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Creep-effects in the analysis of reinforced concrete structures

Berücksichtigung des Kriechens bei der Berechnung von Eisenbetonbauten

Efeitos da fluência no cálculo das estruturas de betão armado

Considération du fluage dans le calcul des structures en béton armé

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1. *General Considerations.*

The creep-sensitivity of a reinforced concrete structure depends on the order of magnitude of two ratios: the ratio between the level of sustained and of total (sustained and transient) compressive stress, and the ratio between the level of total compressive stress and of compressive strength of the concrete. The first ratio is a measure of the «structural» component of the creep-sensitivity, since it depends largely on the type of structure or structural part considered, as well as on its span; the second ratio is a mesure of the «material» component of the creep-sensitivity which depends on the degree of utilization of the material. The larger these ratios over an extensive part of the structure the more significant are the creep-and the associated relaxation-effects in the structural performance, and the more important, therefore, their consideration in the structural analysis.

Sustained stresses are either induced by sustained forces («load--stresses»), or they arise from restrained or from enforced deformation («deformation stresses»). While creep-effects only influence the *distribution* of the load-stresses, they determine the *order of magnitude* of the deformation-stresses [1]. Since in statically indeterminate structures the value of the redundants depend jointly on the acting forces and the specific deformations, the creep-sensitivity of such structures is necessarily higher than that of simple statically determinate structures, in which only load stresses occur. In this respect even simple prestressed reinforced concrete structures are obviously to be considered as internally indeterminate, since their stresses depend jointly on the acting external forces and on the deformation, both enforced and restrained, of the reinforcement. Strictly speaking the same consideration would apply to the internal stress-distribution in any simply reinforced concrete structure, which is the result of the external forces and of the joint deformation of concrete and reinforcement.

Creep, relaxation and structural damping occuring in the same material under different loading conditions are, in general, different aspects of the same phenomenon of visco-elastic response: creep represents the deformational response to sustained forces or stresses, relaxation the stress-response to sustained deformation or strain, and damping the energy-loss associated with processes of transformation of potential and kinetic energy. The parameters describing the various aspects are therefore closely interrelated and can be derived from the same set of physical assumptions concerning the mechanical response of the material, provided this response is linear. This condition represents a rather serious limitation with respect to the practical use for conversion of the results of one type of test into another of the interrelation between creep-, relaxation-, and damping parameters, since the mechanical response of most structural materials, and of concrete in particular, is linear only within the range of relatively low stresses.

The conventional analysis of reinforced concrete structures is based on two assumptions: under service conditions the response of the structure as a whole to applied external forces is elastic, while the resistance close to failure of the individual sections of the structure is determined on the basis of an essentially plastic response of both the concrete and the reinforcement. In fact, proposals have recently been made to abondon the first assumption completey in favor of the second one, and to base the analysis of reinforced concrete structures on the same principles of plastic «limit design» that have been developed for mild steel structures. (2) It seems that such proposals overemphasize the apparent similarty of the «plastic» response in concrete and in metals, particularly mild steel, and completely disregard the fundamental, very serious differences.

It is by now quite well established that the mechanical response of the concrete itself is neither elastic nor plastic, but highly time-sensitive and thus essentially visco-elastic. The apparent elasticity of reinforced concrete under relatively low stresses of moderate duration is the result of both the high coefficient of viscosity of the concrete and the elastic restraining action of the reinforcement. The apparent plasticity under high stresses is the result of the pronounced non-linearity with respect to stress, of the creep-rate of the concrete. This non-linearity appears to be primarily a manifestation of the progressive internal disruption of the material, starting considerably below the conventional compressive strength, rather than of «plastic flow» of a type occuring in metals which is not associated with internal disruption. The effect on the failure load of the structure of this apparent «plasticity» of the concrete will therefore be quite different from that of plasticity in metal structures, unless the percentage of reinforcement is so small that the carrying capacity of the reinforced section is determined only by the yield-stress of the reinforcing steel which is reached at a very low concrete stress. Even then, however, the fracture resistance of the section is rapidly attained as the reinforcement yields, the concrete stress increases towards the crushing strength of the concrete while the cracks open and spread, reducing the compression zone in this operation of a «plastic hinge». It is therefore rather doubtful whether any indeterminate reinforced concrete structure will actually survive the formation of more than a single «plastic hinge».

While for structures of low creep-sensitivity-ratios the above assumptions, although rather crude, result in fairly adequate procedures of design and analysis, the analysis of highly creep-sensitivie structures and structural parts such as flat long-span arches, long-span frames, struts and columns and, particularly, pre-stressed concrete structures, requires the consideration of the actual visco-elastic response of the material, both with respect to service conditions and to failure.

2. *Creep of Concrete.*

The visco-elastic response of concrete is generally determined by creep-tests. Since they are much simpler to perform than relaxation or damping tests, the respective parameters are usually deduced from the creep parameters. Systematic creep-tests so far performed have been limited almost exclusively to relatively low ranges of stress, within which the relation between creep or creep-rate and stress is practically linear (3). Creep-rates at constant stress decrease with time and tend towards zero after several years presumably as a result of the completion of the crystallization process within the cement paste, which finally forms a 3-dimensional elastic network practically blocking further creep. Numerous attempts have been made to represent the observed creep-time-relation in analytical form mostly by the fitting of arbitrary time-function to the test results.

In a more recent investigation at Columbia University supported by the New Jersey State Highway Department an attempt has been made to derive a creep equation by interpreting a series of creep-tests on the basis of a few relatively simple physical assumptions. (4) These creep tests were performed on concrete cylinders 10 inch high and 3 and 4 inches in diameter, loaded at an age of approximately 28 days and subjected to stress levels varying roughly between 15 and 65 percent of the 28-day compressive strength f_c. An attempt to impose a stress-level of 0.8 f_c failed, since the cylinders cracked or disintegrated explosively within a few days after loading.

Considering concrete as a heterogeneous material made up of the practically elastic sand and stone aggregate suspended in or held together by the highly viscous cement paste, the deformational response of this material to an applied force may be assumed to depend significantly on the volume concentration of the solid aggregate, varying between that of a «suspension» in which the aggregate of relatively low volume concentration increases the coefficient of viscosity of the cement paste in some relation to its volume concentrations (5), and that of a granular mass of aggreate the voids of which are filled by the cement paste. The over-all deformation of such a material will therefore vary between that of an essentially visco-elastic, though not necessarily linear, «suspension», the response of which is determined primarily by the response of the cement paste, and that of a granular mass with internal friction the contact areas of which are «lubricated» by the void-filling cement paste, which also provides the cohesion within the material.

The three concrete mixes which have been subjected to creep tests have therefore been made to vary essentially with respect to the volume concentration of the aggregate: Mix A represents a dense, highly viscous suspension of 0.37 volume concentration of aggregate and of a weight ratio 1:1 of cement and aggregate; Mix C represents a concrete of maximum volume concentration of aggregate of 0.73 and of weight ratio 1:6, while Mix B represents an intermediate condition between the above two extremes with volume concentration of 0.55 and weight ratio 1:2.5. Because of the workability of the concrete the water cement ratio (by weight) could not be kept constant, but varied from 0.31 for Mix A to 0.44 for Mix B and 0.53 for Mix C. Since the compressive strength decreases with increasing water cement ratio the 3 mixes showed considerable variation in strength. Table I summarizes the results of the compressive strength tests on the 3 inch, 4 inch and standard 6 inch cylinders. The compressive strength was determined at various periods both for specimens that had been kept unloaded and for those that had supported sustained loads. Thus the ageing and «stress-consolidation» effects on compressive strength can be separated. With the exception of the cylinders subject to creep tests for which only two replications were available, since two parallel, nominally identical creep tests were always performed, each figure represents the mean of at least 3 individual test results.

Parallel creep tests were performed under continually sustained and under periodically removed constant stress levels in order to observe both creep and creep recovery. The maximum duration of the tests is roughly 400 days. It is assumed that most of the creep in concrete occurs within the first year; the expected total creep has been predicted on the basis of the trend established over the test period and checked by comparison with ratios of creep after six months and one year to total creep reported by different investigators (3).

The elastic modulus was carefully determined on 3 inch, 4 inch and 6 inch cylinders at various times by fast unloading from relatively low stress-levels. The recorded values differ quite significantly from the elastic moduli obtained by conventional procedures. The observed values of the elastic moduli are summarized in Table II.

The creep tests were performed under controlled conditions of temperature (70°F) and humidity (60 %). The shrinkage of control-specimens was continually recorded and the amount of shrinkage substracted from the total deformation before the true creep-curves were drawn.

In order to reproduce and represent the observed creep-and creep-recovery curves at the various applied stress-levels by analytical expressions and to identify the different components of the total deformation, a mechanical model was conceived, consisting of the following model-elements:

(a) a Maxwell element, consisting of a linear spring and of a dashpot, non-linear with respect to both force and time, coupled in series, which represents the long-time visco-elastic response in shear resulting in irrecoverable creep (Fig. 1a);

(b) a Kelvin or Voigt element, consisting of a linear spring and dashpot coupled in parallel, which represents the visco-elastic interaction of solid and fluid phases resulting in recoverable creep or «delayed elasticity» (Fig. 1b);

(c) two Kelvin elements coupled in series, both non-linear with respect to force and representing, respectively, the short-time «consolidation» effects of irrecoverable pore-water motion towards the surface and evaporation, and the effect of permanent set due to initial, internal, partially destructive readjustments within the granular mass of the concrete (Fig. 1c).

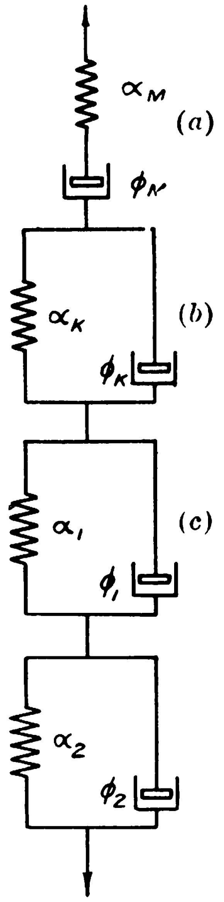


FIG. 1. Complete Structural Model of Concrete

Elements *(a)* and *(b)* respond to increasing and decreasing forces; elements *(c)* are assumed to respond to increasing forces only. This is necessary to reproduce the fact that neither the internal readjustment nor the pore-water motion and drying out effects can generally be reversed by reversing the applied forces, although, particularly in the case of the latter, a significant reversal migth be expected on removal of the forces followed by release of the pore-water overpressure, if evaporation at the surface were completely prevented, as in the case of imperviously covered or submerged concrete.

Since the well-known size and surface effects in creep, i. e., the reduction of the creep-rate with increasing minimum dimension of the specimen, as well as with increasing moisture content, is probably related to the pore-water motion and evaporation, this effect would be associated only with model-elements *(c)*, while the response of elements *(a)* and *(b)* might be considered essentially unaffected by size and surface-condition of the concrete specimens or structural parts. Since most of the creep observations suggest a rather significant reduction with specimen-size and with moisture content of the total creep-rate, at least at the relatively low stress-levels at which most observations have been made (a ratio of roughly 2.0 between creep-rates on 4 in. dia. and on 10 in. dia. cylinders is generally assumed to indicate the order of magnitude of the size-effect), it appears that the reduction, with specimen size, of the deformation-rates of model-elements *(c)* alone would have to be a multiple of the observed reduction of the toal creep-rate, to reproduce test results on large specimens. This is not unreasonable since under low applied stress levels the permanent set appears to be relatively small in relation to the short-time creep-effects due to pore-water motion and subsequent evaporation; thus a large or a completely submerged small specimen may show only the permanent set contribution to the creep reproduced by model-elements *(c)*, which is only a fraction of the total contribution observed on a small specimen with freely drying surface.

The results of this investigation were evaluated on the basis of the proposed 4 model elements, which are coupled in series; the total deformation at any moment is thus represented by the sum of the deformations of the elements. The relative importance, at different stress-levels and different times, of the contributions associated with each of the four model elements is illustrated in TABLE III, based on the Columbia University creep-investiation (4).

Identifying displacement x and force P in the model with uniaxial strain ε and uniaxial constant stress σ in the creep specimen, the following relations define the 4 types of contribution to the total deformation:

$$\varepsilon_M = \alpha_M f_c\left(\frac{\sigma}{f_c}\right) + C_M \tau_M (1 - e^{-t/\tau_M}) . \exp\left[a\left(\frac{\sigma}{f_c}\right)^b\right] = \tag{1}$$

$$= \int \alpha_M f_c\left(\frac{\dot{\sigma}}{f_c}\right) dt + \int \Phi_M f_c\left(\frac{\sigma}{f_c}\right) dt$$

for the Maxwell model, where Φ_M is the fluidity (inverse viscosity coefficient) which is a function of σ and τ;

$$\varepsilon_K = \alpha_K f_c\left(\frac{\sigma}{f_c}\right) (1 - e^{-t/\tau_K}) \tag{2}$$

for the reversible Kelvin model, and

$$\varepsilon_1 + \varepsilon_2 = \sum_{i=1,2} \alpha_i f_c\left(\frac{\sigma}{f_c}\right) (1 - e^{-t/\tau_i}) \tag{3}$$

for the non-reversible Kelvin models. The dot represents differentiation with respect to time.

The coefficients α represent spring constants (inverse moduli in uniaxial stress) the coefficients τ retardation-times. Of the cofficients those with subscript M and K are constants, while α_i and τ_i are simple (linear) functions of stress determined so as to reproduce the creep-test results as closely as possible. Table IV summarizes the best values of the constant parameters of these equations for each of the three mixes, as well as values of the stress dependent parameters at 2 stress-levels. These parameters could be expressed as relatively simple functions of the volume concentration of the aggregate l.c. and the water-cement ratio w_c.

Since the main purpose of the tests is to determine the general stress-non-linearity of the creep of concrete at moderately high stress levels, it can not be expected that the same equations (1) to (3) will automatically represent the response of the material at the low stress levels at which the linearity with respect to stress of this response appears to be well established. Thus it is necessary to introduce for the range of low stresses truly linear model equations instead of eqs. (1) to (3), with constant parameters derived from the condition that the linear equations should be tangents to the non-linear relations (1) to (3), so that their points of contact represent the constant stress level at which, for each mix, deviation from linear response would start. It has been found that these «stress limits of linear response» σ_o are time-dependent, particularly within the range of very short times (~50 days) where they may be as low as $\sigma_o = 0.10\ f_c$; for long times, however, they tend towards a more or less time-insensitive value of roughly 0.25 f_c which, in general, is not too far removed from the maximum sustained compressive stress in reinforced concrete structures under service conditions. Thus, visco-elastic analysis under such conditions may be based on the following linear model equation for the Maxwell model, which replaces the non-linear equation (1), extending it towards the reference point ($\sigma = 0$, $\varepsilon = 0$):

$$\text{(1a)} \qquad \varepsilon_M = \alpha_M\, f_c \left(\frac{\sigma}{f_c}\right) + C_M\, C_O\, \tau_M \left(1 - e^{-t/\tau_M}\right) \left(\frac{\sigma}{f_c}\right)$$

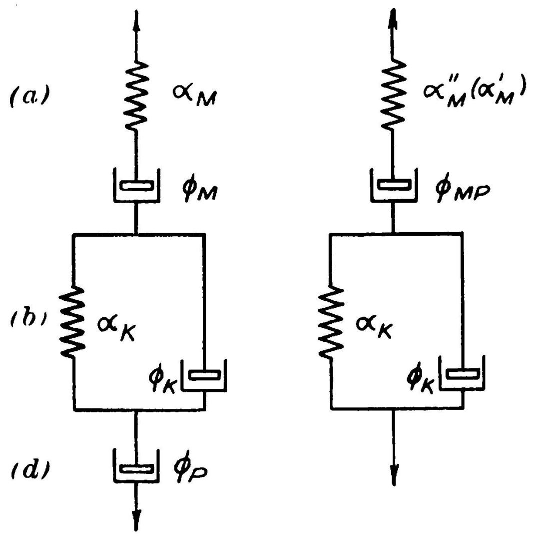
while the other two equations have the same form as (2) and (3), but with constant coeficients. Table V summarizes the values of the parameters of the linear equations for the three mixes.

Comparing the «relaxation time» τ_M with the retardation times τ_K, τ_1, τ_2 it appears that the duration of the long-term creep governed by τ_M is of a different order of magnitude than the duration of the deformation processes governed by τ_K and τ_1, τ_2. The actual value of τ_M depends strongly on the cement quality.

For small specimens with freely drying surfaces for which the deformation due to pore-water motion governed by τ_1 is not only of relatively short duration but irrecoverable, the increasing deformation of both Kelvin elements *(c)* could be reproduced by a non-linear time-and stress dependent dashpot with the model equation (for constant stress)

$$\text{(4)} \qquad \dot{\varepsilon}_P = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 = \left[\Phi_1 e^{-t/\tau_1} + \Phi_2 e^{-t/\tau_2}\right] f_c \left(\frac{\sigma}{f_c}\right) = \Phi_P\, f_c \left(\frac{\sigma}{f_c}\right)$$

where the component «fluidities» $\Phi_1 = \alpha_1/\tau_1$ and $\Phi_2 = \alpha_2/\tau_2$. In this case creep-recovery is limited and of (short) duration geverned by τ_K For large, imperviously covered or submerged specimens or concrete parts creep recovery would be more significant and of longer duration since it would be governed largely by τ_1. Thus the four-element model representation of the deformation response of concrete shown in Fig. 1 could be somewhat simplified by replacing one or both of the elements *(c)* by a dashpot the fluidity of which, above the linear stress-limit, would be a function of stress and time, below this limit a function of time alone. The simplest representation would therefore be a model obtained by a coupling in series of the Maxwell element *(a)*, the Kelvin element *(b)*, and the dashpot *(d)* replacing both elements *(c)*, as shown in Fig. 2A. This model is equivalent to a model consisting of a Maxwell element



A *B*

FIG. 2. Simplified Structural Models of Concrete

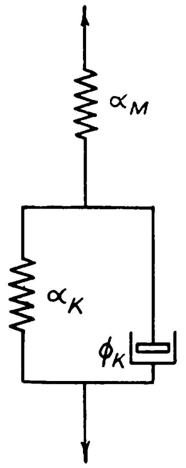


FIG. 3. Structural Model of Completely Hardened Concrete

combining the fluidities of both dashpots, coupled in series with the Kelvin element *(b)*; this latter model can therefore be considered as the simplest possible representation of the creep-equation of concrete (Fig. 2B). The combined fluidity of the Maxwell model in the non-linear range is

$$\Phi_{MP} = \frac{1}{f_c} C_M e^{-t/\tau_M} \left(\frac{\sigma}{f_c}\right)^{-1} \exp\left[a\left(\frac{\sigma}{f_c}\right)^b\right] + \Phi_P \tag{5}$$

in the linear range

$$\Phi_{MP} = \left[\frac{1}{f_c} C_M C_O e^{-t/\tau_M} + \Phi_P\right] \tag{5a}$$

The initial combined fluidity in the linear range is therefore $\Phi_{MPO} = [C_M C_O/f_c + \Phi_1 + \Phi_2]$; for the three mixes accorring to Table V this value is of the order of magnitude 0.8×10^{-8}, 1.6×10^{-8}, 1.4×10^{-8} for the mixes A, B and C respectively. Hence the respective coefficients of viscosity η_{MP} are 7.5×10^{17}, 3.8×10^{17} and 4.1×10^{17} poises. For the Maxwell element alone $\Phi_{MO} = 0.22 \times 10^{-8}$, 0.43×10^{-8} and 0.52×10^{-8} while $\eta_{MO} = 2.7 \times 10^{18}$, 1.4×10^{18} and 1.2×10^{18} poises respectively.

After a period t of about $t \sim 3 - 4\ \tau_M$ the value of the fluidity Φ_{MP} is practically zero; the Maxwell model thus degenerates into a linear spring which, coupled in series with the remaining single Kelvin element, governs the deformational response of the finally hardened concrete (Fig. 3).

The complete model equations are obtained by combining the relation

$$\dot{\varepsilon} = \dot{\varepsilon}_M + \dot{\varepsilon}_K + \dot{\varepsilon}_1 + \dot{\varepsilon}_2 \tag{6}$$

with the respective equations (1) to (5) in such a way as to obtain differential equations involving ε, (σ/f_c) and their time-derivatives.

3. *Basis of Visco-Elastic Analysis of Linear Structures*

The general differential equation of visco-elastic response that is obtained by combining all non-linear model equations according to eq. (6) is non-linear in stress and has coefficients that are variable in time, which makes its actual use for the solution of problems with time-dependent stresses prohibitive. However, a number of simplifications can be obtained by considering that in most practical problems either the response of certain of the model elements is simplified because the time-scale of the imposed load or enforced deformation is much shorter or much longer than the times τ governing the various model responses, or the variation with time of the applied loads or enforced deformations is either rapid enough to disregard the time dependence of the parameters or slow enough to disregard the effect of this variation.

Because of the relatively short values of τ_K, τ_1 and τ_2 in relation to τ_M, the effect of the change with time of ε_p according to eq. (4) can be disregarded with respect to analysis of sustained loads and deformations and the final value $\varepsilon_p = \varepsilon_{p\infty}(\sigma)$ introduced as an immediate response. The effect of Φ_K can also be neglected and the actual response of the element *(b)* replaced by its long-time elastic response α_K. Hence, for the analysis of the effect of sustained loads and deformations the single Maxwell element, with $\alpha'_M = \alpha_M + \alpha_K$ and Φ_M according to eq. (5), with $\Phi_p = 0$, would represent the response of the concrete closely enough, provided that $\varepsilon_{p\infty}(\sigma)$ is added at time $t = 0$ to any deformation. The final irrecoverable value of this deformation is obtained from eq. (3) by using the stress-dependent spring constants α_1 and α_2, blocked on stress reversal. Hence

$$\varepsilon_{p\infty}(\sigma) = (\alpha_1 + \alpha_2)\, f_c \left(\frac{\sigma}{f_c}\right) \tag{7}$$

The apparent spring constant of the Maxwell element under increasing loads or stresses could therefore be introduced as $\alpha''_M = \alpha'_M + \alpha_1 + \alpha_2$, while for decreasing loads the actual spring constant α'_M would operate.

For loads of relatively short duration, variation with time of parameters can be disregarded. If their duration is short in relation to τ_K, τ_1 and τ_2 the Maxwell element with α_M and Φ_M will provide the total response; if their duration is of the order of magnitude of τ_K, the elements *(a)* and *(b)* will respond jointly. For loads producing stress below σ_o this response is linear, so that linear visco-elastic structural equations are obtained.

For a linear visco-elastic response, such as that of the simple Maxwell type obtained by differentiating eq. (1a) for constant stress

$$\dot{\varepsilon}_M = \alpha_M \dot{\sigma} + \frac{1}{f_c} C_M C_o e^{-t/\tau_M} \sigma = \alpha_M \dot{\sigma} + \Phi_M(t)\sigma, \qquad \text{for } \sigma < \sigma_o, \tag{8}$$

the differential equation for the visco-elastic beam is obtained by applying the same differential operation to the equation of the elastic beam that produces eq. (8) from the elastic equation $\varepsilon = \alpha\sigma$ (1). Considering the transverse load p, axial force P and inertia term *m*, the following linear visco-elastic beam equation is immediately obtained by this analogy:

$$E_M I \dot{w}_{xxxx} = \left(\frac{\partial}{\partial t} + \frac{1}{\tau_M}\right)(p \pm P w_{xx} + m\ddot{w}) \tag{9}$$

where the subscript x denotes differentiation with respect to the coordinate, and the dot differentiation with respect to time. The modulus $E_M = 1/\alpha_M$, the time $\tau_M = \alpha_M/\Phi_M$. For loads of short duration eq. (9) has constant coefficients, while for sustained loads $\tau_M = \tau_M(t)$. The standard equations for bending, buckling and transverse vibration are obtained simply by putting two of the three right-hand terms respectively equal to zero.

If the model elements *(a)* and *(b)* act jointly, (Fig. 2B), the respective differential equation obtained from eq. (6) has the form:

$$\alpha_M[\ddot{\sigma} + (k/\tau_K)\dot{\sigma} + (1/\tau_M\tau_K)\sigma] = \ddot{\varepsilon} + (1/\tau_K)\dot{\varepsilon} \tag{10}$$

where $k = (1 + \tau_K/\tau_M + \alpha_K/\alpha_M)$. Hence the respective beam equation:

$$E_M I\,[\ddot{w}_{xxxx} + (1/\tau_K)\dot{w}_{xxxx}] = \left[\frac{\partial^2}{\partial t^2} + k/\tau_K \frac{\partial}{\partial t} + 1/\tau_M\tau_K\right](p \pm P w_{xx} + m\ddot{w}) \tag{11}$$

where $E_M = 1/\alpha_M$.

Solutions of eqs. (9) and (11) or parts thereof, derived by eliminating all but one load-effect, are obtained by introducing solutions of the form

$$w(x, t) = \sum_{n=1}^{\infty} X_n(x) \cdot T_n(t) \tag{12}$$

wich, in case of constant coefficients, transforms the partial differential equations into separate equations for X_n (x) and T_n (t).

The non-linear equations can not be established by such simple analogy between the stress-strain relations and the structural equations. Some simple effects of non-linearity can, however, be obtained by evaluating the non-linear equation of the Maxwell model *(a)* with some of the constants shown in Table IV for conditions of simple bending and of uni-axial relaxation. This has been done with the constants pertaining to mix B (1:2.5); the results are presented in Figs. 4 for simple bending and in Fig. 5 for relaxation.

The non-linear compressive stress-distributions shown for different maximum fiber stresses in Fig. 4 have been obtained on the basis of the Navier-Bernoulli assumption of beam theory modified for linear strain-rate. It appears that the non-linearity of the stress-distribution in bending is the more pronounced the higher the extreme fiber stress. As this stress tends towards the uniaxial compressive strength, the stress-distribution becomes almost indistinguishable from that of an ideal elastic plastic material, a fact which suggests that this idealization represents the failure conditions in the bent concrete sections reasonably well, while the linear visco-elastic response represents the service conditions. This conclusion applies to a relatively early age of the concrete during which creep has not yet been blocked by complete hardening. The diagrams have been drawn under the assumption of homogeneous material, with fixed neutral axis. In reinforced concrete with cracking tension zone the neutral axis will move upward as the fiber stress increases.

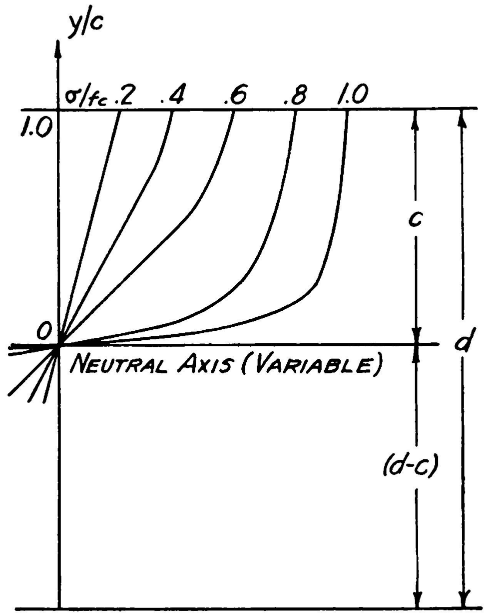


FIG. 4. Distribution of Compressive Stress in Bending as Fuction of Extreme Fiber Stress (drawn on invariable neutral axis)

In Fig. 5 the differences are shown between the stress-relaxation processes representing the response of the non-linear Maxwell model to initial (elastic) strain, producing an initial (elastic) stress σ_i, and the relaxation of an equivalent linear model. In both cases the fluidity decreases exponentially with time. Comparison of the respective curves indicates the sharply increased relaxation of the initial stress with increasing initial stress level $\sigma_i > \sigma_0$. It appears that as a result of the non-linearity an initial stress $\sigma_i = 0.8\,f_c$ would, after a certain time, be reduced to a stable level that does not differ very significantly from the stable level attained by an initial stress $\sigma_i = 0.4 f_c$, while the assumption of linearity suggests the existence of large differences in the stable stress-levels remaining from various levels of initial stress. Thus the application of high concrete prestress defeats its purpose, a fact which theoretically limits the level of prestress that can be permanently sustained in prestressed concrete structures.

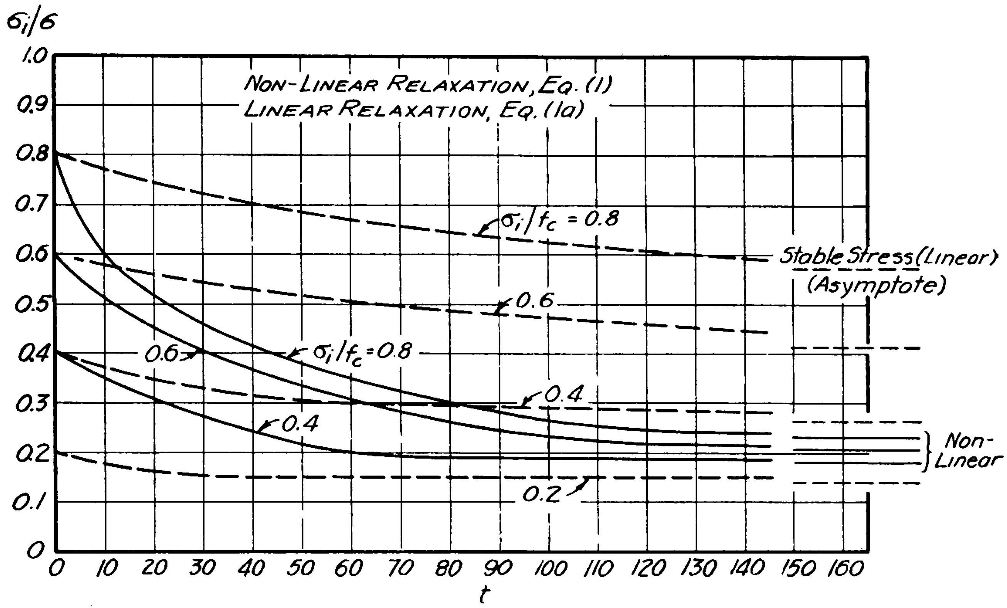


FIG. 5 Stress-Relaxation Diagrams in the Non-linear Range Compared to Diagrams Obtained by Assumption of Linearity

TABLE I

Compressive Strength

Diameter, inches	Loading Conditions	Compressive Strength, psi					
		Mix A (1:1)		Mix B (1:2.5)		Mix C (1:6)	
		28 days	200 days	28 days	200 days	28 days	200 days
3	Unloaded	8940	7900	6490	5760	6100	4740
	Sustained Load						
	$\sigma/f_c = .20$	-	7960	-	6000	-	6160
	$\sigma/f_c = .40$	-	8090	-	6450	-	5920
	$\sigma/f_c = .60$	-	8290	-	5720	-	6250
4	Unloaded	8560	8280	6020	6470	4940	5630
	Sustained Load						
	$\sigma/f_c = .17$	-	9410	-	-	-	-
	$\sigma/f_c = .34$	-	10,420	-	-	-	-
	$\sigma/f_c = .26$	-	-	-	7250	-	-
	$\sigma/f_c = .52$	-	-	-	7410	-	-
	$\sigma/f_c = .32$	-	-	-	-	-	6080
	$\sigma/f_c = .64$	-	-	-	-	-	6420
6	Unloaded	-	-	5920	6660	4740	5530

TABLE II
Modulus of Elasticity

Diameter, inches	Loading Conditions	Modulus of Elasticity x 10^6 psi					
		Mix A (1:1)		Mix B (1:2.5)		Mix C (1:6)	
		28 days	200 days	28 days	200 days	28 days	200 days
3	Sustained Load						
	$\sigma/f_c = .20$	-	4.10	-	3.65	-	5.07
	$\sigma/f_c = .40$	-	4.57	-	4.38	-	3.60
	$\sigma/f_c = .60$	-	4.74	-	4.16	-	3.79
4	Sustained Load						
	$\sigma/f_c = .41$	-	4.26	-	-	-	-
	$\sigma/f_c = .54$	-	-	-	4.43	-	-
	$\sigma/f_c = .66$	-	-	-	-	-	4.45
6	Unloaded	4.44	4.15	4.76	4.62	5.46	5.07

TABLE III
Contribution of Various Model Elements to Creep, in 10^{-5} in./in.

Mix	Cylinder dia.	4 in.				3 in.					
	Period	28 days		250 days		28 days			250 days		
	Stress Level	$.17f_c$	$.34f_c$	$.17f_c$	$.34f_c$	$.20f_c$	$.40f_c$	$.60f_c$	$.20f_c$	$.40f_c$	$.60f_c$
1:1	Model (a) ...	6.1	13.8	36.2	81.9	9.2	16.6	36.4	25.2	71.9	157.9
	ε_1	14.0	31.8	18.2	46.5	3.8	24.4	53.5	3.8	27.4	80.5
	Model (c) ...										
	ε_2	8.8	27.8	8.8	27.8	0	5.5	47.6	0	5.5	47.6
	Model (b) ...	9.4	18.7	9.4	18.7	10.9	21.9	32.9	10.9	21.9	32.9
	Total ...	38.3	92.1	72.6	174.9	23.9	68.4	170.4	39.9	126.7	318.9
	Stress Level	$.26f_c$	$.52f_c$	$.26f_c$	$.52f_c$	$.20f_c$	$.40f_c$	$.60f_c$	$.20f_c$	$.40f_c$	$.60f_c$
1:2.5	Model (a) ...	11.7	25.3	53.1	114.9	13.4	23.2	51.7	40.5	83.1	185.2
	ε_1	17.1	52.3	17.8	62.3	8.1	28.0	60.3	8.1	29.8	74.2
	Model (c) ...										
	ε_2	6.8	40.3	6.8	40.3	0	11.3	46.4	0	11.3	46.4
	Model (b) ...	11.0	22.0	11.0	22.0	9.2	18.5	27.8	9.2	18.5	27.8
	Total ...	46.6	139.9	88.7	239.5	30.7	81.0	186.2	57.8	142.7	333.6
	Stress Level	$.315f_c$	$.63f_c$	$.315f_c$	$.63f_c$	$.20f_c$	$.40f_c$	$.60f_c$	$.20f_c$	$.40f_c$	$.60f_c$
1:6	Model (a) ...	6.8	16.1	44.1	104.5	14.4	23.8	52.8	37.4	66.4	147.7
	ε_1	12.1	48.5	12.4	61.8	5.0	21.4	48.8	5.0	22.8	61.7
	Model (c) ...										
	ε_2	5.0	47.0	5.0	47.0	0	9.2	41.3	0	9.2	41.3
	Model (b) ...	7.4	14.8	7.4	14.8	6.8	13.6	20.4	6.8	13.6	20.4
	Total ...	31.3	126.4	68.9	228.1	26.2	68.0	163.3	49.2	112.0	271.1

TABLE IV — *Non-Linear Coefficients*

Mix		1:1		1:2.5		1:6	
Stress	psi	3650	5475	2640	3970	2480	3720
	$\%f_o$	40	60	40	60	40	60
$\alpha_M \times 10^6$, $in^2 \times lb^{-1}$		.223	.223	.247	.247	.246	.246
$C_M \times 10^5$, day^{-1}		.468	.468	.666	.666	.748	.748
τ_M, days (order of magn.)		200		100		100	
a		5	5	5	5	5	5
b		3	3	3	3	3	3
$\alpha_K \times 10^6$, $in^2 \times lb^{-1}$		.060	.060	.070	.070	.055	.055
$\alpha_1 \times 10^6$, $in^2 \times lb^{-1}$		.075	.147	.113	.187	.092	.166
$\alpha_2 \times 10^6$, $in^2 \times lb^{-1}$		.015	.087	.043	.117	.037	.111
τ_K, (order of magn.)				1 - 2 days			
τ_1, days		15	25	10	17	10	18
τ_2, (order of magn.)				1 - 2 days			
$\Phi_K \times 10^8$, $in^2 \times lb^{-1} \times day^{-1}$		3.75	3.75	6.36	6.36	5.00	5.00
$\Phi_1 \times 10^8$, $in^2 \times lb^{-1} \times day^{-1}$		.572	.572	1.121	1.121	.927	.927
$\Phi_2 \times 10^8$, $in^2 \times lb^{-1} \times day^{-1}$		1.15	3.34	4.30	6.90	3.70	6.17

TABLE V — *Linear Coefficients*

Mix		1:1	1:2.5	1:6
Diameter ins.		3	3	3
Stress	psi	1825	1330	1245
	$\%f_o$	20	20	20
Ultimate strength, f_c psi		8940	6490	6100
$\alpha_M \times 10^6$, $in^2 \times lb^{-1}$		.223	.247	.246
$C_M \times 10^5$, day^{-1}		.468	.666	.748
C_0		4.20	4.20	4.20
τ_M, days (order of magn)...		200	100	100
$\alpha_K \times 10^6$, $in^2 \times lb^{-1}$		.060	.070	.055
$\alpha_1 \times 10^6$, $in^2 \times lb^{-1}$		.021	.061	.040
$\alpha_2 \times 10^6$, $in^2 \times lb^{-1}$		0	0	0
τ_K, days			1 - 2 days	
τ_1, days			5 - 10 days	
τ_2 days		0	0	0
$\Phi_K \times 10^8$, $in^2 \times lb^{-1} \times day^{-1}$		3.75	6.36	5.00
$\Phi_1 \times 10^8$, $in^2 \times lb^{-1} \times day^{-1}$		.572	1.121	.927
$\Phi_2 \times 10^8$, $in^2 \times lb^{-1} \times day^{-1}$		0	0	0

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SUMMARY

The general creep sensitivity of reinforced concrete structures is discussed in the light of the assumed nature of creep of concrete, as deduced from recent experimental investigations at Columbia University. Methods of visco-elastic analysis of such structures are related to the actual response of the concrete to sustained loads, and some effects of the non-linearity of this response are illustrated.

ZUSAMMENFASSUNG

Das Kriechproblem bei Eisenbetonbauten wird unter Berücksichtigung der neusten an der Columbia-Universität durchgeführten Versuche über den Kriechvorgang im Beton behandelt.

Die nach der elasto-plastischen Theorie ausgeführten Berechnungen werden mit dem tatsächlichen Verhalten des Betons infolge äusserer Belastung verglichen; ferner werden einige Auswirkungen des nichtlinearen Verhaltens behandelt.

RESUMO

O autor discute o problema da fluência em estruturas de betão armado partindo das teorias sobre a natureza da fluência do betão, deduzidas de investigações experimentais recentes efectuadas na Universidade de Columbia. Estabelece-se a ligação entre os métodos de cálculo dessas estruturas baseados na teoria da visco-elasticidade, e o comportamento efectivo do betão submetido a cargas prolongadas. Exemplificam-se alguns efeitos da não linearidade desse comportamento.

RÉSUMÉ

L'auteur discute le problème du fluage dans les structures en béton armé en partant des théories concernant la nature du fluage du béton, déduites de recheches expérimentales récentes effectuées à l'Université de Columbia. Il établit le rapport qui existe entre les méthodes de calcul des structures fondées sur la théorie visco-élastique et le comportement réel du béton soumis à des charges de longue durée et donne des exemples de certains effets de la non-linéarité de ce comportement.

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