esforços inicialmente calculados. E dada a grande dificuldade de evitar tais inconvenientes só um caminho seguro existe: considerar, no projecto e na construção, para a carga permanente, articulações provisórias que obriguem a ponte a funcionar em condições de isostaticidade.

Mas se por uma razão qualquer o mal já está feito, que não exista pelo menos o receio de lhe dar o remédio já indicado: cortar a ponte em certas zonas onde deveria ser nulo o momento flector ou onde mais convenha e voltar depois a ligá-la novamente, soldando as armaduras e refazendo o betão demolido.

### 2.3 — *Betonagem duma estrutura monolítica constituída por vários elementos*

Quando a superestrutura é formada por vários elementos constitutivos que sejam obrigados a um funcionamento de conjunto também o plano

de betonagem pode ter fundamental importância. É necessário que ao «fechar» um certo elemento ou parte de elemento não existam esforços nos restantes elementos da «malha» que se completa, particularmente se esta é triangulada e que possam ser prejudiciais em virtude das cargas que faltam não poderem já contribuir para os anular ou reduzir, como sucede nos bow-string, vigas Langer, Nielsen, e em tipos particulares de arco de tímpanos vazados.

Seja o caso da ponte em arco de tabuleiro superior de tímpanos vazados — arco esbelto e tabuleiro rígido, como se adoptou na *ponte de Barca d'Alva* (Fig. 12).

Este tipo de ponte é particularmente vantajoso por permitir a construção com um cimbres muito ligeiro pelo facto dos muretes e do tabuleiro,

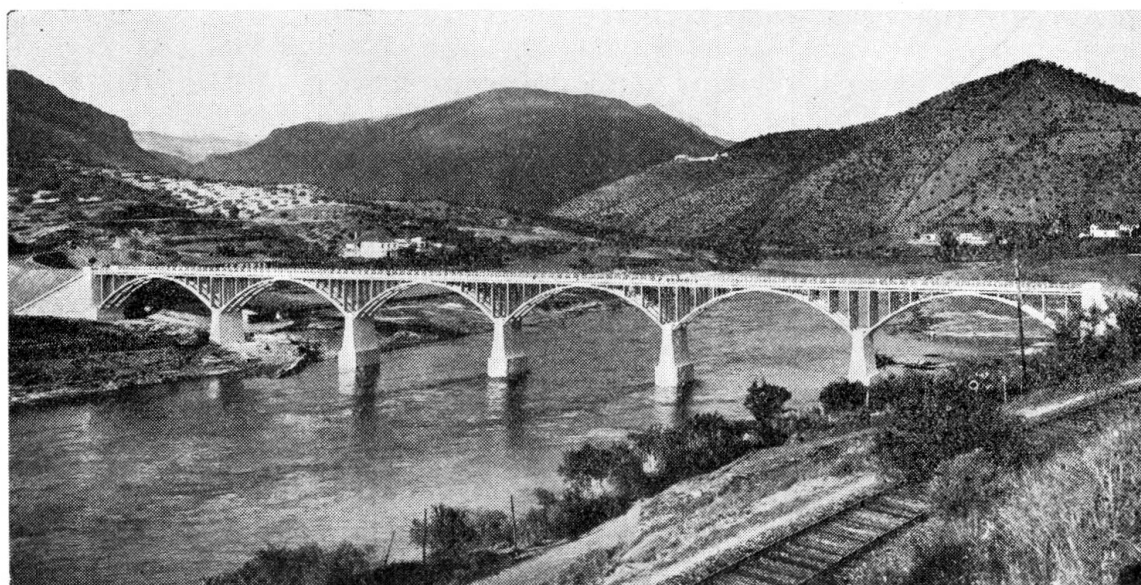


FIG. 12. Ponte de Barca d'Alva, do tipo arco esbelto e tabuleiro rígido

que mais pesam, poderem executar-se já com o arco a servir de cavalete, desde que se distribuam convenientemente as zonas de betonagem.

De qualquer modo, suponhamos que se executa o tabuleiro pela ordem indicada na Fig. 13 começando pela zona do fecho. Quando estiverem executados os troços 1, 2, 3, 4, 5 e 6, o arco funciona como elemento isolado apenas com uma perturbação na zona do fecho (visto que o betão aderindo ao do arco já trabalha em conjunto para as cargas 2, 3, 4, 5 e 6.

Mas, algumas horas depois de se betonarem as zonas 7 e 8, por exemplo, já o arco tem os seus movimentos condicionados ao tabuleiro nas zonas entre os troços 2 e 3. Então, as cargas adicionais 9 e 10, de algumas dezenas de toneladas, não poderão colaborar integralmente nesses troços do arco visto ser agora o tabuleiro a absorver a maior parte dos momentos flectores.

Se no projecto se havia admitido — como geralmente sucede — que a directriz do arco é o antifunicular do carga permanente, o momento que foi para o tabuleiro nas zonas entre 2 e 3 proveniente das tais cargas 9 e

10 fará imensa falta ao arco por não permitir a centragem da linha das pressões, por hipótese admitida nos cálculos.

Em conclusão, a execução da obra tem que corresponder ao cálculo ou este ao processo de execução. No presente caso, em que se adoptou

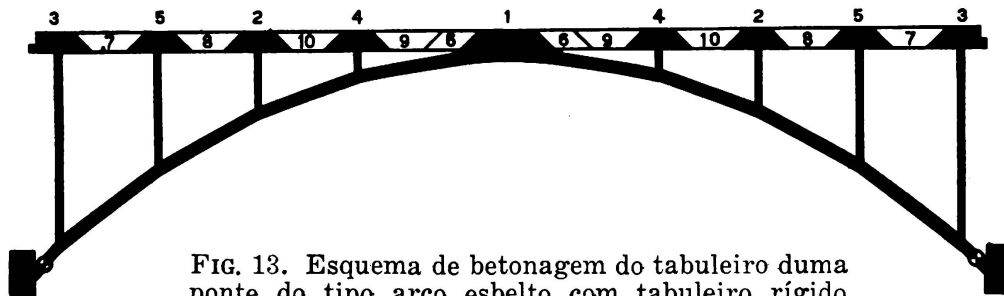


FIG. 13. Esquema de betonagem do tabuleiro duma ponte do tipo arco esbelto com tabuleiro rígido

para directriz o antifunicular da carga permanente dever-se-á «fechar» simultaneamente todos os troços 7, 8, 9 e 10, quer por camadas quer preferivelmente a toda a altura em fracções da largura.

### 3 - A CONTRACÇÃO DIFERENCIAL E OS SEUS EFEITOS NAS CONSTRUÇÕES MONOLÍTICAS

O betão é um material com «vida própria» de características bem conhecidas, função do tempo, para a mesma génese. Além das características mecânicas as mais importantes são a *contração de presa*, dum modo geral prejudicial e a *fluência*, fenómeno de auto-adaptação ao mínimo esforço, quase sempre favorável.

Sendo pois a contração uma função do tempo, traduzida por uma curva O, a, b, c, d com a configuração da Fig. 14, resulta que, se sobre um certo betão já resistente de  $n$  dias de idade, executarmos um novo troço de betão este não pode contrair livremente segundo a curva O' a' b' c' d'.

Suponhamos que não existe a fluência e que os betões resistem, o novo às tracções e o mais idoso às compressões (este sem dúvida resistirá). Ao fim de  $m$  dias a junta de betonagem do betão velho encurtou de  $m_1$  e o novo betão que devia ter encurtado se fosse livre de  $m''_1$  o'' encurtou apenas  $m'_1$  o'', isto é, ficou submetido a uma tensão correspondente a  $m''_1 - m'_1$ . Gerar-se-ão, portanto, na superfície de betonagem tensões de escorregamento e perpendicularmente tracções no betão mais jovem e compressões correspondentes ao encurtamento adicional  $m'_1$  no betão mais idoso.

Estas tensões raramente provocam a rotura tanto mais que são atenuadas pela fluência, principalmente a do jovem betão. Contudo,

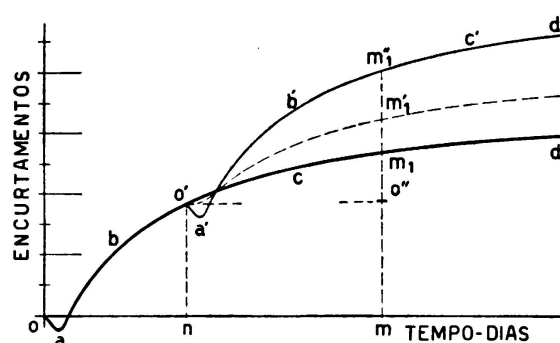


FIG. 14. Variação da contração do betão com a idade — Contração diferencial



tratando-se de juntas de betonagem muito extensas — no caso das barragens, dos elementos de pontes muito largas — tal rotura poderá surgir se não se adoptam disposições construtivas adequadas como é, com efeito, suficientemente conhecido dos engenheiros da especialidade.

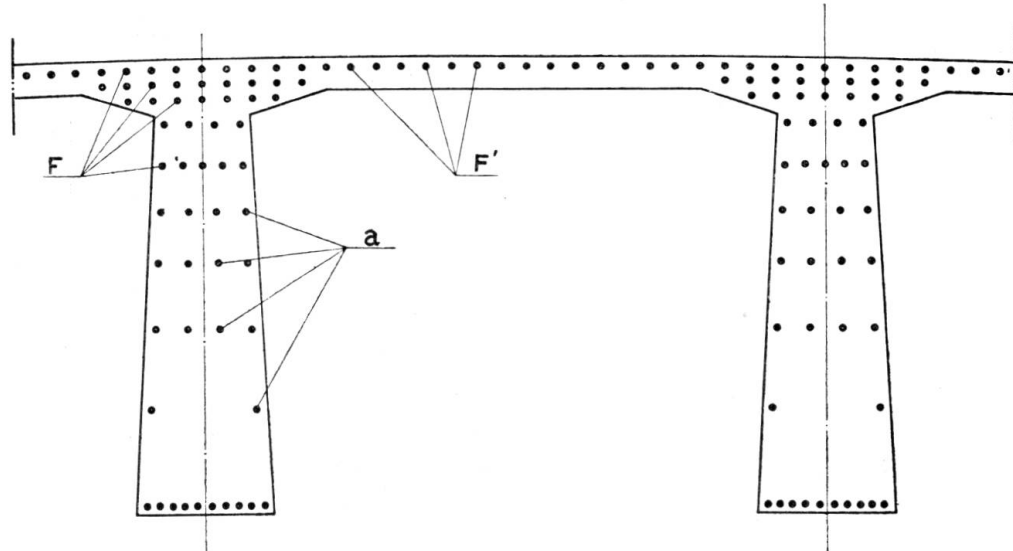


FIG. 15. Esquema da distribuição das armaduras nas zonas dos apoios das vigas contínuas, em «T»

Um outro aspecto do problema diz respeito à diferença da contracção verificada entre as zonas com grande percentagem de armaduras e das zonas fracamente armadas, embora da mesma idade, efeito nem sempre devidamente tido em conta nos projectos, fenómeno importante, sem dúvida, para as peças de grande comprimento e de apreciável altura das secções transversais ou de grande largura com armadura concentrada em restritas zonas.

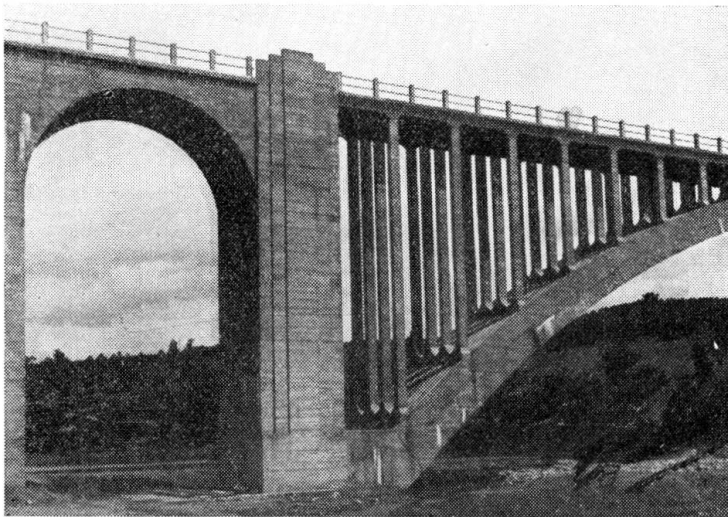


FIG. 16. Ponte em arco, betonado por anéis

Referimo-nos às vigas altas armadas à tracção e à compressão (para ter em conta a alteração dos esforços com as posições das sobrecargas) com reduzidas armaduras nas almas ou a grandes

vãos de laje dos tabuleiros vigados nas zonas dos momentos negativos das vigas "T". É pois necessário colocar armaduras *a* nas almas (Fig. 15) e distribuir as armaduras de tracção das vigas, pela alma *F* e pela laje, *F'*. Se assim não está indicado nos desenhos deverá o construtor

chamar a atenção do autor do projecto ou das entidades da fiscalização dos perigos que poderão advir.

Tais dispositivos para reduzir os efeitos da contracção diferencial consistem, no primeiro caso, na alternância, das betonagens, no endentamento das camadas, no emprego de varões de travamento, caixas, saliências ou simples pedras entre os dois betões, na adopção de armaduras longitudinais colocadas nos troços mais jovens paralelamente às juntas de betonagem e próximo destas, etc., além dos aconselháveis tratamentos dos betões (mantê-los húmidos, empregar a refrigeração) e do emprego de cimentos especiais como os de baixo ou moderado calor de hidratação. No segundo, além dos cuidados já enumerados, na distribuição racional das armaduras pela secção da peça.

No caso das pontes o fenómeno surge mais particularmente nos elementos seguintes:

- a) Nas juntas de ligação das «camadas» dos arcos quando se adopta este tipo de construção, o que se considera altamente defeituoso mesmo quando se endentam as camadas, semelhantemente à clássica técnica usada nas abóbodas de cantaria de pedra natural.

Na Fig. 16 duma ponte onde foi seguida esta técnica a 2.<sup>a</sup> camada desligou da 1.<sup>a</sup> pelo menos em certas zonas como se constata pelas próprias escorrências calcáreas das juntas.

Entre as várias camadas dum arco construído por anéis desenvolvem-se tensões que esquematicamente se indicam na Fig. 17 e que se sobrepõem às do funcionamento «arco».

Um caso semelhante ao da construção dos arcos por camadas é o dos muros de tímpano sobre as abóbodas.

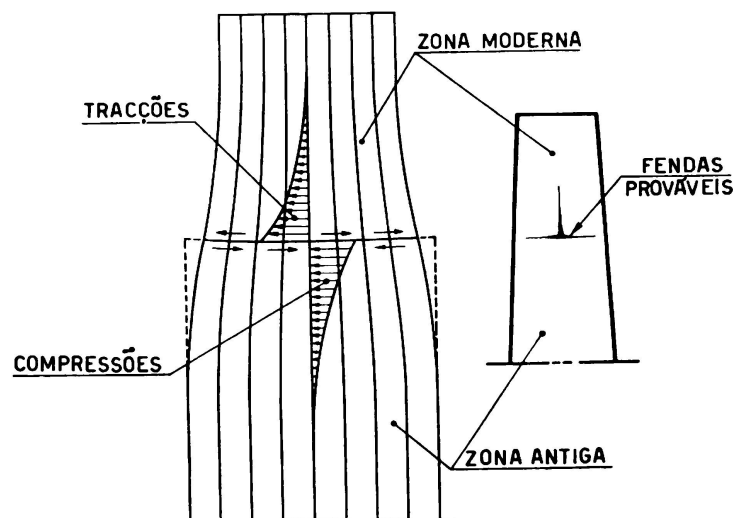


FIG. 18. Tensões devidas à contracção diferencial num pilar. Fendas prováveis

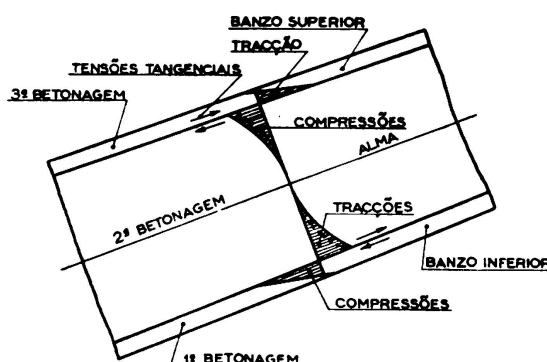


FIG. 17. Tensões devidas à contracção diferencial num arco betonado por anéis

- b) Nas juntas de betonagem dum mesmo elemento quando betonado simultaneamente em toda a área das secções normais à directriz, tal como nos grandes pilares ou na construção de arcos por aduelas.



Neste caso, o problema não tem a gravidade do caso anterior, à excepção dos depósitos para líquidos, porque mesmo que se dê a rotura da junta, por deslizamento, ela continuará a trabalhar em boas condições por estarem em geral em funcionamento de compressão. O fenómeno



FIG. 19. Efeitos da contracção diferencial na ligação duma abóboda aos encontros

pode esquematizar-se na Fig. 18. O troço mais antigo na ligação ao moderno é obrigado a comprimir-se e este a dilatar donde resulta, na junta, um estado de tensão tangencial e nas secções paralelas à directriz um estado de tensão normal, de tracção no betão novo e de compressão no betão velho.

Se a superfície tem uma grande área, da ordem dos 100 m<sup>2</sup>, ou um grande lado, 20 ou 30 m, quase certo é dar-se a rotura quando o intervalo nas betonagens vai além de 15 a 30 dias se não se tomam as disposições

atrás indicadas. A rotura dá-se como indica a Fig. 18 sendo numerosíssimos os exemplos que se poderiam apresentar.

O caso mais característico é o da ligação das abóbodas largas aos maciços dos encontros (Fig. 19). Estes são construídos em geral com uns meses de avanço de modo que o betão da abóboda tende a contrair sobre um maciço de grande volume, já quase completamente contraído.

Se a abóboda não possui armaduras transversais suficientes é praticamente inevitável a rotura, para larguras de abóboda superiores a 20 m. Constatamos uma rotura deste tipo na Passagem Superior da Cruz das Oliveiras na Auto-estrada Lisboa-Estádio, apenas com 15 m de largura de abóboda, a qual tem muito reduzida armadura transversal.

Não é pois possível, sem perigo, projectar uma larga abóboda de betão simples, pois que a esses efeitos adicionam-se os das sobrecargas e dos impulsos dos muros de tímpano.

#### 4 - A DESIGUALDADE DE TEMPERATURAS ENTRE AS DIVERSAS FIBRAS DO MATERIAL DAS SECÇÕES

Pode dizer-se que é corrente, no cálculo de pontes, ter-se em conta os efeitos das variações de temperatura uniforme, quando as estruturas são hiperestáticas. Mais raramente se consideram as variações de temperatura desigual dos elementos componentes de uma obra e só muito excepcionalmente se tem em conta a diferença de temperatura entre as diversas fibras do material das secções transversais das estruturas.

Este último efeito é, contudo, de alta importância em certas obras directamente expostas às radiações solares. As mais afectadas são, sem dúvida, os tabuleiros de pontes (na construção civil os terraços) do tipo hiperestático (lajes contínuas ou lajes nervuradas hiperestáticas) como é do conhecimento geral. Todos os engenheiros civis sabem calcular estes efeitos pelo menos para certas hipóteses particulares como a da variação linear da temperatura nas secções transversais.

Mas o que nem sempre se tem presente é a ordem de grandeza das tensões geradas e muito especialmente o «sentido» dos esforços gerados.

Nos tabuleiros do tipo viga contínua, laje nervurada ou laje sem nervuras, para fraca camada protectora isolante como é a dos pavimentos modernos das pontes (betão betuminoso de 3 a 5 cm de espessura), a variação de temperatura pode atingir, no nosso clima 10 a 20° C, consoante a região. Do meio dia às 3 da tarde natural é verificar-se em certos dias de verão tais variações de temperatura no sentido positivo, quer dizer, encontrar-se a face superior a 40° C e a face inferior a 25° C, o que não sucede em sentido inverso. De facto ao desaparecer o sol essa diferença estará praticamente reduzida a zero não fazendo o fresco da noite com que exista uma diferença negativa de mais do que  $\frac{1}{4}$  ou  $\frac{1}{5}$  daquele valor.

O que sucederá pois?

Se chamarmos positivos os momentos flectores que geram tracções nas fibras inferiores somos levados a concluir que os efeitos do sol sobre os tabuleiros contínuos de pontes provocam momentos flectores positivos acentualmente mais intensos do que momentos negativos. Haverá portanto um apreciável aumento de tensões nas zonas dos vãos, tensões que serão em muitos casos da ordem das provocadas pelas cargas permanentes com uma correspondente diminuição de esforços nas zonas dos apoios.

Em conclusão, se tais efeitos não foram considerados no projecto deverá o construtor ao elaborar o plano de betonagem, tomar o cuidado de executar a superestrutura de modo que ela fique com momentos positivos menores do que previu o cálculo tanto mais que os efeitos da fluência são geralmente do mesmo sinal da variação diferencial de temperatura. Foi esta razão pela qual, no exemplo atrás citado das pontes do Cávado e do Caldo, decidimos anular totalmente os momentos a meio do vão provenientes da carga permanente à custa do corte temporário dos tabuleiros.

## RESUMO

No presente artigo indicam-se os cuidados a ter na construção das grandes pontes de betão armado tendo em vista a eliminação total ou parcial dos esforços secundários desenvolvidos nas estruturas durante as suas diferentes fases de execução em particular provenientes do sistema de betonagem e em função das deformações dos cavaletes e da contracção diferencial do betão.

Analisa-se, também, o fenómeno da alteração da distribuição dos esforços principais desenvolvidos nas estruturas proveniente das mesmas causas e, ainda, da desigualdade de temperatura entre as diversas fibras do material.

Ilustra-se o artigo com os casos concretos:

- a) Arco abatido do tipo aligeirado (ponte da Foz do Sousa).
- b) Pórtico especial com vigas de altura muito variável (ponte de Coimbra).

- c) Arco múltiplo esbelto de tabuleiro rígido (ponte de Barca d'Alva).
- d) Viga contínua de alma vasada sobre pilares ôcos de alvenaria (pontes do Cávado e do Caldo sobre a albufeira da Caniçada, da Hidro Eléctrica do Cávado).

### R É S U M É

L'auteur décrit les précautions à prendre lors de la construction des ponts en béton armé de grandes dimensions en vue de l'élimination totale ou partielle des efforts secondaires qui prennent naissance au cours des différentes phases d'exécution; il traite plus particulièrement le cas des efforts secondaires dûs au système de bétonnage adopté et en fonction des déformations des supports de coffrages et du retrait du béton.

Il étudie également le phénomène de la modification de la distribution des contraintes principales dues aux mêmes causes et aux différences de température entre les différentes fibres du matériau.

Il cite les exemples pratiques suivants:

- a) Arc surbaissé allégé (Pont de Foz do Sousa).
- b) Portique spécial à poutre de hauteur très variable (Pont de Coimbra).
- c) Arc multiple mince à tablier rigide (Pont de Barca d'Alva).
- d) Poutre continue en caisson sur piliers creux en maçonnerie (Ponts du Cávado et du Caldo sur la retenue de Caniçada de l'aménagement Hydro-Electrique du Cávado).

### S U M M A R Y

The author describes practical methods applied to large reinforced concrete bridge construction in order to eliminate in part, or totally, secondary efforts developed in the course of construction; the author deals more particularly with those secondary efforts which are caused by the concrete laying methods adopted and depending on the trestles' deformations and shrinkage of concrete.

Modification of the repartition of a bridge's principal stresses due to the same factors and to differences of temperature between the various fibers of the concrete is also dealt with.

The following practical examples are mentioned:

- a) Lightened flat arch (Foz do Sousa Bridge).
- b) Special frame structure with large variations of beam height (Coimbra bridge).
- c) Multiple thin arch with rigid platform (Barca d'Alva bridge).
- d) Continuous box web beam on hollow masonry columns (Cávado and Caldo bridges on the Caniçada empoundment lake of the Cávado Hydro-Electric Scheme).

## ZUSAMMENFASSUNG

In der vorliegenden Arbeit werden die Massnahmen gezeigt, welche bei der Konstruktion von grossen Brücken in Eisenbeton getroffen werden müssen, um die Sekundärkräfte im Bauwerk vollständig oder teilweise auszuschalten. Diese treten während den verschiedenen Phasen des Baues auf, insbesondere infolge des Betoniervorganges, der Formänderungen des Lehrgerüsts und des Schwindens von Beton.

Untersucht wird ferner die Aenderung der Hauptbeanspruchungen, welche aus denselben Gründen und infolge der Temperaturunterschiede in den verschiedenen Teilen des Bauwerks auftritt.

Die Arbeit wird illustriert durch folgende konkrete Fälle:

- a) Flacher Bogen in aufgeflöster Bauweise (Bücke von Foz do Sousa).
- b) Rahmenbrücke mit veränderlicher Querschnittshöhe der Balken (Brücke von Coimbra).
- c) Mehrfacher schlanker Bogen mit Versteifungsträger (Brücke von Barca d'Alva).
- d) Durchlaufender Balken mit unterbrochenem Steg über Hohl Pfeilern aus Mauerwerk (Brücken von Cávado und Caldo über den Caniçada-Stausee des Kraftwerks Cávado).

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## **Vla3**

**Slide-forms in concrete bridge column construction**

**Gleitschalungen für Betonsäulen bei Brückenbauten**

**Cofragens deslisantes para a construção de pilares de pontes**

**Coffrages glissants pour la construction de piliers de ponts**

**GEORG ENSKOG**

*Civil Engineer*

Bromma

For high bridge columns of concrete the costs for forms and scaffolding for shuttering and casting in situ are rather high compared with the costs for the reinforcing steel and the concrete which are the main components of the construction. The support-columns on concrete arches with large spans have to be made as slender as possible which makes the use of concrete of a high quality necessary. A great deal is expected also in regard to the look of the columns. Previously used, conventional timber or steel shuttering for columns hamper the handling of the concrete in the narrow form and for the erection of the shuttering a comparatively large number of skilled hands is needed.

Through the simplified method for slide-form concreting — the Concretor System — developed in Sweden during the last decade, large savings in material and labour costs can be made, and at the same time the concrete will be easily accessible for handling, whereby the possibilities of making high quality concrete probably are better than by any other method. A profitable use of this new method, however, is subject to a suitable design of the construction and a comparatively large height of the columns.

The first bridge building in Sweden at which slide-forms were used would be the railway bridge over the river Vindelälven up north, constructed in the years 1950 to 1952 by Nya Asfalt AB for the Royal Board of Swedish State Railways. This bridge comprises an arch span, 112 metres of length (fig. 1) and approach viaducts, supported by pairs of columns with circular cross section, spaced 14.35 metres (fig. 6). The height of the columns varies from 9.7 to 22.8 metres. The total length of the bridge is 242 metres

When in late autumn 1952 the work was started, heavy increases in prices, caused by the war in Korea, prevailed. It was, therefore, to the advantage of the client as well as of the contractor to keep the quantity

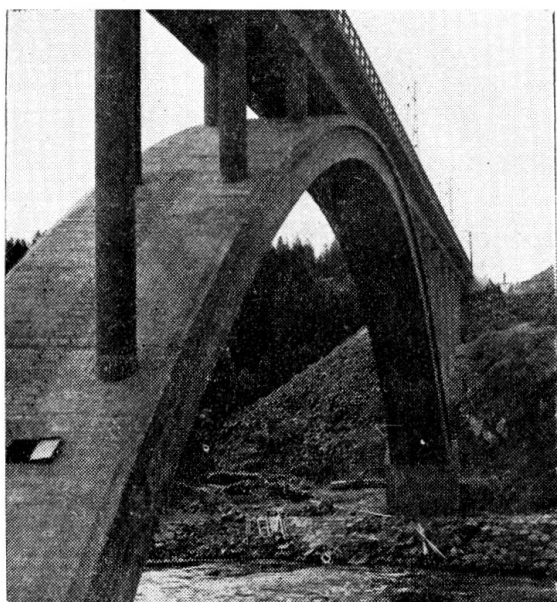


FIG. 1. Railway bridge over the river Vindelälven in North Sweden

of material to be used for temporary constructions as low as possible. The contractor, together with AB Byggförbättringar of Stockholm who originated the Concretor Method, made out a working programme for the use of slide-forms. The programme was approved by the client and all the bridge columns, except the shortest ones at both sides of the crown of the arch span, were constructed by the Concretor Method. The forms were of sheet steel and of a height of 1.2 metres (fig. 3). The jacking equipment consists of a number of hydraulic jacks, here four, on which the form equipment is suspended. These jacks are connected through oil pipes with a small electric powered oil pump. When the pump is started oil is pressed to the jacks which all raise

simultaneously. When needed the jacks may be operated individually, thus allowing for adjustment in case of deviation from the vertical line.

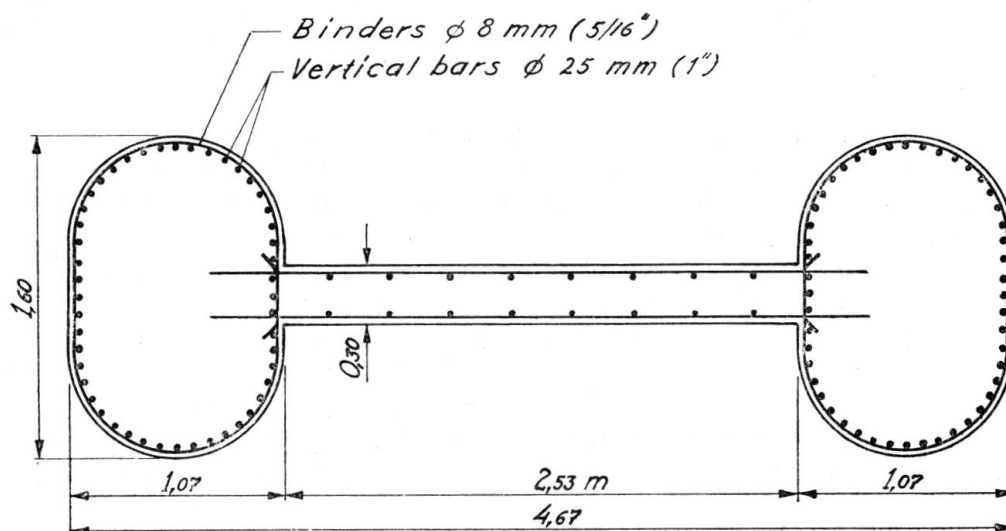


FIG. 2. Cross section of an abutment column

Each jack «climbs» on a smooth steel rod 25 millimetres of diameter and each raise is 25 millimetres.

The lifting speed depends amongst others on the growing strength of the concrete, and was in this case 3 to 4 metres per 24 hours. The



quality of the concrete used for columns and all other parts of the concrete construction, with the exception of foundations and abutments was K 400 (min. ultimate strength  $400 \text{ kg/cm}^2$  or  $5700 \text{ lb/in}^2$  after 28 days). The reinforcing steel consists of deformed bars of the quality Ks 40 with a yield ratio of  $40 \text{ kg/mm}^2$  ( $57000 \text{ lb/in}^2$ ).

The columns on the arch abutments (fig. 2) are heavier than the rest of the columns. On account of the new method the sharp angles between the columns proper and the connecting wall have to be rounded, which merely adds to the structural strength. Furthermore, the binders, being placed in the course of the pouring, need have two splices, making each unit consist of two U-shaped parts instead of one oval.

For constructive reasons the vertical reinforcement of the columns has to be made using as long bars and as few splices as possible. The maximum length of the bars is 10 metres. It was therefore, necessary to build, at the side of a column to be constructed, an auxiliary scaffold fitted with extending fixtures for keeping the reinforcing bars in position. The steel scaffolding used will be seen from fig. 4 and 5. If it is possible to limit the length of the reinforcing bars to 5 to 6 metres they may be kept in position by fitting similar fixtures to vertical masts, attached to the slideform. The reinforcing bar positioning fixtures will then follow the upward movement of the slide form, and no scaffolding will be needed at the side of the column.

When the first column was cast the night temperature was rather low and calcium chloride was added to the concrete so that the lifting speed would not have to be reduced too much. When the slideform had been raised somewhat above the position shown on fig. 6 a suspended scaffolding was attached as indicated on fig. 3 to make possible inspection of the slide-form itself and of the consistency of the concrete underneath the slide-form. The suspended scaffolding is used also for finishing the concrete surface. Great demands are made on the appearance of the

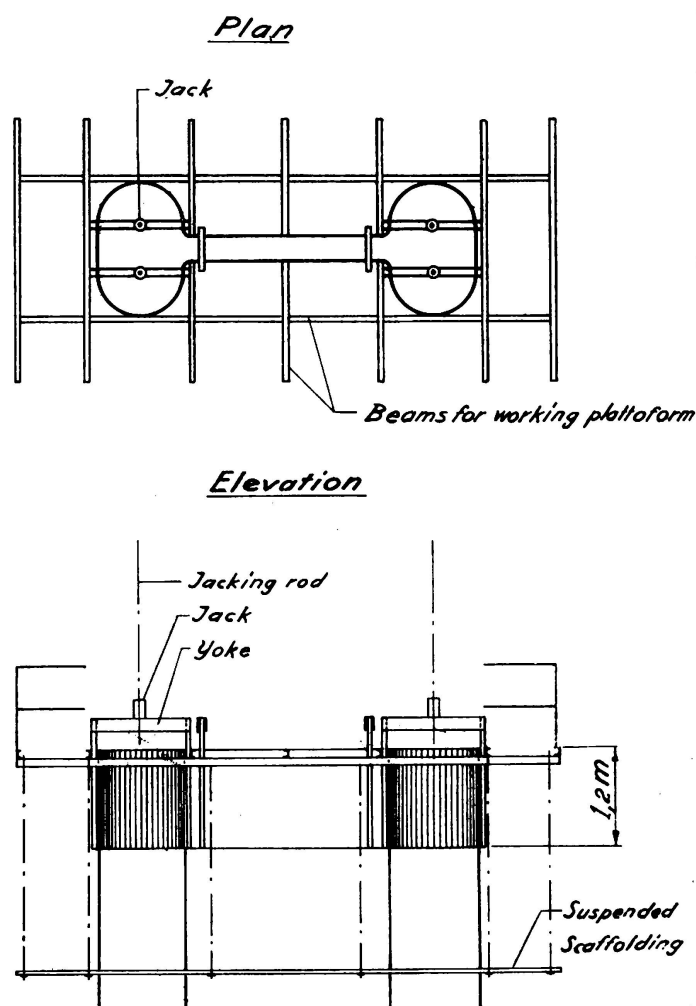


FIG. 3. Slide-form for abutment column

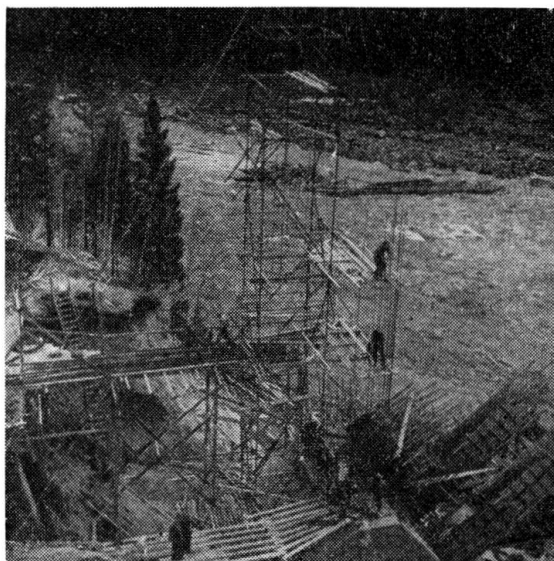


FIG. 4. Auxiliary steel tubular scaffolding at an abutment column. The assembly of the sliding form is in progress

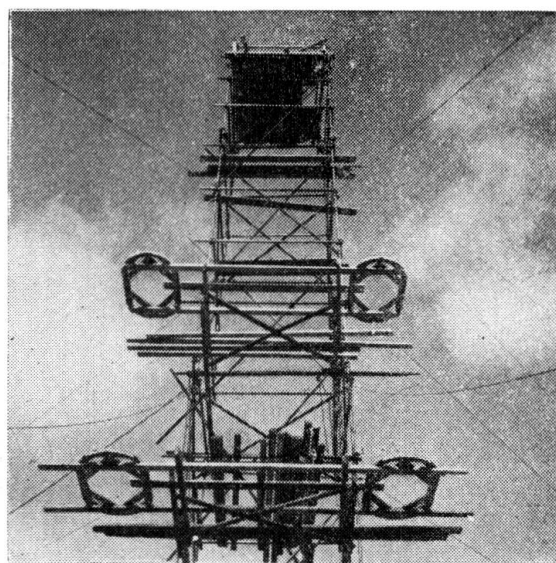


FIG. 5. Auxiliary scaffolding with fixtures for the reinforcement, seen from below

concrete surfaces of bridges and therefore the aftercasting treatment is of main importance. During the period following the casting all slide-formed surfaces are getting flamy or striped depending upon the age and the humidity of the concrete. If the after-casting treatment is properly

done by skilled workmen these flaws normally will disappear when the concrete has dried up. A slight rubbing of the surface with a broom should be sufficient, whereas the rubbing with felt is not recommended. The after-casting treatment of course includes water spraying and protection of fresh concrete surfaces, amongst others, against insolation.

The rest of the columns are circular and placed in pairs. The viaduct columns are 1.0 metre and the columns on the arch are 0.9 metres of diameter. Slide-forms and working platform used for these

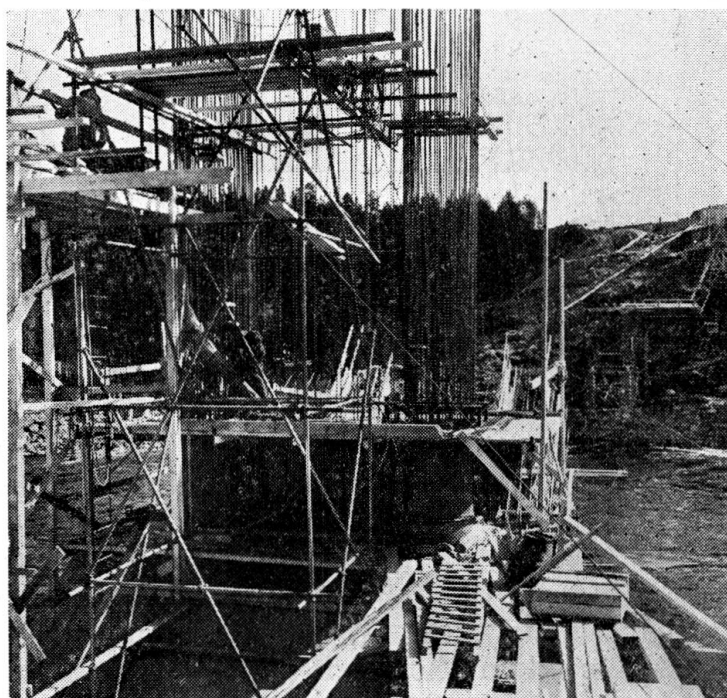


FIG. 6. Concreting of an abutment column by means of a sliding form. The raising of the form has recently begun. Photo. Eric Rosenberg

columns were similar to the equipment used for the abutment columns. Consequently the slide-forms for a pair of columns were connected through the working platform. In these columns too the binders were made in halves.

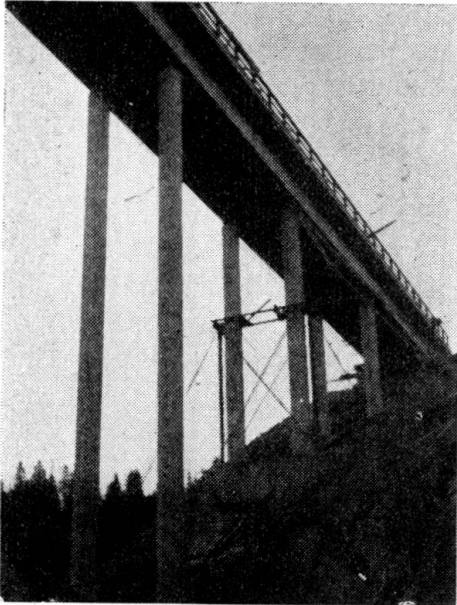


FIG. 7. Guying of a column

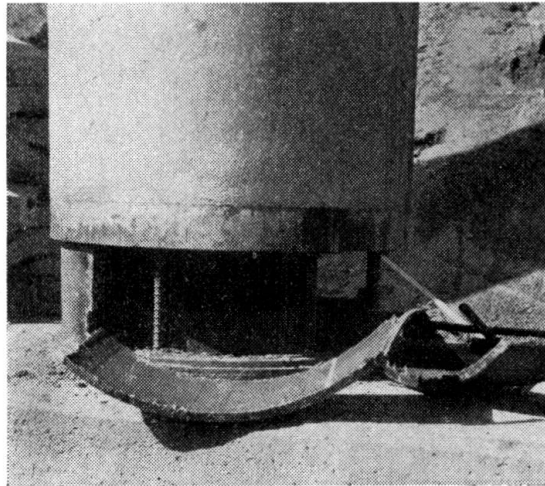


FIG. 8. Provisional arrangement used for a row of columns

During the construction work the columns are fixed only in the foundation, and the longer columns therefore are exposed to buckling until they are fixed in the bridge deck. Until the concrete is strong enough the columns are even susceptible to wind stress. Several columns, therefore, had to be guyed during the casting and the guying had to remain until the bridge deck was concreted. Before the critical height of the columns was reached a guy bridge as shown on fig. 7, was mounted right below the suspended scaffolding. The guy bridge consists of rings around the columns connected by a steel frame. The guy bridge was supported by guys resting on the foundation and braced by steel rods with tightening nuts at the end brackets.

To make possible adjustments made necessary by eventual future sinking, columns not having foundations on rock were supported on the foundation by bearings of steel.

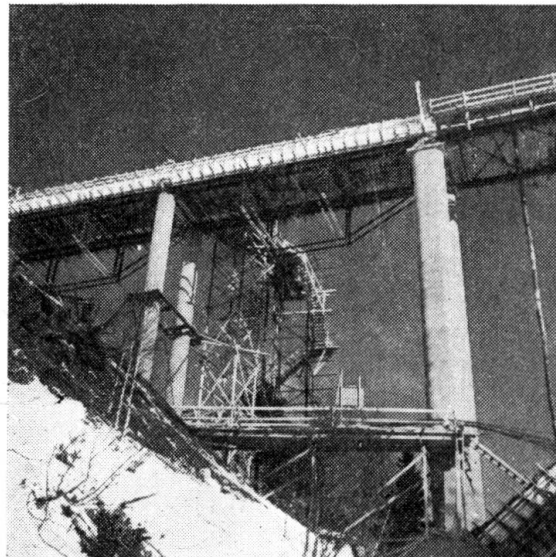


FIG. 9. Steel trusses between columns, concreted by means of slide-forms. The hoist and tower at the concrete mixing plant is seen in the background

When casting these columns the foundation and the columns were provisionally connected by means of reinforcing irons passing through holes in the plates of the bearing, and a supporting ring of steel was inserted between the plates. After concreting the bridge deck the provisional parts were removed (fig. 8).

In order to fully utilize the rationalization implied in the use of slide forms, conventional scaffolding close to, or between, the columns

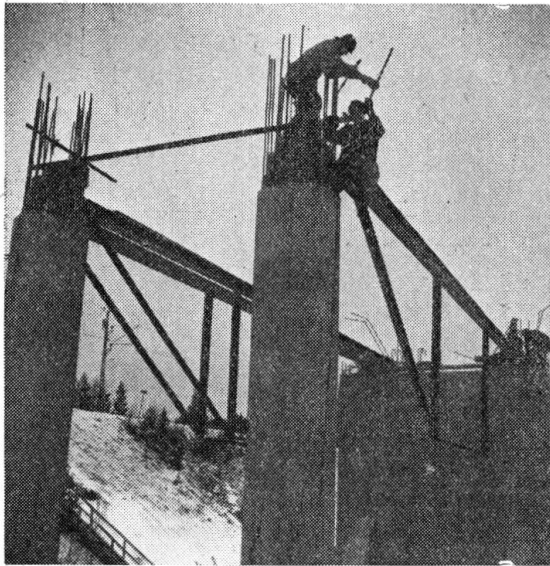


FIG. 10. Mounting of steel trusses on the top of a column

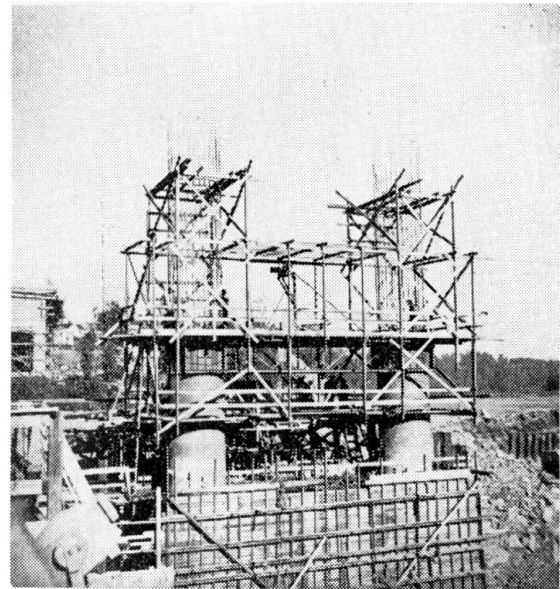


FIG. 11. Columns under construction for a highway bridge across the river Indalsäven at Bergforsen, Sweden

for carrying the form work for the superstructure under construction should, if possible, be avoided. At the construction of the bridge here described, the space of 14.35 metres between the viaduct columns was overbridged by steel trusses (fig. 9). The steel trusses were placed on sand boxes, located in cupels at the top of the columns (fig. 10). The area of the remaining part of the cross section of the top of the column was sufficient for taking the full bridge deck and traffic load.

The cupels were filled out with concrete afterwards. Compared with conventional scaffolding a further advantage of these steel trusses will be that they make possible to compute deformations with a high grade of accuracy.

When this bridge building had been completed a new contract for a similar bridge over the river Indalsälven at Ragunda was placed with Nya Asphalt AB. With exception for the differences in regard to foundations and the length of the columns, the main difference between these bridges is that the Ragunda Bridge has one additional viaduct span at each side of the river. In view of the good result obtained in the construction of the Vindelälven Bridge the client, the Royal Board of Swedish State Railways, again agreed to the suggestion that slide-forms



should be used for the casting of the columns and the slide-form equipment previously used was used once more.

Owing to delays one of the abutment columns had to be concreted in the winter. Before starting the casting a heating shelter was built around and to the full height of the column. Hot air was blown in at the base of the shelter. Because the extending reinforcing irons will cool down the concrete just poured it is, unfortunately, not possible to use

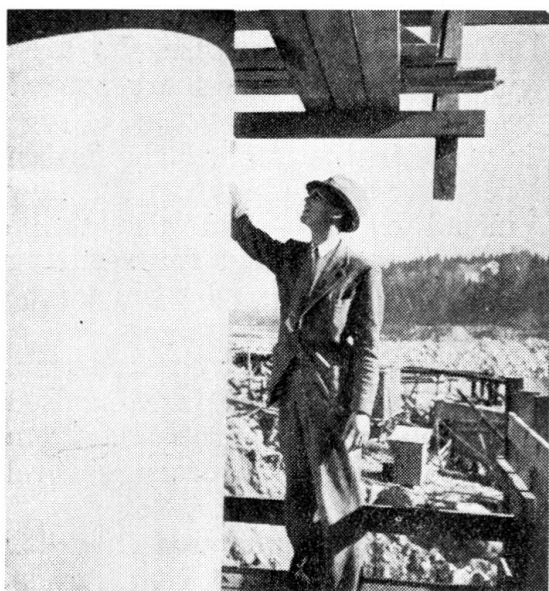


FIG. 12. Examination of the concrete surface of a column at Bergforsen



FIG. 13. Concrete viaduct at Bergforsen, Sweden

the simpler method of hanging tarpaulins from the working platform of the slide-form equipment. Great attention and foresight is called for at such a casting in wintertime.

The same method, as described above, was used also in the construction of the highway bridge over the river Indalsälven at Bergforsen, which was built in the years 1953-1954. The columns of this bridge are 2.0 metres of diameter, and the quality of concrete used was K 300. Fig. 11 shows a pair of columns under construction and fig. 12 a detail of the concrete surface. Even here a pair of columns had to be cast at a low temperature.

The columns of a concrete approach viaduct (fig. 13) placed in rows of three, were also constructed with the aid of slide-forms. The columns are 0.7 metres of diameter. All columns in one row were cast at the same time from a common working platform.

Here too, there were two lifting jacks in use on each column. Because of the small diameter the columns will be susceptible to eccentric loads on the working platform. When constructing similar groups of columns by the aid of slide-forms columns less than 0.65 – 0.70 metres of diameter, therefore, would not be recommended.

## SUMMARY

For high bridge columns of concrete the costs for forms and scaffolding are rather high. Through slide-form concreting large savings in material and labour costs can be made and the concrete will be easily accessible to handling.

For a concrete railway bridge, constructed in North Sweden from 1950 to 1952, all support-columns were concreted by means of slide-forms in accordance with the Concretor method. The bridge comprises an arch span, 112 m in length and approach viaducts, whose piers are spaced 14.35 m.

A description of the sliding forms is given in the paper. The design of the abutment columns was modified so as to be adapted to the new method of construction. The binders in all columns had also to be modified. It was necessary to build an auxiliary scaffold for securing the vertical reinforcement bars in position, since these bars should be as long as possible.

The columns are fixed only in the foundations during construction. Therefore, the columns, which are of considerable length, are exposed to buckling in the course of construction, and are not protected from buckling until they are fixed to the bridge deck. For this reason, several columns had to be guyed during the slide casting.

An account is given of the provisional arrangements used in order to render possible the concreting of columns with steel bearings at the base.

Steel trusses resting on the tops of the columns carried the form for the superstructure. These trusses contributed to the utilisation of the possibilities of rationalising the construction produce, since conventional scaffolding was entirely dispensed with in this manner.

For another similar railway bridge, constructed from 1953 to 1955 the columns are concreted by means of slide-forms. One of the columns of this bridge was built in the winter, then the temperature was permanently below 0° C.

The same method has also been used for the columns of a highway bridge.

## ZUSAMMENFASSUNG

Die Gerüst- und Schalungskosten bei längeren Brückensäulen sind ausgesprochen hoch. Bei Anwendung der Gleitschalung lassen sich bedeutende Material- und Arbeitskosten einsparen; auch wird die Zugänglichkeit für die Betonbearbeitung verbessert.

Bei einer in Nordschweden in den Jahren 1950-52 gebauten Eisenbahnbrücke aus Stahlbeton wurden alle Stütz-Säulen mit Hilfe von Gleitschalungen nach dem Concretorsystem betoniert. Die von einem Bogen überspannte Mittelöffnung dieser Brücke ist 112 m lang, und die Stützen der seitlichen Anschlussviadukte liegen je 14,35 m auseinander.

Die Bemessung der Widerlager-Säulen wurde der neuen Betoniermethode angepasst. Auch die Bügel wurden entsprechend ausgebildet. Um die möglichst langen Vertikaleisen in ihrer richtigen Lage festzuhalten, musste ein besonderes Hilfsgerüst erstellt werden.

Während des Baues sind die ausgesprochen langen Säulen nur am untern Ende eingespannt; demnach sind sie dem Knicken ausgesetzt, bis sie mit der Fahrbahnplatte verbunden werden können. Aus diesem Grunde mussten verschiedene Säulen während des Betonierens seitlich abgestützt werden.

Es folgt eine Beschreibung über die Einrichtungen, die das Betonieren von Säulen mit Stahllagern am untern Säulenende ermöglichten.

Ueber den Säulenköpfen aufliegende Stahltragwerke dienten zur Unterstützung der Fahrbahnschalung und hatten wesentlichen Anteil an der Verwirklichung dieser rationelleren Bauweise.

Bei einer ähnlich gearteten, im 1955 fertiggestellten Eisenbahnbrücke werden die Säulen ebenfalls mit Hilfe der Gleitschalung betoniert. Eine dieser Säulen wurde im Winter erstellt, als die Temperatur ständig unter 0° C lag.

Das gleiche Verfahren kam auch bei Strassenbrücken-Säulen zur Anwendung.

#### RESUMO

O custo dos andaimes e das cofragens dos pilares de pontes de betão de grande altura é bastante elevado. O emprego de cofragens deslizantes não só permite realizar importantes economias de material e mão de obra como também facilita a manutenção do betão.

Na construção de uma ponte de caminho de ferro no Norte da Suécia, de 1950 a 1952, betonaram-se todos os pilares de apoio com o auxílio de cofragens deslizantes segundo o método «Concretor». A referida ponte compõe-se de um arco de 112 m. de vão e dois viadutos marginais cujos pilares têm uma espaçamento de 14,35 m.

O autor descreve as cofragens deslizantes. O projecto dos pilares dos encontros foi modificado de modo a adaptar-se ao referido método de construção. Os estribos também foram modificados em todos os pilares. Tornou-se ainda necessário construir um andaime auxiliar para suportar as armaduras verticais que deviam ser o mais compridas possível.

Durante a construção, os pilares só estavam ligados às fundações, ficando portanto, devido à sua grande altura, submetidos à encurvadura até ao momento de virem ligar ao tabuleiro. Por esta razão, alguns pilares tiveram de ser suportados durante a betonagem.

O autor relata as disposições provisórias adoptadas para tornar possível a betonagem dos pilares com articulações metálicas na base.

Vigas metálicas trianguladas, apoiadas no topo dos pilares, suportavam a cofragem da superestrutura. Essas vigas contribuíram para o aproveitamento das possibilidades de racionalização do método de construção, pois permitiram dispensar completamente os andaimes habituais.

Na construção de outra ponte semelhante executada de 1953 a 1955 foram os pilares também betonados com cofragens deslizantes. Um dos



pilares foi inteiramente betonado durante o inverno com temperaturas permanentemente negativas.

O mesmo método foi também utilizado para construir os pilares de uma ponte de estrada.

#### R É S U M É

Le prix des échaffaudages et des coffrages des piliers de ponts en béton de grande hauteur est assez élevé. L'emploi de coffrages glissants permet non seulement une sérieuse économie de matériaux et de main d'oeuvre mais encore rend la manutention du béton plus aisée.

Lors de la construction d'un pont-rail dans le Nord de la Suède, de 1950 à 1952, tous les piliers d'appui ont été bétonnés à l'aide de coffrages glissants selon la méthode «Concretor». Ce pont comprend un arc de 112 m de portée et des viaducs d'approche dont les piliers sont espacés de 14.35 m.

L'auteur décrit les coffrages glissants. Le projet des piliers des naissances a été modifié afin de pouvoir les adapter à la nouvelle méthode de construction. Les étriers de tous les piliers ont également dû être modifiés. Il a été nécessaire de construire un échafaudage auxiliaire pour guider les armatures verticales qui devaient être aussi longues que possible.

Pendant la construction, les piliers n'étaient fixés qu'aux fondations, ce qui, du fait de leur grande hauteur, les soumettait au flambement jusqu'au moment de les relier au tablier. Pour cette raison, certains piliers ont dû être étayés pendant le bétonnage.

L'auteur décrit les dispositions provisoires adoptées pour rendre possible le bétonnage des piliers munis d'articulations métalliques à leur base.

Des poutres métalliques triangulées, appuyées sur le sommet des piliers, soutenaient les coffrages de la superstructure. Ces poutres ont contribué à l'utilisation des possibilités de rationalisation du mode de construction en permettant d'éviter entièrement les échafaudages conventionnels.

Des coffrages glissants ont été également utilisés dans la construction d'un autre pont-rail exécuté de 1953 à 1955. L'un des piliers a été complètement bétonné en plein hiver par des températures constamment en dessous de zéro.

La même méthode a également été employée pour la construction des piliers d'un pont-route.

## **Vla4**

### **Berechnung der Stahlbeton Fertigteil-Konstruktionen verbunden an Ort und Stelle**

**Cálculo de estruturas de betão armado com elementos  
prefabricados, ligados no local da obra**

**Calcul des constructions en béton armé exécutées en  
éléments préfabriqués montés sur place**

**Calculation of reinforced concrete structures incorporating  
precast elements erected at the site**

BÉLA GOSCHY

Budapest

#### **1.) Einleitung.**

Die Wirkungsweise der an Ort und Stelle verbundenen Stahlbeton-Fertigteil-Systeme unterscheidet sich von den monolythischen Stahlbeton-Konstruktionen und dadurch wird die Statik und Bemessung der Stahlbeton-Verbundkonstruktionen wesentlich beeinflusst. Ausserdem müssen noch die Zeitveränderlichkeit, die plastischen Verformungen (Kriechen und Schwinden) des Betons, sowie die Montage dieser Konstruktionen bei den Rechnungen beachtet werden. Die Untersuchung dieses Verhaltens ist das Ziel der vorliegenden Arbeit.

Die Stahlbeton-Fertigteilbauten können ohne Gerüst, als selbsttragende Tragwerke ausgebildet werden. In diesem Falle wird das Gesamtgewicht (Fertigteil- und Ortsbetongewicht) allein von dem *Fertigteil* getragen. Sie können aber völlig oder nur teilweise auf Gerüst gebaut werden, in diesem Falle wird das Gesamtgewicht, einschliesslich die Nutzlast von dem *Verbund-System* getragen. Die Rechnungsverfahren werden auf diese zwei Grundfälle ausgerichtet.

Der mit dem Ortsbeton zusammenwirkende Fertigteilbalken bildet innerlich ein statisch unbestimmtes System. Bei der Vermittlung der unbekannten statischen Grössen -wie es bei den statisch unbestimmten Stahlbetonsystemen üblich ist- rechnen wir mit dem homogenen Betonquerschnitt.

**2.) Bezeichnungen.**

|                                               |                                                  |
|-----------------------------------------------|--------------------------------------------------|
| $F_1, J_1, W_1, i_1^2 = \frac{J_1}{F_1}$      | Querschnittsgrössen des Fertigteils              |
| $F_2, J_2, W_2, i_2^2 = \frac{J_2}{F_2}$      | Querschnittsgrössen des Ortsbetonteils           |
| $P_o, M_o$                                    | äussere Druckkraft, Biegemoment                  |
| $P_1, M_1, N_o$                               | Last- und Momentenanteil des Fertigteils         |
| $P_2, M_2, N_o$                               | Last- und Momentenanteil des Ortsbetonteils      |
| $P_B, M_B$                                    | Bruchkraft, Bruchmoment                          |
| $\varphi_1, \varphi_{1,t}; \varphi_{1\infty}$ | Kriechzahl des Fertigteils                       |
| $\varphi_2, \varphi_{2,t}; \varphi_{2\infty}$ | Kriechzahl des Ortsbetons                        |
| $k_1, k_2$                                    | Prismenfestigkeit des Betons                     |
| $\sigma_f$                                    | Fliessgrenze des Stahls                          |
| $\sigma_{10}, \sigma_{20}$                    | zulässige Betondruckspannung bei mittigem Druck  |
| $\sigma_{1b}, \sigma_{2b}$                    | zulässige Betondruckspannung bei reiner Biegung. |

**3.) Einfluss des Kriechens und des Schwindens.**

Die Fertigbauteile und die an Ort und Stelle betonierten Trägereile kriechen nach verschiedenen Gesetzen, die von dem Betonalter und der Betongüte, wie auch von dem Beginn der Belastung abhängig sind.

Der Stahlbeton-Fertigteil beginnt mit  $\Delta t$  früher zu kriechen als der Ortsbeton, nach folgendem Gesetz:

$$\varphi_1, t_1 = \varphi_{1\infty} (1 - e^{-t_1}) \quad (1)$$

Die Kriechlinie des Ortsbetons hat die folgende Form:

$$\varphi_2, t_2 = \varphi_{2\infty} (1 - e^{-t_2}) \quad (2)$$

Die beiden Zeitpunkte stehen in folgendem Zusammenhang:

$$t_1 = t_2 + \Delta t$$

Wenn wir den Zeitpunkt der Wirkung der ständigen Last mit  $t_{10}$  bzw.  $t_{20}$  bezeichnen, so kriechen die einzelnen Trägereile nach:

$$\bar{\varphi}_1, t_1 = \varphi_1, t_1 - \varphi_1, t_{10} = \varphi_{1\infty} (e^{-t_{10}} - e^{-t_1}) \quad (3)$$

$$\bar{\varphi}_2, t_2 = \varphi_2, t_2 - \varphi_2, t_{20} = \varphi_{2\infty} (e^{-t_{20}} - e^{-t_2}) \quad (4)$$

Da zwischen den beiden Funktionen eine Affinität besteht, kann man die Kriechfunktion des Fertigteils als Funktion des Ortsbetonteils ausdrücken:

$$t_{10} = t_{20} + \Delta t; \nu = \frac{\varphi_{1\infty}}{\varphi_{2\infty}} \text{ und } \mathcal{H} = e^{-\Delta t}$$

Die Funktionen des Kriechens:

$$\bar{\varphi}_1, t_1 = \nu \mathcal{H} \varphi_{2\infty} (e^{-t_{20}} - e^{-t_2}) = \nu \mathcal{H} \bar{\varphi}_2, t_2 \quad (5)$$

$$\bar{\varphi}_2, t_2 = \varphi_{2\infty} (e^{-t_{20}} - e^{-t_2}) \quad (6)$$

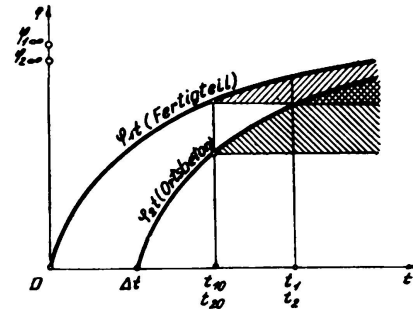


ABB. 1. Kriechkurven der Trägereile. Die Belastung beginnt zur Zeit  $t_{10}$  bzw.  $t_{20}$

Bei der Berücksichtigung des Kriechens kann man zwei Sonderfälle unterscheiden:

- a.) Ist der Fertigteil nicht mehr kriechfähig, so erleidet er unter der ständigen Last keine Verformungen.

In diesem Falle ist  $\Delta t = \infty$  und  $\bar{\varphi}_1, t_1 = 0$

$$\bar{\varphi}_2, t_2 = \varphi_{2\infty} (e^{-t_{20}} - e^{-t_2})$$

- b.) Wird der Ortsbeton sofort nach dem Erhärten belastet, dann ist  $t_{10} = \Delta t$  und  $t_{20} = 0$ , und die Kriechfunktionen:

$$\bar{\varphi}_1, t_1 = \nu \mathcal{H} \varphi_{2\infty} (1 - e^{-t_2}) = \nu \mathcal{H} \bar{\varphi}_2, t_2 \quad (5-a.)$$

$$\bar{\varphi}_2, t_2 = \varphi_{2\infty} (1 - e^{-t_2}) \quad (6-a.)$$

Bei der Berücksichtigung des Kriechens werden wir die Änderung des  $E_b$ -als unbedeutende vernachlässigen.

Das Schwinden ist eine dem Kriechen affine Erscheinung; so können wir die Schwindfunktion proportional der Kriechfunktion auffassen. Das

Verhältnis zwischen den beiden Funktionen ist durch  $\frac{\varepsilon_{\infty}}{\varphi_{\infty}}$  gegeben, wo  $\varepsilon_{\infty} = \omega T^0$  ist. Das Schwindmass wird wegen der Schwindbewehrung vermindert angenommen.

Die Schwindfunktion des Fertigteilträgers:

$$\bar{\varepsilon}_1, t_1 = \nu \mathcal{H} \frac{\varepsilon_{\infty}}{\varphi_{1\infty}} \bar{\varphi}_2, t_2 = \mathcal{H} \bar{\varepsilon}_2, t_2 \quad (7)$$

Die Schwindfunktion des Ortsbetonträgerteils:

$$\bar{\varepsilon}_2, t_2 = \frac{\varepsilon_\infty}{\varphi_{2\infty}} \bar{\varphi}_2, t_2 \quad (8)$$

#### 4.) Die Bestimmung der inneren Kräfte des Verbundquerschnittes.

Bei der Berechnung der Trägereile nehmen wir einen vollkommenen Verbund an.

##### 4.1 Mittig gedrückte Verbundquerschnitte.

4.11 Die Kraft  $P_0$  greift nur am Fertigteil an. In dem Zeitpunkte  $t = 0$  wirkt auf den Fertigteilträger die Teilkraft,  $P_1 = P_0$  und auf den Ortsbetonträger die Teilkraft  $P_2 = 0$ .

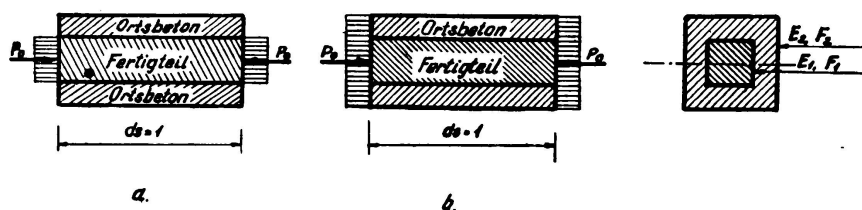


ABB. 2. Zusammengesetzter und an Ort und Stelle verbundener Druckstab. Abb. 3-a. Die Last  $P_0$  wird nur von dem Fertigteil getragen. Abb. 3-b. Die Last  $P_0$  wird von dem Verbundträger getragen

4.12 Infolge der Wirkung der Dauerlast  $P_0$  treten plastische Verformungen auf. Die Kräfteumlagerung im Verbundquerschnitt bestimmen wir durch die Auflösung des Verbundes zwischen dem Fertigteil und Ortsbeton und durch die Einführung der unbekannten Kraft  $X_t$ . Dann können wir die zwischen den zwei Teilen entstehende relative Verschiebung auf eine Säulenlänge  $ds = 1$ , zwischen dem Zeitpunkt  $t$  bis zu  $t + dt$ , als Null annehmen.

Die Formänderungsgleichung ist:

$$X_t \left( a_{11}^1 \frac{d\varphi_1}{dt_2} + a_{11}^2 \frac{d\varphi_2}{dt_2} \right) + \frac{dX_t}{dt_2} \left( a_{11}^1 + a_{11}^2 \right) + a_{10}^1 \frac{d\varphi_1}{dt_2} = 0$$

Die einzelnen Werte:

$$a_{11}^1 = a_{11}^1 + a_{10}^2 = \frac{1}{E_1 F_1} + \frac{1}{E_2 F_2}$$

$$a_{10}^1 = a_{10}^1 + a_{10}^2 = \frac{P_0}{E_1 F_1} + 0$$

Die Gleichung wird mit  $dt_2/d\varphi_2$  multipliziert; dann durch die Einsetzung von  $\bar{\varphi}_1, t_1 = \nu \mathcal{H} \bar{\varphi}_2, t_2$  bekommen wir:

$$X_t \left( \nu \mathcal{H} a_{11}^1 + a_{11}^2 \right) + \frac{dX_t}{d\varphi_2} \left( a_{11}^1 + a_{11}^2 \right) + \nu \mathcal{H} a_{10}^1 = 0$$

Die Einführung der folgenden Beziehungen:

$$\alpha = \frac{\nu \mathcal{H} a_{11}^1 + a_{11}^2}{a_{11}}; \quad \beta = \frac{\nu \mathcal{H} a_{10}^1 + a_{10}^2}{a_{11}} = -\bar{X}$$

$$\bar{X} = \nu \mathcal{H} P_0 \frac{E_2 F_2}{E_1 F_1 + E_2 F_2}$$

$$\text{gibt } \frac{dX_t}{d\varphi_2} + \alpha X_t - \bar{X} = 0 \quad (9)$$

Die Lösung der Gleichung (9):

a.) Wenn  $t_{20} \neq 0$  (mit Schonzeit)

$$X_t = \frac{\bar{X}}{\alpha} \left( 1 - \lambda e^{-\alpha \varphi_2 \infty} \right) \quad (9-a.)$$

$$\lambda = e^{+\alpha \varphi_2 t_{20}}$$

b.) Wenn  $t_{20} = 0$  (ohne Schonzeit)

$$X_t = \frac{\bar{X}}{\alpha} \left( 1 - e^{-\alpha \varphi_2 \infty} \right) \quad (9-b.)$$

Zur Zeit  $t = \infty$  entsteht im Ortsbeton die Druckkraft  $P_{2t} = X_t$  und in dem Fertigteilträger die Druckkraft  $P_{1t} = P_0 - X_t$ .

4.13 Die Kraft  $P_0$  greift den Verbundquerschnitt an. Auf Grund der elastischen Formänderungen wird zur Zeit  $t = 0$  der Fertigteilträger mit dem Kraftanteil  $P_1$  und der Ortsbetonträger mit  $P_2$  belastet:

$$P_1 = P_0 \frac{E_1 F_1}{E_1 F_1 + E_2 F_2}; \quad P_2 = P_0 \frac{E_2 F_2}{E_1 F_1 + E_2 F_2}$$

4.14 Infolge der Dauerbelastung  $P_0$  entstehen plastische Verformungen, sowohl in Fertigteilträger wie auch im Ortsbetonträger.

Die Kraft  $X_t$  kann man nach dem Verfahren 4.12 bestimmen. Das Belastungsglied in Funktion der Zeit:

$$a_{10}^2 \frac{d\varphi_2}{dt_2} - a_{10}^1 \frac{d\varphi_1}{dt_2} = \frac{P_2}{E_2 F_2} \frac{d\varphi_2}{dt_2} - \frac{P_1}{E_1 F_1} \frac{d\varphi_1}{dt_2} = \frac{P_0}{E_1 F_1 + E_2 F_2} (1 - \nu \mathcal{H}) \frac{d\varphi_2}{dt_2}$$

$$\beta = \frac{a_{10}^2 - \nu \mathcal{H} a_{10}^1}{a_{11}} = -\bar{X}; \quad \bar{X} = (1 - \nu \mathcal{H}) \frac{P_0}{E_1 F_1 + E_2 F_2} : \frac{1}{E_1 F_1} + \frac{1}{E_2 F_2}$$



Die Lösung der Gleichung gibt für  $X_t$  folgende Werte:

a.) Wenn  $t_{20} \neq 0$  (mit Schonzeit)

$$X_t = \frac{\bar{X}}{\alpha} \left( 1 - \lambda e^{-\alpha \varphi_{2\infty}} \right)$$

b.) Wenn  $t_{20} = 0$  (ohne Schonzeit)

$$X_t = \frac{\bar{X}}{\alpha} \left( 1 - e^{-\alpha \varphi_{2\infty}} \right)$$

Zur Zeit  $t = \infty$  wird der Fertigteilträger mit  $P_{1t} = P_1 + X_t$ , und der Ortsbetonträger mit  $P_{2t} = P_2 - X_t$  belastet.

4.15 Die Berechnung des Schwindens:

Das relative Schwinden zwischen den Trägern im Zeitpunkte  $t = \infty$  ist  $\varepsilon = \varepsilon_\infty (1 - e^{-t_{10}})$ , wenn der Altersunterschied der zwei Trägereile  $\Delta t = t_{10}$  ist.

Die Schwindkraft wird durch das Kriechen wesentlich vermindert und im Zeitpunkte  $t = \infty$  erreicht sie den Wert nach der Gleichung:

$$\begin{aligned} X a_{11} + a_{10} &= 0; \quad a_{11}^1 + a_{11}^2 = \frac{1}{E_1 F_1} + \frac{1}{E_2 F_2} \\ a_{10} &= \varepsilon_\infty (1 - e^{-t_{10}}) = \varepsilon_\infty (1 - e^{-\Delta t}); \quad X = \varepsilon_\infty (1 - \mathcal{K}); \quad \frac{E_1 F_1 \times E_2 F_2}{E_1 F_1 + E_2 F_2} \\ X_t &= \frac{X}{\alpha \varphi_{2\infty}} \left( 1 - e^{-\alpha \varphi_{2\infty}} \right) \end{aligned} \quad (10)$$

#### 4.2 Biegung des zusammengesetzten Balkens.

Bei Biegung des zusammengesetzten Balkens werden zwei Fälle untersucht:

- a.) die Schwerlinien der beiden Querschnitte fallen zusammen,
- b.) die Schwerlinien der beiden Querschnitte sind verschieden.

Die Untersuchung der Träger des Falles a.) können wir nach Punkt 4.1 ausführen, wenn wir in die Formeln statt  $P$ ,  $M$  und statt  $F$ ,  $J$  einsetzen.

Untersuchung der Träger von Punkt b.):

4.21 Das Biegemoment  $M_0$  belastet den Verbundträger.

Die Berechnung der Innenkräfte zur Zeit  $t = 0$ , können wir in folgenden Schritten durchführen (Bezeichnungen nach Abb. 3):

a.) Berechnung der Null-Linie des Verbundträgers:

$$a_2 = a \frac{E_1 F_1}{E_1 F_1 + E_2 F_2}; \quad a_1 = a - a_2$$

b.) Bestimmung der Kernpunkte unter der Voraussetzung, dass die Spannungen in der neutralen Achse gleich Null sind.

$$b_2 = \frac{i_2^2}{a_2}; \quad b_1 = \frac{i_1^2}{a_1}$$

c.) Berechnung des Hebelarmes des inneren Kräftepaars:

$$c = a + b_2 + b_1$$

d.) Bestimmung der inneren Kräfte:  $N_0 = M_0 / c$ ;  $M_1 = N_0 b_1$ ;

$$M_2 = N_0 b_2$$

e.) Bemessung der Trägereile. Im Besitz der Normalkräfte und Biegemoment wird der Stahlbeton-Fertigteil als ausmittig gezogener und der Ortsbetonträger als ausmittig gedrückter Balken bemessen.

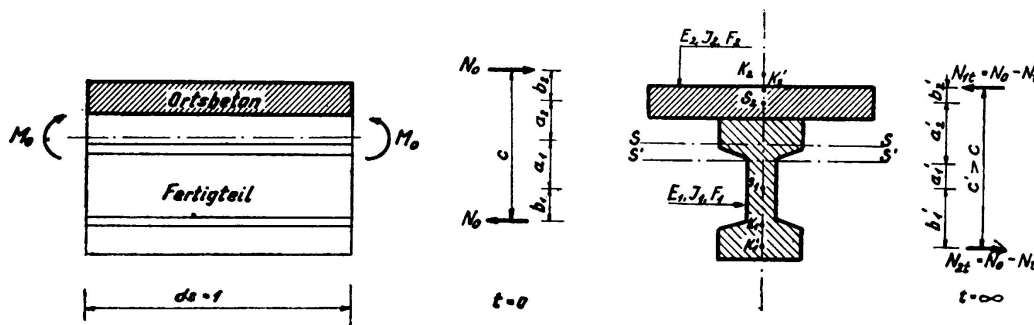


ABB. 3. Innere Kräfte in einem Stahlbeton-Fertigteilträger mit Druckplatte zur Zeit  $t=0$  und  $t=\infty$

4.22 Die plastischen Verformungen verursachen eine Umlagerung der inneren Kräfte. Die Veränderung der Normalkraft wird aus der Formänderungsgleichung gerechnet:

$$\begin{aligned} N_t \left( \frac{1}{E_1 F_1} \frac{d\varphi_1}{dt_2} + \frac{1}{E_2 F_2} \frac{d\varphi_2}{dt_2} \right) + \frac{dN_t}{dt_2} \left( \frac{1}{E_1 F_1} + \frac{1}{E_2 F_2} \right) = \\ = N_0 \left( \frac{1}{E_1 F_1} \frac{d\varphi_1}{dt_2} + \frac{1}{E_2 F_2} \frac{d\varphi_2}{dt_2} \right) - \frac{M_1 a}{E_1 J_1} \frac{d\varphi_1}{dt_2} \end{aligned}$$

Mit  $N_0 = \frac{M_0}{c}$  und  $M_1 = N_0 b_1$

Die Lösung der Differenzialgleichung:

$$N_t = \left( 1 - \lambda e^{-\alpha \varphi_2 \infty} \right) \frac{\beta}{\alpha} N_0 \quad (11)$$

$$\text{Wenn } \alpha = \frac{\nu \mathcal{H} a_{11}^1 + a_{11}^2}{a_{11}^1 + a_{11}^2}; \quad \beta = \alpha - \alpha'; \quad \alpha' = \frac{\nu \mathcal{H} a_{10}^1}{a_{11}^1 + a_{11}^2};$$

$$a_{10}^1 = \frac{ab_1}{E_1 J_1}; \quad \beta = (1 - \nu \mathcal{H}) \frac{E_1 F_1}{E_1 F_1 + E_2 F_2}$$

Zur Zeit  $t = \infty$  wird die Normalkraft:  $N_{1t} = N_{2t} = N_o - N_t$

Da das Biegemoment  $M_o$  unverändert bleibt, wächst der Hebelarm von  $c$  zu  $c'$ . Die Grösse des  $c'$  ist durch die Beziehung gegeben:

$$c' N_{1t} = c N_o = M_o \text{ und davon } c' = \frac{M_o}{N_{1t}}$$

Infolge des Kriechens verschiebt sich die Null-Linie des zusammengesetzten Querschnittes in Richtung des Fertigteils und die Lage der Kernpunkte ändert sich nach Abb. 3. Die Umlagerung der charakteristischen Punkte des Querschnittes kann man leicht bestimmen aus den Beziehungen:

$$a + b'_1 + b'_2 = c' \quad (12)$$

$$a = a'_1 + a'_2 \quad (13)$$

$$a'_1 b'_1 = i_1^2 \quad (14)$$

$$a'_2 b'_2 = i_2^2 \quad (15)$$

Zur Zeit  $t = \infty$  sind also die inneren Kräfte:

$$N_{1t} = N_{2t}; \quad M_{1t} = N_{1t} b'_1 \text{ und } M_{2t} = N_{1t} b'_2$$

4.23 Die Last wird nur von dem Stahlbeton-Fertigteil getragen. Die Belastung des Stahlbeton-Fertigteiles wird zur Zeit  $t = 0$   $M_1 = M_o$  und der an Ort und Stelle betonierte Trägereil wird unbelastet,  $M_2 = 0$ .

4.24 Um die inneren Kräfte infolge plastischer Verformungen zu bestimmen, lösen wir die Differenzialgleichung:

$$N_t \left( \frac{1}{E_1 F_1} \frac{d\varphi_1}{dt_2} + \frac{1}{E_2 F_2} \frac{d\varphi_2}{dt_2} \right) + \frac{dN_t}{dt_2} \left( \frac{1}{E_1 F_1} + \frac{1}{E_2 F_2} \right) =$$

$$= \frac{M_o a}{E_1 J_1} \frac{d\varphi_1}{dt_2} = \frac{N_o b_1 a}{E_1 J_1} \frac{d\varphi_1}{dt_2}; \quad N_o = \frac{M_o}{b_1}$$

$$\text{wird } N_t = \left( 1 - e^{-\alpha \varphi_2 \infty} \right) \frac{\beta}{\alpha} N_o \quad (16)$$

$$\beta = \nu \mathcal{H} \text{ aus } \beta = \alpha' = \frac{\nu \mathcal{H} a_{10}^1}{a_{11}^1 + a_{11}^2}; \quad a_{10}^1 = \frac{ab_1}{E_1 J_1}$$

Die Kraftanteile des Verbundquerschnittes zur Zeit  $t = \infty$  werden: im Ortsbeton die Druckkraft  $N_t$  und das Moment  $M_{2t} = + N_t b_2$ ; im Fertigteil die Zugkraft  $N_t$  und das Moment  $M_{1t} = M_o - N_t (a + b_2)$ .

4.25 Infolge Schwindens des Betons entsteht laut Punkt 4.15 eine Normalkraft  $X$  und ein Moment  $Xa$ . Die Randspannungen werden dann im Stahlbeton-Fertigteil:

$$\sigma_1 = - \frac{X}{F_1} + \frac{Xa}{cF_1} + \frac{Xab_1}{c W_1} \quad (17)$$

und im Ortsbeton:

$$\sigma_2 = + \frac{X}{F_2} - \frac{Xa}{cF_2} - \frac{Xab_2}{c W_2} \quad (18)$$

Die Schwindspannungen werden durch die plastischen Verformungen wesentlich vermindert (Gl. 10) und zur Zeit  $t = \infty$  erreichen sie die Werte:

$$\sigma_1 = - \frac{X_t}{F_1} + \frac{M_t}{cF_1} + \frac{M_t b_1}{c W_1} \quad (19)$$

$$\sigma_{2t} = + \frac{X_t}{F_2} - \frac{M_t}{cF_2} - \frac{M_t b_2}{c W_2} \quad (20)$$

wo  $X_t = \frac{X}{\alpha \varphi_{2\infty}} \left( 1 - e^{-\alpha \varphi_{2\infty}} \right)$  und  $M_t = X_t a$ .

## 5.) Bruchberechnung zusammengesetzter Stahlbeton-Konstruktionen. Bruchsicherheit.

### 5.1 Mittig belastete Säulen (ohne Knickgefahr).

Die Bruchlast eines symmetrisch zusammengesetzten und auf mittigen Druck beanspruchten Stabes wird durch das bekannte Additionsgesetz berechnet. Die Bruchlast des zusammengesetzten Stabes ist gleich der Summe der Bruchlasten der einzelnen Stäbe.

$$P_B = P_{1B} + P_{2B}, \text{ wo } P_{1B} = k_1 F_{1b} + \sigma_t F_{1s} \quad (21)$$

$$P_{2B} = k_2 F_{2b} + \sigma_f F_{2s} \quad (22)$$

Ferner werden wir nur mit dem Betonquerschnitt rechnen; dementsprechend sind die Bruchlasten:

$$P_{1B} = k_1 F_1; \quad P_{2B} = k_2 F_2$$

Der Druckstab wird auf Grund der zulässigen Last  $P_o$  bemessen.

$$P_o = P_{10} + P_{20} = \frac{k_1}{n} F_1 + \frac{k_2}{n} F_2 = \sigma_{10} F_1 + \sigma_{20} F_2$$

Die Bruchsicherheit ist:  $n = \frac{P_B}{P_0}$ .

Belasten wir nun den Fertigteil mit einer Druckkraft  $P_1$ , werden infolgedessen die Spannungen im Fertigteil:  $\sigma_1 = \frac{P_1}{F_1}$ .

Bestimmen wir in diesem Zustande die Drucklast  $P_0'$  des Verbundquerschnittes sodass infolge der Lasten  $P_0'$  und  $P_1$  die Spannungen im Fertigteil den zulässigen Wert  $\sigma_{10}$  erreichen sollen. Zur Vereinfachung der Berechnung nehmen wir an, dass der Beton ein idealplastischer Baustoff ist. Wegen  $P_0' = P_{10}' + P_{20}'$ , ist

$$\sigma_1 + \frac{P_0'}{F_1} \cdot \frac{E_1 F_1}{E_1 F_1 + E_2 F_2} = \sigma_{10}, \text{ oder}$$

$$P_1 + P_0' \cdot \frac{E_1 F_1}{E_1 F_1 + E_2 F_2} = P_{10}; \quad P_0' = (P_{10} - P_1) \frac{E_1 F_1 + E_2 F_2}{E_1 F_1}$$

Um die zulässige Last  $P_0$  des Verbundquerschnittes zu erreichen, steigern wir die Last mit  $\Delta P$ . Der Wert von  $\Delta P$  hängt von der zulässigen Spannung  $\sigma_{20}$  des an Ort und Stelle betonierten Trägereiles ab, — und sie ist proportional mit der Vorbelastung  $P_1$ .

$$\Delta P + P_0' \cdot \frac{E_2 F_2}{E_1 F_1 + E_2 F_2} = P_{20};$$

$$\Delta P = P_{20} - \frac{E_2 F_2}{E_1 F_1} (P_{10} - P_1)$$

Die Bruchsicherheit in dem auf zulässige Spannungen bemessenen Querschnitt wird nun:

$$n' = n \left( 1 + \frac{\Delta P}{P_b} \right) \quad (23)$$

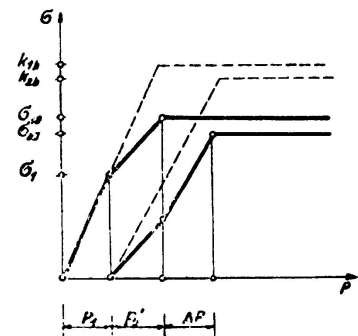


ABB. 4. Verlauf der Druckspannungen in einem zusammengesetzten Querschnitt mit Vorbelastung  $P_1$ .

5.2 Bruchberechnung des auf Biegung beanspruchten Verbundträgers, wenn die Trägereile dieselbe Schwerlinie haben.

Das Betonmoment des Verbundquerschnittes ist bei reiner Biegung im Bruchzustand:  $M_b = M_{1B} + M_{2B}$ ; nach den Versuchsergebnissen:

$$M_{1B} = k_1 b_1 h_1^2 / 3 \quad (24)$$

$$M_{2B} = k_2 b_2 h_2^2 / 3 \quad (25)$$

Um die Größenordnung der Sicherheit zu bestimmen, werden wir ein solches Bemessungsverfahren benutzen, das die Elastizitätstheorie und die Bruchtheorie vereinigt. Zu diesem Zwecke werden wir die Versuchsergebnisse der Bruchtheorie auf die Elastizitätstheorie übertragen.

Das Verfahren ist  $n$ -frei und auch bei homogenen Querschnitten brauchbar. Die Richtigkeit des Verfahrens wurde bei Rechteckquerschnitten in vollem Masse bewiesen <sup>(1)</sup>.

Da wir die Rechnungen auf den homogenen Betonquerschnitt beziehen, wird das Bruchmoment beider Trägereile:

$$M_{1B} = K_1 W_1; \quad M_{2B} = K_2 W_2$$

Rechteckquerschnitte haben den Formfaktor  $k/K$  laut den Versuchen von der Grössenordnung 0,5-0,7.

Das zulässige Betonmoment ist:

$$M_o = M_{1o} + M_{2o} = \frac{K_1}{n} W_1 + \frac{K_2}{n} W_2 = \sigma_{1b} W_1 + \sigma_{2b} W_2$$

Der Sicherheitsfaktor ist:  $n = \frac{M_B}{M_o}$ .

Wenn der Stahlbeton-Fertigteil eine Vorbelastung  $M_1$  bekommen hat, kann man wie im Punkt 5.1 den Wert  $\Delta M$  bestimmen.

$$\Delta M = M_{2o} - \frac{E_2 J_2}{E_1 J_1} (M_{1o} - M_1)$$

Und so erhalten wir die Bruchsicherheit des Verbundträgers, bemessen auf Grund der Elastizitätstheorie:

$$n' = n \left( 1 + \frac{\Delta M}{M_b} \right) \quad (26)$$

Bemerkung: Die Bruchsicherheit ändert sich, wenn wir beim Bemessen den Einfluss des Schwindens und Kriechens auch in Rechnung setzen.

5.3 Bruchberechnung des auf Biegung beanspruchten Verbundträgers, wenn die Trägereile verschiedene Schwerlinien haben. Bruchsicherheit.

Nach der Plastizitätstheorie ist der Additionssatz auch für das Bruchmoment gültig. Das Bruchmoment des Stahlbetonverbund-Plattenbalkens ist, wenn  $x = 0.5 h$  und  $v$  (die Plattendicke)  $< x$ , und  $b_1$  konstant ist:

$$M_B = M_{1B} + M_{2B} = k_1 b_1 \left( \frac{h}{2} - v \right) \frac{3h - 2v}{4} + k_2 b_2 v \left( h - \frac{v}{2} \right)$$

$$\text{wenn } h = h_1 + v \text{ und } v = \alpha h_1; \quad h = h_1 (1 + \alpha)$$

$$M_{1B} = k_1 b_1 h_1^2 \left( \frac{3 - 2\alpha - \alpha^2}{8} \right); \quad M_{2B} = k_2 b_2 v^2 \left( \frac{1 + 0.5\alpha}{\alpha} \right)$$

<sup>(1)</sup> Hognestad, E.: Univ. of Illinois Bull. Series n°. 399, vol. 49, n°. 22.

Da die zulässigen Spannungen  $\sigma_{10} = \frac{k_1}{n}$ ;  $\sigma_{20} = \frac{k_2}{n}$  sind, die zulässigen Momente:

$$M_{10} = \sigma_{10} b_1 h_1^2 \left( \frac{3 - 2\alpha - \alpha^2}{8} \right) \quad (27)$$

$$M_{20} = \sigma_{20} b_2 v^2 \left( \frac{1 + 0.5\alpha}{\alpha} \right) \quad (28)$$

Die Berechnung wird an ausmittig belasteten Querschnitten gezeigt. Da zwischen der Normalkraft  $N_B$  und dem Moment  $M_B$  im Bruchzustand ein geradliniger Zusammenhang besteht, können wir das Gesetz in die Form bringen:

$$\frac{N}{N_{1B}} + \frac{M}{M_{1B}} = 1; \quad \frac{N}{N_{2B}} + \frac{M}{M_{2B}} = 1$$

$$\frac{N}{\sigma_{10} F_1} + \frac{M}{\sigma_{1b} W_1} = 1; \quad \frac{N}{\sigma_{20} F_2} + \frac{M}{\sigma_{2b} W_2} = 1$$

Wenn  $\frac{\sigma_{10}}{\sigma_{1b}} = \frac{\sigma_{20}}{\sigma_{2b}} = 0,5$  dann

$$M_{10} = \sigma_{1b} W_1 \frac{1}{1 + \frac{h_1}{3b_1}} = \sigma_{1b} \frac{b_1 h_1^2}{6} \left( \frac{1}{1 + \frac{h_1}{3b_1}} \right) \quad (29)$$

$$M_{20} = \sigma_{2b} W_2 \frac{1}{1 + \frac{v}{3b_2}} = \sigma_{2b} \frac{b_2 v^2}{6} \left( \frac{1}{1 + \frac{v}{3b_2}} \right) \quad (30)$$

Die Bedingung, dass die Biegemomente aus der Gl. (27, 29) und (28, 30) gleich sein müssen, gibt den Wert des Formfaktors  $\beta_1$  und  $\beta_2$

$$\beta_1 = \frac{\sigma_{10}}{\sigma_{1b}} = \frac{4}{3} \frac{1}{3 - 2\alpha - \alpha^2} \frac{1}{1 + \frac{h_1}{3b_1}}$$

$$\beta_2 = \frac{\sigma_{20}}{\sigma_{2b}} = \frac{\alpha}{6(1 + 0.5\alpha)} \frac{1}{1 + \frac{v}{3b_2}}$$



Die auf Grund der Elastizitätstheorie bemessenen Träger sind nur bis zum Erreichen der zulässigen Randspannungen in Anspruch genommen. Die Randspannungen:

$$\sigma_{1b} = \frac{\sigma_{10}}{\beta_1} \text{ und } \sigma_{2b} = \frac{\sigma_{20}}{\beta_2}$$

Wenn der Fertigteil infolge Montierung mit  $M_1$  vorbelastet ist, kann die zusätzliche Beanspruchung  $\Delta M$  in der folgenden Form ausgedrückt werden:

$$\Delta M = M_{20} - \frac{4}{3} (M_{10} - M_1) \frac{b_2 + \frac{W_2}{F_2}}{b_1 + \frac{W_1}{F_1}} = M_{20} - \frac{4}{3} (M_{10} - M_1) \frac{b_2 + \frac{v}{6}}{b_1 + \frac{h_1}{6}}$$

Wenn  $b_1 = \frac{h_1}{6}$  ist, so bleibt die Randlinie des Fertigteilträgers spannungslos. In diesem Falle steht die Null-Linie des Verbundträgers in der Grenzlinie der zwei Querschnitte. Dann ist  $b_2 = \frac{v}{6}$  und somit

$\Delta M = M_{20}$ . Wenn  $b_1 > \frac{h_1}{6}$  ist, liegt also die Null-Linie des Verbundträgers in dem Fertigteil, so ist  $\Delta M < M_{20}$ .

Wenn  $b_1 < \frac{h_1}{6}$ , liegt also die Null-Linie in der Druckplatte, ändert sich das Vorzeichen des zweiten Gliedes und  $\Delta M$  wird  $\Delta M > M_{20}$ . Die Zugspannungen mindern den Wert der Vorbelastung im Steg.

Die Werte  $M_{10}$  und  $M_{20}$  werden mit Hilfe der Formfaktoren aus  $M_{1b}$  und  $M_{2b}$  gerechnet, und die Bruchmomente:

$$M_{1B} = n M_{10}; \quad M_{2B} = n M_{20}.$$

## 6.) Statisch unbestimmte Stahlbeton-Verbundkonstruktionen.

Die statisch unbestimmten Stahlbeton-Verbundkonstruktionen kann man mit den bekannten Methoden lösen. Im Rahmen dieser Arbeit werden einige Gesichtspunkte zur Vereinfachung der Berechnung angegeben.

6.1 Die Verformungsgleichungen eines unbestimmten Tragwerkes im Zeitpunkt  $t = 0$ , sind:

$$a_{11} X_1 + a_{21} X_2 + \dots + a_{10} = 0.$$

$$a_{21} X_1 + a_{22} X_2 + \dots + a_{20} = 0. \text{ , usw.}$$

Bei der Ausrechnung der einzelnen Beiwerte werden wir nur die Verformungen eines Trägereils in Betracht nehmen. Z. B.: Beim Verbundquerschnitt aus der Abb. 3. können die Verformungsglieder auf

die Ortsbeton-Druckplatte bezogen werden. In diesem Falle werden die Momentenflächen der virtuellen Kräfte  $X_1 = 1$ , nur auf die Schwerlinie des Ortsbetons bezogen. Diese Tatsache hat nur bei Trägern mit polygonalen oder krummlinigen Achsen (Rahmen und Bogen) Bedeutung, bei Trägern mit geraden Achsen (Durchlaufträger) ist die Beziehungssachse gleichgültig.

Dementsprechend die Belastungsglieder:

$$a_{ik} = \int_0^s \frac{M_i M_{2k}}{E_2 J_2} ds + \int_0^s \frac{N_i N_{2k}}{E_2 F_2} ds$$

$$a_{io} = \int_0^s \frac{M_i M_{20}}{E_2 J_2} ds + \int_0^s \frac{N_i N_{20}}{E_2 F_2} ds$$

$M_i$  und  $M_{2k}$  sind die Momente der virtuellen Kräfte  $X_1 = 1$  und  $X_k = 1$ , bezogen auf die Schwerlinie des Ortsbetonträgers,  $M_{20}$  ist das Aussenmomentanteil am statisch bestimmten Träger. Z. B.: bei Durchlaufträger (Verdrehung an einem Ende des Trägers).

$$a_{ik} = \int_0^1 \frac{M_i M_{2k}}{E_2 J_2} ds = \int_0^1 \frac{M_i M_k b_2}{E_2 J_2 c} ds$$

$$a_{io} = \int_0^1 \frac{M_i M_{20}}{E_2 J_2} ds = \int_0^1 \frac{M_i M_o b_2}{E_2 J_2 c} ds$$

6.2 Die Belastungsglieder des unbestimmten Tragwerkes in der Folge der Zeit werden:

$$a_{ikt} = \int_0^s \frac{M_i M_{2kt}}{E_2 J_2} ds + \int_0^s \frac{N_i N_{2kt}}{E_2 F_2} ds$$

Wenn  $\frac{b'_2}{b_2} \approx 1$  und  $\nu = 1$ , der Wert des auf den Ortsbeton bezogenen Momentes ist.

$$M_{2kt} = N_{2kt} b'_2 \approx N_{2kt} b_2 = M_k \frac{b_2}{c} \left( 1 - \lambda e^{-\alpha \varphi_{2t}} \right) \frac{\beta}{\alpha}$$

$$a_{ikt} = \int_0^s \frac{M_i M_k}{E_2 J_2} \frac{b_2}{c} \left( 1 - \lambda e^{-\alpha \varphi_{2t}} \right) \frac{\beta}{\alpha} ds$$

$$a_{i0t} = \int_0^s \frac{M_i M_o}{E_2 J_2} \frac{b_2}{c} \left( 1 - \lambda e^{-\alpha \varphi_{2t}} \right) \frac{\beta}{\alpha} ds$$

Die Formänderungsgleichung zwischen den Zeitpunkten  $t$  und  $t + dt$ :

$$\begin{aligned} a_{11} \frac{d\varphi_2}{dt_2} X_{1t} + a_{11} \frac{dX_{1t}}{dt_2} + a_{12} \frac{d\varphi_2}{dt_2} X_{2t} + \\ + a_{12} \frac{dX_{2t}}{dt_2} + \dots + a_{10} \frac{d\varphi_2}{dt_2} = 0. \end{aligned}$$

Wo z. B.:

$$a_{ii} \frac{d\varphi_2}{dt_2} = \int_0^s \frac{M_i M_i}{E_2 J_2} \frac{b_2}{c} \lambda \alpha e^{-\alpha \varphi_{2t}} \frac{\beta}{\alpha} \frac{d\varphi_2}{dt_2} ds$$

### 6.3 Bruch des statisch unbestimmten Tragwerkes.

Das statisch unbestimmte Tragwerk wird auf Grund der Elastostatik bemessen, mit Rücksicht auf die Bauweise oder Montage und Ueberlagerung der inneren Kräfte.

Bei der Untersuchung der statisch bestimmten Träger im Gebrauchszustande, haben wir darauf hingewiesen, dass die Berechnung der Vorbelastung, des Kriechens und des Schwindens in manchen Fällen, zu einem höheren Sicherheitsgrad führt. Bei statisch unbestimmten Tragwerken ist auch die unbestimmte Grösse eine Funktion der Zeit. Bei freigebauten Teilen, wo die gesamte Last von dem statisch bestimmten Fertigteil getragen wird, entstehen nach dem Verbund negative Momente und die positiven Momente vermindern sich. Bei auf Rüstung gebauten Tragwerken geht das umgekehrt, an der Stelle der frischen Verbindung werden die negativen Momente vermindert.

Die Berücksichtigung dieser Beanspruchungen und Spannungsumlagerungen beeinflusst aber den Bruchzustand des Trägers nicht, wenn wir den Bruch des Trägers mit der Bildung der Fliessgelenke identisch nehmen.

### 7.) Schlusswort.

Die Bedeutung der Stahlbeton-Verbundkonstruktionen wird von Tag zu Tag grösser infolge der rapiden Entwicklung der Fertigteil-Bauten. Das Bestreben, ein durch Versuche bestätigtes Bemessungsverfahren zu entwickeln, ist von wirtschaftlicher Bedeutung. Durch die Versuche wird es möglich sein, mehrere Fragen zu beleuchten, z. B.: die Frage des Verbundes der zwei Betonteile, der Schubsicherheit, des Einflusses der Zeit, der Bemessung des Plattenbalkens und der Schubbewehrung, usw.

Die versuchsmässige Auswertung obengenannter Fragen ist im Gange und die Ergebnisse werden im Schlussbericht veröffentlicht.

### ZUSAMMENFASSUNG

Der Zweck der vorliegenden Arbeit ist, einen Ueberblick der Berechnung der Stahlbeton-Fertigteilkonstruktionen -die nachträglich an Ort und Stelle verbunden sind- zu geben. Es wird die Bestimmung der Teilkkräfte einzelner Trägereile, bei verschiedenen Lastfällen untersucht.

Die statisch bestimmten Tragwerke werden in zwei Gruppen analysiert: Systeme mit gleicher Schwerachse und Systeme mit verschiedenen Schwerachsen. Da beim Brückenbau und Hochbau der letztere Fall immer häufiger angewendet wird, wollen wir ein vereinfachtes Rechenungsverfahren zeigen, womit die Spannungsumlagerungen leicht ermittelt werden können.

Im Rahmen der Arbeit wird der Einfluss des Kriechens und des Schwindens, -hauptsächlich mit Rücksicht auf die Schonzeit und den Zeitalterunterschied beider Trägereile-, in Betracht genommen.

Die Frage der Bemessung der Stahlbeton-Verbundkonstruktionen wird ebenfalls behandelt. Es wird ein Bemessungsverfahren vorgeschlagen, das die Ergebnisse der elastischen und plastischen Theorien vereinigt und das durch zahlreiche Versuche bestätigt wurde. Auf dieser Grundlage kann der Sicherheitsfaktor beim Bruch der nach der Elastizitätstheorie bemessenen Verbundsysteme leicht ausgerechnet werden. Die Berechnung des unbestimmten Verbundsystemes wird auch behandelt, besonders mit Rücksicht auf den Einfluss der Zeitveränderlichkeit auf die einzelnen Belastungsglieder.

### RESUMO

O autor dá uma ideia geral do cálculo das estruturas de betão armado com elementos prefabricados montados no local da obra e analisa a determinação dos esforços nas vigas para diversos casos de carga.

As estruturas isostáticas dividem-se em dois grupos: as co-axiais e as de eixo distinto. Sendo o último o caso mais corrente nos edifícios e nas pontes, o autor expõe um método de cálculo simples que permite determinar facilmente a repartição das tensões em estruturas desse tipo.

O autor também considera o efeito da fluência e da contracção do betão, especialmente no que diz respeito à diferença de idade de dois elementos de uma viga.

Estuda-se igualmente o caso das construções compostas de betão armado. O autor propõe um método de cálculo que combina os resultados das teorias elástica e plástica e que foi verificado em numerosas experiências.

Este método permite determinar facilmente o coeficiente de segurança à rotura de sistemas compostos calculados pela teoria da elasticidade.

O cálculo dos sistemas compostos indeterminados também é discutido, considerando particularmente a influência das variações das cargas elementares com o tempo.

### R É S U M É

L'auteur donne un aperçu général du calcul des constructions en béton armé exécutées en éléments pré-fabriqués montés sur place. Il étudie la détermination des efforts dans les poutres pour divers cas de charge.

Les structures isostatiques peuvent se diviser en deux groupes: les structures co-axiales et celles à axes distincts. Les dernières étant plus courantes dans le bâtiment et les ponts, l'auteur expose une méthode de calcul simple qui permet de déterminer facilement la répartition des contraintes dans des constructions de ce genre.

L'auteur considère aussi l'influence du fluage et du retrait, surtout en ce qui concerne deux éléments d'une même poutre, d'âges différents.

Les cas des structures composées en béton armé est également étudié. L'auteur propose une méthode de calcul qui combine les résultats des théories élastique et plastique et qui a été vérifiée par de nombreuses expériences. Cette méthode permet de déterminer facilement la sécurité à la rupture de systèmes composés calculés par la théorie de l'élasticité. Le calcul des systèmes composés indéterminés est aussi traité, en considérant surtout l'influence de la variation des charges élémentaires avec le temps.

### S U M M A R Y

The author gives a general survey of the calculation methods used in reinforced concrete structures incorporating precast units assembled at the site. He discusses the determination of forces in the members for various types of load.

Statically determinate structures are divided into two groups: coaxial systems and systems with distinct axis. Owing to the latter being more commonly found in buildings and bridges, the author describes a method of analysis through which stress repartition is easily determined for that type of structure.

The author also considers the influence of concrete creep and shrinkage, as related to the differences of age between two precast elements.

Composite reinforced concrete structures are also considered. A calculation method is proposed which is based upon the results of the elastic and plastic theories and has been verified by a number of tests. This method enables to easily determine the failure safety coefficient of composite structures calculated by the theory of elasticity. Indeterminate composite structure calculation is discussed, especially regarding the influence of the variation of load elements with time factor.

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## **VI a5**

### **The vibration of precast concrete elements for post-stressed bridge members**

### **Des Vibrieren von Beton-Fertigteilen für nachträglich zusammenengespannte Brückenglieder**

### **Vibração de peças de betão prefabricadas para elementos de ponte post-esforçados**

### **Vibration d'éléments préfabriqués pour membrures de pont précontraintes sur place**

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Bridges constructed of post-stressed concrete units have certain advantages over other types of construction. The more important being the small amount of maintenance required and the possibility of manufacture at a convenient site, reducing the erection work to placing the units on a skeleton scaffold and stressing the bars or wires linking them together. These advantages are especially important in areas of rapid development such as West Africa, and areas of high rainfall requiring much maintenance of steel structures.

Concrete of high strength is always specified for this type of construction and the usual method of obtaining this high strength is to use a mix of low water content compacted with the aid of vibration. Vibration may be applied by:—

1. Vibrating tables.
2. Shutter vibrators.
3. Immersion vibrators.

All these may be used for compacting precast post-tensioned units, the method chosen depending on the weight of the casting and its dimensions. Vibrating tables are usually limited in the weight of concrete which may be compacted upon them; this is less important in the case of electro-magnetic tables since any number of these may be placed side by side to share the load and to run in phase with each other.



Mechanically driven tables owing to their inability to run in phase with controlled amplitude cannot be used in this way. Immersion and shutter vibrators are used in sufficient numbers to achieve compaction without consideration of phase difference.

The requirements are a casting which is completely homogeneous with uniform strength and modulus of elasticity throughout. Unfortunately these are rarely achieved, it is therefore proposed to examine the problem and discuss its solution.

The problem approximates to that of obtaining uniform strength throughout the casting. It has been shown elsewhere (1) that for vibrated concrete there is a minimum acceleration below which compaction does not take place. It should be emphasised that this minimum acceleration is the acceleration of the concrete and not that of the vibrating unit or mould. The minimum value of acceleration for compaction is of the order  $2\frac{1}{2}g$  to  $3g$  dependent upon the frequency of the vibration and the workability of the mix. It may be stated therefore that to achieve full compaction throughout the casting the acceleration of every point within the casting must be equal to or exceed the minimum value.

In a large casting the acceleration at any point varies according to the distance of the point from the source of vibration and to the proximity of the side of the mould (2). Thus the lowest value of acceleration occurs near the centre of a casting vibrating on a vibrating table. It might be argued that if the acceleration at this point is greater than  $3g$  then all would be well. In this connection it is necessary to consider a further point. An increase in acceleration results in a reduction in the time of vibration necessary to achieve compaction, therefore those parts of a casting having the highest acceleration will be compacted before those with a lower acceleration. It can readily be seen therefore that vibration should be continued for such time as is necessary for full compaction at the lowest acceleration.

The concrete adjacent to the sides of the mould and on the surface of the casting will have a greater acceleration than that at the centre and will be compacted first, its appearance will not then be a true indication of the completeness of vibration of all parts of the casting. If no account is taken of the foregoing then a casting may result in which the outer layers are fully compacted whilst the core is not. The conditions are analogous to a case hardened steel and the casting will not fulfill the requirements stated previously because its strength will vary. The much publicised danger of OVER VIBRATION must be considered in this context. If a mix is suitable for vibration then after full compaction has taken place no further increase in strength will result however long the period of vibration nor will it be in any way adversely affected. The troubles of overvibration and segregation only result from a badly designed mix.

Concrete mixes must be designed for vibration, the ordinary methods of design for hand placing being inadequate and in some cases harmful when applied to concrete which is to be vibrated. In addition to the segregation a further problem must be borne in mind when designing a mix, namely «Rotational instability» (3). The phenomenon is caused by one part of the mould having a slightly higher acceleration than

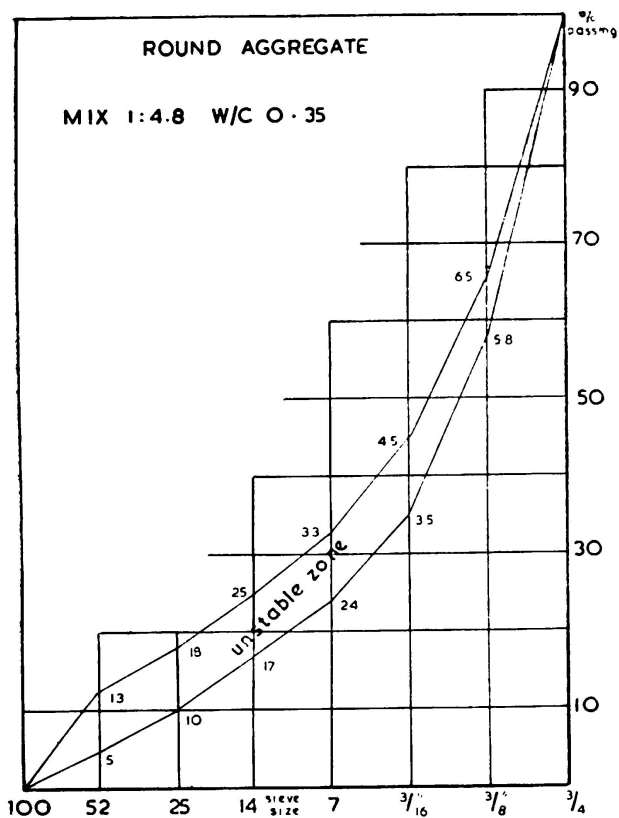


FIG. 1. Limit curve for rotational instability at accelerations up to 18g with rounded river gravel

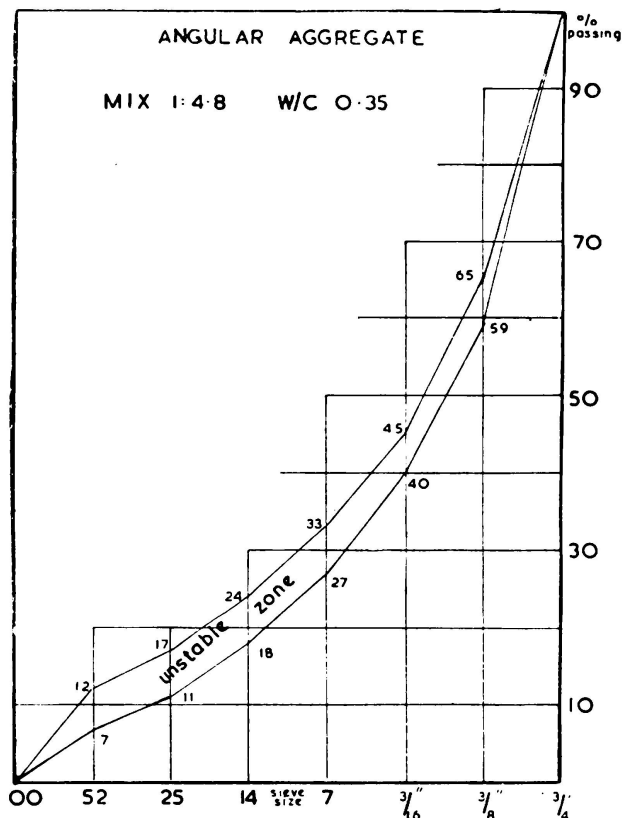


FIG. 2. Limit curve for rotational instability at accelerations up to 18g with angular granite aggregate

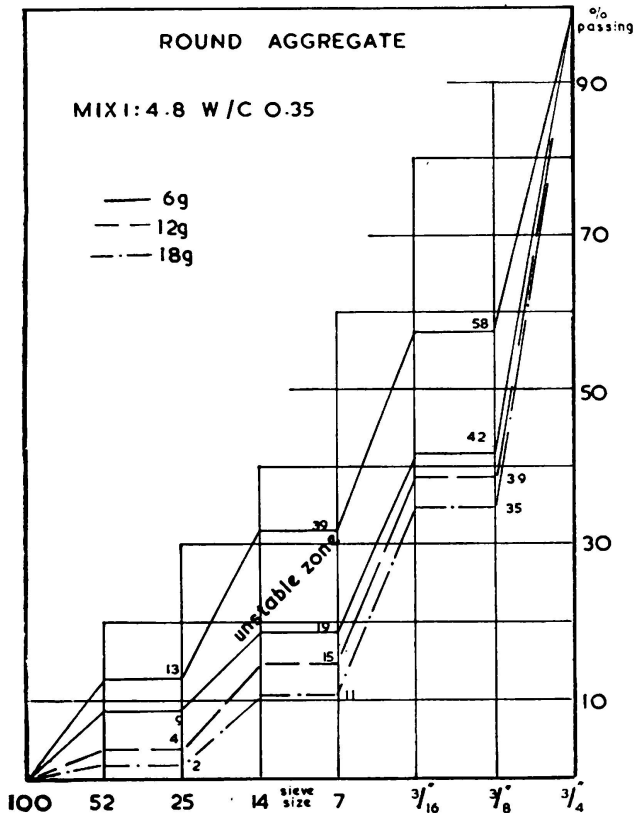


FIG. 3. Limit curve for rotational instability at accelerations up to 18g with gap graded rounded river gravel

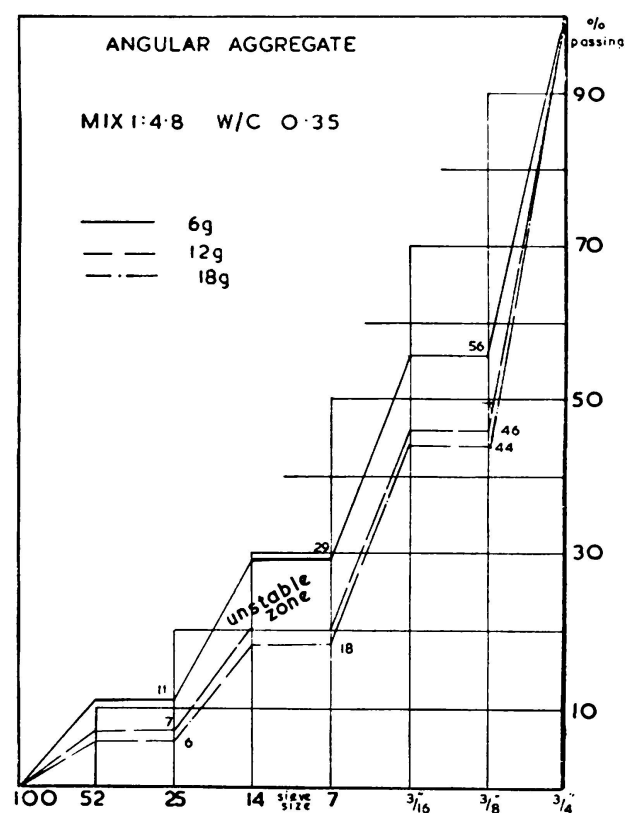


FIG. 4. Limit curve for rotational instability at accelerations up to 18g with gap graded angular granite aggregate

other parts, in practice this, it will be agreed, is inevitable. This variation of acceleration results in a couple tending to rotate the concrete mix about a horizontal axis. If rotation does take place one part of the plastic concrete rises above the level of the remainder and a depression occurs diametrically opposite. The concrete rotates at a rate of several revolutions per minute sucking in air at the depression and forcing air hubbles into the body of the casting; compaction is never achieved and this may result in the reduction of strength by 50 % without any surface indication on the resulting casting. This phenomenon is also caused by unsuitable grading, thus the grading of a vibrated mix should be considered more closely than that for hand placed concrete. Besides considering ultimate density its internal frictional resistance must be high to eliminate Rotational instability; this may be accomplished by using the minimum quantity of fine sand i. e. that passing a N° 52 B. S. sieve.

It has frequently been claimed as an advantage of post tensioned concrete that every component is tested whilst stressing takes place. This is only partly true since only if the factor of safety of the concrete is reduced below unity does failure take place. If the strength is reduced, as it may well be, by 30 % by any of the foregoing then the stress factor of safety may be reduced from say 3 to 2 and test cubes or cylinders will not indicate such reductions since they are small enough for full compaction to be achieved under all save the most extreme conditions of instability or segregation.

The difficulties have now been described. What are the remedies?

- I. Ensure that adequate power for vibration is applied to obtain an acceleration at any point in a casting greater than 3 g. A rough guide to the power required is to allow 1 watt per pound of concrete plus the energy required to vibrate the table mould etc. without any concrete (4).
- II. Vibration must be continued for a sufficient time for full compaction of the whole casting. Say 120 % of the time at which the emission of bubbles from the surface ceases.
- III. Design the concrete mix for vibration using fig. 1, 2, 3, or 4.

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#### SUMMARY

This paper considers the problems encountered in the manufacture of precast units for post tensioned bridge members of high strength with the aid of vibration. The desiderata are stated together with difficulties and the means of overcoming them to achieve the desired results.

**ZUSAMMENFASSUNG**

Die Arbeit behandelt die Probleme bei der Herstellung von vibrierten Fertigteilen, die später zu Brückenteilen zusammengespannt werden. Die auftretenden Schwierigkeiten sowie die Möglichkeiten zu deren Überwindung und somit zur Erreichung der gewünschten Ergebnisse werden beschrieben.

**R E S U M O**

O autor considera os problemas relativos à fabricação, com o emprego de vibração, de elementos préfabricados para vigas de pontes de grande resistência preesforçadas no local. Indica também as condições a que devem obedecer, bem como as dificuldades, e meios de as resolver, de modo a obter os resultados procurados.

**R É S U M É**

L'auteur considère les problèmes posés par la fabrication, à l'aide de la vibration, d'éléments préfabriqués pour poutres de ponts précontraintes sur place. Il décrit les desiderata ainsi que les difficultés, et les moyens de les résoudre, pour obtenir les résultats recherchés.

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## **Vlb1**

### **Losses of cable force at prestressing**

#### **Spannungverluste in den Kabeln bei der Vorspannung**

#### **Perdas de tensão nos cabos de uma estrutura em betão preesforçado**

#### **Pertes de tension dans les câbles d'un ouvrage en béton précontraint**

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One of the most important conditions for calculating prestressed concrete is to have a good knowledge of the different forces which are conveyed into the prestressing cable. To form an opinion of the future existence of the construction it is also necessary to know in which way the forces are going to change after some time.

The cable force at prestressing, checked on the jack or calculated through the extension of the cable is only very exceptionally constant along the length of the cable but changes because of losses of different nature. These losses can be divided as follows: jack losses, losses along the length of the cable, and losses when, and after anchoring the cable. A more accurate division is shown in table 1.

Losses  $c$ ,  $\mu$  and  $k$  are usually measured by pulling a cable with a jack at one end and checking the force transmitted to the other end. This can be also done by simultaneously measuring the force and the resulting extension of the cable at one end for one loading and unloading cycle. The result given by the first measurement is the total loss, while the second one gives a relation between the final force after losses have taken place and this force's distribution along the cable. These results give an approximate idea of the value of the area of force x cable length. For the determination of the constants for the different losses, this method is scarcely suitable except for cables with comparatively sharp curves. A combination of the different methods will of course give the best results.

TABLE 1

| Losses of cable forces in prestressed concrete                                 |                       |
|--------------------------------------------------------------------------------|-----------------------|
| Jack losses ... ..                                                             | $C = C_1 + C_2 + C_3$ |
| Friction in the jack ... ..                                                    | $C_1$                 |
| Friction at the thread angle for attaching to jack. On the jack ... ..         | $C_2$                 |
| Friction at the thread angle for attaching to jack. In the anchorage ... ..    | $C_3$                 |
| Friction losses due to the cable's line shape...                               | $\mu$                 |
| Losses due to deformation of the cable tubes...                                | $k$                   |
| Friction caused by unintended wave shape                                       | $k_1$                 |
| Adhesion in a compressed tube ... ..                                           | $k_2$                 |
| Adhesion or increased friction due to leaking of concrete into the tube ... .. | $k_3$                 |
| Losses due to cable slip at anchoring ... ..                                   | $\delta_1$            |
| Losses after the anchoring... ..                                               | $\Delta\sigma$        |
| Creep in the cable steel ... ..                                                | $\Delta_1\sigma$      |
| Shrinkage and creeping of concrete ... ..                                      | $\Delta_{2,3}\sigma$  |
| Temporary variations such as changes in temperature and load ... ..            | $\Delta_{4,5}\sigma$  |

***The theory of friction losses at cable curves.***

The calculation of friction losses is normally based upon the rope theory. In average structures, curve radii are large enough to justify complete disregard of the effect of the stiffness of prestressed reinforcement.

When a rope is pulled over a curve and bent with the contact angle  $\alpha$ , the traction  $Z$  is reduced owing to the friction between the rope and the slab (Fig. 1). The contact stress is  $p = \frac{Z}{r}$  for a radius  $r$ . If the friction coefficient is  $\mu$ , then the traction  $Z$  is reduced of  $dZ = \mu p r \cdot d\alpha$  along the length  $r \cdot d\alpha$

$$\begin{aligned} \therefore dZ &= \mu p r \cdot d\alpha = Z \cdot \mu \cdot d\alpha \\ \therefore Z &= Z_1 \cdot e^{-\mu\alpha} \end{aligned} \quad (1)$$

if  $\alpha$  is measured from the end 1, where the force is  $Z_1$ .

The variation of length, due to the loading and subsequent unloading to 0 tension of a curved prestressing cable may be calculated, by means of a simple integration, with this well known formula.



When *loading* from 0 to  $P_0$  the extension is

$$\delta = \int_0^\alpha \frac{Z}{EA} \cdot r \cdot d\alpha_x = \frac{Pl}{EA} \cdot \frac{1 - e^{-\mu\alpha}}{\mu\alpha} = \frac{Pl}{EA} \cdot \left[ 1 - \frac{\mu\alpha}{2} + \frac{(\mu\alpha)^2}{6} - \dots \right] \quad (2)$$

where  $\alpha$  is the total angle variation,  $\alpha_x$  is the variation up to an arbitrary point and  $l = r \alpha$ . The final form of this expression is obtained by developing  $e^{-\mu\alpha}$  in series.

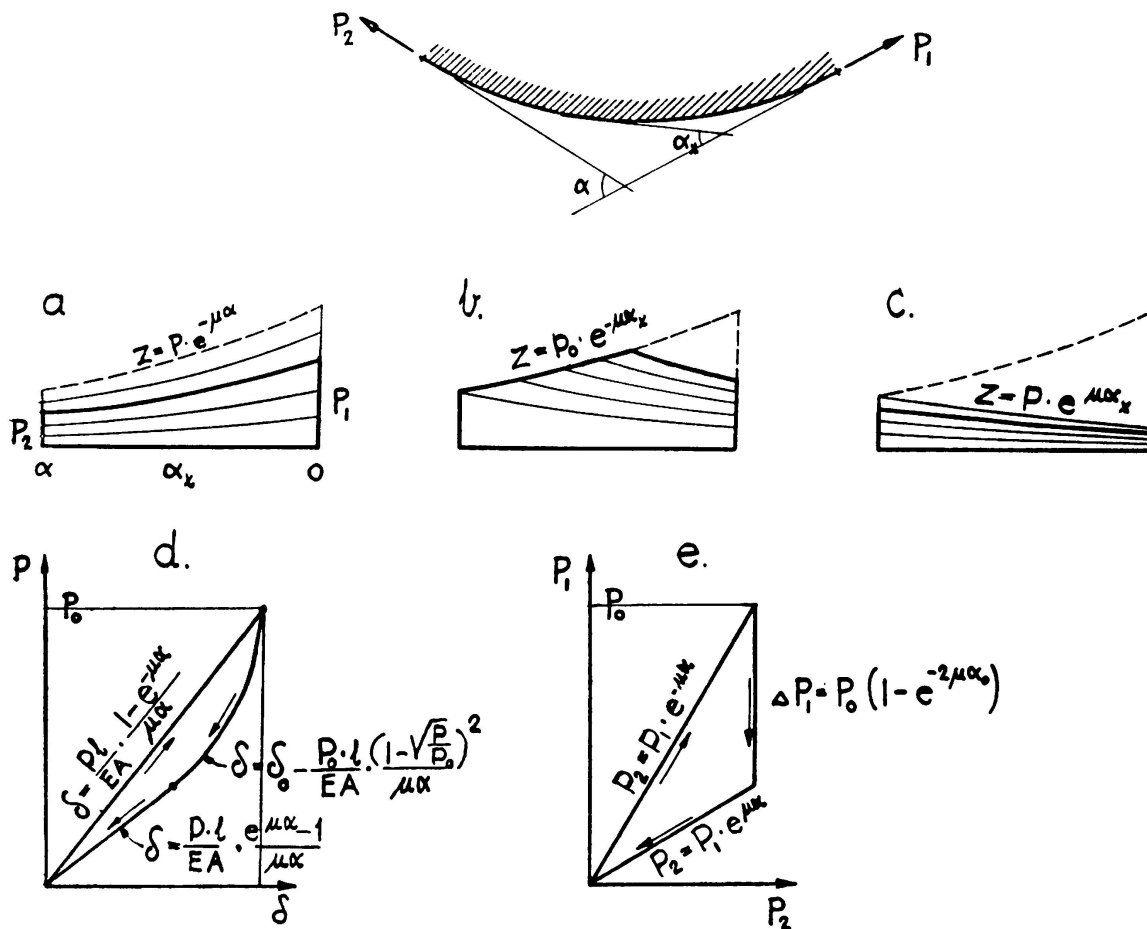


FIG. 1. The theory of friction losses at a cable curve. a) Variation of the cable stress along the length of contact. Rope friction. Increasing of the force  $P_1$  from 0 to  $P_0$ . b) Discharging from  $P_0$  to  $P_0 \cdot e^{-2\mu\alpha}$ . c) Further discharging from  $P_0 \cdot e^{-2\mu\alpha}$  to 0. d) Relation between stress and strain at the end of the cable curve for one loading and unloading cycle. e) Relation between the force  $P_1$  applied at one end of the cable curve and the force  $P_2$  acting at the other end, for one loading and unloading cycle.

When *unloading*, the theoretical sequence must be divided into two stages. The first one is when the force  $P$  is reduced from  $P_0$  to  $P_0 \cdot e^{-2\mu\alpha}$ , under which the force at the fixed end will not be reduced. The second stage, is when the force  $P$  is reduced from  $P_0 \cdot e^{-2\mu\alpha}$  to 0, the reduction at the fixed end being proportional.

Calling  $\delta_0$  the extension due to  $P_0$ , the residual extension after the first stage of the unloading is:

$$\delta = \delta_0 - \frac{Pl}{EA} \left( 1 - \sqrt{\frac{P}{P_0}} \right)^2 = \delta_0 - \Delta\delta \quad (3)$$

When changing over to the second unloading stage, i. e. when  $P = P_0 \cdot e^{-2\mu\alpha}$ , the negative term reaches its maximum value

$$\Delta\delta_\alpha = \delta_0 (1 - e^{-\mu\alpha}) \quad (4)$$

Along the second unloading stage, i. e. from  $P_0 \cdot e^{-2\mu\alpha}$  to 0, the residual extension is:

$$\delta = \frac{Pl}{EA} \cdot \frac{e^{\mu\alpha} - 1}{\mu\alpha} = \frac{Pl}{EA} \cdot \left[ 1 + \frac{\mu\alpha}{2} + \frac{(\mu\alpha)^2}{6} + \dots \right] \quad (5)$$

At the change-over point between the two unloading stages the  $\delta$  curves are tangent to each other and the residual extensions is  $\delta = \delta_0 \cdot e^{-\mu\alpha}$

It will be noted that, in all the formulae for  $\delta$ , either  $r$  or  $l = r \cdot \alpha$  are found, besides  $\alpha$ . From this it is seen that the variation of length cannot, as is the case for the friction loss, be independent of the curve's shape.

The above expressions giving a relation between  $P$  and  $\delta$  for one loading and unloading sequence are shown in Fig. 1 - d; Fig. 1 - e shows the relation between the external force  $P_1$  and the force  $P_2$  at the fixed end.

To illustrate the theoretical curves, results of earlier experiments [4] carried out during 1951 and 1952 according to a program drawn up by me, are also shown. This example has been chosen because the prestressing cable, in this case, is curved practically along its whole length and the shape, consequently, corresponds very nearly to the above mentioned theoretical curves.

Fig. 2 shows the dimensions and shape of the prestressing cable and its stress-strain diagram; its main interest lies in the fact that it also shows the measured results under symmetrical as well as one sided loading cycles. Under the measured curves are the corresponding theoretically calculated ones for different values of  $\mu$ . The likely value for  $\mu$  is, of course, the one which gives the best resemblance between the measured and calculated curves. The theoretical curves are calculated with a probable jack loss of 3 %, i. e.  $c = 0.97$ . Experiments made with the press in KTH's Laboratory for Building Statics, have shown losses as small as 1 % for new jacks and as high as 8 % for jacks that have been in use. Experiments at the site, extending two jacks against each other, have shown losses from 1 to 6 %. It follows that 3 % must probably be a reasonable average value.

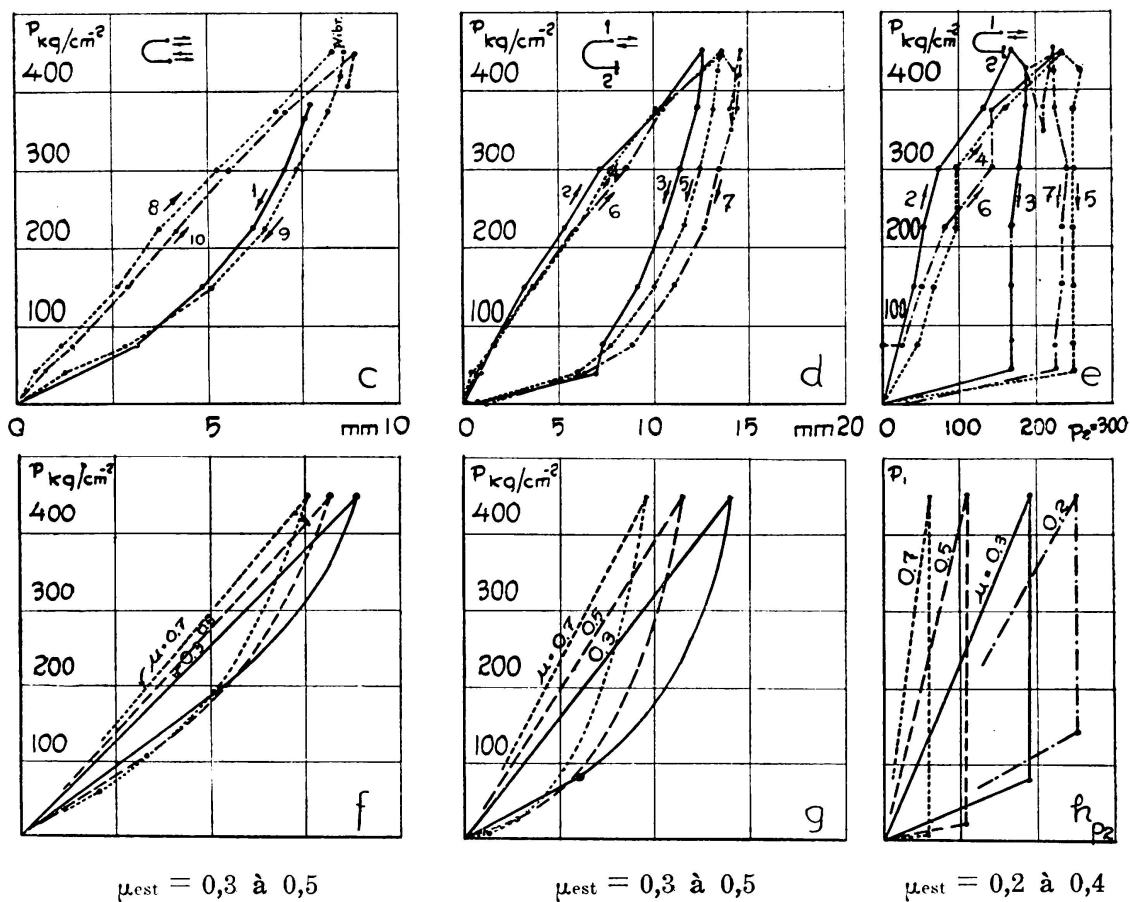
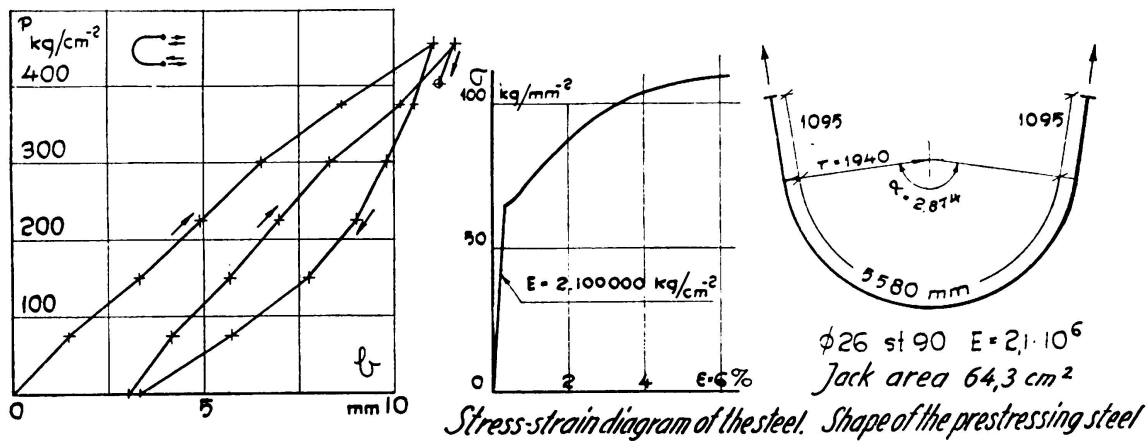


FIG. 2. Results of tests with prestressed bars of almost semicircular shape (the Fig. is from the article Ref. 4). b)  $P$ - $\delta$ -diagram at the first loading and subsequent unloading and reloading. Symmetrical tension from both ends. c) Repeated loading and unloading according to b. d) Repeated loading and unloading at tension from one end. e)  $P_1$ - $P_2$ -diagram corresponding to d. f, g, h) Theoretical curves corresponding to the measured c, d, e.  $\mu_{est}$  is estimated by comparison between the figures for various presumed  $\mu$  values.

Any loss  $k$  due to unintentional deformation of the cable tubes was in this case, not separately determined but related to  $\mu$ . The length being small and the steel in the tube stiff, the effect of  $k$  ought to be small.

It may be observed in Fig. 2 - b that the first loading gives a far greater extension than the later cycles. This can partly be explained by the fact that in this case, due to the small bending radius, the cable tubes are likely to buckle at the bend causing the formation of folds. When the cable is stretched these folds are pressed down and this deformation appears as an extension of the cable. To a certain extent it is, however, a real extension presumably owing to the fact that the cable at the bend has been loaded above the flow limit, and a levelling by sheer traction gives a greater stretching than normal. The curve corresponding to the first loading is consequently markedly set apart from the following cycles. These also show, however, considerable spreading, which indicates that the value of  $\mu$  is not wholly fixed. Any marked reduction of  $\mu$  as the number of loading cycles increases is hardly shown although such a reduction seems obvious.

*Experimental results observed during the construction of two bridges.*

In 1953 and 1954 two bridges were built for the Port of Gothenburg, over the Rosenlund canal, and a number of measurements were made to determine the prestressing losses and the deformations. Both bridges are freely-supported beam bridges and the main beams are prestressed according to Freyssinet's system, (see Fig. 3).

At the first bridge the measurements were made by determining the force and corresponding cable extension at one cable end only. The determination of the constants gave the results [4]  $\mu = 0,5$ ,  $c = 0,90$  and  $k = 65 \text{ kg/m}$  for 26 ton or  $k = 0,003^{1/m}$ , which  $k$  corresponds to an imagined change of direction of  $1/2^\circ$  per m. Judging among other things, from the shape of the prestressing bars selected for the investigation and from the spreading of the test results, it seems that the sharing of the losses amongst those represented by  $c$  and by  $k$  is not very accurate.

During the tests carried out in 1954 in connection with the construction of the other twin bridge, the steel cables were therefore especially selected for tes-

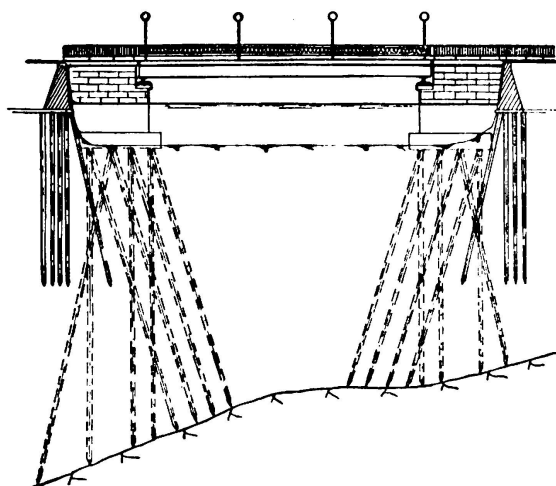


FIG. 3. Bridge over Rosenlund canal, Gothenburg. Span 18,5 m.

ting so that the resulting measurements should allow a separate determination of the different influences. The measurements were, again, carried out so as to include also the determination of the force at the fixed cable end.

Fig. 1 shows the variation curve of the force along a continuous cable bend for a loading and unloading cycle. The shape of the interesting cables in the bridge is, however, the one shown in Fig. 4. By combination

of curve parts similar to those of Fig. 1, it is possible to obtain the curves of Fig. 4.

#### Jack losses.

Jack losses are measured from the extension of a free cable pulled at both ends through anchor blocks. This system simultaneously allows a control of the modulus of elasticity  $E$ .

The jack loss for a Freyssinet-jack is composed of an internal loss due to the relative movements of its different parts and of losses due

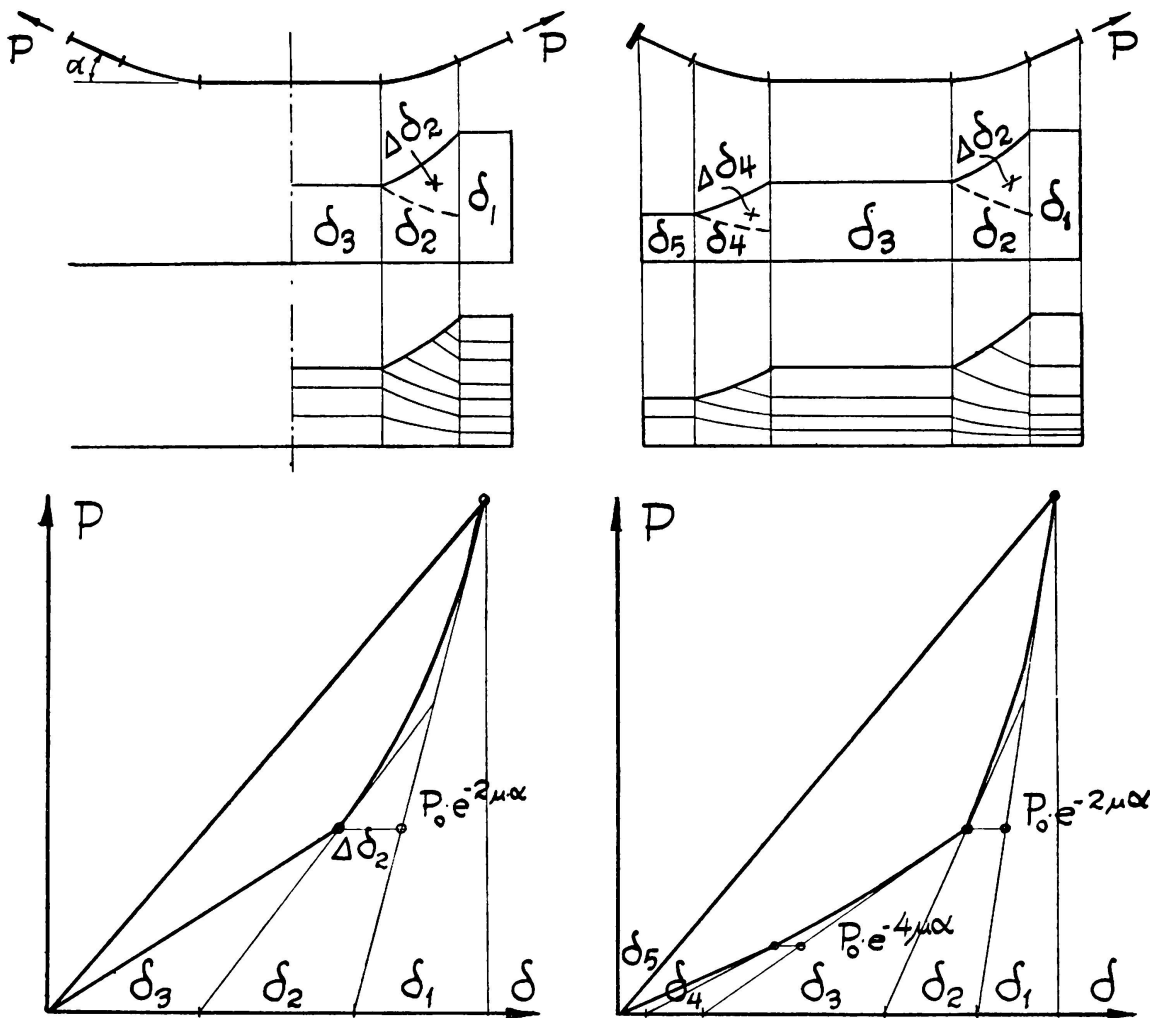


FIG. 4. Curves corresponding to fig. 1 d for cables with straight middle parts.

to the bending of the threads, both at the jack and at the re-bending in the anchor block. Fig. 4 (right) may also show the stretching of a free cable, if it is assumed that the total jack losses are represented by one cable bend.

Fig. 5 - a shows measurement curves for two loading and unloading cycles, jack 1 being the only one operating. Similar curves are obtained

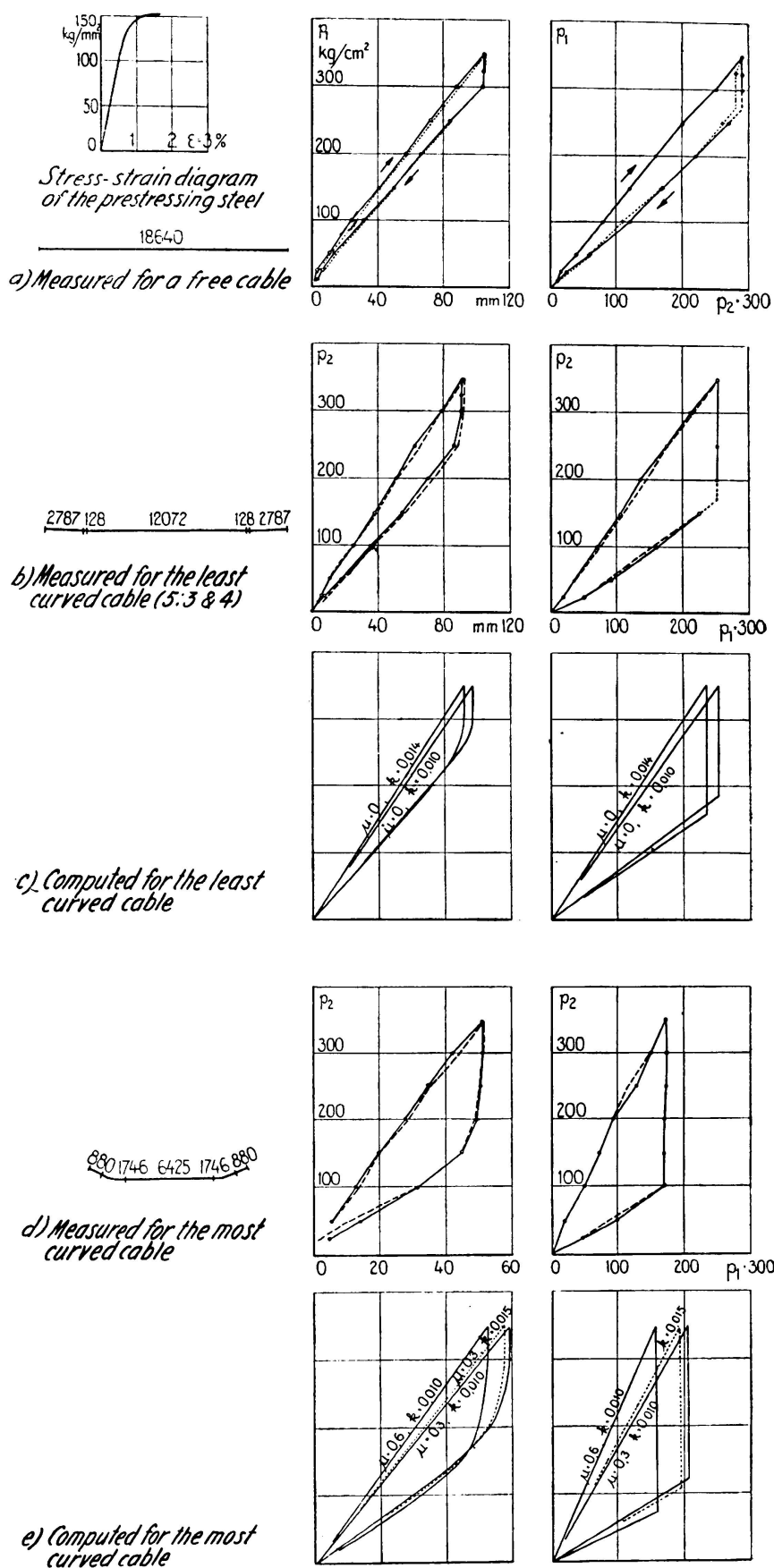


FIG. 5. Measured and theoretically computed  $p-\delta$  and  $p_1-p_2$  curves of loading and unloading for the different cables.

a) Measured for a free cable.

b) Measured for the least curved cable (5:3 and 4).

c) Computed for the least curved cable.

d) Measured for the most curved cable (5:16 and 13).

e) Computed for the most curved cable.

for jack 2. The cables consist of 12  $\phi$  5,0 mm and the jack's piston area is 81,07 cm<sup>2</sup>.

The curves show fairly regular readings. For  $p_1 = 350$  at the measured values for both experiments is  $p_2 = 290$  at, the jack loss in both cases should therefore be  $\sqrt{290:350} = 0,910$  or 9,0 %. When unloading, the value of  $p_2$  changes first when  $p_1$  is reduced respectively to 272 and 275, and the corresponding losses are  $\sqrt{272:290} = 0,968$  and  $\sqrt{275:280} = 0,991$ . More satisfactory values of  $\sqrt[4]{272:350} = 0,939$  and  $\sqrt[4]{275:350} = 0,941$ , are reached if the total jack force, necessary to change the force at the fixed end, is considered for the calculation. The following values are obtained from the diagram corresponding to jack 2,

|            |                             |       |
|------------|-----------------------------|-------|
| loading:   | $\sqrt{310:350} = 0,942$    | 5,8 % |
|            | $\sqrt{305:350} = 0,933$    | 6,7 % |
| unloading: | $\sqrt{270:310} = 0,933$    | 6,7 % |
|            | $\sqrt{262:305} = 0,927$    | 7,3 % |
| total:     | $\sqrt[4]{270:350} = 0,935$ | 6,5 % |
|            | $\sqrt{262:750} = 0,930$    | 7,0 % |

If all the 12 values are used in order to calculate the average value and mean error, the loss is

$$1 - c = 6,2 \% \pm 0,6 \%$$

Losses can also be determined from the extension diagram shown in Fig. 5. The vertical drop between both curves represents very accurately the double jack loss, as the extension depends mainly on the extension of the straight part of the cable.

The correction, which depends on the camber of the unloading curve, is slight as the camber is only a function of the extension of the jack sections represented in Fig. 4 by  $\delta_1 + \delta_2$  and  $\delta_4 + \delta_5$ . The greatest of these values, i. e.  $\delta_1 + \delta_2$  is only about

$$0,96 \cdot 350 \cdot 81,07 \cdot \frac{665}{2,1 \cdot 10^6 \cdot 2,35} = 3,85 \text{ mm},$$

while the extension of the straight part corresponding to  $\delta_3$  is approximately 100 mm.

The values obtained are

$$c = \sqrt{304:350} = 0,933 \quad 6,7 \%$$

$$\sqrt{305:350} = 0,935 \quad 6,5 \%$$

The values obtained from the diagram corresponding to jack 2 are:

$$c = \sqrt{293:350} = 0,916 \quad 8,4 \%$$

$$\sqrt{298:350} = 0,923 \quad 7,7 \%$$



The resulting mean value is 7,3 %, which is fairly larger than the one obtained from the proportion between the forces. In the subsequent calculations the assumed likely value is 7,0 %.

A control of E gives in the first experiment

$$E = \frac{P_1}{\delta A} = \frac{0,93 \cdot 81,07 \cdot 350 \cdot 1864,0}{(10,8 - 0,735) \cdot 2,35} = 2\,070\,000 \text{ kg/cm}^2$$

and the corresponding average value is  $E = 2,05 \cdot 10^6$ .

*Friction losses owing to unintended wave-shaped cable tubes.*

The loss along the cable is assumed to be  $e^{-(\mu\alpha + kx)}$ , where the exponent's first term is the normal loss due to friction in the curves of the cable. The other term is assumed to depend on the fact that the reinforcement is involuntarily wave-shaped while being erected. The stays cannot be made very tight and, while vibrating the concrete for instance, the cables may happen to be further displaced.  $k$  is calculated here as a constant along the length of the cable  $x$ .  $k$  could be assumed to be smaller in the bends, where the stays are usually tighter; however, the risk of leakage is perhaps somewhat greater in curves and  $k$  in that case ought to be larger.

Experiments have been carried out with the least bent cable for two loading and unloading cycles with  $p_1$  in operation; similar experiments have also been carried out with  $p_2$  in operation. Fig. 5-b shows two average cycles.

The diagram  $p_1 - p_2$  gives the proportion between these forces when loading and unloading as well as the drop corresponding to the change of loading. By comparing Fig. 1 with Fig. 4 it appears that the proportion is  $c^2 \cdot e^{-2\mu\alpha} \cdot e^{-kl}$  and the drop the square of this expression. Introducing the different points of the diagram the result will be: loading  $242:350 = 0,720$ ; unloading  $168:250 = 0,672$ ; and from the drop  $\sqrt{168:350} = 0,693$ .

A combination of the results also taken from the other experiments gives the following values:

| loading        | unloading      | drop           |
|----------------|----------------|----------------|
| 0,685<br>0,657 | 0,768          | 0,707          |
| 0,720<br>0,720 | 0,672<br>0,672 | 0,693<br>0,693 |
| 0,700<br>0,685 | 0,833<br>0,833 | 0,755<br>0,755 |

If all values are given the same weight, the mean value is  $0,723 \pm 0,012$ .

If the values of the known magnitudes are  $c = 0,93$ ,  $\alpha = 1^\circ 50' = 0,032$  radius, and  $l = 17,9$  m, and if  $\mu$  is assumed to have probable

limit values, then, for  $\mu = 0$ , the value  $k = 0,010$  and for  $\mu = 0,6$  the value  $k = 0,008$ . The influence of  $\mu$  in this almost straight cable is thus relatively small and it may be assumed that the likely value of the losses due to the cable's wave shape is

$$k = 0,009 \text{ } 1/\text{m}$$

The theoretical curves in Fig. 5 - c, also show that a value of about  $k = 0,01$  is in agreement with the  $p - \delta$  curves.

**Friction losses owing to the curves of the cable.**

For the most bent cable where the factor  $e^{-\mu\alpha}$  dominates, measurements have been carried out, similar to those already described for the least bent one. Two average cycles are shown in Fig. 5-d.

A combination of all the values of  $p_1/p_2$  as given by the diagram of  $p_1 - p_2$  gives the following results:

| loading | unloading | drop    |
|---------|-----------|---------|
| 0,528   | (0,303)   | (0,378) |
| 0,528   | 0,572     | 0,534   |
| 0,494   | 0,588     | 0,534   |
| 0,500   | 0,588     | 0,534   |
| 0,528   | 0,572     | 0,534   |
| 0,542   | 0,572     | 0,534   |

If both values within brackets are excluded, the average value is  $0,542 \pm 0,007$ . This corresponds to  $c^2 \cdot e^{-2\mu\alpha} \cdot e^{-kl}$ ; introducing  $c = 0,93$ ,  $\alpha = 25^\circ = 0,436$  radius,  $k = 0,009$ , and  $l = 11,677$  m in the expression, a value  $\mu = 0,415$  is obtained.

The theoretical curves in Fig. 5-e also show that the pair of values  $k = 0,010$ ,  $\mu = 0,40$  to  $0,45$  are in agreement with the  $p - \delta$  curves. It should be noted that for instance  $k = 0,020$ ,  $\mu = 0,35$  would also be satisfactory but would not suit the straightest cable where even for  $\mu = 0$ , it would be hardly possible to assume a value higher than  $k = 0,015$ .

Regarding the calculation of the friction losses, it should be noted that the  $p_1 - p_2$  diagrams give the total force loss. Calculations based on the rope equation give, in this case, very satisfactory values for different angles. The shape of the  $p - \delta$  diagrams give a certain amount of control as to whether or not the distribution along the length of the cable really follows the rope equation. The correspondence of the shapes of the measured curves and the theoretically calculated ones is quite satisfactory. Both experiments described above — one with Dywidag-steel as shown on Fig. 2 and one with Freyssinet-cables as shown on Fig. 5 — give, however, measured diagrams somewhat more «full»

than the calculated ones. The fact that  $\mu$  at unloading is larger than under normal conditions as long as the load is high possibly explains this difference. For a smaller load, the  $p_1 - p_2$  diagrams show that the value of  $\mu$  is normal. It may not be entirely impossible to imagine that the particles, pressed hard against the cable tubes, are directed in the direction of the stress. The friction would then have to be quite large to make them return to their original position as long as stress pressure was high.

**Cable slip at the anchoring.**

When the jack is released and the cable force is transferred to the anchoring, there is a certain slipping of the cable in relation to the anchor block. This will cause a certain reduction of the force in the cable but, as can be seen from Fig. 1 and 4, this generally occurs only

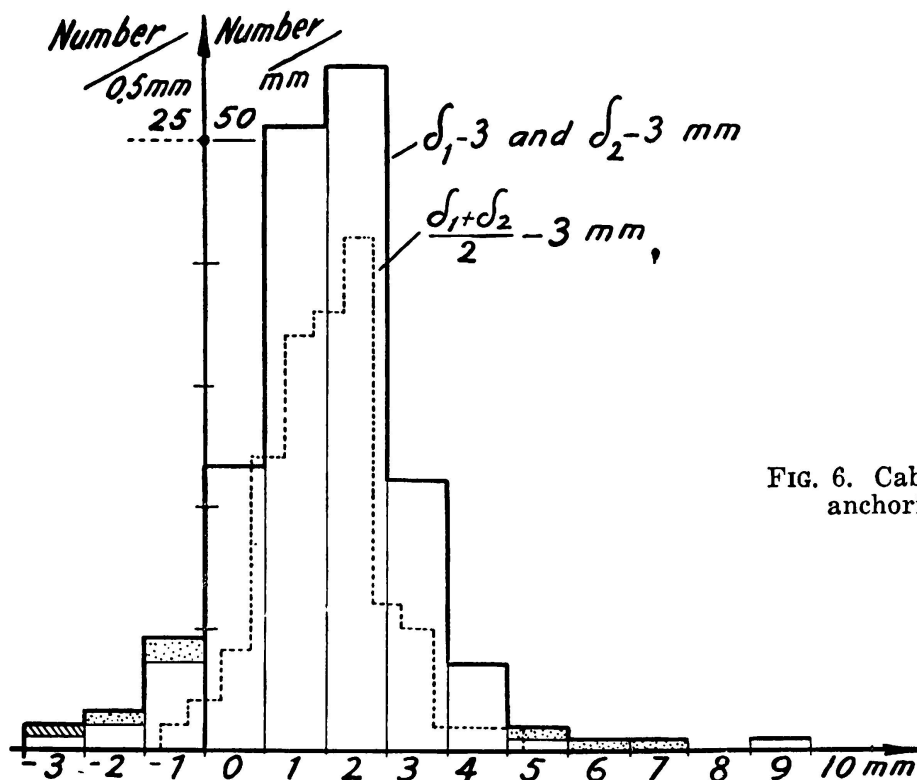


FIG. 6. Cable slip at anchorings.

at the ends. It is however possible, due to the injections for instance, for a certain adjustment of the different forces to occur.

The lock slipping being rather small, as is shown below, its effect is only noticeable in very short cables.

The lock slipping at both ends was measured in both bridges for each one of the 180 cables. The results of these 360 measurements are shown in Fig. 6. In this case the displacement of a point on the cable at a distance of 60 cm from the external surface of the concrete was

measured and reduced by 3 mm corresponding to a shortening of 3,0 to 3,5 mm of the projecting cable end while unloading.

Some results seem to indicate a negative lock slipping. If, however, it is remarked that, in some of the cases, the sliding at the other end of the cable is unusually large, it is obvious that a certain movement of the cable over to one side has taken place. Such pairs of values are marked with dotted areas in the diagram. To eliminate the influence of such effects, the mean value of the readings at both ends of each cable was considered. The corresponding results have been indicated in Fig. 6 with a dotted line. It may be seen that the lock slip is usually 1 or 2 mm. Only in a few cases does this value raise above 3 mm. The mean value is approximately  $\delta_1 = 1,5$  mm or probably even less if it is considered that the extension of the projecting cable is in some cases as high as 3,5 mm, while for the diagram it is assumed to be 3,0 mm.

#### *Measurement of permanent set.*

There is a further alteration of the force in the cable after the anchoring has been carried out, which is principally due to the shrinkage and creep (plastic flow) of the concrete. This is the reason why it is necessary to use relatively high tensile steel with high stresses in order to have sufficient effective stress left, even after the permanent set of the concrete has taken place.

In order to check these losses, the variation of length and bending of the bridge were measured. The measuring began at the same time as the bridge was being prestressed and the resulting diagrams in Fig. 7, show how closely the upward bending follows the stretching of the cables. After a few months, part of the permanent set has already taken place. The diagram also shows that the measured values depend, to some extent upon air temperature changes and upon rain, both of which in combination can give rapid cooling.

Only the main lines of the calculation principle of these deformations are given here. From these deformations it is then possible to deduce the prestressing losses. Secondary influences such as the reduction of the creep due to the decrease of stress in the steel or the influence of the reinforcement upon the shrinkage, are not treated.

#### *Shrinkage and creeping of the concrete.*

I have chosen to base the following calculation of the shrinkage and creep of the concrete upon the calculations I had the pleasure of carrying out for I. Häggbom as a complement to his description [5], at the Congress in 1948, of the large concrete arch of Sandö-bridge in Sweden. At the time the transformation of the results obtained from testing upon concrete prisms in the laboratory to the actual arch with its box section of concrete slabs, was undertaken following an idea brought forward in an article by R. W. Carlson [6]. In it he points out the possibility of using an equation corresponding to the heat equation

in order to judge time effect upon shrinkage of concrete objects of different size and form.

The same equation is also valid for electric fields and diffusion of gases; Terzaghi and Fröhlich [7, 8] for instance have also used it for the

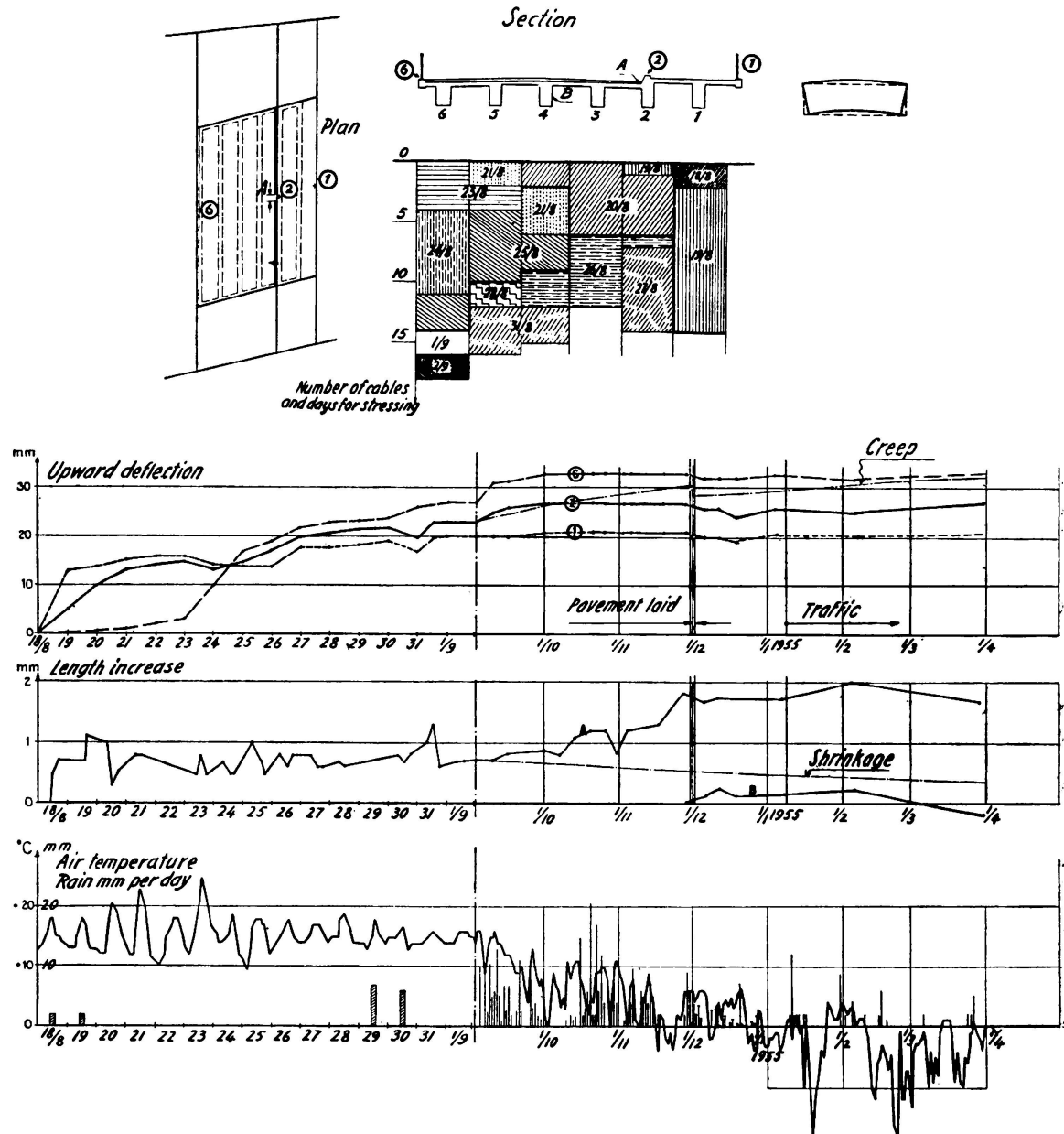


FIG. 7. Days for prestressing of the cables. Deformation-time diagram.

calculation of void water streaming in clay and for the subsequent calculation of the corresponding settling. Because of the strong resemblance existing between that problem and the one treated herewith, the equation is taken in the form given by those authors for parallel plane clay layers.

$$\frac{k}{v \gamma} \cdot \frac{\partial^2 w}{\partial z^2} = \frac{\partial w}{\partial t} \quad (6)$$

Here,  $w$  is the hydrostatic overpressure at a point with coordinate  $Z$ , at the time  $t$ .  $k$  is the water perviousness,  $v$  is the specific void water loss, and  $\gamma$  is the volume weight.

For three-dimensional problems,  $\frac{\partial^2 w}{\partial z^2}$  has to be changed for the differential parameter of Laplace,  $\Delta^2 w$ .

The equation is dealt with in detail in different treatises [9, 10] on heat, and reference is only made here to the simple solution valid for hydrostatic pressure which is sinus-shaped at time  $t_0 = 0$ . After time  $t$  has elapsed, it has been uniformly diminished by means of the multiplier  $e^{-\beta T}$ , where  $\beta$  is a constant and  $T = at/d^2$  (7)

in which  $d$  is the thickness of the layer and  $a = \frac{k}{v\gamma}$ . Using the same symbol  $\mu$  for the time function of settlement, as in the above-mentioned paper [5], one has  $\mu = 1 - e^{-\beta T}$ .

If the original hydrostatic pressure has a form other than the simple sinus-shape, it can, for instance, be expressed by means of Fourier series, and the solution can be written in a generalized form:

$$\mu = 1 - \sum A_n \cdot e^{-\beta_n \cdot T} \quad (8)$$

If instead of a slab one has a long prism, this could then be mathematically treated as being composed of two slabs, one for each direction and, in the case of a square prism the solution is

$$\mu^{pr} = 1 - (\sum A_n \cdot e^{-\beta_n \cdot T})^2 \quad (9)$$

The curves for shrinkage  $K$  and creep  $f$ , which were measured for the prisms at Sandö and are intended to be used here as it is plausible that the concrete is similar, are given in Fig. 13 of the above-mentioned article [5]. They are easy to express approximately, by means of equation (9):

$$\mu_K^{pr} = 1 - e^{-8\beta T}, \quad \mu_f^{pr} = 1 - \frac{2}{3} \cdot e^{-\beta T} - \frac{1}{3} \cdot e^{-32\beta T} \quad (10, 11)$$

With  $t$  in years,  $h$  in cm and, if  $a$  is a constant for the concrete used, then constant  $\beta \cdot a = 138 \text{ cm}^2 \text{ year}$ .

This expression is easily made valid for the case of a slab by means of the relation between equations (9) and (8):

$$\mu_K^s = 1 - e^{-4\beta T}, \quad \mu_f^s \approx 1 - \frac{4}{5} \cdot e^{-\beta/2 \cdot T} - \frac{1}{5} \cdot e^{-32\beta T} \quad (10-a, 11-a)$$

From this and assuming the shrinkage to be

$$K = \frac{100-R}{100} \cdot k \cdot \mu_K, \text{ and the creep to be } f = \sigma \cdot F \cdot \mu_1 \quad (12, 13)$$

where  $R$  = relative humidity,  $k = 0,88 \cdot 10^{-3}$ ,  $\sigma$  = stress in  $\text{kg/cm}^2$  and  $F = 0,94 \cdot 10^{-5}$ , the sinking the aforesaid bridge crown was calculated. In the the above-mentioned paper [5] it was seen that calculated and measured sinking were almost exactly the same, 17 cm, 5 years after the crown was built. This corresponds to a theoretical sinking of 28 cm counted from the time, some months earlier, when the watering was ceased and the load brought on. After another 5 years the measured value was 19 cm and the corresponding calculated value was 20 cm. The difference is relatively still less when the theoretical total values 30 and 31 cm are compared. It may also be seen that the values are high up in an extrapolated, and thus very uncertain, part of the  $\mu_{pr}$ -curve.

This extremely good agreement is probably due to a great extent, to the fact that, in this case proof-objects and actual construction have rather similar cross dimensions and about the same load and variation of relative humidity. For more than 20 years it has been pointed out [11] that shrinkage and creep should not be calculated totally independently; this is possibly connected with the very curved shape of the elasticity curve of concrete. It has also been pointed out by G. Pickett [12], that surface resistance should not be neglected in diffusion or heat equations. The above deduction, where no surface resistance has been taken into account, corresponds to a Pickett's factor  $B = \infty$ . As a matter of fact,  $B = 2$  should give a little better agreement if the «diffusion constant» is similarly altered. The difference is, however, extremely slight and it seems possible to choose any value  $B$  as long as the other constant is equivalently adopted. It therefore, seems necessary to take  $B$  into account only when comparing tests of various dimensions and loadings.

In the case of the Pustervik bridge, the simplified equations (12) and (13), with time functions (10-a) and (11-a), are applied independently of each other for the calculation of camber and length variation.

The time function must be altered for application to bending. In this case, the parts nearest to the surface must, of course, play a most significant part and the creep function for a bended slab will be rather similar to the one for a prism with centric pressure. An estimated curve for the upward bending due to creep where the irregular shape of the slab and the beam were considered, is drawn on Fig. 7 as a thin line.

The length variation at point A due to creep, is composed of compression with its  $\mu_r$ , and of bending with its faster increasing  $\mu_{rb}$ ; the shrinkage is to be added to both of these. In Fig. 7 only the shrinkage is drawn as a thin line merely in order to give an idea of its magnitude.

It may be seen that there is not as yet good agreement between these uncorrected calculated values and the measurings. The most important influence to be added is presumably the creep of the steel.



**Temperature variations.**

Temperature variations also have a great influence upon the measurements, in spite of the fact that most of the length variations due to temperature are eliminated by measuring with a steel bar placed under the paving, directly upon the concrete surface. The temperature differences ought therefore to be small and the length variations ought not to differ greatly.

In order to give at least a qualitative picture of the conditions for day and night variation of temperature, some diagrams are included, even though they are primarily established for concrete cisterns and dams where the temperature changes are rather large on one side and small on the other. They are, however, also valid for relative conditions between surface and centre of a rather thick concrete construction or, as in this case, for a bridge deck the upper surface of which is exposed to sun and rain, whilst the conditions underneath are more constant.

Fig. 8 shows the temperature conditions in a concrete wall, one surface of which is exposed to a sinus-shaped temperature change with the amplitude  $\mathcal{V}_{10}$  and the period  $T$ . For an infinitely thick wall the solution [9] is

$$\mathcal{V} = \mathcal{V}_{10} \cdot e^{-kx} \left( \cos \frac{2\pi t}{T} - kx \right), \quad k = \sqrt{\frac{\pi}{aT}} \quad (14)$$

Here the temperautre conductive power  $a$  is  $2 \cdot 10^{-3}$  to  $3,5 \cdot 10^{-3}$  for concrete. In Fig. 8, conditions corresponding to a wall with a thickness  $d$  such that  $kd = 4$  are shown; for day and night variation this corresponds, to a 0,5 to 0,6 m thick wall, which is enough for agreement with the above formula.

M. Ritter [13] has given a solution for the case of a wall with a smaller thickness and under the condition that it is the temperature of the surroundings which varies. From this I have calculated the diagrams in Fig. 9 and 10. They show length variations and bending of a slab for sinus-variations of temperature; they also show how the amplitude on the concrete surface is less than that of the surrounding air.

For the actual Pustervik-bridge, no detailed calculations have yet been concluded; it has nevertheless been noticed that the day and night movements, as measured and drawn in Fig. 7, agree in principle with what was to be expected from temperature variations and rain showers.

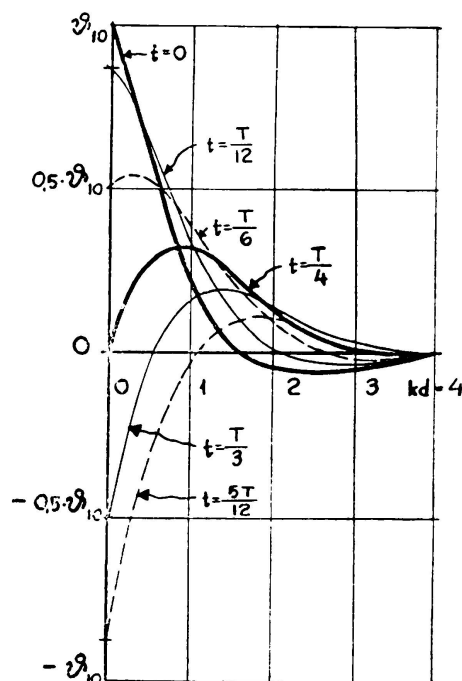


FIG. 8. Temperature conditions for variable surface temperature.

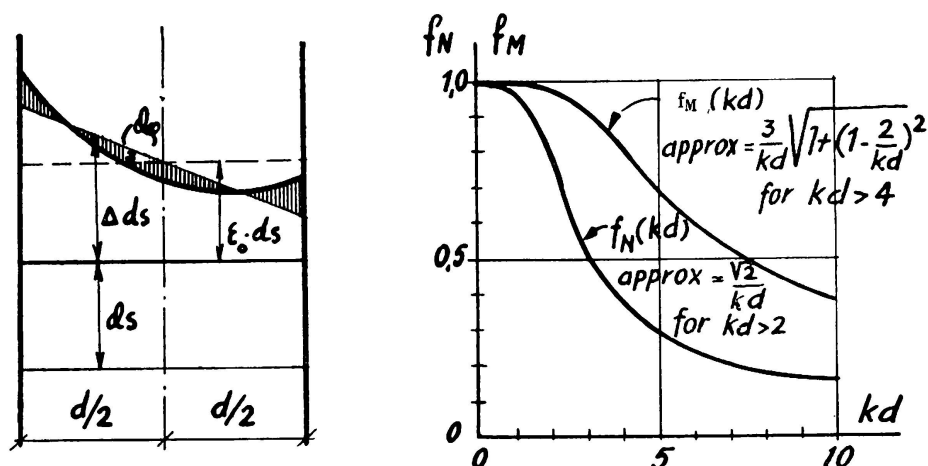


FIG. 9. Length variation and bending of a slab for variable surface temperature. Max. value of the mean temperature  $\vartheta_{mo} = \frac{\vartheta_{10} + \vartheta_{20}}{2}$ .  $f_N(kd)$

$$\text{Max. value of the bending } \left( \frac{d\varphi}{ds} \right) = \frac{\vartheta_{10} - \vartheta_{20}}{d} \cdot w \cdot f_M(kd)$$

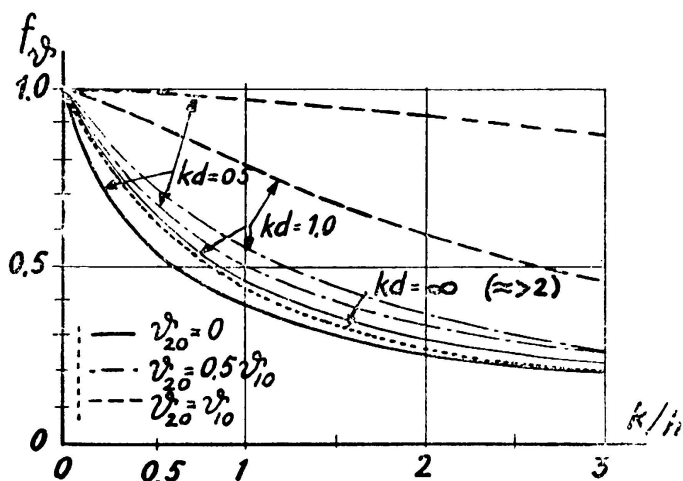


FIG. 10. Amplitude of the surface temperature  $\vartheta_{10}$  if the amplitude of the surrounding air is  $\vartheta_{ao}$

$$\vartheta_{10} = \vartheta_{ao} \cdot f_\nu \quad \text{Here } f_\nu = f_\nu \left( \frac{k}{h}, kd, \frac{\vartheta_{20}}{\vartheta_{10}} \right) \text{ where } h = \frac{\alpha}{\lambda}$$

and  $\alpha = 3$  at calm,  $= 10$  for gentle wind and  $= 500$  for rain and storm.

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### SUMMARY

Prestressing force losses have been calculated and measured for two prestressed concrete bridges in Gothenburg. The bridges were built according to the Freyssinet system. The measurements were carried out according to two methods:

— By applying a force with a jack at one end of the cable and measuring the force transmitted to the other end.

— By measuring both the force and corresponding extension at one end only, the other end being fixed.

The latter is particularly satisfactory for very curved cables and is applied when only one end is accessible. The combination of both these methods gives a certain control of the distribution of the prestressing force along the cable.

Results of these measurements showed jack losses to be of 6,5 to 7 %, which corresponds to  $c = 0,93$ . This not only includes jack losses proper, but also stretching losses at the bending of the threads, both at the jack and in the anchor block.

The friction loss in the cable curves corresponds to  $\mu = 0,42$  in the expression  $e^{-\mu\alpha}$ .

The friction losses owing to unintended wave-shaped cable tubes correspond to  $k = 0,009$  in the expression  $e^{-kx}$  where "x" is the measured length of the cable, in meters.

A previous measurement [4] had given approximately the same or a little higher  $\mu$  but a different ratio between "c" and "k".

The slip at anchoring was approximately 1,5 mm and exceeded only very occasionally 3,0 mm.

Measurement of permanent set is being carried on for the purpose of checking the calculation of the corresponding losses of prestressing force. Some basic equations for shrinkage and creep are given. The measurements for the hitherto rather short period of time show little agreement, but this probably is due, partly to certain subsidiary influences such as the irregular shape of the bridge and to the reinforcement, and mainly to the influence of the creeping of the prestressed steel.

## ZUSAMMENFASSUNG

Zwei nach dem System Freyssinet vorgespannte Betonbrücken in Göteborg dienten zur Untersuchung der Spannungsverluste. Die Durchführung der beiden Messungen erfolgte teils nach dem üblichen Verfahren, indem eine Kraft mit Hilfe einer Spannvorrichtung am einen Kabelende zur Wirkung gebracht und die übertragene Kraft am andern Ende bestimmt wurde, im andern Fall indem die Spannung und die entsprechende Dehnung nur an einem Ende unter Festhaltung des andern Endes gemessen wurde. Die letztere Methode ist besonders für stark gekrümmte Kabel angebracht und praktisch unumgänglich, wenn nur ein Ende erfasst werden soll. Bei der Kombination beider Verfahren kann die Spannungsverteilung längs des Kabels kontrolliert werden.

Aus den Untersuchungen ergab sich ein Spannpressenverlust von 6,5 bis 7 %, entsprechend einem  $c$ -Wert von 0,93. Dieser Verlust umfasst den eigentlichen innern Pressenverlust neben den Streck-Verlusten beim Umbiegen der Drähte am Spannschloss und im Verankerungsblock.

Der Reibungsverlust in den gekrümmten Kabeln entspricht einem  $\mu$  - Werte von 0,42 für den Ausdruck  $e - \mu x$ .

Die Reibungsverluste infolge unbeabsichtigter, unregelmässiger Form der Kabelrohre entsprechen einem  $k$  - Wert von 0,009 für den Ausdruck  $e^{-kx}$  wobei "x" die Kabellänge in m, der Kurve nach gemessen, bedeutet.

Eine frühere Messung [4] ergab angenähert gleiche oder etwas höhere  $\mu$  - Werte, aber eine andere Proportion " $c:k$ ".

Die Verschiebung bei der Verankerung betrug angenähert 1,5 mm und schlug nur gelegentlich bis zu 3 mm aus.

Die Messungen der bleibenden Formänderungen werden weitergeführt mit dem Zweck, die Berechnung der entsprechenden Verluste bei der Vorspannung nachzuprüfen. Einige Grundgleichungen für das Schrumpfen und das Kriechen liegen vor. Die Messungen während der bisherigen kurzen Zeitspanne stimmen mit den Berechnungen schlecht überein; der Grund ist wahrscheinlich bei den Nebeneinflüssen zu suchen, z. B. bei der unregelmässigen Form der Brücke und der Bewehrung, besonders aber beim Einfluss des Kriechens im vorgespannten Stahl.

## RESUMO

Mediram-se e calcularam-se as perdas de tensão em duas pontes de betão preesforçado situadas em Gotemburgo. Estas duas obras tinham sido executadas de acordo com sistema Freyssinet.

As medições foram efectuadas por dois métodos diferentes:

- Esticando os cabos com um macaco num dos extremos e medindo a tensão transmitida ao outro.
- Fixando um dos extremos e medindo no outro a força exercida e a extensão resultante.

Este último método dá resultados satisfatórios especialmente no caso de cabos fortemente encurvados e também se emprega quando só uma das extremidades é acessível. A combinação destes dois métodos permite controlar a distribuição da tensão ao longo do cabo.

Os resultados destas medições mostram que as perdas de tensão nos macacos são da ordem de 6,5 a 7 % o que corresponde a  $c = 0,93$ . Este valor compreende, além das perdas devidas aos próprios macacos, as que são causadas pela encurvadura dos cabos no macaco e no cunho de amarração.

As perdas por fricção, devidas à encurvadura do cabo, correspondem a  $\mu = 0,42$  na expressão  $e^{-\mu\alpha}$ ,

As perdas por fricção, devidas a uma forma ondulada accidental do tubo de protecção do cabo, correspondem a  $k = 0,009$  na expressão  $e^{-kx}$  em que "x" é o comprimento do cabo expresso em metros. Em medições anteriores [4] tinham-se obtido, para  $\mu$ , valores semelhantes ou ligeiramente superiores; no entanto, a relação entre "c" e "k" era diferente.

O escorregamento nas amarrações era de cerca de 1,5 mm e só raramente ultrapassou 3 mm.

Estão actualmente em curso medições das deformações permanentes que se destinam a verificar as perdas de tensão correspondentes.

O relatório inclui igualmente, algumas equações de base relativas à contracção e à fluência.

Os resultados das medições efectuadas durante o espaço de tempo relativamente curto que medeia desde que as pontes foram postas em tensão não correspondem exactamente aos valores dados pelos cálculos, o que pode talvez ser atribuído em parte a certos factores secundários tais como a forma irregular da ponte, a armadura e principalmente à fluência do aço em tracção.

## R É S U M É

Les pertes de tension ont été mesurées et calculées sur deux ponts en béton pécontraint situés à Goteborg. Ces deux ouvrages ont été construits selon le système Freyssinet.

Les mesures ont été effectuées suivant deux méthodes différents:

- En tendant les câbles, à l'aide d'un vérin, à l'une de leurs extrémités et en mesurant l'effort transmis à l'autre.
- En fixant l'une des extrémités et en mesurant à l'autre la force exercée et l'extension résultante.

Cette dernière méthode donne des résultats particulièrement satisfaisants dans le cas de câbles à forte courbure et s'applique également quand seule l'une des extrémités est accessible. La combinaison de ces deux méthodes permet de contrôler la distribution de la tension le long du câble.

Les résultats de ces mesures montrent que les pertes de tension aux vérins sont de l'ordre de 6,5 à 7 % ce qui correspond à  $c = 0,93$ . Ce chiffre comprend, non seulement les pertes dues aux verins eux-mêmes,

mais aussi celles dues à la courbure des fils dans le vérin et dans les cônes d'ancrage.

Les pertes par friction dues à la courbure du câble sont données, pour  $\mu = 0,42$ , par l'expression  $e^{-\mu x}$ .

Les pertes par friction dues à une forme ondulée accidentelle des tubes de protection des câbles sont données, pour  $k = 0,009$ , par l'expression  $e^{-kx}$ , où "x" est la longueur mesurée du câble exprimée en mètres. Une mesure antérieure [4] avait donné pour  $\mu$  des résultats semblables ou légèrement supérieurs, mais le rapport entre "c" et "k" était alors différent.

Le glissement aux ancrages était de l'ordre de 1,5 mm. et ne dépassait que rarement 3 mm.

Des mesures des déformations permanentes sont en cours dans le but de vérifier les pertes de tension correspondantes.

Le rapport comprend également quelques équations de base relatives au retrait et au fluage.

Les mesures effectuées pendant le temps relativement court qui s'est passé, ne donnent pas encore une correspondance très rigoureuse avec le calcul, ce qui peut, probablement, non seulement être attribué en partie à certaines influences secondaires telles que la forme irrégulière du pont et l'armature, mais surtout à l'influence du fluage de l'acier tendu.

## **Vlb2**

**Nouveaux types d'armature précontrainte**

**New types of prestressing cable**

**Neue Typen von Vorspannarmierungen**

**Novos tipos de armadura para betão preesforçado**

PROF. J. COURBON

*Ingénieur en Chef des Ponts et Chaussées*

Paris

### **A) ARMATURE SOUPLE PRETEDUE**

#### **I – Description de l'armature souple prétendue**

L'armature souple prétendue système CHALOS comprend trois éléments essentiels :

- a) un élément tendu : *la gaine*, formé de torons d'acier tréfilé enroulés en hélice sur un tube en acier flexible.
- b) un support provisoire comprimé, *l'âme*, formé de courts noyaux en acier, capable d'équilibrer la traction de la gaine.
- c) des organes de jonction aux extrémités de l'armature, les *culots*, permettant de reporter la traction de la gaine sur l'âme pendant la construction, puis sur le béton à précontraindre, après extraction de l'âme.

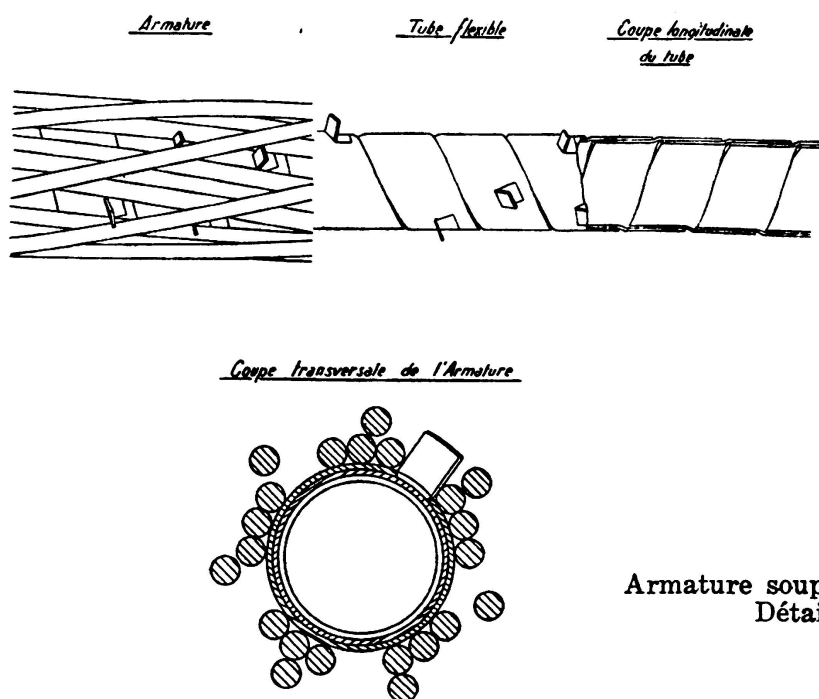
L'armature souple précontrainte est donc capable de conserver une tension constante et connue pendant un temps suffisamment long pour permettre son transport, sa mise en place et le bétonnage des éléments sur lesquels sera reportée sa tension après durcissement du béton.

L'armature souple présente tous les avantages des armatures tendues à l'avance sur ancrages fixes en ce qui concerne l'adhérence au béton et la connaissance précise des forces de précontrainte. Le procédé utilisé pour sa mise en tension permet d'obtenir une tension uniforme le long de l'armature connue avant emploi et susceptible même d'être modifiée avant le bétonnage. Cette uniformité de la tension n'est pas altérée par les



déformations élastiques ou permanentes que l'armature doit subir pour être disposée suivant le tracé prescrit. La robustesse de l'armature, mise en place dans les coffrages avant bétonnage, permet la pervibration dans la masse, au contact même de l'armature. Il en résulte un enrobage total de l'armature, constituant la plus sûre garantie pour sa conservation.

L'armature souple prétendue, bien que se rattachant à la classe des armatures mises en tension sur ancrages fixes avant coulage du béton, ne présente pas l'inconvénient d'immobiliser le moule pendant le durcisse-



ment du béton. Grâce au support provisoire intérieur, le matériel nécessaire à la mise en tension n'est immobilisé que quelques heures par armature. Il serait du reste difficile d'obtenir au moyen d'ancrages fixes, sans dépense excessive, des forces de précontrainte aussi importantes que celles mises en oeuvre par l'emploi simultané de plusieurs armatures souples prétendues (90 tonnes par armature).

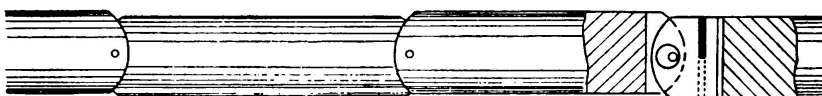
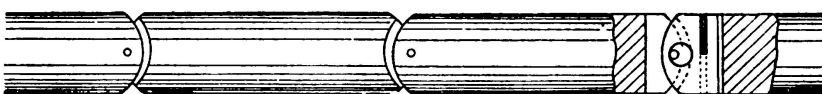
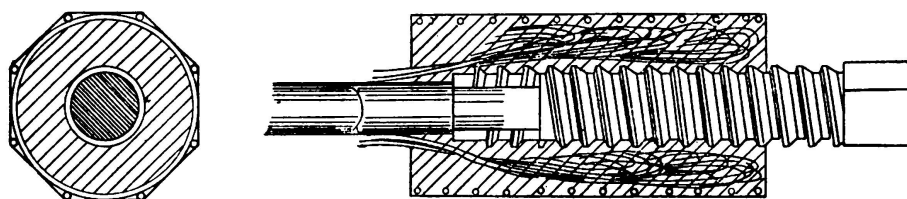
La fabrication des armatures souples précontraintes exige des fils d'acier de haute qualité. Enfin, leur mise en oeuvre impose aux armatures des efforts de traction supérieure à ceux qu'elle supportera en service. Cet exemple assez rare d'un essai total portant sur tous les éléments mis en service sans aucune exception, constitue une amélioration non négligeable du facteur sécurité.

## II - Description des éléments de l'armature souple prétendue

a) *Gaine*: L'élément tendu de l'armature est constitué par deux nappes de torons enroulés en hélice de sens opposé sur un tube support. Les pas et le nombre des torons de chaque nappe sont déterminés de façon que s'équilibrent les couples de torsion de chacune des nappes tendues

- 1) *Les torons*: Les torons sont formés de fils de 2 à 4 m/m de diamètre en acier tréfilé d'une résistance à la rupture de 180/200 kg/mm<sup>2</sup> et d'un allongement de rupture appréciable. La gaine toute entière est écrouie à un taux moyen de 160 kg/mm<sup>2</sup> à 170 kg/mm<sup>2</sup> environ. Cet écrouissage qui se fait progressivement, permet d'obtenir un ensemble dont les défor-

## Armature souple prétendue Détails

AMEVue en planVue de profil Ame compriméeVue de profil au cours d'une extractionCULOOT D'ANCRAGE

mations sont rigoureusement proportionnelles à la tension lorsque le taux de traction de l'acier varie de 0 à 160 kg/mm<sup>2</sup>.

Il est nécessaire qu'il y ait une possibilité de déformation plastique assez importante, pour permettre d'égaliser les tensions dans les différents cablots et d'aboutir à un ensemble homogène. Pratiquement, les fils employés ont un allongement à la rupture de l'ordre de 2 %.

- 2) *Tube support*: Le tube support a un diamètre intérieur de 45,6 mm. Il est flexible et étanche. Pour l'obtenir, on enroule en hélice un feuillard de 80 mm de largeur, en acier doux. Avant son enroulage, le feuillard porte des saillies ou «taquets» obtenues par poinçonnage et qui définissent les hélices d'enroulement des torons.

Les taquets sont, l'autre part, indispensables pour maintenir en place les torons, lorsque l'armature en charge est disposée suivant un tracé courbe. En effet, les taquets empêchent les torons de glisser les uns par rapport aux autres.

b) *Ame*: L'âme est l'organe qui sert à maintenir la tension de la gaine depuis la mise en tension de l'armature jusqu'au moment de mise en précontrainte du béton.

Elle est formée d'une chaîne d'éléments cylindriques en acier dur trempé de 44 mm de diamètre et de 160 mm de longueur. Chaque élément est terminé par deux surfaces cylindriques de 30 mm de rayon. Ces surfaces sont convexes pour un élément (élément mâle) et concaves pour l'élément suivant (élément femelle).

Dans l'armature en charge, les éléments s'appuient les uns sur les autres et chacun d'eux peut éprouver un déplacement angulaire par rapport à un élément voisin en glissant sur la surface d'appui.

Les éléments de l'âme ont un diamètre inférieur au diamètre intérieur du tube. Ce jeu est nécessaire pour une introduction et une extraction faciles. Ce jeu ne nuit en rien à la stabilité de l'armature en charge, ni à sa stabilité pendant les manipulations.

c) *Organes de jonction*: Les organes de jonction aux extrémités de l'armature ont un rôle double: reporter sur l'âme la traction des câbles de la gaine pendant la période de la mise en tension de l'armature à la précontrainte du béton, et ensuite reporter sur le béton durci la tension de la gaine au cas où, pour une raison quelconque, l'adhérence des torons de la gaine au béton à précontraindre ferait défaut.

Les organes de jonction sont des prismes en béton fretté de 0,50 m de longueur et dont la section transversale est un octogone inscrit dans un cercle de 0,21 m de diamètre. Les fils des torons de la gaine sont ancrés dans ce béton qui fait office de culot d'ancrage.

Au moment de la mise en tension de l'armature, la tension des cablots est reportée sur l'âme par un tirefond en acier vissé dans le béton du culot d'ancrage. L'empreinte du tirefond se fait au moment du bétonnage du culot. L'extrémité du tirefond étant légèrement engagée dans le tube support pour éviter toute entrée de mortier dans le tube. On retire le tirefond après prise du béton. Les blocs d'ancrage sont ainsi traversés longitudinalement par un tube d'un diamètre suffisant pour permettre l'introduction et l'extraction des éléments de l'âme.

L'extrémité du tirefond a une section carrée qui permet, soit de le visser pour le mettre en contact avec l'âme soit de le dévisser sous charge pour reporter la tension de l'armature sur le béton en déchargement l'âme.

Le pas et le tracé des filets du tirefond ont été choisis de manière à permettre un dévissage facile du tirefond sous la poussée de l'âme, sans toutefois que celui-soit réversible.

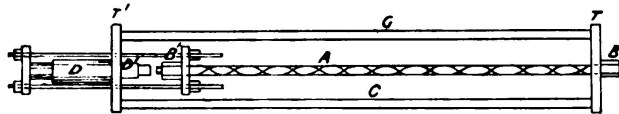
### III – Mise en tension de l'armature

La mise en tension de l'armature se fait dans un «banc de mise en tension» à l'aide de deux vérins: l'un servant à tendre les torons et l'autre

plus petit à exercer sur les éléments de l'âme une poussée par l'intermédiaire du tirefond du culot d'ancrage.

Le banc de mise en tension se compose essentiellement de deux poutres parallèles (INP ou ILA) entretoisées et terminées par deux traverses T et T'.

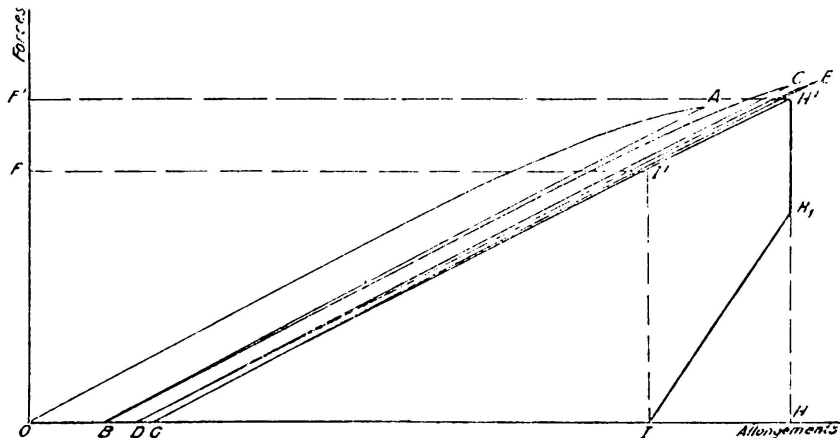
L'armature terminée avec les tirefonds vissés jusqu'au contact de l'âme est placée sur le banc de mise en tension entre les deux poutres C. L'un des culots d'ancrage C est rendu solidaire d'une traverse d'extrémité T. Le deuxième culot d'ancrage est saisi par un palonnier qui reçoit la poussée du vérin D prenant appui sur la traverse d'extrémité T' du banc à travers laquelle passe de palonnier.



Un vérin auxiliaire D' prenant appui sur l'autre face de la traverse T' permet d'exercer une poussée sur l'âme par intermédiaire du tirefond du culot d'ancrage.

Les différentes phases de l'opération sont les suivantes :

- Ecrouissage de l'armature à l'aide du vérin D. On soumet l'armature à une série d'extensions dépassant la limite élastique des aciers, suivies de décharges complètes jusqu'à ce que les allongements de l'armature et les efforts auxquels elle est soumise soient proportionnels de zéro à une valeur correspondant à une contrainte moyenne des torons de 160 kg/mm<sup>2</sup>. Les cycles correspondants représentés sur le diagramme sont : OAB, BCD, DEG.
- L'armature étant devenue un système élastique, on tend l'armature à une tension F' intérieure au domaine élastique. L'opération se traduit sur le graphique par la droite G H'.



- A l'aide du vérin D' on exerce sur l'âme une compression Q par l'intermédiaire du tirefond qui est vissé au fur et à mesure que la poussée du vérin D' augmente. La compression de l'âme est à chaque instant très sensiblement égale à la poussée du vérin D', le vissage du tirefond ne pouvant se faire que si la différence

entre la compression et la poussée est très faible. On effectue ainsi une substitution d'appui de la tension de la gaine sans variation de cette tension, ce qui se traduit sur le graphique par la droite verticale  $H H'$ .

- d) On décharge ensuite lentement le vérin D. La tension de la gaine décroît de la valeur  $F'$  à la valeur  $F$  tandis que la compression de l'âme augmente de la valeur  $Q$  à la valeur  $F$ . C'est la partie H, I du graphique.

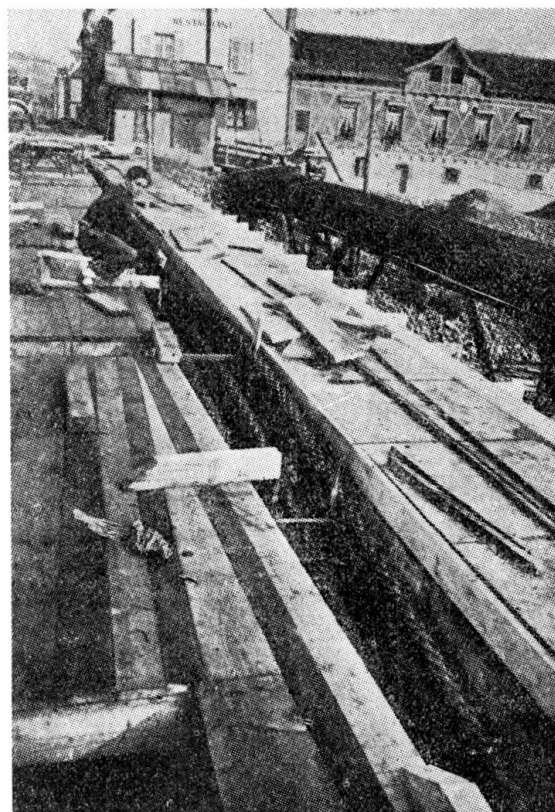
La tension finale de l'armature est donnée par l'ordonnée  $I I'$ .

#### B) OUVRAGES CONSTRUITS AVEC EMPLOI D'ARMATURE SOUPLE

##### *Pont sur la Fecht à Ingersheim (Haut-Rhin)*

Ce pont a été construit en 1950 et a permis pour la première fois, l'emploi de l'armature souple prétendue. C'est pour cette raison que les caractéristiques choisies n'ont rien d'exceptionnellement osé.

Il est constitué par deux travées continues de 24,50 m de portée. Sa hauteur constante d'une culée à d'autre est de 1,62 m. Il supporte une chaussée de 6 m et deux trottoirs de 1,50 m, de largeur.



Pont d'Ingersheim. Courbure des armatures de précontrainte sur

##### *Pont sur le Canal du Loing à la Genevraye (Seine-et-Marne)*

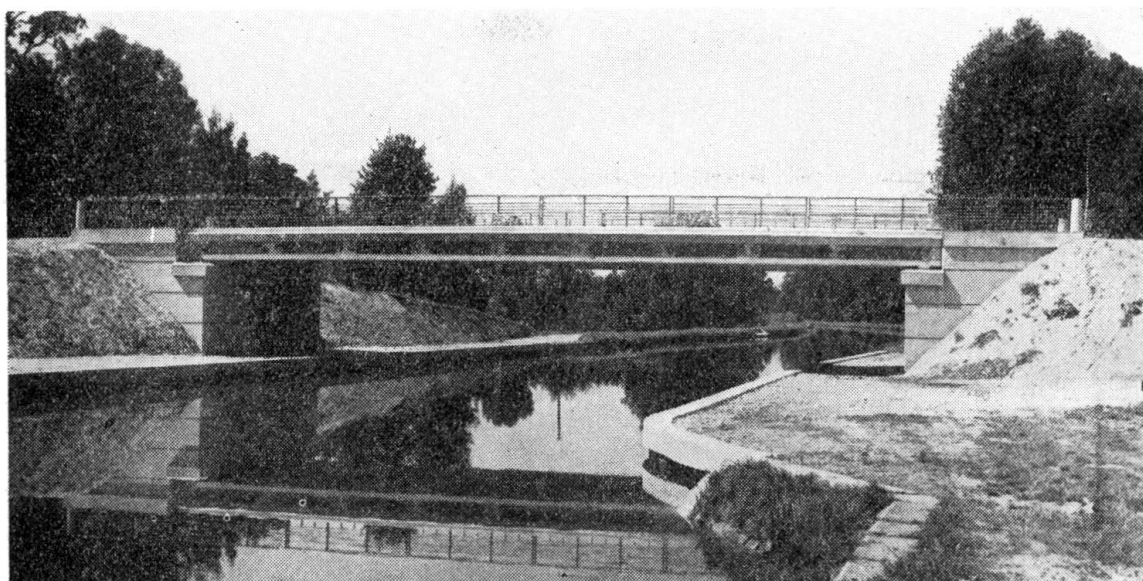
Cet ouvrage est un pont dalle de 24 m de portée dont l'épaisseur varie de 0,79 m sur les culées à 0,85 m à la clé. L'éclancement est de 1/28. Il supporte une chaussée de 5,50 m et deux trottoirs de 1 m.

Il est précontraint dans le sens longitudinal et dans le sens transversal par des armatures souples prétendues. Les culots d'ancrage des armatures transversales sont visibles sur la photographie.

##### *Pont sur la Seine à Chartrettes (Seine-et-Marne)*

C'est un cantilever à articulation centrale dont les portées sont 29,46 m, 55,68 m et 29,46 m. Il a été construit sur les piles et culées





Pont de la Genevraye sur le Canal du Loing

existantes, ce qui a nécessité la construction de contrepoids aux extrémités des travées de rive.

Le tracé de l'intrados a été déterminé en fonction des gabarits de navigation imposés pendant et après la construction.

Il supporte une chaussée de 6 m et deux trottoirs de 1 m.

Les deux poutres consoles ont été construites l'une après l'autre en remployant le même cintre. Elles ont été réunies après mise en précon-



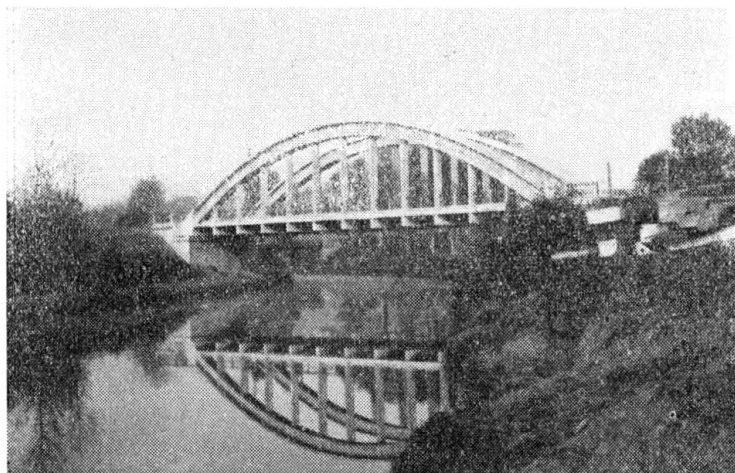
Pont de Chartrettes, sur la Seine

trainte par l'articulation centrale qui ainsi ne subit d'efforts que sous les surcharges.

*Pont sur le Canal de Nantes à Brest Redon (Ille & Vilaine)*

Cet ouvrage est un bow-string de 42,50 m de portée.

Les deux arcs sont en forme de double T dont la hauteur est de 1,30, les âmes et les ailes ayant la même épaisseur 0,25.



Pont sur le Canal de Nantes à Brest à Redon  
(Ille & Vilaine)

Le tirant est constitué par la dalle de 0,25 d'épaisseur qui s'étend sous la chaussée et les trottoirs. Les 12 armatures de précontrainte sont placées dans la dalle et réparties à raison de 4 sous chaque arc et 4 sous la chaussée. De cette manière, la poussée maximum est inférieure à la somme des tensions des armatures placés au

droit de l'arc. Les cisaillements longitudinaux dans la dalle sont ainsi réduits à une valeur extrêmement faible.

Les culées sont fondées sur le schiste à faible profondeur.

*Pont sur la Garonne à Ondes (Hte Garonne)*

L'ouvrage est constitué par cinq travées indépendantes de 39,20 m de portée, la hauteur des poutres est de 2,35. La dalle sous chaussée et trottoirs est supportée par 4 poutres longitudinales réunies par sept entretoises est précontraintes chacune par six armatures souples.

Les piles et culées sont fondées sur des caissons descendus par havage sur le tuf dur. Les caissons des culées sont en béton armé. Les caissons des piles sont à double paroi métalli-



Pont d'Ondes sur la Garonne



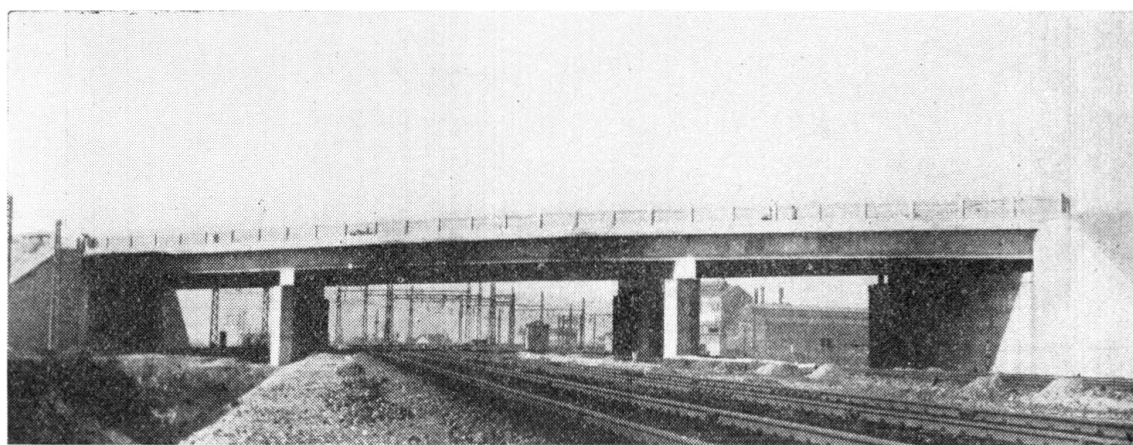
ques. Ils ont été mis en place au blondin et lestés par du béton coulé entre les deux parois.

L'ouvrage a été construit sur cintre en bois à l'avancement.

*Pont sur le Boulevard de Ceinture à St Fons — Lyon (Rhône)*

L'ouvrage comprend trois travées continues dont les portées biaises sont 23,80 – 36,50 – 23,80. La largeur totale de l'ouvrage est de 25,50 m. Il est coupé dans le sens longitudinal par un joint placé dans l'axe de l'ouvrage. L'angle du biais est de  $42^{\circ} 35'$ .

Chaque moitié supporte une chaussée de 8 m, un trottoir extérieur de 2,75 m et un trottoir central de 2 m. L'ouvrage est du type caisson,



Pont sur le Boulevard de Ceinture à St Fons — Lyon (Rhône)

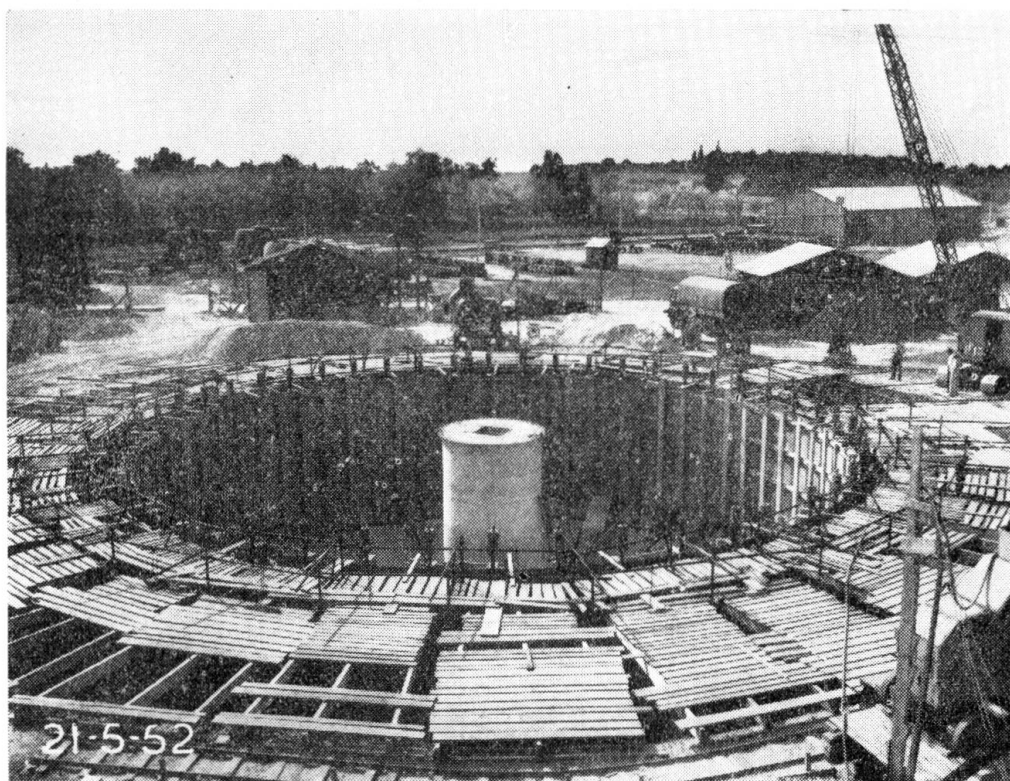
la dalle supérieure ayant une épaisseur de 0,18 m et la dalle inférieure une épaisseur de 0,07 m. Les deux dalles sont réunies par six poutres longitudinales précontraintes chacune par six armatures souples en travée de rive. Des entretoises espacées de 6 m environ réunissent les poutres transversalement. Elles sont armées par des aciers doux ordinaires.

Par suite du biais, l'ouvrage n'a qu'un seul point fixe, les autres appuis ayant des articulations sphériques.

*Réservoir à hydrocarbures (Maître de l'oeuvre Marine Nationale)*

C'est un réservoir de 2 700 m<sup>3</sup> pour hydrocarbure de forme torique. La paroi cylindrique extérieure est précontrainte par des armatures verticales et horizontales en forme de demi-cercle. La partie supérieure et la partie inférieure sont des calottes toriques qui s'appuient sur un pilier central et la paroi cylindrique extérieure.

Ce réservoir a subi avec succès les essais d'étanchéité à l'eau et aux hydrocarbures avec suppression de 400 g/cm<sup>2</sup> et dépression de 20 g/cm<sup>2</sup>. Il est actuellement en service.



Réservoir à hydrocarbures de 2700 m<sup>3</sup>

### C) NOUVELLE ARMATURE DE PRECONTRAINTE

La SOCIÉTÉ DES GRANDS TRAVAUX DE MARSEILLE a mis au point un nouveau type d'armature de précontrainte mise en tension sur le béton durci.

Ces armatures sont constituées par des torons d'acier tréfilé dur maintenus en tension par des ancrages métalliques d'un type breveté.

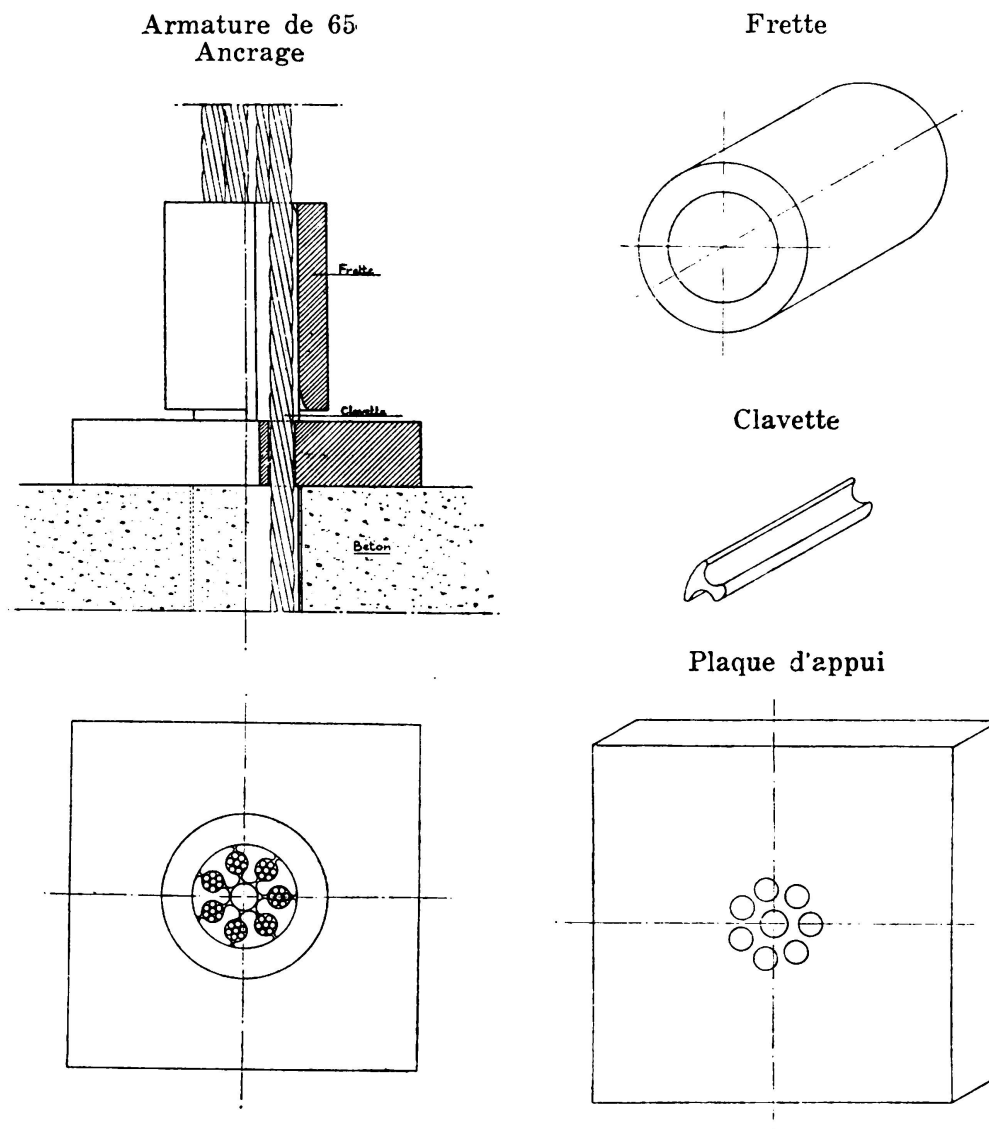
#### *Acier de précontrainte*

Les armatures sont constituées par un certain nombre de torons de 7 fils, dont le diamètre peut varier de 3 à 3,6 mm. Le nombre de torons est déterminé par la force que l'on désire obtenir. Les deux types principaux d'armatures comprennent 4 et 7 torons. Une armature de 4 torons de 7 fils de 3,6 permet une tension initiale de 35 t. Une armature de 7 torons de 7 fils de 3,6 permet une tension initiale de 65 t.

Dans les deux cas, ces torons sont câblés ensemble de façon à obtenir un ensemble n'ayant aucune tendance à un écrasement. Le câble obtenu présente cependant une très grande souplesse et permet les tracés les plus sinueux. L'acier employé pour les torons est de l'acier tréfilé de la nuance 160/180 ou même 180/200.

*Ancrages de l'armature (voir dessin)*

L'ancrage est un organe métallique constitué par une frette cylindrique maintenant des clavettes en acier mi-dur serrées sur les torons. A la mise en tension, les clavettes sont matricées sur les torons par la frette cylindrique extérieure. Les clavettes reportent la tension des fils



sur le béton par appui sur une plaque d'acier permettant la répartition des pressions sur une surface suffisante.

Les clavettes, dont le nombre pour un ancrage donné est égal au nombre des torons, sont profilées suivant une surface cylindrique d'un diamètre légèrement inférieur à celui des torons et pour la face extérieure suivant une surface cylindrique également.

La frette est un cylindre creux dont la partie inférieure présente un évasement torique permettant l'introduction des clavettes. Elle est enfoncée à force et exerce alors un serrage suffisant pour entraîner une pénétration des fils torons dans le métal des clavettes.

### *Mise en tension de l'armature*

L'armature est mise en tension par traction à ses extrémités au moyen de vérins annulaires à deux corps opposés. Les torons de l'armature sont enfilés dans le tube intérieur du vérin et ancrés à la partie arrière du vérin par un ancrage provisoire.

La mise en tension s'effectue en prenant appui sur le béton durci à précontrainte. Le corps arrière du vérin tend les câbles tenus dans l'ancrage provisoire. Après écrouissage et tension à la valeur désirée, la frette est enfoncée sur les clavettes qui occupent déjà leur fonction définitive par le corps avant du vérin. A l'écrouissage, la tension des torons est portée à  $150 \text{ kg/mm}^2$ , après plusieurs montées à des valeurs inférieures et descente complète. Ces opérations ont pour but d'égaliser des tensions entre les différents torons de l'armature. Après avoir été maintenue pendant quelques minutes à  $150 \text{ kg/mm}^2$  environ, la tension est ramenée à la valeur désirée.

Après mise en tension, on injecte un coulis de ciment dans la gaine occupée par l'armature, par l'intermédiaire d'un orifice traversant l'organe d'ancrage.

### *Qualités de l'armature*

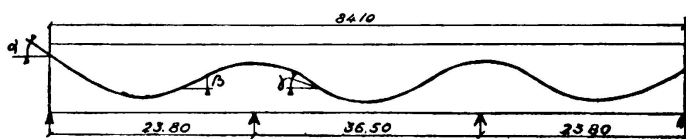
En dehors de ses qualités de robustesse, de facilité d'emploi et d'économie, l'armature présente trois caractéristiques essentielles :

- Absence totale de glissement de l'armature dans son ancrage au moment de la mise en tension ;
- Coefficient de frottement très faible (inférieur à 0,08) ;
- Possibilité de mise en précontrainte par tronçons en laissant aux câbles la longueur nécessaire pour la mise en précontrainte définitive.

On peut montrer l'absence de glissement de l'armature dans son ancrage au moment de la mise en tension de la manière suivante : la frette cylindrique extérieure est bloquée sur les clavettes par le corps avant du vérin. Si on place entre la frette et le vérin une plaque d'un diamètre légèrement inférieur au câble de précontrainte, le câble est érafflé en surface par cette plaque et on mettrait en évidence s'il existait le déplacement relatif des fils et des clavettes.

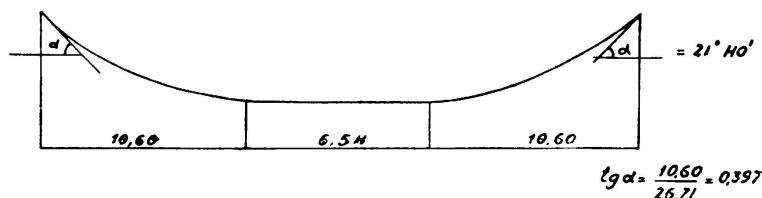
Diverses expériences ont été faites pour déterminer la valeur du coefficient de frottement. On s'est servi pour ces expériences du tube support d'armature souple prétendue décrit par ailleurs. Ces armatures avaient été utilisées pour la précontrainte d'ouvrages d'art. Les expériences ont été faites avant l'injection de ces armatures.

La première expérience a eu lieu au pont de St Fons, ouvrage à trois travées continues. Le tracé de l'armature était le suivant :



|                                                                                                   |         |
|---------------------------------------------------------------------------------------------------|---------|
| Longueur totale ... ..                                                                            | 84,10 m |
| Somme des angles en valeur absolue ... ..                                                         | 93° 44' |
| Rapport moyen des tensions entre les manomètres placés à chaque extrémité après correction ... .. | 0,93    |
| Valeur du coefficient de frottement ... ..                                                        | 0,045   |

La seconde expérience a eu lieu au Pont d'Ondes, sur la Garonne. Le tracé de l'armature était le suivant :



|                                                                                                   |         |
|---------------------------------------------------------------------------------------------------|---------|
| Longueur totale ... ..                                                                            | 27,74 m |
| Somme des angles en valeur absolue ... ..                                                         | 43° 20' |
| Rapport moyen des tensions entre les manomètres placés à chaque extrémité après correction ... .. | 0,94    |
| Valeur du coefficient de frottement ... ..                                                        | 0,08    |

La valeur très faible de ce coefficient est due en grande partie à la conception même de l'armature. L'emploi de torons câblés formant un ensemble robuste facilite un glissement l'armature n'étant en contact avec la gaine que par points. L'ensemble des torons n'a pas tendance à s'écraser dans les parties courbes.

La troisième qualité de cette armature est la possibilité qu'elle procure de mettre en précontrainte un élément par tronçons pouvant servir à l'assembler sur un autre tronçon. Il suffit pour cela de laisser au câble une longueur suffisante, au-delà de l'ancrage provisoire sur le vérin. On voit toutes les ressources que procure cette possibilité pour la préfabrication.

Enfin, employée simultanément avec l'armature souple prétendue, elle permet d'obtenir des unités de l'ordre de 150 t dans un espace très réduit et d'une manière très économique, le tube support de l'armature souple procurant une gaine gratuite à cette nouvelle armature.

### R É S U M É

La «Société des Grands Travaux de Marseille» a mis au point deux systèmes d'armature de précontrainte.

L'armature souple prétendue (système «chalos») qui comprend trois éléments essentiels :

- un élément tendu, la gaine, formé de torons d'acier tréfilé, enroulés sur un tube,
- un support provisoire comprimé, l'âme, formé de courts noyaux d'acier,
- des organes de jonction aux extrémités, les culots.

Les avantages sont: l'adhérence parfaite au béton, la connaissance précise des forces de précontrainte, l'uniformité de la tension le long de l'armature qui est conservée quelles que soient les déformations imposées à l'armature pour obtenir la tracé prescrit.

La fabrication et la mise en place des armatures ne fait appel qu'à un matériel simple et immobilisé peu de temps.

De nombreux ouvrages construits avec ce type d'armature sont en service et donnent entière satisfaction.

Un second type d'armature de précontrainte qu'on met en tension en prenant appui sur le béton à précontraindre est constitué par des torons d'acier tréfilé, ancrés par matriçage de clavettes à l'intérieur d'une frette cylindrique.

Les qualités sont notamment:

- l'absence totale de glissement de l'armature dans son ancrage au moment de la mise en tension,
- un coefficient de frottement très faible (inférieur à 0,08).

Ces deux types d'armatures peuvent être utilisés simultanément, le tube de l'armature souple procurant une gaine gratuite à la seconde.

#### S U M M A R Y

The «Société des Grands Travaux de Marseille» has devised two new types of prestressing cables.

One of them is the «pre-tensioned flexible reinforcement» with three basic components:

- a stretched element or «sheath», composed of wire drawn steel strands twisted around a tube,
- a temporary compressed support or «core» composed of short steel stubs,
- end anchorages.

It insures an excellent steel-concrete bond, an exact knowledge of prestressing forces and a uniform tension along the cable, whatever its shape in the structure.

Manufacturing and installation requires only a simple equipment in use for a short time.

There are many structures in service built by this process, their behaviour being very satisfactory.

The other type of reinforcement, which is prestressed by using the concrete as a support, is composed of wire drawn steel strands, anchored by steel keys in a cylindrical end-piece.

It is completely slip-proof at the anchorage at prestressing and its friction coefficient is extremely low, inferior to 0,08).

Both types of reinforcement can be used simultaneously, the tube of the flexible cable then becoming a sheath for the second.

### ZUSAMMENFASSUNG

Die Société des Grands Travaux de Marseille hat zwei Systeme von Vorspannarmierungen herausgebracht.

Die biegsame Armierung mit vorausgehender Vorspannung, die aus drei Hauptteilen besteht:

- ein gespannter Teil, ein «Kabel», bestehend aus Litzen von gezogenem Stahldraht, die auf einer Röhre aufgewickelt sind,
- eine Stahl-Kern als vorübergehende, gedrückte Abstützung aus kurzen Stücken,
- die Verbindungsteile an den Enden, die Anker.

Die Vorteile sind die folgenden: die vollkommene Haftung mit dem Beton, die genaue Kenntnis der Vorspannkkräfte, die Gleichförmigkeit der Spannung längs der Armierung, die unabhängig von all den für die vorgeschriebene Linienführung der Armierung auferlegten Abbiegungen, erhalten bleibt.

Die Herstellung und Montage der Armierung verlangt nur einfache, Kurzfristig verwendete Hilfsmittel.

Zahlreiche Bauwerke mit diesem Vorspannsystem stehen heute zur vollen Zufriedenheit im Gebrauch.

Ein zweiter Typ einer Vorspannarmierung, die unter Abstützung gegen den vorzuspannenden Beton gespannt wird, besteht aus gezogenen Stahldraht-Litzen, die mit Keilen im Innern eines zylindrischen Stahlrings verankert werden.

Die Merkmale sind im besondern:

- das vollständige Fehlen eines Gleitens der Armierung in der Verankerung im Zeitpunkt der Vorspannung.
- ein sehr kleiner Reibungskoeffizient (kleiner als 0,08).

Diese zwei Armierungstypen können auch gemeinsam verwendet werden, indem das Rohr der biegsamen Armierung als Hülle für die zweite dient.

### RESUMO

A «Société des Grands Travaux de Marseille» elaborou dois novos tipos de armaduras para betão preesforçado.

Pode citar-se em primeiro lugar a «armadura flexível em pré-tensão» que compreende três elementos essenciais:

- um elemento em tensão, ou «bainha», formado por fios de aço trefilados enrolados em torno de um tubo,



- um suporte provisório comprimido ou «alma» formada por curtos cilindros de aço,
- órgãos de união nas extremidades ou ancoragens.

Este tipo de armadura permite garantir uma aderência perfeita com o betão, o conhecimento exacto das forças de pré-tensão e a uniformidade da tensão ao longo do cabo, que se mantém sejam quais forem as deformações impostas à armadura para seguir o traçado prescrito.

A fabricação e a colocação das armaduras exigem apenas um material simples e imobilizado durante tempo.

Numerosas obras construídas por este processo estão já em serviço, e o seu comportamento tem sido satisfatório.

O outro tipo de armadura, que é preesforçada tomando apoio sobre o betão, é constituído por fios de aço trefilados, fixados por chavetas num cilindro fretado.

Este tipo de armadura garante uma ausência total de escorregamento na ancoragem quando é posta em tensão e um coeficiente de atrito muito baixo (inferior a 0,08).

Os dois tipos de armadura podem empregar-se simultaneamente, formando o tubo da armadura flexível bainha para o segundo tipo.